Supply Chain Game Theory Network Modeling Under Labor Constraints: Applications to the COVID-19 Pandemic

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Acknowledgments

This presentation is dedicated to essential workers, who sustained us in the COVID-19 pandemic. I also acknowledge all the freedom-loving people on the planet, including those fighting for their freedom in Ukraine.

It's All About People

A major research theme of ours in the COVID-19 pandemic is the inclusion of labor in supply chains, using optimization and game theory. This research is also very relevant with Russia's war against Ukraine.

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In the End, It's All About People

COVID-19 vaccine production reveals dependency on supply chains, labor workforce in the U.S.

By Anna Nagurney

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The COVID-19 pandemic has dramatically revealed how dependent we are on supply chains and the availability of labor. Without the human element, meatpacking plants cannot function; fresh produce cannot be picked; grocery stores cannot be shelved; PPEs cannot be produced and distributed; and products cannot be delivered to our homes through e-commerce. Also, COVID-19 vaccine production may lack the human resources to ensure product quality and efficacy, as well as its distribution and ultimate administration into our arms. Without healthcare workers to administer COVID-19 vaccines, the battle against the coronavirus cannot be won. Many hospitals are already short-staffed because of the pandemic.

Outline of Presentation

- **Background and Motivation**
- **Game Theory Supply Chain Network Models with Labor**
- **The Algorithm**
- **Numerical Examples**
- **Some Additional Research**

Game Theory Supply Chain Network Modeling with Labor

This presentation is based on the paper, **"Supply Chain Game Theory Network Modeling Under Labor Constraints: Applications to the Covid-19 Pandemic," A. Nagurney,** *European Journal of Operational Research* **293(3), (2021), pp 880-891**, in which a game theory model for supply chains with labor was constructed, under three different sets of constraints, building on our previous work.

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Supply chain game theory network modeling under labor constraints: Applications to the Covid-19 pandemic

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ABSTRACT

The Covid-19 pandemic has brought attention to supply chain networks due to disruptions for many reasons, including that of labor shortages as a consequences of illnesses, death, risk mitigation, as well as travel restrictions. Many sectors of the economy from food to healthcare have been competing for workers, as a consequence. In this paper, we construct a supply chain game theory network framework that captures labor constraints under three different scenarios. The appropriate equilibrium constructs are defined, along with their variational inequality formulations. Computed solutions to numerical examples inspired by shortages of migrant labor to harvest fresh produce; specifically, blueberries, in the United States, reveal the impacts of a spectrum of disruptions to labor on the product flows and the profits of the firms in the supply chain network economy. This research adds to the literature in both economics and operations research.

1. Introduction

The Covid-19 pandemic, officially declared by the World Health Organization on March 11, 2020, has demonstrated dramatically the importance of the health of workers to the global economy (Organization, 2020). Product supply chains as varied as those for Personal Protective Equipment (PPEs) (Burki, 2020) and other medical supplies (Ranney, Griffieth, & Jha, 2020), toilet paper and cleaning supplies (Gao, 2020), meat (Corkery & Yaffe-Bellany, 2020) and fresh produce (Knight, 2020; Laborde, Martin, Swinnen, & Vos, 2020), and even blood (Nagurney, 2020a), have been disrupted for reasons including that of the reduction of labor availability because of illnesses and death of workers, along with workers' fear of contracting the disease. Some workers have not been able to travel for seasonal employment because of travel restrictions to mitigate the spread of the coronavirus that causes the Covid-19 disease, resulting in both economic and personal losses (IHS-Markit, 2020). According to Elflein (2020), as of August 14, 2020, Covid-19 had spread to six continents, with over 750,000 succumbing to the coronavirus that causes the Covid-19 disease. Just over 2 months after, as of October 27, 2020, there were 1,160,421 documented deaths globally due to Covid-19 (JHU-CSSE2020, 2020). The negative impact of this global healthcare and economic crisis has affected different tiers of supply chains and associated network economic activities of production, transportation, storage, and distribution.

