

# Supply Chain Network Design of a Sustainable Blood Banking System

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23rd Annual POMS Conference  
Chicago, Illinois - April 20-23, 2012

# Acknowledgments

This research was supported by the John F. Smith Memorial Fund at the University of Massachusetts Amherst. This support is gratefully acknowledged.

The authors acknowledge *Mr. Len Walker*, the Director of Business Development for **the American Red Cross Blood Services** in the greater Boston area, and *Dr. Jorge Rios*, the Medical Director for the American Red Cross Northeast Division Blood Services, for sharing valuable information on the subject, and enlightening thoughts on the model.

# Outline

- Background and Motivation
- Some of the Relevant Literature
- The Sustainable Blood Banking System Supply Chain Network Design Model
- The Algorithm and Explicit Formulae
- Numerical Examples
- Summary and Conclusions



This talk is based on the paper:

Nagurney, A., and Masoumi, A.H. (2012), Supply Chain Network Design of a Sustainable Blood Banking System. *Sustainable Supply Chains: Models, Methods and Public Policy Implications*, Boone, T., Jayaraman, V., and Ganeshan, R., Editors, International Series in Operations Research & Management Science 174, pp49-70, Springer, London, England.

where additional background as well as references can be found.

# Background and Motivation

Blood service operations are a key component of the healthcare system all over the world.



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A blood donation occurs when a person **voluntarily** has blood drawn and used for transfusions.

An event where donors come to give blood is called a **blood drive**. This can occur at a blood bank but they are often set up at a location in the community such as a shopping center, workplace, school, or house of worship.

# Background and Motivation: Types of Donation

- **Allogeneic (Homologous)**: a donor gives blood for storage at a blood bank for transfusion to an unknown recipient.
- **Directed**: a person, often a family member, donates blood for transfusion to a specific individual.
- **Autologous**: a person has blood stored that will be transfused back to the donor at a later date, usually after surgery.

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In the developed world, most blood donors are unpaid volunteers who give blood for an **established community supply**. In poorer countries, donors usually give blood when family or friends need a transfusion.

# Background and Motivation: Screening

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The donor must answer questions about **medical history** and take a short physical examination to make sure the donation is not hazardous to his or her health.

If a potential donor does not meet these criteria, they are **deferred**. This term is used because many donors who are ineligible may be allowed to donate later.

# More on Blood Donation

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The blood is usually stored in a flexible plastic bag that also contains **certain chemicals**. This combination keeps the blood from **clotting** and preserves it during storage.

The US does not have a centralized blood donation service. The **American Red Cross** collects a little less than half of the blood used, the other half is collected by independent agencies, most of which are members of **America's Blood Centers**. The **US military** collects blood from service members for its own use, but also draws blood from the civilian supply.

# Background and Motivation

Over **39,000 donations** are needed everyday in the United States, alone, and the blood supply is frequently reported to be just **2 days** away from running out (American Red Cross).

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Of 1,700 hospitals participating in a survey in 2007, a total of 492 reported cancellations of elective surgeries on one or more days due to blood shortages.

Hospitals with as many days of surgical delays as 50 or even 120 have been observed (Whitaker et al. (2007)).

# Background and Motivation

The hospital cost of a unit of red blood cells in the US had a **6.4%** increase from 2005 to 2007.

In the US, this criticality has become more of an issue in the **North-eastern** and **Southwestern** states since this cost is meaningfully higher compared to that of the Southeastern and Central states.



# Life Cycles of Blood Products

The collected blood is usually stored as separate components.

- Platelets: the longest shelf life is **7 days**.
- Red Blood Cells: a shelf life of **35-42 days** at refrigerated temperatures
- Plasma: can be stored frozen for up to **one year**.

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It is **difficult to have a stockpile of blood to prepare for a disaster**.

After the 9/11 terrorist attacks, it became clear that **collecting during a disaster was impractical** and that efforts should be focused on maintaining an adequate supply at all times.

# Background and Motivation

In 2006, the national estimate for the number of units of blood components outdated by blood centers and hospitals was **1,276,000** out of 15,688,000 units.

Hospitals were responsible for approximately **90% of the outdates**, where this volume of medical waste imposes discarding costs to the already financially-stressed hospitals (The New York Times (2010)).



# Background and Motivation

While many hospitals have their waste burned to avoid polluting the soil through landfills, the incinerators themselves are one of the nations leading sources of toxic pollutants such as **dioxins** and **mercury** (Giusti (2009)).

Healthcare facilities in the United States are **second** only to the food industry in producing waste, generating more than 6,600 tons per day, and more than **4 billion pounds annually** (Fox News (2011)).



## Some of the Relevant Literature

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# Sustainable Blood Banking System

## Supply Chain Network Design Model

We developed a generalized network model for design/redesign of the complex supply chain of human blood, which is a life-saving, **perishable** product.

More specifically, we developed a multicriteria **system-optimization** framework for a regionalized blood supply chain network.

# Sustainable Blood Banking System

## Supply Chain Network Design Model

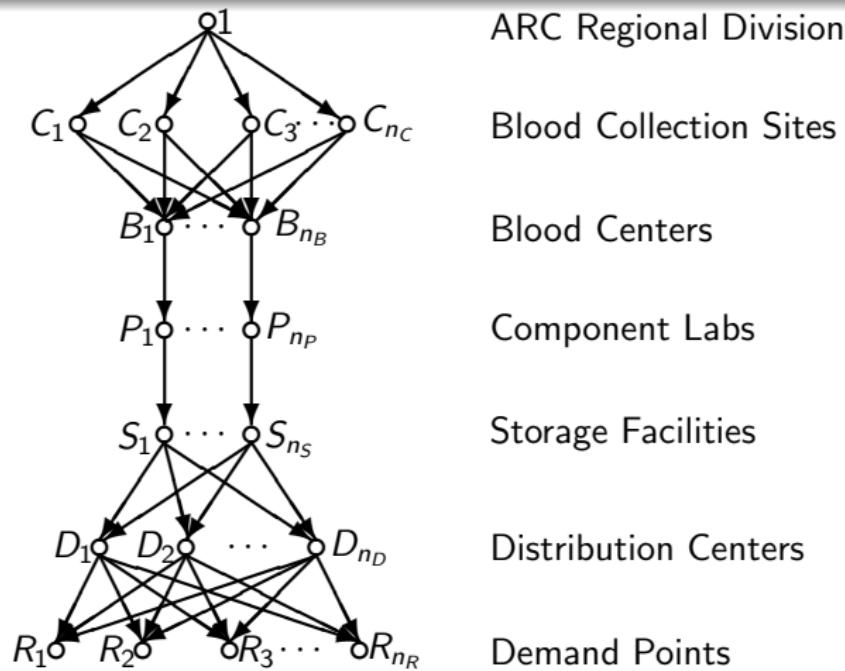
We assume a **network topology** where the top level (origin) corresponds to the organization; i.e., the regional division management of the American Red Cross. The bottom level (destination) nodes correspond to the demand sites - typically the hospitals and the other surgical medical centers.

