Supply Chain Network Operations Management of a Blood Banking System with Cost and Risk Minimization

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22nd Annual POMS Conference
Reno, Nevada
April 29 - May 2, 2011

Acknowledgments

This research was supported by the John F. Smith Memorial Fund at the University of Massachusetts Amherst. This support is gratefully acknowledged.

The authors acknowledge *Mr. Len Walker*, the Director of Business Development for the American Red Cross Blood Services in the greater Boston area, and *Dr. Jorge Rios*, the Medical Director for the American Red Cross Northeast Division Blood Services, for sharing valuable information on the subject, and enlightening thoughts on the model.

Outline

- Background and Motivation
- Some of the Relevant Literature
- The Supply Chain Network Model of a Blood Banking System
- Numerical Examples
- The Algorithm and Explicit Formulae
- Summary and Conclusions


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Background and Motivation

Blood service operations are a key component of the healthcare system all over the world.

Over 39,000 donations are needed everyday in the United States, alone, and the blood supply is frequently reported to be just 2 days away from running out (American Red Cross).

Background and Motivation

Of 1,700 hospitals participating in a survey in 2007, a total of 492 reported cancellations of elective surgeries on one or more days due to blood shortages.

Hospitals with as many days of surgical delays as 50 or even 120 have been observed.

In 2006, the national estimate for the number of units of blood components outdated by blood centers and hospitals was 1,276,000 out of 15,688,000 units (Whitaker et al. (2007)).

Background and Motivation

The hospital cost of a unit of red blood cells in the US had a 6.4% increase from 2005 to 2007.

In the US, this criticality has become more of an issue in the Northeastern and Southwestern states since this cost is meaningfully higher compared to that of the Southeastern and Central states.

Background and Motivation

Hospitals were responsible for approximately 90% of the outdates, where this volume of medical waste imposes discarding costs to the already financially-stressed hospitals (The New York Times (2010)).

Background and Motivation

The health care facilities in the United States are second only to the food industry in producing waste, generating more than 6,600 tons per day, and more than 4 billion pounds annually (Fox News (2011)).

Some of the Relevant Literature

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Regionalized Blood Supply Chain Network Model

We developed a generalized network optimization model for the complex supply chain of human blood, which is a life-saving, perishable product.

More specifically, we developed a multicriteria system-optimization framework for a regionalized blood supply chain network.

Regionalized Blood Supply Chain Network Model

We assume a network topology where the top level (origin) corresponds to the organization; .i.e., the regional division management of the American Red Cross. The bottom level (destination) nodes correspond to the demand sites - typically the hospitals and the other surgical medical centers.

The paths joining the origin node to the destination nodes represent sequences of supply chain network activities that ensure that the blood is collected, tested, processed, and, ultimately, delivered to the demand sites.

Components of a Regionalized Blood Banking System

- ARC Regional Division Management (Top Tier),

Blood Collection

- Blood Collection Sites (Tier 2), denoted by: $C_1, C_2, \ldots, C_{n_C}$,

Shipment of Collected Blood

- Blood Centers (Tier 3), denoted by: $B_1, B_2, \ldots, B_{n_B}$,

Testing and Processing

- Component Labs (Tier 4), denoted by: $P_1, P_2, \ldots, P_{n_P}$,

Storage

- Storage Facilities (Tier 5), denoted by: $S_1, S_2, \ldots, S_{n_S}$,

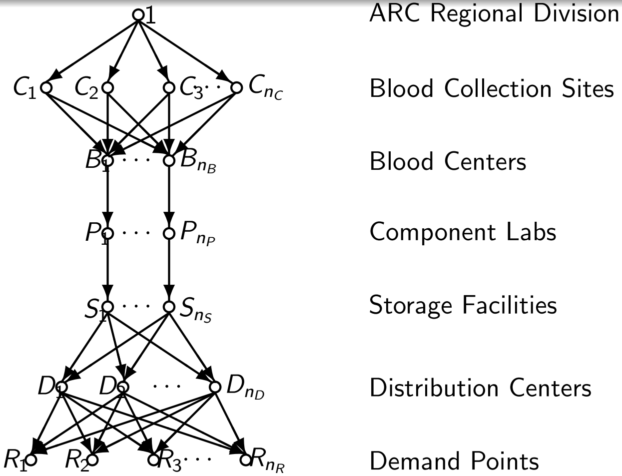
Shipment

- Distribution Centers (Tier 6), denoted by: $D_1, D_2, \ldots, D_{n_D}$,

Distribution

- Demand Points (Tier 7), denoted by: $R_1, R_2, \ldots, R_{n_R}$

Supply Chain Network Topology for a Regionalized Blood Bank



Graph $G = [N, L]$, where N denotes the set of nodes and L the set of links.