Labor has emerged as an essential factor in the functioning of supply chains in the pandemic. Without farmers, food cannot be produced. Without workers to harvest, food may end up rotting in the fields or be discarded (see Associated Press, 2020); without freight service provision food cannot be delivered to food processors; without food processors food cannot be processed, resulting in additional waste (see Polansek & Huffstutter, 2020). Ultimately, freight service provision is also essential for deliveries to retail outlets for purchase by consumers or even for direct transport to consumers using, for example, electronic commerce. Freight service workers have also been impacted by Covid-19 (Parker, 2020). Employees at distribution centers have been felled by Covid-19 (Daniels, 2020), as well, and Covid-19 has further exacerbated worker shortages at a time of greater demand because of electronic commerce (see Hardwick, 2020). Shortages of labor in manufacturing facilities were an issue even earlier on in February 2020 in China, since that country was impacted first by the coronavirus, which was to have originated in Wuhan, with implications for global supply chains (see Bloomberg, 2020). Labor shortages arose due to illnesses, need for social distancing in facilities, as well as adherence to quarantines. Each link of a product supply chain requires labor, and, therefore, reduction in labor capacity can propagate through paths of a supply chain network, affecting product flows as well as prices (see, e.g., OECD, 2020). Furthermore, unhealthy workers cannot be fully productive.

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Background and Motivation

Background and Motivation

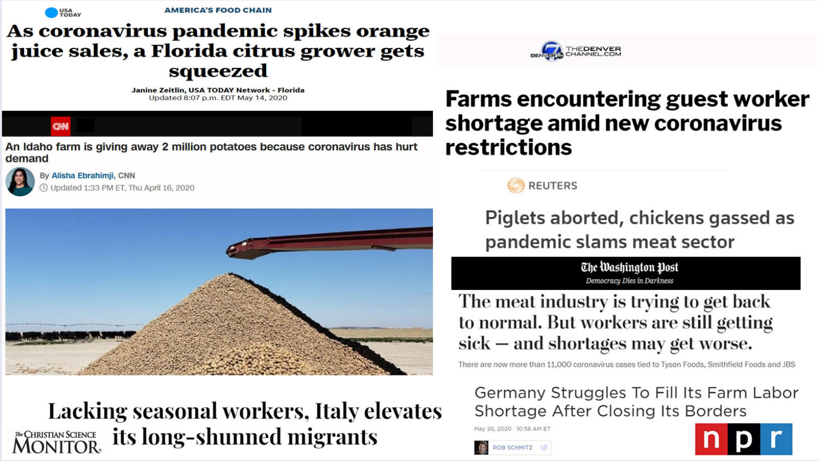
The COVID-19 pandemic has demonstrated the importance of supply chains and their effective and efficient operation. The reasons for disruptions have been multifaceted with shocks both on the demand side as well as on the supply side and challenges associated with transport.

A major characteristic of the pandemic has been that of labor shortages. Workers throughout the pandemic have been falling ill; many, sadly, lost their lives, whereas others chose to switch jobs or to leave the labor force.

Various countries imposed restrictions further impeding the flow of workers.

Background and Motivation

Employers have had difficulties recruiting workers not only with advanced technical skills but also those with low and middle level skills.



Game Theory Supply Chain Network Models with Labor

The first set of constraints on labor yields a **Nash Equilibrium model**.

Two other sets of constraints have labor being shared among the competing supply chain networks of firms/organizations, in which case the governing concept is that of a **Generalized Nash Equilibrium** (rather than a Nash Equilibrium).

The research adds to modeling methodology as well as applications.