The **paths** joining the origin node to the destination nodes represent sequences of supply chain network activities that ensure that the blood is collected, tested, processed, and, ultimately, delivered to the demand sites.

# Components of a Regionalized Blood Banking System

- ARC Regional Division Management (Top Tier),  
*Blood Collection*
- Blood Collection Sites (Tier 2), denoted by:  $C_1, C_2, \dots, C_{n_C}$ ,  
*Shipment of Collected Blood*
- Blood Centers (Tier 3), denoted by:  $B_1, B_2, \dots, B_{n_B}$ ,  
*Testing and Processing*
- Component Labs (Tier 4), denoted by:  $P_1, P_2, \dots, P_{n_P}$ ,  
*Storage*
- Storage Facilities (Tier 5), denoted by:  $S_1, S_2, \dots, S_{n_S}$ ,  
*Shipment*
- Distribution Centers (Tier 6), denoted by:  $D_1, D_2, \dots, D_{n_D}$ ,  
*Distribution*
- Demand Points (Tier 7), denoted by:  $R_1, R_2, \dots, R_{n_R}$

## Supply Chain Network Topology for a Regionalized Blood Bank



Graph  $G = [N, L]$ , where  $N$  denotes the set of nodes and  $L$  the set of links.

# Sustainable Blood Banking System

## Supply Chain Network Design Model

Our formalism is that of **multicriteria optimization**, where the organization seeks to determine the optimal levels of blood processed on each supply chain network link coupled with the optimal levels of capacity escalation/reduction in its blood banking supply chain network activities,

*subject to:*

the minimization of the **total cost** associated with its various activities of blood collection, shipment, processing and testing, storage, and distribution, in addition to the **total discarding cost** as well as the minimization of the **total supply risk**, subject to the **uncertain demand** being satisfied as closely as possible at the demand sites.

# Notation

- $c_a$ : the unit operational cost on link  $a$ .
- $\hat{c}_a$ : the total operational cost on link  $a$ .
- $f_a$ : the flow of whole blood/red blood cell on link  $a$ .
- $p$ : a path in the network joining the origin node to a destination node representing the activities and their sequence.
- $w_k$ : the pair of origin/destination (O/D) nodes  $(1, R_k)$ .
- $\mathcal{P}_{w_k}$ : the set of paths, which represent alternative associated possible supply chain network processes, joining  $(1, R_k)$ .
- $\mathcal{P}$ : the set of all paths joining node 1 to the demand nodes.
- $n_p$ : the number of paths from the origin to the demand markets.
- $x_p$ : the nonnegative flow of the blood on path  $p$ .
- $d_k$ : the uncertain demand for blood at demand location  $k$ .
- $v_k$ : the projected demand for blood at demand location  $k$ .

# Formulation

## Total Operational Cost on Link $a$

$$\hat{c}_a(f_a) = f_a \times c_a(f_a), \quad \forall a \in L, \quad (1)$$

assumed to be convex and continuously differentiable.

Let  $P_k$  be the probability distribution function of  $d_k$ , that is,  $P_k(D_k) = P_k(d_k \leq D_k) = \int_0^{D_k} \mathcal{F}_k(t) d(t)$ . Therefore,

## Shortage and Surplus of Blood at Demand Point $R_k$

$$\Delta_k^- \equiv \max\{0, d_k - v_k\}, \quad k = 1, \dots, n_R, \quad (2)$$

$$\Delta_k^+ \equiv \max\{0, v_k - d_k\}, \quad k = 1, \dots, n_R, \quad (3)$$

## Expected Values of Shortage and Surplus

$$E(\Delta_k^-) = \int_{v_k}^{\infty} (t - v_k) \mathcal{F}_k(t) d(t), \quad k = 1, \dots, n_R, \quad (4)$$

$$E(\Delta_k^+) = \int_0^{v_k} (v_k - t) \mathcal{F}_k(t) d(t), \quad k = 1, \dots, n_R. \quad (5)$$

## Expected Total Penalty at Demand Point $k$

$$E(\lambda_k^- \Delta_k^- + \lambda_k^+ \Delta_k^+) = \lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+), \quad (6)$$

where  $\lambda_k^-$  is a large penalty associated with the shortage of a unit of blood, and  $\lambda_k^+$  is the incurred cost of a unit of surplus blood.

# Formulation

## Arc Multiplier, and Waste/Loss on a link

Let  $\alpha_a$  correspond to the percentage of loss over link  $a$ , and  $f'_a$  denote the final flow on that link. Thus,

$$f'_a = \alpha_a f_a, \quad \forall a \in L. \quad (7)$$

Therefore, the waste/loss on link  $a$ , denoted by  $w_a$ , is equal to:

$$w_a = f_a - f'_a = (1 - \alpha_a) f_a, \quad \forall a \in L. \quad (8)$$

## Total Discarding Cost function

$$\hat{z}_a = \hat{z}_a(f_a), \quad \forall a \in L. \quad (9)$$

## Non-negativity of Flows

$$x_p \geq 0, \quad \forall p \in \mathcal{P}, \quad (10)$$

## Path Multiplier, and Projected Demand

$$\mu_p \equiv \prod_{a \in p} \alpha_a, \quad \forall p \in \mathcal{P}, \quad (11)$$

where  $\mu_p$  is the throughput factor on path  $p$ . Thus, the projected demand at  $R_k$  is equal to:

$$v_k \equiv \sum_{p \in \mathcal{P}_{w_k}} x_p \mu_p, \quad k = 1, \dots, n_R. \quad (12)$$

## Relation between Link and Path Flows

$$\alpha_{ap} \equiv \begin{cases} \delta_{ap} \prod_{a' < a} \alpha_{a'}, & \text{if } \{a' < a\} \neq \emptyset, \\ \delta_{ap}, & \text{if } \{a' < a\} = \emptyset, \end{cases} \quad (13)$$

where  $\{a' < a\}$  denotes the set of the links preceding link  $a$  in path  $p$ . Also,  $\delta_{ap}$  is defined as equal to 1 if link  $a$  is contained in path  $p$ ; otherwise, it is equal to zero. Therefore,

$$f_a = \sum_{p \in \mathcal{P}} x_p \alpha_{ap}, \quad \forall a \in L. \quad (14)$$

## Total Investment Cost of Capacity Enhancement/Reduction on Links

$$\hat{\pi}_a = \hat{\pi}_a(u_a), \quad (15)$$

where  $u_a$  denotes the change in capacity on link  $a$ , and  $\hat{\pi}_a$  is the total investment cost of such change.

