Competition for Blood Donations

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Outline

1. Background
2. Literature Review
3. Research Questions
4. Competitive Network Model
5. Algorithm and Numerical Examples
6. Summary and Conclusions

Background

- The blood banking industry runs on voluntary donations from altruistic donors.
- Approximately 36,000 units of red blood cells are needed every day in the U.S.
- Nearly 21 million blood components are transfused each year in the U.S.
- One donation can potentially save up to three lives.

Background

- An estimated **38%** of the U.S. population is eligible to donate blood at any given time.
- Less than **10%** of that eligible population actually donates blood each year.
- The different blood service organizations have to compete for this limited donor pool in order to meet the demand.

Background

Examples of competition:

- Blood Centers of the Pacific versus BloodSource in Sonoma County in 2011.
- Suncoast Communities Blood Bank versus Florida Blood Services in Sarasota, Florida in 2012.
- Blood Assurance versus Medic Regional Blood Center in in Tennessee in 2015.

Rival blood banks vie for donors in Sonoma County

Bundled up against air conditioning, John Giampoli of Santa Rosa uses an ice pack to keep bruising to a minimum after donating platletts at Blood Centers of the Pacific, formerly Blood Bank of the Redwoods, Tuesday August 16, 2011 in Santa Rosa, at right is nurse assistant Jessica Ryan. (Kent Porter / Press Democrat) 2011

Background

How to motivate and retain donors?

- Operational factors: **satisfaction from the blood donation process, convenience, location of facilities, wait times, treatment by staff** of the organization collecting blood are some of these factors.
- These factors can be aggregated and termed as the **quality of services** offered by the blood service organizations.

Literature Review

- **Gillespie and Hillyer (2002)** found that negative donation experiences (comfort, convenience of the process, and treatment by the staff) account for 6-19 percent attrition for all donors and 20 to as high as 41 percent of the dropout rate for first time donors.
- **Charbonneau et al. (2015)** found that for a significant percentage of whole blood donors, lapsed whole blood donors and plasma/platelet donors too much waiting time is a deterrent.
- **Nguyen et al. (2008), Aldamiz-echevarria and Aguirre-Garcia (2014)** point out personal experience and satisfaction from the blood donation process, image or awareness of the impact of the organization collecting blood, as significant factors in donor motivation and retention.

Research Questions

- In a competing scenario, what should be the **service quality levels** at the blood collection sites run by different blood service organizations?
- How do **varying service quality levels** affect the **amount of blood collected** from different collection sites?

Competitive Network Model

- There are m blood service organizations responsible for collection of blood,testing, processing, and distribution to hospitals and other medical facilities. A typical blood service organization is denoted by i.
- There are n regions in which blood collection can take place. A typical collection region is denoted by j.

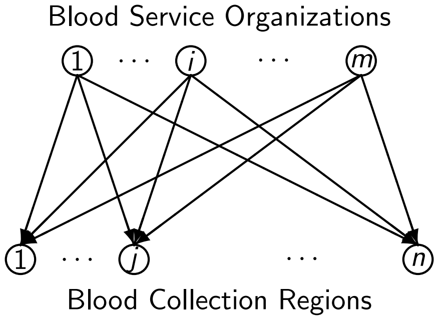


Figure: The Network Structure of the Game Theory Model for Blood Donations

Competitive Network Model

- In this game theory model the blood service organizations compete for blood donations.
- The blood service organizations have, as their strategic variables, the **quality of services** that they provide donors at their collection sites in the regions.