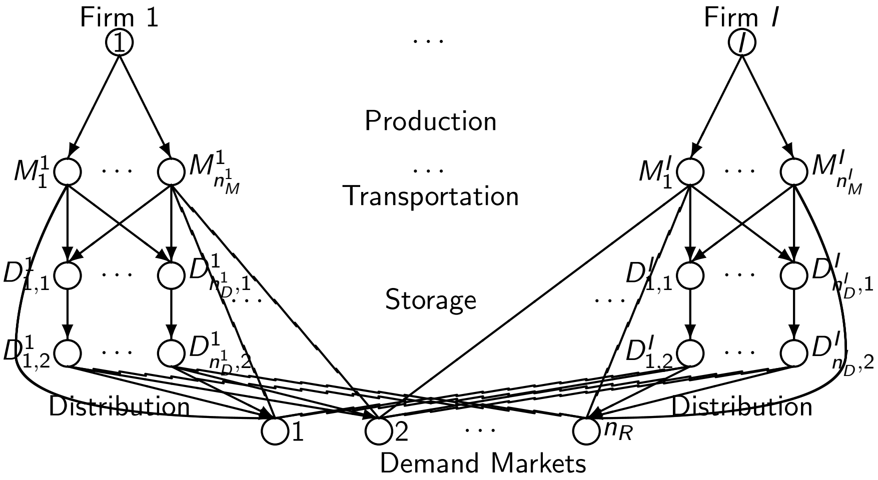


Figure: The Supply Chain Network Topology of the Game Theory Models with Labor

Game Theory Supply Chain Network Model Notation

Table: Game Theory Supply Chain Network Notation

Notation	Definition
L^i	The set of links in firm i's supply chain network, with L being all the links.
$G = [N, L]$	the graph of the supply chain network consisting of all nodes N and all links L.
P_k^i	set of paths in firm i's supply chain network terminating in demand market k; $\forall i, k$.
P^i	set of all n_{P^i} paths of firm i; $i = 1, \ldots, I$.
P	set of all n_P paths in the supply chain network economy.
x_p; $p \in P_k^i$	nonnegative flow on path p originating at firm node i and terminating at k; $\forall i, k$. Group firm i's path flows into vector $x^i \in R_+^{n_{P^i}}$. Then group all firms' path flows into vector $x \in R_+^{n_P}$.
f_a	nonnegative flow of the product on link a, $\forall a \in L$. Group all link flows into vector $f \in R_+^{n_L}$.
l_a	labor on link a (usually denoted in person hours).
α_a	positive factor relating input of labor to output of product flow on link a, $\forall a \in L$.
$\bar{l}_a$	bound on the availability of labor on link a under Scenario 1, $\forall a \in L$
$\bar{l}^t$	bound on labor availability for tier t activities under Scenario 2. $T+1$ is electronic commerce tier.
$\bar{l}$	bound on labor availability under Scenario 3.
d_{ik}	demand for the product of firm i at demand market k; $\forall i, k$. Group $\{d_{ik}\}$ elements for firm i into vector $d^i \in R_+^{n_R}$ and all demands into vector $d \in R_+^{I \times n_R}$.
$\hat{c}_a(f)$	total operational cost associated with link a, $\forall a \in L$.
π_a	cost of a unit of labor on link a, $\forall a$.
$\rho_{ik}(d)$	demand price function for the product of firm i at demand market k; $\forall i, k$.

For each firm i; $i = 1, \ldots, I$, we must have that:

$$\sum_{p \in P_k^i} x_p = d_{ik}, \quad k = 1, \ldots, n_R. \tag{1}$$

The path flows must be nonnegative; that is, for each firm i; $i = 1, \ldots, I$:

$$x_p \geq 0, \quad \forall p \in P^i. \tag{2}$$

The link flows of each firm i; $i = 1, \ldots, I$, are related to the path flows as:

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L^i, \tag{3}$$

where $\delta_{ap} = 1$, if link a is contained in path p, and 0, otherwise. We now discuss how labor is related to product flow.

$$f_a = \alpha_a l_a, \quad \forall a \in L^i, \quad i = 1, \ldots, I. \tag{4}$$

The utility function of firm i, U^i; $i = 1, \ldots, I$, is the profit, given by the difference between its revenue and its total costs:

$$U^i = \sum_{k=1}^{n_R} \rho_{ik}(d) d_{ik} - \sum_{a \in L^i} \hat{c}_a(f) - \sum_{a \in L^i} \pi_a l_a. \qquad (5a)$$

The functions U_i; $i = 1, \ldots, I$, are assumed to be concave, with the demand price functions being monotone decreasing and continuously differentiable and the total link cost functions being convex and also continuously differentiable.

The Optimization Problem of Each Firm

The optimization problem of each firm i; $i = 1, \ldots, I$, is:

$$\text{Maximize} \quad \sum_{k=1}^{n_R} \rho_{ik}(d) d_{ik} - \sum_{a \in L^i} \hat{c}_a(f) - \sum_{a \in L^i} \pi_a l_a, \qquad (5b)$$

subject to: (1), (2), (3), and (4).