## Capacity Adjustments Constraints

$$f_a \leq \bar{u}_a + u_a, \quad \forall a \in L, \quad (16)$$

and

$$-\bar{u}_a \leq u_a, \quad \forall a \in L, \quad (17)$$

where  $\bar{u}_a$  denotes the nonnegative existing capacity on link  $a$ .

# Cost Objective Function

## Minimization of Total Costs

$$\begin{aligned} \text{Minimize} \quad & \sum_{a \in L} \hat{c}_a(f_a) + \sum_{a \in L} \hat{z}_a(f_a) + \sum_{a \in L} \hat{\pi}_a(u_a) \\ & + \sum_{k=1}^{n_R} (\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)), \end{aligned} \quad (18)$$

subject to: constraints (10), (12), (14), (16), and (17).

# Supply Side Risk

One of the most significant challenges for the ARC is to **capture the risk** associated with different activities in the blood supply chain network. Unlike the demand which can be projected, albeit with some uncertainty involved, the amount of donated blood at the collection sites has been observed to be **highly stochastic**.

## Risk Objective Function

$$\text{Minimize} \quad \sum_{a \in L_1} \hat{r}_a(f_a), \quad (19)$$

where  $\hat{r}_a = \hat{r}_a(f_a)$  is the total risk function on link  $a$ , and  $L_1$  is the set of blood collection links.

# The Multicriteria Optimization Formulation

$\theta$ : the weight associated with the risk objective function, assigned by the decision maker.

## Multicriteria Optimization Formulation in Terms of **Link Flows**

$$\begin{aligned}
 \text{Minimize} \quad & \sum_{a \in L} \hat{c}_a(f_a) + \sum_{a \in L} \hat{z}_a(f_a) + \sum_{a \in L} \hat{\pi}_a(u_a) \\
 & + \sum_{k=1}^{n_R} (\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)) + \theta \sum_{a \in L_1} \hat{r}_a(f_a), \quad (20)
 \end{aligned}$$

subject to: constraints (10), (12), (14), (16), and (17).

# The Multicriteria Optimization Formulation

## Multicriteria Optimization Formulation in Terms of Path Flows

$$\begin{aligned}
 & \text{Minimize} \quad \sum_{p \in \mathcal{P}} (\hat{C}_p(x) + \hat{Z}_p(x)) + \sum_{a \in L} \hat{\pi}_a(u_a) \\
 & + \sum_{k=1}^{n_R} (\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)) + \theta \sum_{p \in \mathcal{P}} \hat{R}_p(x), \quad (21)
 \end{aligned}$$

subject to: constraints (10), (12), (16), and (17).

The **total costs on path  $p$**  are expressed as:

$$\hat{C}_p(x) = x_p \times C_p(x), \quad \forall p \in \mathcal{P}_{w_k}; k = 1, \dots, n_R, \quad (22a)$$

$$\hat{Z}_p(x) = x_p \times Z_p(x), \quad \forall p \in \mathcal{P}_{w_k}; k = 1, \dots, n_R, \quad (22b)$$

$$\hat{R}_p(x) = x_p \times R_p(x), \quad \forall p \in \mathcal{P}_{w_k}; k = 1, \dots, n_R, \quad (22c)$$

The **unit cost functions on path  $p$**  are, in turn, defined as below:

$$C_p(x) \equiv \sum_{a \in L} c_a(f_a) \alpha_{ap}, \quad \forall p \in \mathcal{P}_{w_k}; k = 1, \dots, n_R, \quad (23a)$$

$$Z_p(x) \equiv \sum_{a \in L} z_a(f_a) \alpha_{ap}, \quad \forall p \in \mathcal{P}_{w_k}; k = 1, \dots, n_R, \quad (23b)$$

$$R_p(x) \equiv \sum_{a \in L_1} r_a(f_a) \alpha_{ap}, \quad \forall p \in \mathcal{P}_{w_k}; k = 1, \dots, n_R. \quad (23c)$$

# Formulation: Preliminaries

It is proved that the partial derivatives of expected shortage at the demand locations with respect to the path flows are derived from:

$$\frac{\partial E(\Delta_k^-)}{\partial x_p} = \mu_p \left[ P_k \left( \sum_{p \in \mathcal{P}_{w_k}} x_p \mu_p \right) - 1 \right], \quad \forall p \in \mathcal{P}_{w_k}; k = 1, \dots, n_R. \quad (24a)$$

Similarly, for surplus we have:

$$\frac{\partial E(\Delta_k^+)}{\partial x_p} = \mu_p P_k \left( \sum_{p \in \mathcal{P}_{w_k}} x_p \mu_p \right), \quad \forall p \in \mathcal{P}_{w_k}; k = 1, \dots, n_R. \quad (24b)$$

# Formulation: Lemma

The following lemma was developed to help us calculate the partial derivatives of the cost functions:

## Lemma 1

$$\frac{\partial(\sum_{q \in \mathcal{P}} \hat{C}_q(x))}{\partial x_p} \equiv \sum_{a \in L} \frac{\partial \hat{c}_a(f_a)}{\partial f_a} \alpha_{ap}, \quad \forall p \in \mathcal{P}_{w_k}; k = 1, \dots, n_R, \quad (25a)$$

$$\frac{\partial(\sum_{q \in \mathcal{P}} \hat{Z}_q(x))}{\partial x_p} \equiv \sum_{a \in L} \frac{\partial \hat{z}_a(f_a)}{\partial f_a} \alpha_{ap}, \quad \forall p \in \mathcal{P}_{w_k}; k = 1, \dots, n_R, \quad (25b)$$

$$\frac{\partial(\sum_{q \in \mathcal{P}} \hat{R}_q(x))}{\partial x_p} \equiv \sum_{a \in L_1} \frac{\partial \hat{r}_a(f_a)}{\partial f_a} \alpha_{ap}, \quad \forall p \in \mathcal{P}_{w_k}; k = 1, \dots, n_R. \quad (25c)$$

# Variational Inequality Formulation: Feasible Set and Decision Variables

Let  $K$  denote the feasible set such that:

$$K \equiv \{(x, u, \gamma) | x \in R_+^{n_p}, (17) \text{ holds, and } \gamma \in R_+^{n_L}\}. \quad (26)$$

Our multicriteria optimization problem is characterized, under our assumptions, by a convex objective function and a convex feasible set.