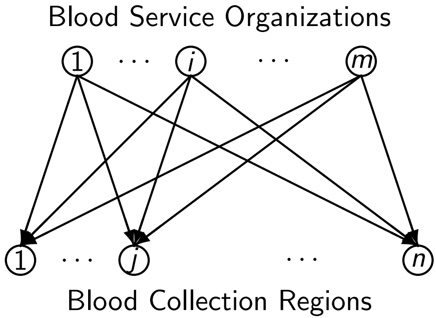


Figure: The Network Structure of the Game Theory Model for Blood Donations

Competitive Network Model

Quality Constraint

There is a non-negative lower bound and a positive upper bound on the quality of service, Q_{ij}, that i provides in region j such that:

$$\underline{Q}_{ij} \leq Q_{ij} \leq \bar{Q}_{ij}, \quad j = 1, \ldots, n. \tag{1}$$

Cost of Collection

Each blood service organization i encumbers a total cost $\hat{c}_{ij}$ associated with collecting blood in region j, where

$$\hat{c}_{ij} = \hat{c}_{ij}(Q), \quad j = 1, \ldots, n, \tag{2}$$

where $\hat{c}_{ij}$ is assumed to be convex and continuously differentiable for all i, j.

Competitive Network Model

Monetized Service Utility

Each blood organization, i, enjoys a utility associated with the service given by:

$$\omega_i \sum_{j=1}^{n} \gamma_{ij} Q_{ij}, \tag{3}$$

where the ω_i and the γ_{ij}s; $j = 1, \ldots, n$, take on positive values.

Competitive Network Model

Blood Donations

Each blood service organization i receives a volume of blood donations in region j, denoted by P_{ij}; $j = 1, \ldots, n$, where

$$P_{ij} = P_{ij}(Q), \tag{4}$$

where each P_{ij} is assumed to be concave and continuously differentiable.

Revenue

Each blood service organization i achieves revenue that is associated with its blood collection activities over the time horizon, given by

$$\pi_i \sum_{j=1}^{n} P_{ij}(Q) \tag{5}$$

where π_i is an average price for blood (typically, measured in pints) for blood service organization i; $i = 1, \ldots, m$.

Optimization Problem

Each blood service organization i seeks to maximize its transaction utility, U_i. Hence, the optimization problem is as follows:

$$\text{Maximize} \quad U_i = \pi_\mathbf{i} \sum_{\mathbf{j=1}}^{\mathbf{n}} \mathbf{P_{ij}(Q)} + \omega_\mathbf{i} \sum_{\mathbf{j=1}}^{\mathbf{n}} \gamma_\mathbf{ij} \mathbf{Q_{ij}} - \sum_{\mathbf{j=1}}^{\mathbf{n}} \mathbf{\hat{c}_{ij}(Q)} \tag{6}$$

subject to (1).

- **Revenue**
- **Monetized altruism**
- **Total cost**

Definition 1: Nash Equilibrium for Blood Donations

A service quality level pattern $Q^* \in K$ is said to constitute a Nash Equilibrium in blood donations if for each blood service organization $i; i = 1, \ldots, m,$

$$U_i(Q_i^*, \hat{Q}_i^*) \geq U_i(Q_i, \hat{Q}_i^*), \quad \forall Q_i \in K^i, \tag{7}$$

where

$$\hat{Q}_i^* \equiv (Q_1^*, \ldots, Q_{i-1}^*, Q_{i+1}^*, \ldots, Q_m^*). \tag{8}$$

- According to (7), a Nash Equilibrium is established if no blood service organization can improve upon its transaction utility by altering its quality service levels, given that the other organizations have decided on their quality service levels.

Theorem 1: Variational Inequality Formulation of the Nash Equilibrium for Blood Donations

A quality service level pattern $Q^* \in K$ is a Nash Equilibrium according to Definition 1 if and only if it satisfies the variational inequality problem:

$$-\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{\partial U_i(Q^*)}{\partial Q_{ij}} \times (Q_{ij} - Q_{ij}^*) \geq 0, \quad \forall Q \in K \tag{9}$$

or, equivalently, the variational inequality:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\sum_{k=1}^{n}\frac{\partial \hat{c}_{ik}(Q^*)}{\partial Q_{ij}} - \omega_i\gamma_{ij} - \pi_i\sum_{k=1}^{n}\frac{\partial P_{ik}(Q^*)}{\partial Q_{ij}}\right] \times \left[Q_{ij} - Q_{ij}^*\right] \geq 0, \forall Q \in K. \tag{10}$$

Competitive Network Model

We can put the variational inequality formulations of the Nash Equilibrium problem into standard variational inequality form (see Nagurney (1999)), that is: determine $X^* \in \mathcal{K} \subset R^N$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{11}$$

where F is a given continuous function from $\mathcal{K}$ to R^N and $\mathcal{K}$ is a closed and convex set.