Labor Scenario 1 – A Bound on Labor on Each Supply Chain Network Link

In Scenario 1, the additional constraints on the fundamental model are:

$$l_a \leq \bar{l}_a, \quad \forall a \in L. \tag{6}$$

Labor Scenario 2 – A Bound on Labor on Each Tier of Links in the Supply Chain Network

In Scenario 2, firms are faced with the ff. additional constraints:

$$\sum_{a \in L^1} l_a \leq \bar{l}^1, \tag{7, 1}$$

$$\sum_{a \in L^2} l_a \leq \bar{l}^2, \tag{7, 2}$$

and so on, until

$$\sum_{a \in L^{T+1}} l_a \leq l^{T+1}. \tag{7, T+1}$$

Labor Scenario 3 – A Single Labor Bound on Labor for All the Links in the Supply Chain Network

Scenario 3 may be interpreted as being the least restrictive of the scenarios considered here in that labor can be transferable across different activities of production, transportation, storage, and distribution. In Scenario 3, in addition to constraints (1) through (4), the firms are now faced with the following single constraint:

$$\sum_{a \in L} l_a \leq \bar{l}. \tag{8}$$

Recall that x^i denotes the vector of strategies, which are the path flows, for each firm i; $i = 1, \ldots, I$. We can redefine the utility/profit functions $\tilde{U}^i(x) \equiv U^i$; $i = 1 \ldots, I$ and group the profits of all the firms into an I-dimensional vector $\tilde{U}$, such that

$$\tilde{U} = \tilde{U}(x). \tag{9}$$

Objective function (5b), in lieu of the above, can now be expressed as:

$$\text{Maximize } \tilde{U}^i(x) = \sum_{k=1}^{n_R} \tilde{\rho}_{ik}(x) \sum_{p \in P_k^i} x_p - \sum_{a \in L^i} \tilde{c}_a(x) - \sum_{a \in L^i} \frac{\pi_a}{\alpha_a} \sum_{p \in P} x_p \delta_{ap}. \tag{10}$$

Scenario 1 Nash Equilibrium Conditions

We define the feasible set K_i for firm i:

$K_i \equiv \{x^i | x^i \in R_+^{n_{P^i}}, \frac{\sum_{p \in P^i} x_p \delta_{ap}}{\alpha_a} \leq \bar{l}_a, \forall a \in L^i\}$, for $i = 1, \ldots, I$.

Also, we define $K \equiv \prod_{i=1}^{I} K_i$.

In Scenario 1, each firm competes noncooperatively until the following equilibrium is achieved.

Definition: Supply Chain Network Nash Equilibrium for Scenario 1

A path flow pattern $x^* \in K$ is a supply chain network Nash Equilibrium if for each firm i; $i = 1, \ldots, I$:

$$\tilde{U}^i(x^{i*}, \hat{x}^{i*}) \geq \tilde{U}^i(x^i, \hat{x}^{i*}), \quad \forall x^i \in K_i, \tag{11}$$

where $\hat{x}^{i*} \equiv (x^{1*}, \ldots, x^{i-1*}, x^{i+1*}, \ldots, x^{I*})$.

Variational Inequality Formulations

Applying the classical theory of Nash equilibria and variational inequalities, under our imposed assumptions on the underlying functions, it follows that (cf. Gabay and Moulin (1980) and Nagurney (1999)) the solution to the above Nash Equilibrium problem (see Nash (1950, 1951)) coincides with the solution of the variational inequality problem: determine $x^* \in K$, such that

$$-\sum_{i=1}^{I} \langle \nabla_{x^i} \tilde{U}^i(x^*), x^i - x^{i*} \rangle \geq 0, \quad \forall x \in K, \tag{12}$$

where $\langle \cdot, \cdot \rangle$ represents the inner product in the corresponding Euclidean space, which here is of dimension n_P, and $\nabla_{x^i} \tilde{U}^i(x)$ is the gradient of $\tilde{U}^i(x)$ with respect to x^i.

We introduce Lagrange multipliers λ_a associated with constraint (6), $\forall a \in L$ and group the Lagrange multipliers for each firm i's network L^i into the vector λ^i. Group all such vectors for firms into vector $\lambda \in R_+^{n_L}$. Define feasible sets: $K_i^1 \equiv \{(x^i, \lambda^i) | (x^i, \lambda^i) \in R_+^{n_{P^i}+n_{L^i}}\}$; $i = 1, \ldots, I$, and $K^1 \equiv \prod_{i=1}^{I} K_i^1$.