Regionalized Blood Supply Chain Network Model

The formalism is that of multicriteria optimization, where the organization seeks to determine the optimal levels of blood processed on each supply chain network link subject to the minimization of the total cost associated with its various activities of blood collection, shipment, processing and testing, storage, and distribution, in addition to the total discarding cost as well as the minimization of the total supply risk, subject to the uncertain demand being satisfied as closely as possible at the demand sites.

Notation

- c_a: the unit operational cost on link a.
- $\hat{c}_a$: the total operational cost on link a.
- f_a: the flow of whole blood/red blood cell on link a.
- p: a path in the network joining the origin node to a destination node representing the activities and their sequence.
- w_k: the pair of origin/destination (O/D) nodes $(1, R_k)$.
- $\mathcal{P}_{w_k}$: the set of paths, which represent alternative associated possible supply chain network processes, joining $(1, R_k)$.
- $\mathcal{P}$: the set of all paths joining node 1 to the demand nodes.
- n_p: the number of paths from the origin to the demand markets.
- x_p: the nonnegative flow of the blood on path p.
- d_k: the uncertain demand for blood at demand location k.
- v_k: the projected demand for blood at demand location k.

Formulation

Total Operational Cost on Link a

$$\hat{c}_a(f_a) = f_a \times c_a(f_a), \qquad \forall a \in L, \tag{1}$$

assumed to be convex and continuously differentiable.

Let P_k be the probability distribution function of d_k, that is, $P_k(D_k) = P_k(d_k \leq D_k) = \int_0^{D_k} \mathcal{F}_k(t)d(t)$. Therefore,

Shortage and Surplus of Blood at Demand Point R_k

$$\Delta_k^- \equiv \max\{0, d_k - v_k\}, \qquad k = 1, \ldots, n_R, \tag{2}$$

$$\Delta_k^+ \equiv \max\{0, v_k - d_k\}, \qquad k = 1, \ldots, n_R, \tag{3}$$

Expected Values of Shortage and Surplus

$$E(\Delta_k^-) = \int_{v_k}^{\infty} (t - v_k)\mathcal{F}_k(t)d(t), \qquad k = 1, \ldots, n_R, \tag{4}$$

$$E(\Delta_k^+) = \int_{0}^{v_k} (v_k - t)\mathcal{F}_k(t)d(t), \qquad k = 1, \ldots, n_R. \tag{5}$$

Expected Total Penalty at Demand Point k

$$E(\lambda_k^- \Delta_k^- + \lambda_k^+ \Delta_k^+) = \lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+), \tag{6}$$

where λ_k^- is a large penalty associated with the shortage of a unit of blood, and λ_k^+ is the incurred cost of a unit of surplus blood.

Formulation

Arc Multiplier, and Waste/Loss on a link

Let α_a correspond to the percentage of loss over link a, and f'_a denote the final flow on that link. Thus,

$$f'_a = \alpha_a f_a, \qquad \forall a \in L. \tag{7}$$

Therefore, the waste/loss on link a, denoted by w_a, is equal to:

$$w_a = f_a - f'_a = (1 - \alpha_a) f_a, \qquad \forall a \in L. \tag{8}$$

Total Discarding Cost function

$$\hat{z}_a = \hat{z}_a(f_a), \qquad \forall a \in L. \tag{9}$$

Non-negativity of Flows

$$x_p \geq 0, \qquad \forall p \in \mathcal{P}, \tag{10}$$

Path Multiplier, and Projected Demand

$$\mu_p \equiv \prod_{a \in p} \alpha_a, \qquad \forall p \in \mathcal{P}, \tag{11}$$

where μ_p is the multiplier corresponding to the loss on path p. Thus, the projected demand at R_k is equal to:

$$v_k \equiv \sum_{p \in \mathcal{P}_{w_k}} x_p \mu_p, \qquad k = 1, \ldots, n_R. \tag{12}$$