We group the path flows, the link flows, and the projected demands into the respective vectors  $x$ ,  $f$ , and  $v$ . Also, the link capacity changes are grouped into the vector  $u$ . Lastly, the Lagrange multipliers corresponding to the links capacity adjustment constraints are grouped into the vector  $\gamma$ .

# Variational Inequality Formulation (in Term of Path Flows)

## Theorem 1

Our multicriteria optimization problem, subject to its constraints, is equivalent to the variational inequality problem: determine the vector of optimal path flows, the vector of optimal capacity adjustments, and the vector of optimal Lagrange multipliers  $(x^*, u^*, \gamma^*) \in K$ , such that:

$$\begin{aligned}
 & \sum_{k=1}^{n_R} \sum_{p \in \mathcal{P}_{w_k}} \left[ \frac{\partial(\sum_{q \in \mathcal{P}} \hat{C}_q(x^*))}{\partial x_p} + \frac{\partial(\sum_{q \in \mathcal{P}} \hat{Z}_q(x^*))}{\partial x_p} + \lambda_k^+ \mu_p P_k \left( \sum_{p \in \mathcal{P}_{w_k}} x_p^* \mu_p \right) \right. \\
 & \quad \left. - \lambda_k^- \mu_p \left( 1 - P_k \left( \sum_{p \in \mathcal{P}_{w_k}} x_p^* \mu_p \right) \right) + \sum_{a \in L} \gamma_a^* \delta_{ap} + \theta \frac{\partial(\sum_{q \in \mathcal{P}} \hat{R}_q(x^*))}{\partial x_p} \right] \times [x_p - x_p^*] \\
 & + \sum_{a \in L} \left[ \frac{\partial \hat{\pi}_a(u_a^*)}{\partial u_a} - \gamma_a^* \right] \times [u_a - u_a^*] + \sum_{a \in L} \left[ \bar{u}_a + u_a^* - \sum_{p \in \mathcal{P}} x_p^* \alpha_{ap} \right] \times [\gamma_a - \gamma_a^*] \geq 0, \\
 & \quad \forall (x, u, \gamma) \in K. \tag{27}
 \end{aligned}$$

# Variational Inequality Formulation (in Terms of Link Flows)

## Theorem 1 (cont'd)

The variational inequality mentioned, in turn, can be rewritten in terms of link flows as: determine the vector of optimal link flows, the vectors of optimal projected demands and the link capacity adjustments, and the vector of optimal Lagrange multipliers  $(f^*, v^*, u^*, \gamma^*) \in K^1$ , such that:

$$\begin{aligned}
 & \sum_{a \in L} \left[ \frac{\partial \hat{c}_a(f_a^*)}{\partial f_a} + \frac{\partial \hat{z}_a(f_a^*)}{\partial f_a} + \gamma_a^* + \theta \frac{\partial \hat{r}_a(f_a^*)}{\partial f_a} \right] \times [f_a - f_a^*] \\
 & + \sum_{a \in L} \left[ \frac{\partial \hat{\pi}_a(u_a^*)}{\partial u_a} - \gamma_a^* \right] \times [u_a - u_a^*] + \sum_{k=1}^{n_R} [\lambda_k^+ P_k(v_k^*) - \lambda_k^- (1 - P_k(v_k^*))] \times [v_k - v_k^*] \\
 & + \sum_{a \in L} [\bar{u}_a + u_a^* - f_a^*] \times [\gamma_a - \gamma_a^*] \geq 0, \quad \forall (f, v, u, \gamma) \in K^1, \quad (28)
 \end{aligned}$$

where  $K^1$  denotes the feasible set as defined below:

$$K^1 \equiv \{(f, v, u, \gamma) | \exists x \geq 0, (12), (14), \text{ and } (17) \text{ hold, and } \gamma \geq 0\}. \quad (29)$$

# The Solution Algorithm

The realization of **Euler Method** for the solution of the sustainable blood bank supply chain network design problem governed by the developed variational inequalities induces subproblems that can be solved explicitly and in closed form.

At iteration  $\tau$  of the Euler method one computes:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau} F(X^{\tau})), \quad (30)$$

where  $P_{\mathcal{K}}$  is the **projection** on the feasible set  $\mathcal{K}$ , and  $F$  is the function that enters the standard form variational inequality problem.

# Explicit Formulae for the Euler Method Applied to Our Variational Inequality Formulation

$$\begin{aligned}
 x_p^{\tau+1} = & \max\{0, x_p^\tau + a_\tau (\lambda_k^- \mu_p (1 - P_k (\sum_{p \in \mathcal{P}_{w_k}} x_p^\tau \mu_p)) - \lambda_k^+ \mu_p P_k (\sum_{p \in \mathcal{P}_{w_k}} x_p^\tau \mu_p)) \\
 & - \frac{\partial (\sum_{q \in \mathcal{P}} \hat{C}_q(x^\tau))}{\partial x_p} - \frac{\partial (\sum_{q \in \mathcal{P}} \hat{Z}_q(x^\tau))}{\partial x_p} - \sum_{a \in L} \gamma_a^\tau \delta_{ap} - \theta \frac{\partial (\sum_{q \in \mathcal{P}} \hat{R}_q(x^\tau))}{\partial x_p} \}, \\
 & \forall p \in \mathcal{P}_{w_k}; k = 1, \dots, n_R; \tag{31a}
 \end{aligned}$$

$$u_a^{\tau+1} = \max\{-\bar{u}_a, u_a^\tau + a_\tau (\gamma_a^\tau - \frac{\partial \hat{\pi}_a(u_a^\tau)}{\partial u_a})\}, \quad \forall a \in L; \tag{31b}$$