Competitive Network Model

Existence

Existence of a solution Q^* to variational inequality (9) and also (10) is guaranteed from the standard theory of variational inequalities (cf. Nagurney (1999)) since the function $F(X)$ that enters the variational inequality is continuous and the feasible set K is compact.

Uniqueness

If $F(X)$ is strictly monotone, that is:

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle > 0, \quad \forall X^1, X^2 \in \mathcal{K}, X^1 \neq X^2, \tag{12}$$

then the equilibrium solution X^* and, hence, Q^* is unique.

Lagrange Analysis

Lagrange multipliers: λ_{ij}^1 associated with lower bound and λ_{ij}^2 associated with upper bound on quality level.

Marginal Loss:

If $\bar{\lambda}_{ij}^1 > 0$ and, hence, $Q_{ij}^* = \underline{Q}_{ij}$ and $\bar{\lambda}_{ij}^2 = 0$, then we get that

$$-\frac{\partial U_i(Q^*)}{\partial Q_{ij}} = \left[\sum_{k=1}^{n} \frac{\partial \hat{c}_{ik}(Q^*)}{\partial Q_{ij}} - \omega_i \gamma_{ij} - \pi_i \sum_{k=1}^{n} \frac{\partial P_{ik}(Q^*)}{\partial Q_{ij}}\right] = \bar{\lambda}_{ij}^1,$$

$$i = 1, \ldots, m; j = 1, \ldots, n,$$

Marginal Gain:

If $\bar{\lambda}_{ij}^2 > 0$ and, hence, $Q_{ij}^* = \bar{Q}_{ij}$ and $\bar{\lambda}_{ij}^1 = 0$, we have that

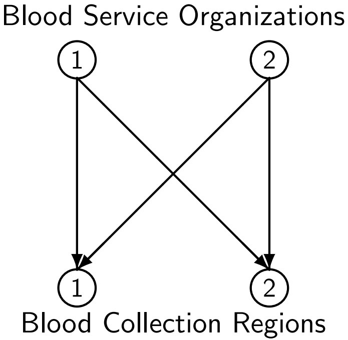
$$-\frac{\partial U_i(Q^*)}{\partial Q_{ij}} = \left[\sum_{k=1}^{n} \frac{\partial \hat{c}_{ik}(Q^*)}{\partial Q_{ij}} - \omega_i \gamma_{ij} - \pi_i \sum_{k=1}^{n} \frac{\partial P_{ik}(Q^*)}{\partial Q_{ij}}\right] = -\bar{\lambda}_{ij}^2,$$

$$i = 1, \ldots, m; j = 1, \ldots, n.$$

Illustrative Example: Example 1

- The American Red Cross (cf. Arizona Blood Services Region (2016)) issued a call for donations.
- Low supply of blood due to seasonal colds and flu and the devastating impact of Hurricane Matthew.
- On October 8, 2016, Hurricane Matthew made landfall that affected such states as Florida, Georgia, and the Carolinas, and disrupted blood donations in many locations in the Southeast of the United States.
- We focus on Tucson, Arizona, where the American Red Cross has held recent blood drives at multiple locations and where there are also competitors for blood, including the United Blood Services.

Example 1



A month of collection of whole blood cells is considered. According to Meyer (2017), Executive Vice President of the American Red Cross, productive Red Cross sites collect, on the average, 700-840 whole blood units a month.