Relation between Link and Path Flows

$$\alpha_{ap} \equiv \begin{cases} \delta_{ap} \prod_{a'<a} \alpha_{a'}, & \text{if } \{a' < a\} \neq \emptyset, \\ \\ \delta_{ap}, & \text{if } \{a' < a\} = \emptyset, \end{cases} \tag{13}$$

where $\{a' < a\}$ denotes the set of the links preceding link a in path p. Also, δ_{ap} is defined as equal to 1 if link a is contained in path p; otherwise, it is equal to zero. Therefore,

$$f_a = \sum_{p \in \mathcal{P}} x_p \; \alpha_{ap}, \qquad \forall a \in L. \tag{14}$$

Cost Objective Function

Minimization of Total Costs

$$\text{Minimize} \quad \sum_{a\in L} \hat{c}_a(f_a) + \sum_{a\in L} \hat{z}_a(f_a) + \sum_{k=1}^{n_R} (\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)) \tag{15}$$

subject to: constraints (10), (12), and (14).

Supply Side Risk

One of the most significant challenges for the ARC is to capture the risk associated with different activities in the blood supply chain network. Unlike the demand which can be projected, albeit with some uncertainty involved, the amount of donated blood at the collection sites has been observed to be highly stochastic.

Risk Objective Function

$$\text{Minimize} \quad \sum_{a \in L_1} \hat{r}_a(f_a), \tag{16}$$

where $\hat{r}_a = \hat{r}_a(f_a)$ is the total risk function on link a, and L_1 is the set of blood collection links.

The Multicriteria Optimization Formulation

θ: the weight associated with the risk objective function, assigned by the decision maker.

Multicriteria Optimization Formulation in Terms of Link Flows

$$\text{Minimize} \quad \sum_{a\in L}\hat{c}_a(f_a)+\sum_{a\in L}\hat{z}_a(f_a)+\sum_{k=1}^{n_R}(\lambda_k^- E(\Delta_k^-)+\lambda_k^+ E(\Delta_k^+))$$

$$+\theta\sum_{a\in L_1}\hat{r}_a(f_a) \tag{17}$$

subject to: constraints (10), (12), and (14).

The Multicriteria Optimization Formulation

Multicriteria Optimization Formulation in Terms of Path Flows

$$\text{Minimize} \quad \sum_{p \in \mathcal{P}} \left(\hat{C}_p(x) + \hat{Z}_p(x) \right) + \sum_{k=1}^{n_R} \left(\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+) \right)$$

$$+ \theta \sum_{p \in \mathcal{P}} \hat{R}_p(x) \qquad (18)$$

subject to: constraints (10) and (12).

The total costs on path p are expressed as:

$$\hat{C}_p(x) = x_p \times C_p(x), \qquad \forall p \in \mathcal{P}_{w_k}; k = 1, \ldots, n_R, \tag{19a}$$

$$\hat{Z}_p(x) = x_p \times Z_p(x), \qquad \forall p \in \mathcal{P}_{w_k}; k = 1, \ldots, n_R, \tag{19b}$$

$$\hat{R}_p(x) = x_p \times R_p(x), \qquad \forall p \in \mathcal{P}_{w_k}; k = 1, \ldots, n_R, \tag{19c}$$

The unit cost functions on path p are, in turn, defined as below:

$$C_p(x) \equiv \sum_{a \in L} c_a(f_a)\alpha_{ap}, \qquad \forall p \in \mathcal{P}_{w_k}; k = 1, \ldots, n_R, \tag{20a}$$

$$Z_p(x) \equiv \sum_{a \in L} z_a(f_a)\alpha_{ap}, \qquad \forall p \in \mathcal{P}_{w_k}; k = 1, \ldots, n_R, \tag{20b}$$

$$R_p(x) \equiv \sum_{a \in L_1} r_a(f_a)\alpha_{ap}, \qquad \forall p \in \mathcal{P}_{w_k}; k = 1, \ldots, n_R. \tag{20c}$$

Formulation: Preliminaries

It is proved that the partial derivatives of expected shortage at the demand locations with respect to the path flows are derived from:

$$\frac{\partial E(\Delta_k^-)}{\partial x_p} = \mu_p \left[P_k \left(\sum_{p \in \mathcal{P}_{w_k}} x_p \mu_p \right) - 1 \right], \forall p \in \mathcal{P}_{w_k}; k = 1, \ldots, n_R. \quad (21)$$