Theorem: Alternative VI of Nash Equilibrium for Scenario 1

The supply chain network Nash Equilibrium satisfying the Definition 3.1 is equivalent to the solution of the variational inequality: determine vectors of path flows and Lagrange multipliers, $(x^*, \lambda^*) \in K^1$, *where:*

$$\sum_{i=1}^{m}\sum_{k=1}^{n_R}\sum_{p\in P_k^i}\left[\frac{\partial \tilde{C}_p(x^*)}{\partial x_p}+\sum_{a\in L^i}\frac{\lambda_a^*}{\alpha_a}\delta_{ap}+\sum_{a\in L^i}\frac{\pi_a}{\alpha_a}\delta_{ap}-\tilde{\rho}_{ik}(x^*)-\sum_{l=1}^{n_R}\frac{\partial\tilde{\rho}_{il}(x^*)}{\partial x_p}\sum_{q\in P_l^i}x_q^*\right]\times[x_p-x_p^*]$$

$$+\sum_{a\in L}\left[\bar{l}_a-\frac{\sum_{p\in P}x_p^*\delta_{ap}}{\alpha_a}\right]\times[\lambda_a-\lambda_a^*]\geq 0,\quad \forall (x,\lambda)\in K^1; \qquad (13)$$

where for each path p; $p \in P_k^i$; $i = 1, \ldots, m$; $k = 1, \ldots, n_R$:

$$\frac{\partial \tilde{C}_p(x)}{\partial x_p}\equiv\sum_{a\in L^i}\sum_{b\in L^i}\frac{\partial \hat{c}_b(f)}{\partial f_a}\delta_{ap}, \qquad (14)$$

$$\frac{\partial\tilde{\rho}_{ik}(x)}{\partial x_p}\equiv\frac{\partial\rho_{il}(d)}{\partial d_{ik}}. \qquad (15)$$

For both Scenarios 2 and 3, we use a refinement of the Generalized Nash Equilibrium, known as a *Variational Equilibrium* to construct variational inequality formulations.

Hence, the labor supply chain network equilibrium models, under three different scenarios of constraints, can be uniformly qualitatively studied and solution to numerical problems, quantitatively computed using rigorous algorithms!

The Algorithm

Realization of the Modified Projection Method Computation Step for VI (13)

Specifically, at iteration τ, we compute each of the path flows $\bar{x}_p^\tau$, $\forall P_k^i$, $\forall i$, $\forall k$, according to:

$$\bar{x}_p^\tau = \max\{0, x_p^{\tau-1} - \beta(\frac{\partial \tilde{C}_p(x^{\tau-1})}{\partial x_p} + \sum_{a\in L^i} \frac{\lambda_a^{\tau-1}}{\alpha_a}\delta_{ap} + \sum_{a\in L^i} \frac{\pi_a}{\alpha_a}\delta_{ap}$$

$$-\tilde{\rho}_{ik}(x^{\tau-1}) - \sum_{l=1}^{n_R} \frac{\partial \tilde{\rho}_{il}(x^{\tau-1})}{\partial x_p} \sum_{q\in P_l^i} x_q^{\tau-1})\} \quad (16)$$

and each of the Lagrange multipliers $\bar{\lambda}_a^\tau$, $\forall a \in L$, according to:

$$\bar{\lambda}_a^\tau = \max\{0, \lambda_a^{\tau-1} - \beta(\bar{l}_a - \frac{\sum_{p\in P} x_p^{\tau-1}\delta_{ap}}{\alpha_a})\}. \quad (17)$$

Realization of the Modified Projection Method Adaptation Step for VI (13)

At iteration τ, we compute each of the path flows x_p^τ, $\forall P_k^i$, $\forall i$, $\forall k$, according to:

$$x_p^\tau = \max\{0, x_p^{\tau-1} - \beta(\frac{\partial \tilde{C}_p(\bar{x}^\tau)}{\partial x_p} + \sum_{a \in L^i} \frac{\bar{\lambda}_a^\tau}{\alpha_a}\delta_{ap} + \sum_{a \in L^i} \frac{\pi_a}{\alpha_a}\delta_{ap} - \tilde{\rho}_{ik}(\bar{x}^\tau)$$

$$- \sum_{l=1}^{n_R} \frac{\partial \tilde{\rho}_{il}(\bar{x}^\tau)}{\partial x_p} \sum_{q \in P_l^i} \bar{x}_q^\tau)\} \tag{18}$$

and each of the Lagrange multipliers λ_a^τ, $\forall a \in L$, according to:

$$\lambda_a^\tau = \max\{0, \lambda_a^{\tau-1} - \beta(\bar{l}_a - \frac{\sum_{p \in P} \bar{x}_p^\tau \delta_{ap}}{\alpha_a})\}. \tag{19}$$

Numerical Examples

Numerical Examples

Our numerical examples are based on disruptions in migrant labor in the blueberry supply chain in the Northeast of the US in the summer of 2020.