Example 1

The blood donation functions for the American Red Cross (BSO 1) are:

$$P_{11}(Q) = 10Q_{11} - Q_{21} - Q_{22} + 130,$$

$$P_{12}(Q) = 12Q_{12} - Q_{21} - 2Q_{22} + 135.$$

The blood donation functions for the United Blood Services (BSO 2) are:

$$P_{21}(Q) = 11Q_{21} - Q_{11} - Q_{12} + 123,$$

$$P_{22}(Q) = 12Q_{22} - Q_{11} - Q_{12} + 135.$$

The utility function components of the transaction utilities of these blood service organizations are:

$$\omega_1 = 9, \quad \gamma_{11} = 8, \quad \gamma_{12} = 9, \quad \omega_2 = 10, \quad \gamma_{21} = 9, \quad \gamma_{22} = 10.$$

Example 1

The total costs of operating the blood collection sites over the time horizon, which must cover costs of employees, supplies, and energy, and providing the level of quality service, are:

$$\hat{c}_{11}(Q) = 5Q_{11}^2 + 10,000, \quad \hat{c}_{12}(Q) = 18Q_{12}^2 + 12,000,$$
$$\hat{c}_{21}(Q) = 4.5Q_{21}^2 + 12,000, \quad \hat{c}_{22}(Q) = 5Q_{22}^2 + 14,000.$$

The bounds on the quality levels are:

$$\underline{Q}_{11} = 50, \bar{Q}_{11} = 80, \quad \underline{Q}_{12} = 40, \bar{Q}_{12} = 70,$$
$$\underline{Q}_{21} = 60, \bar{Q}_{21} = 90, \quad \underline{Q}_{22} = 70, \bar{Q}_{22} = 90.$$

The prices, which correspond to the collection component of the blood supply chain, are: $\pi_1 = 70$ and $\pi_2 = 60$.

Example 1

BSO 1	Solution	**BSO 2**	Solution
Q^*_{11}	77.2	Q^*_{21}	83.3
Q^*_{12}	25.5 (40)	Q^*_{22}	82
$P^*_{11}(Q^*)$	736.7	$P^*_{21}(Q^*)$	922.1
$P^*_{12(Q^*)}$	367.7	$P^*_{22}(Q^*)$	1001.8
$U^*_1(Q^*)$	5,507.20	$U^*_2(Q^*)$	40,285.99

Lagrange Analysis

Since Q^*_{12} is at its lower bound and no quality service levels are at their upper bounds: $\bar{\lambda}^1_{11} = 0$, $\bar{\lambda}^1_{21} = 0$, $\bar{\lambda}^1_{22} = 0$, and $\bar{\lambda}^2_{11} = 0$, $\bar{\lambda}^2_{12} = 0$, $\bar{\lambda}^2_{21} = 0$, $\bar{\lambda}^2_{22} = 0$.

But Q^*_{12} is at its lower bound, so BSO 1 experiences a marginal loss of $\bar{\lambda}^1_{12} = 1359$ in Region 2.

The Algorithm

The Euler Method

An iteration $\tau + 1$ of the Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993), is:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau} F(X^{\tau})),$$

The Euler method, the sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \to 0$, as $\tau \to \infty$.

Explicit Formula for the Euler Method Applied to Blood Donation Service Organization Game Theory Model

Closed form expression for the quality service levels $i = 1, \ldots, m; j = 1, \ldots, n$, at iteration $\tau + 1$:

$$Q_{ij}^{\tau+1} = \max\{\underline{Q}_{ij}, \min\{\bar{Q}_{ij}, Q_{ij}^{\tau} + a_{\tau}(\pi_i \sum_{k=1}^{n} \frac{\partial P_{ik}(Q^{\tau})}{\partial Q_{ij}} + \omega_i \gamma_{ij} - \sum_{k=1}^{n} \frac{\partial \hat{c}_{ik}(Q^{\tau})}{\partial Q_{ij}})\}\}.$$

Example 2

Network topology and data are identical to that of Example 1.

New P_{ij} functions: $\alpha_{ij}\sqrt{P_{ij}}$ for $i = 1, 2$; $j = 1, 2$ with $\alpha_{11} = 50$, $\alpha_{12} = 30$, $\alpha_{21} = 40$, and $\alpha_{22} = 20$.