Similarly, for surplus we have:

$$\frac{\partial E(\Delta_k^+)}{\partial x_p} = \mu_p P_k \left(\sum_{p \in \mathcal{P}_{w_k}} x_p \mu_p \right), \forall p \in \mathcal{P}_{w_k}; k = 1, \ldots, n_R. \quad (22)$$

Formulation: Lemma

The following lemma was developed to help us calculate the partial derivatives of the cost functions:

Lemma 1

$$\frac{\partial \hat{C}_p(x)}{\partial x_p} \equiv \sum_{a \in L} \frac{\partial \hat{c}_a(f_a)}{\partial f_a} \alpha_{ap}, \quad \forall p \in \mathcal{P}_{w_k}; k = 1, \ldots, n_R, \qquad (23a)$$

$$\frac{\partial \hat{Z}_p(x)}{\partial x_p} \equiv \sum_{a \in L} \frac{\partial \hat{z}_a(f_a)}{\partial f_a} \alpha_{ap}, \quad \forall p \in \mathcal{P}_{w_k}; k = 1, \ldots, n_R, \qquad (23b)$$

$$\frac{\partial \hat{R}_p(x)}{\partial x_p} \equiv \sum_{a \in L_1} \frac{\partial \hat{r}_a(f_a)}{\partial f_a} \alpha_{ap}, \quad \forall p \in \mathcal{P}_{w_k}; k = 1, \ldots, n_R. \qquad (23c)$$

Variational Inequality Formulation: Feasible Set and Decision Variables

Let K denote the feasible set such that:

$$K \equiv \{x | x \in R_+^{n_p}\}. \quad (24)$$

Our multicriteria optimization problem is characterized, under our assumptions, by a convex objective function and a convex feasible set.

We group the path flows into the vector x. Also, the link flows, and the projected demands are grouped into the respective vectors f and v.

Variational Inequality Formulation

Theorem 1

The vector x^* is an optimal solution to the multicriteria optimization problem (18), subject to (10) and (12), if and only if it is a solution to the variational inequality problem: determine the vector of optimal path flows $x^* \in K$, such that:

$$\sum_{k=1}^{n_R} \sum_{p \in \mathcal{P}_{w_k}} \left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} + \frac{\partial \hat{Z}_p(x^*)}{\partial x_p} + \lambda_k^+ \mu_p P_k \left(\sum_{p \in \mathcal{P}_{w_k}} x_p^* \mu_p \right) \right.$$

$$\left. -\lambda_k^- \mu_p \left(1 - P_k \left(\sum_{p \in \mathcal{P}_{w_k}} x_p^* \mu_p \right) \right) + \theta \, \frac{\partial \hat{R}_p(x^*)}{\partial x_p} \right] \times [x_p - x_p^*] \geq 0, \; \forall x \in K. \tag{25}$$

Variational Inequality Formulation

Theorem 1 (cont'd)

The variational inequality mentioned, in turn, can be rewritten in terms of link flows as: determine the vector of optimal link flows, and the vector of optimal projected demands $(f^*, v^*) \in K^1$, such that:

$$\sum_{a \in L_1} \left[\frac{\partial \hat{c}_a(f_a^*)}{\partial f_a} + \frac{\partial \hat{z}_a(f_a^*)}{\partial f_a} + \theta \, \frac{\partial \hat{r}_a(f_a^*)}{\partial f_a} \right] \times [f_a - f_a^*]$$

$$+ \sum_{a \in L_1^C} \left[\frac{\partial \hat{c}_a(f_a^*)}{\partial f_a} + \frac{\partial \hat{z}_a(f_a^*)}{\partial f_a} \right] \times [f_a - f_a^*]$$

$$+ \sum_{k=1}^{n_R} \left[\lambda_k^+ P_k(v_k^*) - \lambda_k^- (1 - P_k(v_k^*)) \right] \times [v_k - v_k^*] \geq 0, \qquad \forall (f, v) \in K^1, \tag{26}$$

where K^1 denotes the feasible set as defined below:

$$K^1 \equiv \{(f, v) | \exists x \geq 0, \ (10), (12), \ \text{and} \ (14) \ \text{hold}\}.$$

Standard form of Variational Inequality Formulation (25)

The variational inequality (25) can be put into standard form as follows: determine $X^* \in \mathcal{K}$ such that:

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \qquad \forall X \in \mathcal{K}, \tag{27}$$

If we define the feasible set $\mathcal{K} \equiv K$, and the column vector $X \equiv x$, and $F(X)$, such that:

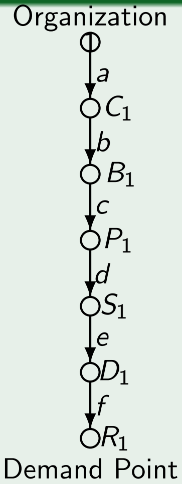
$$F(X) \equiv \left[\frac{\partial \hat{C}_p(x)}{\partial x_p} + \frac{\partial \hat{Z}_p(x)}{\partial x_p} + \lambda_k^+ \mu_p P_k \left(\sum_{p \in \mathcal{P}_{w_k}} x_p \mu_p \right) \right.$$

$$\left. -\lambda_k^- \mu_p \left(1 - P_k \left(\sum_{p \in \mathcal{P}_{w_k}} x_p \mu_p \right) \right) + \theta \, \frac{\partial \hat{R}_p(x)}{\partial x_p}; \; p \in \mathcal{P}_{w_k}; k = 1, \ldots, n_R \right], \tag{28}$$

then (25) can be reexpressed in standard form (27).

Illustrative Numerical Examples

Example 1



Supply Chain Network Topology for Numerical Examples 1 and 2

Example 1 (cont'd)

The total cost functions on the links were:

$$\hat{c}_a(f_a) = f_a^2 + 6f_a, \quad \hat{c}_b(f_b) = 2f_b^2 + 7f_b, \quad c_c(f_c) = f_c^2 + 11f_c,$$

$$\hat{c}_d(f_d) = 3f_d^2 + 11f_d, \quad \hat{c}_e(f_e) = f_e^2 + 2f_e, \quad c_f(f_f) = f_f^2 + f_f.$$

No waste was assumed so that $\alpha_a = 1$ for all links. Hence, all the functions $\hat{z}_a$ were set equal to 0.

The total risk cost function on the blood collection link a was: $\hat{r}_a = 2f_a^2$, and the weight associated with the risk objective, θ, was assumed to be 1.

Example 1 (cont'd)

There is only a single path p_1 which was defined as:
$p_1 = (a, b, c, d, e, f)$ with $\mu_{p_1} = 1$.

Demand for blood followed a uniform distribution on $[0, 5]$ so that:
$P_1(x_{p_1}) = \frac{x_{p_1}}{5}$.

The penalties were: $\lambda_1^- = 100, \quad \lambda_1^+ = 0$.

Example 1: Solution

Substitution of the numerical values, and solving the variational inequality (25) yields:

$$x^*_{p_1} = 1.48,$$

and the corresponding optimal link flow pattern:

$$f^*_a = f^*_b = f^*_c = f^*_d = f^*_e = f^*_f = 1.48.$$

Also, the projected demand is equal to:

$$v^*_1 = x^*_{p_1} = 1.48.$$

Example 2

Example 2 had the same data as Example 1 except that now there was a loss associated with the testing and processing link with $\alpha_c = .8$, and $\hat{z}_c = .5f_c^2$.

Solving the variational inequality (25) yields:

$$x_{p_1}^* = 1.39,$$

which, in turn, yields:

$$f_a^* = f_b^* = f_c^* = 1.39,$$

$$f_d^* = f_e^* = f_f^* = 1.11.$$

The projected demand was:

$$v_1^* = x_{p_1}^* \mu_{p_1} = 1.11.$$

Summary of the Numerical Examples 1 and 2

An Interesting Paradox

Comparing the results of Examples 1 and 2 reveals the fact that when perishability is taken into consideration, with $\alpha_c = .8$ and the data above, the organization chooses to produce/ship slightly smaller quantities so as to minimize the discarding cost of the waste, despite the shortage penalty of λ_1^-.

Summary of the Numerical Examples 1 and 2

Nevertheless, an appropriate increase in the unit shortage penalty cost λ_1^- results in the organization processing larger volumes of the blood product, even exceeding the optimal flow in Example 1, which makes sense intuitively.