The numerical examples investigate:

- Modifications in demand price functions;
- Disruptions in labor on a supply chain network link,

with additional numerical examples presented in the EJOR paper.

Numerical Examples

Examples 1, 2, and 3 have the supply chain network topology given below. There are two competing food firms (blueberry farms), each with two production locations, and with a single distribution center. There are two demand markets. We consider Scenario 1.

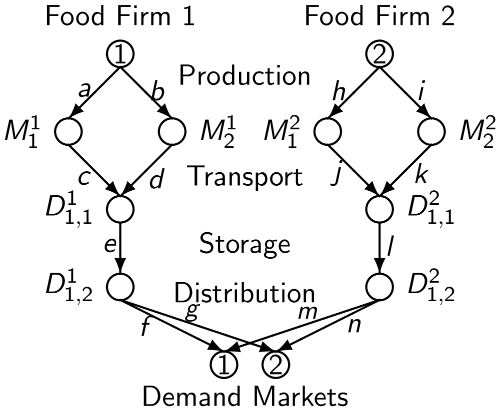


Figure: The Supply Chain Network Topology for the Numerical Examples

Example 1 - Baseline Example

The total operational cost functions for Food Firm 1 on its supply chain network L^1 are:

$$\hat{c}_a(f) = .0006 f_a^2, \quad \hat{c}_b(f) = .0007 f_b^2, \quad \hat{c}_c(f) = .001 f_c^2, \quad \hat{c}_d(f) = .001 f_d^2,$$

$$\hat{c}_e(f) = .002 f_e^2, \quad \hat{c}_f(f) = .005 f_f^2, \quad \hat{c}_g(f) = .005 f_g^2.$$

Also, the total operational costs associated with Food Firm 2's supply chain network L^2 are:

$$\hat{c}_h(f) = .00075 f_h^2, \quad \hat{c}_i(f) = .0008 f_i^2, \quad \hat{c}_j(f) = .0005 f_j^2, \quad \hat{c}_k(f) = .0005 f_k^2,$$

$$\hat{c}_l(f) = .0015 f_l^2, \quad \hat{c}_m(f) = .01 f_m^2, \quad \hat{c}_n(f) = .01 f_n^2.$$

The costs for labor (wages) for Food Firm 1 are:

$$\pi_a = 10, \quad \pi_b = 10, \quad \pi_c = 15, \quad \pi_d = 15, \quad \pi_e = 20, \quad \pi_f = 17, \quad \pi_g = 18,$$

and for Food Firm 2:

$$\pi_h = 11, \quad \pi_i = 22, \quad \pi_j = 15, \quad \pi_k = 15, \quad \pi_l = 18, \quad \pi_m = 18, \quad \pi_n = 18.$$

Example 1 - Baseline Example

The link labor productivity factors for the first firm are:

$$\alpha_a = 24,\ \alpha_b = 25,\ \alpha_c = 100,\ \alpha_d = 100,\ \alpha_e = 50,\ \alpha_f = 100,\ \alpha_g = 100,$$

and for the second firm:

$$\alpha_h = 23,\ \alpha_i = 24,\ \alpha_j = 100,\ \alpha_k = 100,\ \alpha_l = 70,\ \alpha_m = 100,\ \alpha_n = 100.$$

The bounds on labor for the first firm are:

$$\bar{l}_a = 10, \quad \bar{l}_b = 200, \quad \bar{l}_c = 300, \quad \bar{l}_d = 300, \quad \bar{l}_e = 100, \quad \bar{l}_f = 120, \quad \bar{l}_g = 120,$$

and for the second firm:

$$\bar{l}_h = 800, \quad \bar{l}_i = 90, \quad \bar{l}_j = 200, \quad \bar{l}_k = 200, \quad \bar{l}_l = 300, \quad \bar{l}_m = 100, \quad \bar{l}_n = 100.$$

Observe that the labor availability on link a is low. This is done in order to capture a disruption to labor in the pandemic.