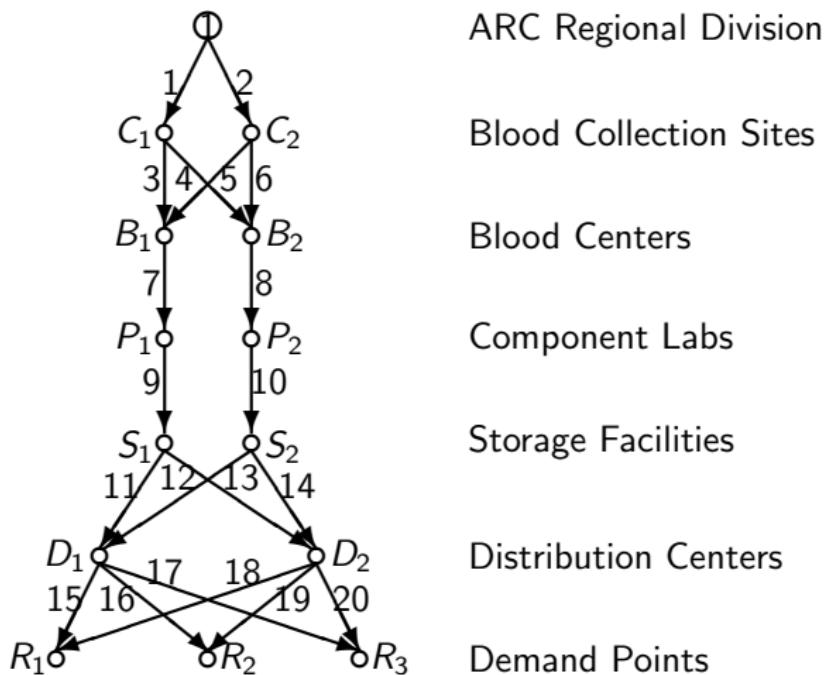
$$\gamma_a^{\tau+1} = \max\{0, \gamma_a^\tau + a_\tau (\sum_{p \in \mathcal{P}} x_p^\tau \alpha_{ap} - \bar{u}_a - u_a^\tau)\}, \quad \forall a \in L. \tag{31c}$$

We applied the above to calculate updated product flows, capacity changes, and Lagrange multipliers during the steps of the Euler Method.

# Illustrative Numerical Examples



# The Supply Chain Network Topology for the Numerical Examples



## Example 1: Design from Scratch

The demands at these demand points followed uniform probability distribution on the intervals [5,10], [40,50], and [25,40], respectively:

$$P_1\left(\sum_{p \in \mathcal{P}_{w_1}} \mu_p x_p\right) = \frac{\sum_{p \in \mathcal{P}_{w_1}} \mu_p x_p - 5}{5}, \quad P_2\left(\sum_{p \in \mathcal{P}_{w_2}} \mu_p x_p\right) = \frac{\sum_{p \in \mathcal{P}_{w_2}} \mu_p x_p - 40}{10},$$

$$P_3\left(\sum_{p \in \mathcal{P}_{w_3}} \mu_p x_p\right) = \frac{\sum_{p \in \mathcal{P}_{w_3}} \mu_p x_p - 25}{15}.$$

$$\lambda_1^- = 2800, \quad \lambda_1^+ = 50,$$

$$\lambda_2^- = 3000, \quad \lambda_2^+ = 60,$$

$$\lambda_3^- = 3100, \quad \lambda_3^+ = 50.$$

$$\hat{r}_1(f_1) = 2f_1^2, \quad \hat{r}_2(f_2) = 1.5f_2^2, \quad \text{and } \theta = 0.7$$

# Solution Procedure

The Euler method for the solution of variational inequality (27) was implemented in **Matlab**. A Microsoft Windows System with a Dell PC at the University of Massachusetts Amherst was used for all the computations. We set the sequence  $a_\tau = .1(1, \frac{1}{2}, \frac{1}{2}, \dots)$ , and the convergence tolerance was  $\epsilon = 10^{-6}$ .

## Example 1 Results: Total Cost Functions and Solution

Link $a$	$\alpha_a$	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	$\hat{\pi}_a(u_a)$	$f_a^*$	$u_a^*$	$\gamma_a^*$
1	.97	$6f_1^2 + 15f_1$	$.8f_1^2$	$.8u_1^2 + u_1$	47.18	47.18	76.49
2	.99	$9f_2^2 + 11f_2$	$.7f_2^2$	$.6u_2^2 + u_2$	39.78	39.78	48.73
3	1.00	$.7f_3^2 + f_3$	$.6f_3^2$	$u_3^2 + 2u_3$	25.93	25.93	53.86
4	.99	$1.2f_4^2 + f_4$	$.8f_4^2$	$2u_4^2 + u_4$	19.38	19.38	78.51
5	1.00	$f_5^2 + 3f_5$	$.6f_5^2$	$u_5^2 + u_5$	18.25	18.25	37.50
6	1.00	$.8f_6^2 + 2f_6$	$.8f_6^2$	$1.5u_6^2 + 3u_6$	20.74	20.74	65.22
7	.92	$2.5f_7^2 + 2f_7$	$.5f_7^2$	$7u_7^2 + 12u_7$	43.92	43.92	626.73
8	.96	$3f_8^2 + 5f_8$	$.8f_8^2$	$6u_8^2 + 20u_8$	36.73	36.73	460.69
9	.98	$.8f_9^2 + 6f_9$	$.4f_9^2$	$3u_9^2 + 2u_9$	38.79	38.79	234.74
10	1.00	$.5f_{10}^2 + 3f_{10}$	$.7f_{10}^2$	$5.4u_{10}^2 + 2u_{10}$	34.56	34.56	375.18
11	1.00	$.3f_{11}^2 + f_{11}$	$.3f_{11}^2$	$u_{11}^2 + u_{11}$	25.90	25.90	52.80
12	1.00	$.5f_{12}^2 + 2f_{12}$	$.4f_{12}^2$	$1.5u_{12}^2 + u_{12}$	12.11	12.11	37.34
13	1.00	$.4f_{13}^2 + 2f_{13}$	$.3f_{13}^2$	$1.8u_{13}^2 + 1.5u_{13}$	17.62	17.62	64.92
14	1.00	$.6f_{14}^2 + f_{14}$	$.4f_{14}^2$	$u_{14}^2 + 2u_{14}$	16.94	16.94	35.88
15	1.00	$.4f_{15}^2 + f_{15}$	$.7f_{15}^2$	$.5u_{15}^2 + 1.1u_{15}$	5.06	5.06	6.16
16	1.00	$.8f_{16}^2 + 2f_{16}$	$.4f_{16}^2$	$.7u_{16}^2 + 3u_{16}$	24.54	24.54	37.36
17	.98	$.5f_{17}^2 + 3f_{17}$	$.5f_{17}^2$	$2u_{17}^2 + u_{17}$	13.92	13.92	56.66
18	1.00	$.7f_{18}^2 + f_{18}$	$.7f_{18}^2$	$u_{18}^2 + u_{18}$	0.00	0.00	1.00
19	1.00	$.6f_{19}^2 + 4f_{19}$	$.4f_{19}^2$	$u_{19}^2 + 2u_{19}$	15.93	15.93	33.86
20	.98	$1.1f_{20}^2 + 5f_{20}$	$.5f_{20}^2$	$.8u_{20}^2 + u_{20}$	12.54	12.54	21.06

## Example 1 Results (cont'd)

The values of the total investment cost and the cost objective criterion were **42,375.96** and **135,486.43**, respectively.