The optimal path flow in Example 2 as a function of the link multiplier and the shortage penalty *simultaneously* can be expressed as:

$$x_{p_1}^* = \frac{5\alpha_c(\lambda_1^- - 14) - 120}{\alpha_c^2(\lambda_1^- + 50) + 65}. \tag{29}$$

Sensitivity Analysis Results

Table 1: Computed Optimal Path Flows $x^*_{p_1}$ and Optimal Values of the Objective Function as α_c and λ_1^- Vary

λ_1^- \ α_c		.2	.4	.6	.8	1
100	$x^*_{p_1}$	0.00	0.58	1.16	1.39	1.48
	OF	250.00	246.96	234.00	218.83	204.24
200	$x^*_{p_1}$	0.88	2.40	2.83	2.77	2.61
	OF	494.19	439.52	376.23	326.94	288.35
300	$x^*_{p_1}$	2.10	3.74	3.86	3.54	3.20
	OF	715.12	581.15	464.85	387.17	331.44
400	$x^*_{p_1}$	3.20	4.76	4.57	4.03	3.55
	OF	914.75	689.71	525.36	425.56	357.63
500	$x^*_{p_1}$	4.20	5.57	5.09	4.37	3.79
	OF	1096.03	775.55	569.30	452.16	375.23

Note: Numbers in red are the paradoxical results.

Sensitivity Analysis Results

Table 1 (cont'd)

λ_1^- \ α_c		.2	.4	.6	.8	1
1000	$x_{p_1}^*$	8.09	7.95	6.41	5.19	4.33
	OF	1799.11	1027.94	681.89	515.88	415.67
2000	$x_{p_1}^*$	12.69	9.80	7.27	5.68	4.65
	OF	2631.32	1224.45	755.64	554.47	439.05
3000	$x_{p_1}^*$	15.33	10.58	7.60	5.86	4.76
	OF	3107.51	1307.25	783.73	568.57	447.39
4000	$x_{p_1}^*$	17.03	11.01	7.77	5.96	4.82
	OF	3415.88	1352.89	798.54	575.88	451.68

The Solution Algorithm

The realization of Euler Method for the solution of the blood bank supply chain management problem governed by variational inequality (25) induces subproblems that can be solved explicitly and in closed form.

At iteration τ of the Euler method one computes:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau} F(X^{\tau})), \tag{30}$$

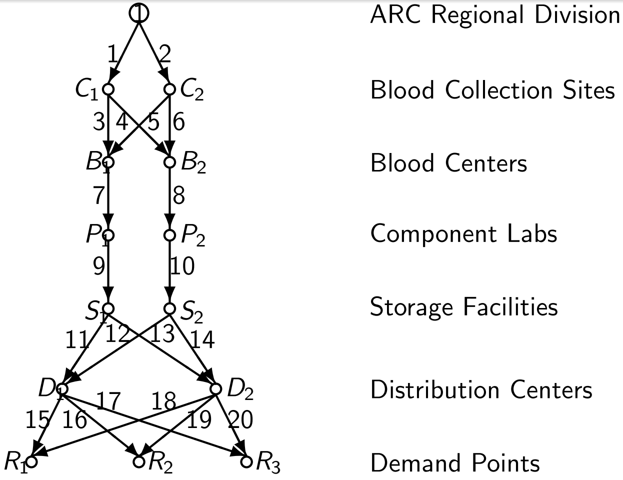
where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$, and F is the function that enters the standard form variational inequality problem (27).

Explicit Formulae for the Euler Method Applied to Our Variational Inequality Formulation

$$x_p^{\tau+1} = \max\{0, x_p^\tau + a_\tau(\lambda_k^- \mu_p(1 - P_k(\sum_{p \in \mathcal{P}_{w_k}} x_p^\tau \mu_p)) - \lambda_k^+ \mu_p P_k(\sum_{p \in \mathcal{P}_{w_k}} x_p^\tau \mu_p)$$
$$- \frac{\partial \hat{C}_p(x^\tau)}{\partial x_p} - \frac{\partial \hat{Z}_p(x^\tau)}{\partial x_p} - \theta \frac{\partial \hat{R}_p(x^\tau)}{\partial x_p})\}, \ \forall p \in \mathcal{P}. \tag{31}$$

The above was applied to calculate the updated product flow during the steps of the Euler Method for our blood banking optimization problem.