BSO 1	Solution	**BSO 2**	Solution
Q^*_{11}	72.43	Q^*_{21}	64.61
Q^*_{12}	**40.00**	Q^*_{22}	**70.00**
$P^*_{11}(Q^*)$	1341.37	$P^*_{21}(Q^*)$	1074.27
$P^*_{12(Q^*)}$	607.74	$P^*_{22}(Q^*)$	587.39
$U^*_1(Q^*)$	67,860.92	$U^*_2(Q^*)$	43,229.16
Revenue	136,437.78	Revenue	99,699.67
Cost	77,031.92	Cost	69,285.48

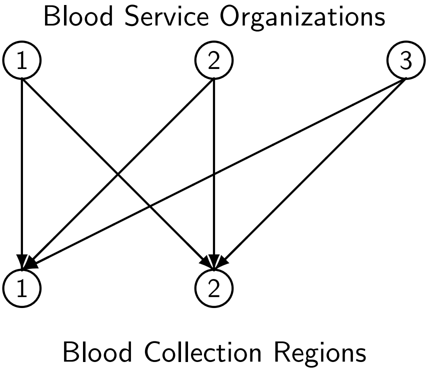
Example 2

Lagrange Analysis

- Q^*_{12} and Q^*_{22} are at their lower bounds.
- Lagrange analysis shows blood service organization 1 suffers a **marginal loss of 737.03** associated with its services in Region 2.
- Blood service organization 2 suffers a **marginal loss of 354.85** associated with its services in Region 2.

Example 3

Here a blood service organization is added to the network.



Example 3

For BSO 1:

$$P_{11}(Q) = 50\sqrt{10Q_{11} - Q_{21} - Q_{22} - .5Q_{31} + 130},$$

$$P_{12}(Q) = 30\sqrt{12Q_{12} - Q_{21} - 2Q_{22} - .3Q_{32} + 135},$$

For BSO 2:

$$P_{21}(Q) = 40\sqrt{11Q_{21} - Q_{11} - Q_{12} - .2Q_{21} + 113},$$

$$P_{22}(Q) = 20\sqrt{12Q_{22} - Q_{11} - Q_{12} - .3Q_{32} + 135}.$$

For BSO 3:

$$P_{31}(Q) = 50\sqrt{11Q_{31} - Q_{21} + 50},$$

$$P_{32}(Q) = 40\sqrt{10Q_{32} - Q_{12} + 2000}.$$

Example 3

$$\omega_3 = 10, \quad \gamma_{31} = 10, \quad \gamma_{32} = 11.$$

Total cost functions given by:

$$\hat{c}_{31}(Q) = 6Q_{31}^2 + 10,000, \quad \hat{c}_{32}(Q) = 5Q_{32}^2 + 12,000.$$

Lower and upper bounds are as follows:

$$\underline{Q}_{31} = 50, \quad \bar{Q}_{31} = 90,$$

$$\underline{Q}_{32} = 40, \quad \bar{Q}_{32} = 80.$$

The price $\pi_3 = 80$.

Example 3

BSO 1	Solution	**BSO 2**	Solution	**BSO 3**	Solution
Q_{11}^*	72.43	Q_{21}^*	64.61	Q_{31}^*	70.73
Q_{12}^*	40	Q_{22}^*	70	Q_{32}^*	66.65
$P_{11}^*(Q^*)$	1318.43	$P_{21}^*(Q^*)$	1059.31	$P_{31}^*(Q^*)$	1381.47
$P_{12}^*(Q^*)$	592.46	$P_{22}^*(Q^*)$	580.15	$P_{32}^*(Q^*)$	2049.99
Revenue	133,762.72	Revenue	98,367.77	Revenue	274,516.72
Cost	77,860.27	Cost	68,644.69	Cost	183,922.09