The computed amounts of projected demand for each of the three demand points were:

$$v_1^* = 5.06, \quad v_2^* = 40.48, \quad \text{and} \quad v_3^* = 25.93.$$

Note that the values of the projected demand were closer to the **lower bounds** of their uniform probability distributions due to the relatively high cost of setting up a new blood supply chain network from scratch.

## Example 2: Increased Penalties

Example 2 had the exact same data as Example 1 with the exception of the penalties per unit shortage which were ten times larger.

$$\lambda_1^- = 28000, \quad \lambda_2^- = 30000, \quad \lambda_3^- = 31000.$$

## Example 2 Results: Total Cost Functions and Solution

Link $a$	$\alpha_a$	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	$\hat{\pi}_a(u_a)$	$f_a^*$	$u_a^*$	$\gamma_a^*$
1	.97	$6f_1^2 + 15f_1$	$.8f_1^2$	$.8u_1^2 + u_1$	63.53	63.53	102.65
2	.99	$9f_2^2 + 11f_2$	$.7f_2^2$	$.6u_2^2 + u_2$	53.53	53.53	65.23
3	1.00	$.7f_3^2 + f_3$	$.6f_3^2$	$u_3^2 + 2u_3$	34.93	34.93	71.85
4	.99	$1.2f_4^2 + f_4$	$.8f_4^2$	$2u_4^2 + u_4$	26.08	26.08	105.34
5	1.00	$f_5^2 + 3f_5$	$.6f_5^2$	$u_5^2 + u_5$	24.50	24.50	50.00
6	1.00	$.8f_6^2 + 2f_6$	$.8f_6^2$	$1.5u_6^2 + 3u_6$	27.96	27.96	86.89
7	.92	$2.5f_7^2 + 2f_7$	$.5f_7^2$	$7u_7^2 + 12u_7$	59.08	59.08	839.28
8	.96	$3f_8^2 + 5f_8$	$.8f_8^2$	$6u_8^2 + 20u_8$	49.48	49.48	613.92
9	.98	$.8f_9^2 + 6f_9$	$.4f_9^2$	$3u_9^2 + 2u_9$	52.18	52.18	315.05
10	1.00	$.5f_{10}^2 + 3f_{10}$	$.7f_{10}^2$	$5.4u_{10}^2 + 2u_{10}$	46.55	46.55	504.85
11	1.00	$.3f_{11}^2 + f_{11}$	$.3f_{11}^2$	$u_{11}^2 + u_{11}$	35.01	35.01	71.03
12	1.00	$.5f_{12}^2 + 2f_{12}$	$.4f_{12}^2$	$1.5u_{12}^2 + u_{12}$	16.12	16.12	49.36
13	1.00	$.4f_{13}^2 + 2f_{13}$	$.3f_{13}^2$	$1.8u_{13}^2 + 1.5u_{13}$	23.93	23.93	87.64
14	1.00	$.6f_{14}^2 + f_{14}$	$.4f_{14}^2$	$u_{14}^2 + 2u_{14}$	22.63	22.63	47.25
15	1.00	$.4f_{15}^2 + f_{15}$	$.7f_{15}^2$	$.5u_{15}^2 + 1.1u_{15}$	9.33	9.33	10.43
16	1.00	$.8f_{16}^2 + 2f_{16}$	$.4f_{16}^2$	$.7u_{16}^2 + 3u_{16}$	29.73	29.73	44.62
17	.98	$.5f_{17}^2 + 3f_{17}$	$.5f_{17}^2$	$2u_{17}^2 + u_{17}$	19.89	19.89	80.55
18	1.00	$.7f_{18}^2 + f_{18}$	$.7f_{18}^2$	$u_{18}^2 + u_{18}$	0.00	0.00	1.00
19	1.00	$.6f_{19}^2 + 4f_{19}$	$.4f_{19}^2$	$u_{19}^2 + 2u_{19}$	18.99	18.99	39.97
20	.98	$1.1f_{20}^2 + 5f_{20}$	$.5f_{20}^2$	$.8u_{20}^2 + u_{20}$	18.98	18.98	31.37

## Example 2 Results (cont'd)

Raising the shortage penalties increased the level of activities in almost all the network links. The new projected demand values were:

$$v_1^* = 9.33, \quad v_2^* = 48.71, \quad \text{and} \quad v_3^* = 38.09.$$

Here the projected demand values were closer to the **upper bounds** of their uniform probability distributions.

Thus, the values of the total investment cost and the cost objective criterion, were **75,814.03** and **177,327.31**, respectively, which were significantly higher than Example 1.

## Example 3: Redesign Problem

The existing capacities for links were chosen **close to the optimal solution** for corresponding capacities in Example 1.

All other parameters were the same as in Example 1.