Example 3



Supply Chain Network Topology for Numerical Example 3

The demands at these demand points followed uniform probability distribution on the intervals [5,10], [40,50], and [25,40], respectively:

$$P_1\Big(\sum_{p\in\mathcal{P}_{w_1}}\mu_p x_p\Big)=\frac{\sum_{p\in\mathcal{P}_{w_1}}\mu_p x_p-5}{5},\quad P_2\Big(\sum_{p\in\mathcal{P}_{w_2}}\mu_p x_p\Big)=\frac{\sum_{p\in\mathcal{P}_{w_2}}\mu_p x_p-40}{10},$$

$$P_3\Big(\sum_{p\in\mathcal{P}_{w_3}}\mu_p x_p\Big)=\frac{\sum_{p\in\mathcal{P}_{w_3}}\mu_p x_p-25}{15}.$$

$$\lambda_1^-=2200,\quad \lambda_1^+=50,$$
$$\lambda_2^-=3000,\quad \lambda_2^+=60,$$
$$\lambda_3^-=3000,\quad \lambda_3^+=50.$$

$$\hat{r}_1(f_1)=2f_1^2,\quad \text{and}\quad \hat{r}_2(f_2)=1.5f_2^2,$$

and $\theta=0.7$.

Solution Procedure

The Euler method (cf. (31)) for the solution of variational inequality (25) was implemented in Matlab. A Microsoft Windows System with a Dell PC at the University of Massachusetts Amherst was used for all the computations. We set the sequence $a_\mathcal{T} = .1(1, \frac{1}{2}, \frac{1}{2}, \cdots)$, and the convergence tolerance was $\epsilon = 10^{-6}$.

Example 3 Results

Table 2: Total Cost and Total Discarding Cost Functions and Solution for Numerical Example 3

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	f_a^*
1	.97	$6f_1^2 + 15f_1$	$.8f_1^2$	54.72
2	.99	$9f_2^2 + 11f_2$	$.7f_2^2$	43.90
3	1.00	$.7f_3^2 + f_3$	$.6f_3^2$	30.13
4	.99	$1.2f_4^2 + f_4$	$.8f_4^2$	22.42
5	1.00	$f_5^2 + 3f_5$	$.6f_5^2$	19.57
6	1.00	$.8f_6^2 + 2f_6$	$.8f_6^2$	23.46
7	.92	$2.5f_7^2 + 2f_7$	$.5f_7^2$	49.39
8	.96	$3f_8^2 + 5f_8$	$.8f_8^2$	42.00
9	.98	$.8f_9^2 + 6f_9$	$.4f_9^2$	43.63
10	1.00	$.5f_{10}^2 + 3f_{10}$	$.7f_{10}^2$	39.51

Example 3 Results

Table 2 (cont'd)

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	f_a^*
11	1.00	$.3f_{11}^2 + f_{11}$	$.3f_{11}^2$	29.68
12	1.00	$.5f_{12}^2 + 2f_{12}$	$.4f_{12}^2$	13.08
13	1.00	$.4f_{13}^2 + 2f_{13}$	$.3f_{13}^2$	26.20
14	1.00	$.6f_{14}^2 + f_{14}$	$.4f_{14}^2$	13.31
15	1.00	$1.3f_{15}^2 + 3f_{15}$	$.7f_{15}^2$	5.78
16	1.00	$.8f_{16}^2 + 2f_{16}$	$.4f_{16}^2$	25.78
17	.98	$.5f_{17}^2 + 3f_{17}$	$.5f_{17}^2$	24.32
18	1.00	$.7f_{18}^2 + 2f_{18}$	$.7f_{18}^2$	.29
19	1.00	$.6f_{19}^2 + 4f_{19}$	$.4f_{19}^2$	18.28
20	.98	$1.1f_{20}^2 + 5f_{20}$	$.5f_{20}^2$	7.29

The computed amounts of projected demand for each of the three demand points were:

$$v_1^* = 6.06, \quad v_2^* = 44.05, \quad \text{and} \quad v_3^* = 30.99.$$

Summary and Conclusions

We developed a supply chain network optimization model for the management of the procurement, testing and processing, and distribution of a perishable product – that of human blood. Our model:

- captures perishability of this life-saving product through the use of arc multipliers;
- it contains discarding costs associated with waste/disposal;
- it handles uncertainty associated with demand points;
- it assesses costs associated with shortages/surpluses at the demand points, and
- it also quantifies the supply-side risk associated with procurement.

Thank You!

For more information, see: http://supernet.som.umass.edu