Lagrange Analysis

$$\bar{\lambda}_{11}^1 = \bar{\lambda}_{11}^2 = 0, \bar{\lambda}_{21}^1 = \bar{\lambda}_{21}^2 = 0, \bar{\lambda}_{31}^1 = \bar{\lambda}_{31}^2 = 0,$$

$$\bar{\lambda}_{32}^1 = \bar{\lambda}_{32}^2 = 0,$$

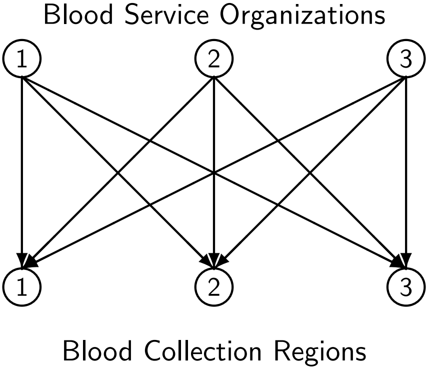
$$\bar{\lambda}_{12}^1 = \mathbf{720.98}, \bar{\lambda}_{12}^2 = 0,$$

and

$$\bar{\lambda}_{22}^1 = \mathbf{351.79}, \bar{\lambda}_{22}^2 = 0.$$

Example 4

In this example an additional collection region is included in the network structure.



Example 4

Input data:

$$\alpha_{13} = 40, \quad \alpha_{23} = 30, \quad \alpha_{33} = 50,$$

$$P_{13}(Q) = 40\sqrt{10Q_{13} - Q_{23} - .2Q_{33} + 150},$$

$$P_{23}(Q) = 30\sqrt{11Q_{23} - Q_{13} - .2Q_{33} + 150},$$

$$P_{33}(Q) = 50\sqrt{10Q_{33} - Q_{23} - .3Q_{13} + 100},$$

$$\hat{c}_{13}(Q) = 100Q_{13}^2 + 15,000, \quad \hat{c}_{23}(Q) = 9Q_{23}^2 + 13000,$$

$$\hat{c}_{33}(Q) = 8Q_{33}^2 + 10000.$$

Lower and upper bounds on the new links to Region 3 given by:

$$\underline{Q}_{13} = 0, \quad \underline{Q}_{23} = 0, \quad \underline{Q}_{33} = 40,$$

$$\bar{Q}_{13} = 60, \quad \bar{Q}_{23} = 70, \quad \bar{Q}_{33} = 90.$$

Example 4

BSO 1	Solution	BSO 2	Solution	BSO 3	Solution
Q^*_{11}	73.57	Q^*_{21}	64.61	Q^*_{31}	70.73
Q^*_{12}	40	Q^*_{22}	70	Q^*_{32}	66.65
Q^*_{13}	36.32	Q^*_{23}	31.51	Q^*_{33}	56.39
$P^*_{11}(Q^*)$	1318.43	$P^*_{21}(Q^*)$	1059.31	$P^*_{31}(Q^*)$	1381.22
$P^*_{12}(Q^*)$	592.46	$P^*_{22}(Q^*)$	580.15	$P^*_{32}(Q^*)$	2,049.99
$P^*_{13}(Q^*)$	867.59	$P^*_{23}(Q^*)$	635.70	$P^*_{33}(Q^*)$	1,246.49
Revenue	194,493.95	Revenue	136,509.97	Revenue	374,216.53
Cost	76,054.13	Cost	93,632.34	Cost	226,36.88

All of the Lagrange multipliers are equal to 0 except for the following: $\bar{\lambda}^1_{12} =$ **720.98**, $\bar{\lambda}^1_{22} =$ **351.79**.

All blood service organizations gain by servicing another region even in the case of competition!

Summary

- In all examples the revenues generated by the blood service organizations exceed costs.
- With the addition of a new blood service organization, revenue for the first two decrease but remain positive.
- Increased competition **increases the total blood collection** although **collections by individual organizations decrease**.
- **Increased competition** can yield **benefits** for blood donors in terms of higher **levels of service quality.**

Conclusions

- Donors respond to the quality of service at blood collection sites.
- We observe the **trade-offs** between cost and quality of service.
- Blood service organizations who do "good," can also be **financially sustainable** even in the face of competition.
- For blood collection regions that have **lower quality levels, internal assessments** can be made by the blood service organizations to figure out the **individual factors** responsible for such low levels such as longer wait time, unfriendly staff, etc.
- In addition to safety measures for blood collection procedure, **attention should be given to service quality aspects of the collection sites**.

Thank you!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