## Example 3 Results: Total Cost Functions and Solution

Link $a$	$\alpha_a$	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	$\hat{\pi}_a(u_a)$	$\bar{u}_a$	$f_a^*$	$u_a^*$	$\gamma_a^*$
1	.97	$6f_1^2 + 15f_1$	$.8f_1^2$	$.8u_1^2 + u_1$	48	54.14	6.14	10.83
2	.99	$9f_2^2 + 11f_2$	$.7f_2^2$	$.6u_2^2 + u_2$	40	43.85	3.85	5.62
3	1.00	$.7f_3^2 + f_3$	$.6f_3^2$	$u_3^2 + 2u_3$	26	29.64	3.64	9.29
4	.99	$1.2f_4^2 + f_4$	$.8f_4^2$	$2u_4^2 + u_4$	20	22.35	2.35	10.39
5	1.00	$f_5^2 + 3f_5$	$.6f_5^2$	$u_5^2 + u_5$	19	20.10	1.10	3.20
6	1.00	$.8f_6^2 + 2f_6$	$.8f_6^2$	$1.5u_6^2 + 3u_6$	21	22.88	1.88	8.63
7	.92	$2.5f_7^2 + 2f_7$	$.5f_7^2$	$7u_7^2 + 12u_7$	44	49.45	5.45	88.41
8	.96	$3f_8^2 + 5f_8$	$.8f_8^2$	$6u_8^2 + 20u_8$	37	41.40	4.40	72.88
9	.98	$.8f_9^2 + 6f_9$	$.4f_9^2$	$3u_9^2 + 2u_9$	39	43.67	4.67	30.04
10	1.00	$.5f_{10}^2 + 3f_{10}$	$.7f_{10}^2$	$5.4u_{10}^2 + 2u_{10}$	35	38.95	3.95	44.70
11	1.00	$.3f_{11}^2 + f_{11}$	$.3f_{11}^2$	$u_{11}^2 + u_{11}$	26	29.23	3.23	7.45
12	1.00	$.5f_{12}^2 + 2f_{12}$	$.4f_{12}^2$	$1.5u_{12}^2 + u_{12}$	13	13.57	0.57	2.72
13	1.00	$.4f_{13}^2 + 2f_{13}$	$.3f_{13}^2$	$1.8u_{13}^2 + 1.5u_{13}$	18	22.05	4.05	16.07
14	1.00	$.6f_{14}^2 + f_{14}$	$.4f_{14}^2$	$u_{14}^2 + 2u_{14}$	17	16.90	-0.10	1.81
15	1.00	$.4f_{15}^2 + f_{15}$	$.7f_{15}^2$	$.5u_{15}^2 + 1.1u_{15}$	6	6.62	0.62	1.72
16	1.00	$.8f_{16}^2 + 2f_{16}$	$.4f_{16}^2$	$.7u_{16}^2 + 3u_{16}$	25	25.73	0.73	4.03
17	.98	$.5f_{17}^2 + 3f_{17}$	$.5f_{17}^2$	$2u_{17}^2 + u_{17}$	14	18.92	4.92	20.69
18	1.00	$.7f_{18}^2 + f_{18}$	$.7f_{18}^2$	$u_{18}^2 + u_{18}$	0	0.00	0.00	1.00
19	1.00	$.6f_{19}^2 + 4f_{19}$	$.4f_{19}^2$	$u_{19}^2 + 2u_{19}$	16	17.77	1.77	5.53
20	.98	$1.1f_{20}^2 + 5f_{20}$	$.5f_{20}^2$	$.8u_{20}^2 + u_{20}$	13	12.10	-0.62	0.00

## Example 3 Results (cont'd)

The optimal Lagrangian multipliers,  $\gamma_a^*$ , i.e., the **shadow prices** of the capacity adjustment constraints, were **considerably smaller** than their counterparts in Example 1.

So, the respective values of the capacity investment cost and the cost criterion were **856.36** and **85,738.13**.

The computed projected demand values:

$$v_1^* = 6.62, \quad v_2^* = 43.50, \quad \text{and} \quad v_3^* = 30.40.$$

## Example 4: Increased Demands

The existing capacities, the shortage penalties, and the cost functions were the same as in Example 3.

However, the demands at the three hospitals were **escalated**, following uniform probability distributions on the intervals  $[10,17]$ ,  $[50,70]$ , and  $[30,60]$ , respectively.

## Example 4 Results: Total Cost Functions and Solution

Link $a$	$\alpha_a$	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	$\hat{\pi}_a(u_a)$	$\bar{u}_a$	$f_a^*$	$u_a^*$	$\gamma_a^*$
1	.97	$6f_1^2 + 15f_1$	$.8f_1^2$	$.8u_1^2 + u_1$	48	65.45	17.45	28.92
2	.99	$9f_2^2 + 11f_2$	$.7f_2^2$	$.6u_2^2 + u_2$	40	53.36	13.36	17.03
3	1.00	$.7f_3^2 + f_3$	$.6f_3^2$	$u_3^2 + 2u_3$	26	35.87	9.87	21.74
4	.99	$1.2f_4^2 + f_4$	$.8f_4^2$	$2u_4^2 + u_4$	20	26.98	6.98	28.91
5	1.00	$f_5^2 + 3f_5$	$.6f_5^2$	$u_5^2 + u_5$	19	24.43	5.43	11.86
6	1.00	$.8f_6^2 + 2f_6$	$.8f_6^2$	$1.5u_6^2 + 3u_6$	21	27.87	6.87	23.60
7	.92	$2.5f_7^2 + 2f_7$	$.5f_7^2$	$7u_7^2 + 12u_7$	44	59.94	15.94	234.92
8	.96	$3f_8^2 + 5f_8$	$.8f_8^2$	$6u_8^2 + 20u_8$	37	50.21	13.21	178.39
9	.98	$.8f_9^2 + 6f_9$	$.4f_9^2$	$3u_9^2 + 2u_9$	39	52.94	13.94	85.77
10	1.00	$.5f_{10}^2 + 3f_{10}$	$.7f_{10}^2$	$5.4u_{10}^2 + 2u_{10}$	35	47.24	12.24	134.64
11	1.00	$.3f_{11}^2 + f_{11}$	$.3f_{11}^2$	$u_{11}^2 + u_{11}$	26	35.68	9.68	20.35
12	1.00	$.5f_{12}^2 + 2f_{12}$	$.4f_{12}^2$	$1.5u_{12}^2 + u_{12}$	13	16.20	3.20	10.61
13	1.00	$.4f_{13}^2 + 2f_{13}$	$.3f_{13}^2$	$1.8u_{13}^2 + 1.5u_{13}$	18	26.54	8.54	32.23
14	1.00	$.6f_{14}^2 + f_{14}$	$.4f_{14}^2$	$u_{14}^2 + 2u_{14}$	17	20.70	3.70	9.40
15	1.00	$.4f_{15}^2 + f_{15}$	$.7f_{15}^2$	$.5u_{15}^2 + 1.1u_{15}$	6	10.30	4.30	5.40
16	1.00	$.8f_{16}^2 + 2f_{16}$	$.4f_{16}^2$	$.7u_{16}^2 + 3u_{16}$	25	30.96	5.96	11.34
17	.98	$.5f_{17}^2 + 3f_{17}$	$.5f_{17}^2$	$2u_{17}^2 + u_{17}$	14	20.95	6.95	28.81
18	1.00	$.7f_{18}^2 + f_{18}$	$.7f_{18}^2$	$u_{18}^2 + u_{18}$	0	0.35	0.35	1.69
19	1.00	$.6f_{19}^2 + 4f_{19}$	$.4f_{19}^2$	$u_{19}^2 + 2u_{19}$	16	21.68	5.68	13.36
20	.98	$1.1f_{20}^2 + 5f_{20}$	$.5f_{20}^2$	$.8u_{20}^2 + u_{20}$	13	14.14	1.14	2.83

## Example 4 Results (cont'd)

A 50% increase in demand resulted in **significant positive capacity changes** as well as **positive flows on all 20 links** in the network.

The values of the total investment function and the cost criterion were **5,949.18** and **166,445.73**, respectively.

The projected demand values were now:

$$v_1^* = 10.65, \quad v_2^* = 52.64, \quad \text{and} \quad v_3^* = 34.39.$$

## Example 5: Decreased Demands

Example 5 was similar to Example 4, but now the **demand suffered a decrease** from the original demand scenario.

The demand at demand points 1, 2, and 3 followed a uniform probability distribution on the intervals [4,7], [30,40], and [15,30], respectively.

## Example 5 Results: Total Cost Functions and Solution

Link $a$	$\alpha_a$	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	$\hat{\pi}_a(u_a)$	$\bar{u}_a$	$f_a^*$	$u_a^*$	$\gamma_a^*$
1	.97	$6f_1^2 + 15f_1$	$.8f_1^2$	$.8u_1^2 + u_1$	48	43.02	-0.62	0.00
2	.99	$9f_2^2 + 11f_2$	$.7f_2^2$	$.6u_2^2 + u_2$	40	34.54	-0.83	0.00
3	1.00	$.7f_3^2 + f_3$	$.6f_3^2$	$u_3^2 + 2u_3$	26	23.77	-1.00	0.00
4	.99	$1.2f_4^2 + f_4$	$.8f_4^2$	$2u_4^2 + u_4$	20	17.54	-0.25	0.00
5	1.00	$f_5^2 + 3f_5$	$.6f_5^2$	$u_5^2 + u_5$	19	15.45	-0.50	0.00
6	1.00	$.8f_6^2 + 2f_6$	$.8f_6^2$	$1.5u_6^2 + 3u_6$	21	18.40	-1.00	0.00
7	.92	$2.5f_7^2 + 2f_7$	$.5f_7^2$	$7u_7^2 + 12u_7$	44	38.99	-0.86	0.00
8	.96	$3f_8^2 + 5f_8$	$.8f_8^2$	$6u_8^2 + 20u_8$	37	32.91	-1.67	0.00
9	.98	$.8f_9^2 + 6f_9$	$.4f_9^2$	$3u_9^2 + 2u_9$	39	34.43	-0.33	0.00
10	1.00	$.5f_{10}^2 + 3f_{10}$	$.7f_{10}^2$	$5.4u_{10}^2 + 2u_{10}$	35	30.96	-0.19	0.00
11	1.00	$.3f_{11}^2 + f_{11}$	$.3f_{11}^2$	$u_{11}^2 + u_{11}$	26	23.49	-0.50	0.00
12	1.00	$.5f_{12}^2 + 2f_{12}$	$.4f_{12}^2$	$1.5u_{12}^2 + u_{12}$	13	10.25	-0.33	0.00
13	1.00	$.4f_{13}^2 + 2f_{13}$	$.3f_{13}^2$	$1.8u_{13}^2 + 1.5u_{13}$	18	18.85	0.85	4.57
14	1.00	$.6f_{14}^2 + f_{14}$	$.4f_{14}^2$	$u_{14}^2 + 2u_{14}$	17	12.11	-1.00	0.00
15	1.00	$.4f_{15}^2 + f_{15}$	$.7f_{15}^2$	$.5u_{15}^2 + 1.1u_{15}$	6	5.52	-0.48	0.63
16	1.00	$.8f_{16}^2 + 2f_{16}$	$.4f_{16}^2$	$.7u_{16}^2 + 3u_{16}$	25	20.68	-2.14	0.00
17	.98	$.5f_{17}^2 + 3f_{17}$	$.5f_{17}^2$	$2u_{17}^2 + u_{17}$	14	16.15	2.15	9.59
18	1.00	$.7f_{18}^2 + f_{18}$	$.7f_{18}^2$	$u_{18}^2 + u_{18}$	0	0.00	0.00	1.00
19	1.00	$.6f_{19}^2 + 4f_{19}$	$.4f_{19}^2$	$u_{19}^2 + 2u_{19}$	16	14.58	-1.00	0.00
20	.98	$1.1f_{20}^2 + 5f_{20}$	$.5f_{20}^2$	$.8u_{20}^2 + u_{20}$	13	7.34	-0.62	0.00

## Example 5 Results (cont'd)

As expected, most of the computed **capacity changes** were **negative** as a result of the diminished demand for blood at our demand points.

The projected demand values were as follows:

$$v_1^* = 5.52, \quad v_2^* = 35.25, \quad \text{and} \quad v_3^* = 23.02.$$

The value of the total cost criterion for this Example was **51,221.32**.

# Summary and Conclusions

we developed a sustainable supply chain network design model for a highly perishable health care product – that of human blood. Our model:

- captures the **perishability** of this life-saving product through the use of arc multipliers;
- contains **discarding costs** associated with waste/disposal;
- determines the **optimal enhancement/reduction of capacities** as well as the determination of the capacities from scratch;
- can capture the cost-related effects of **shutting down** specific modules of the supply chain due to an economic crisis;
- handles **uncertainty** associated with demand points;
- assesses **shortage/surplus penalties** at the demand points, and
- quantifies the **supply-side risk** associated with procurement.

# Thank You!

## The Virtual Center for Supernetwroks

Supernetwroks for Optimal Decision-Making and Improving the Global Quality of Life

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**The Virtual Center for Supernetwroks** at the Isenberg School of Management, under the directorship of Anna Nagurney, the John F. Smith Memorial Professor, is an interdisciplinary center, and includes the Supernetwroks Laboratory for Computation and Visualization.

**Mission:** The mission of the Virtual Center for Supernetwroks is to foster the study and application of supernetwroks and to serve as a resource to academia, industry, and government on networks ranging from transportation, supply chains, telecommunication, and electric power networks to economic, environmental, financial, knowledge and social networks.

**The Applications of Supernetwroks Include:** complex networks and decision-making; critical infrastructure from transportation to electric power and the Internet; financial, economic, and social networks; energy and the environment; global supply chain management; corporate social responsibility; risk management; network vulnerability, resiliency, and performance metrics; ecological networks; humanitarian logistics and healthcare.

[Announcements and Notes from the Center Director](#)  
[Professor Anna Nagurney](#)

Updated: April 13, 2012

### Professor Anna Nagurney's Blog

#### RENEW

Research, Education, Networks, and the World:  
**A Female Professor Speaks**

[Sustaining the Supply Chain](#)

[Mathematical Moments Podcast](#)

### PBS VIDEO

### Publications

[A Paradox of Traffic Planning](#)  
*Environmental Impact Assessment of Transportation Networks with Degradeable Links in an Era of Climate Change*  
 (with Nagurny, Qiang Geng, and Lubomir N. Nagurny)

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