Example 1 - Baseline Example

The demand price functions for Food Firm 1 are:

$$\rho_{11}(d) = -.0001d_{11} - .00005d_{21} + 6, \quad \rho_{12}(d) = -.0002d_{12} - .0001d_{22} + 8.$$

The demand price functions for Food Firm 2 are:

$$\rho_{21}(d) = -.0003d_{21} + 7, \quad \rho_{22}(d) = -.0002d_{22} + 7.$$

The paths are: $p_1 = (a, c, e, f)$, $p_2 = (b, d, e, f)$, $p_3 = (a, c, e, g)$, path $p_4 = (b, d, e, g)$, $p_5 = (h, j, l, m)$, $p_6 = (i, k, l, m)$, $p_7 = (h, j, l, n)$, and $p_8 = (i, k, l, n)$.

Example 1 - Baseline Example

All the Lagrange multipliers are equal to 0.00 except for $\lambda_a^* = 4.925$ with the labor equilibrium value on link a equal to its upper bound of 10.00.

The product prices at equilibrium are:

$$\rho_{11} = 5.97, \quad \rho_{12} = 7.91, \quad \rho_{21} = 6.94, \quad \rho_{22} = 6.96,$$

with equilibrium demands of:

$$d_{11}^* = 172.07, \quad d_{12}^* = 359.15, \quad \rho_{21} = 195.94, \quad \rho_{22} = 197.86.$$

The profit of Food Firm 1 is: 1,671.80 and the profit of Food Firm 2 is: 1,145.06.

Example 2 – Modification of Demand Price Functions

Example 2 has the same data as Example 1 except that we modify the demand price functions for the second firm to include a cross term, so that:

$$\rho_{21}(d) = -.0003d_{21} - .0001d_{11} + 6, \quad \rho_{22}(d) = -.0002d_{22} - .0001d_{12} + 7.$$

The Lagrange multipliers are all equal to 0.00 except for $\lambda_a^* = 4.93$.

The product prices at equilibrium are now:

$$\rho_{11} = 5.97, \quad \rho_{12} = 7.91, \quad \rho_{21} = 6.92, \quad \rho_{22} = 6.92,$$

with the equilibrium demands:

$$d_{11}^* = 172.07, \quad d_{12}^* = 359.16, \quad d_{21}^* = 195.48, \quad d_{22}^* = 196.48.$$

The profit for Food Firm 1 is: 1,671.86 and the profit for Food Firm 2 is: 1,134.61. The profit for Food Firm 1 rises ever so slightly, whereas that for Food Firm 2 decreases.

Example 3 – Disruptions in Storage Facilities

Example 3 has the same data as Example 2 except that we now consider a sizable disruption in terms of the spread of COVID-19 at the distribution centers of both food firms with the bounds on labor corresponding to the associated respective links being reduced to:

$$\bar{l}_e = 5, \quad \bar{l}_l = 5.$$

All computed equilibrium Lagrange multipliers are now equal to 0 except for those associated with the distribution center links, since the equilibrium labor values attain the imposed upper bounds on links e and l, with the respective equilibrium Lagrange multiplier values being:

$$\lambda_e^* = 157.2138, \quad \lambda_l^* = 43.6537.$$

Example 3 – Disruptions of Labor in Storage Facilities

The product prices at equilibrium are now:

$$\rho_{11} = 5.99, \quad \rho_{12} = 7.94, \quad \rho_{21} = 6.94, \quad \rho_{22} = 6.94,$$

with the equilibrium demands:

$$d^*_{11} = 30.03, \quad d^*_{12} = 219.96, \quad d^*_{21} = 174.61, \quad d^*_{22} = 175.39.$$

The profit for Food Firm 1 is now dramatically reduced to 1,218.74 and the profit for Food Firm 2 also declines, but by a much smaller amount, to 1,126.73.

Equilibrium Path Flows

Table: Equilibrium Product Path Flows for Examples 1 Through 3

Equilibrium Product Path Flows	Ex. 1	Ex. 2	Ex. 3
$x^*_{p_1}$	73.23	73.22	15.65
$x^*_{p_2}$	98.85	98.85	14.38
$x^*_{p_3}$	166.77	166.78	110.60
$x^*_{p_4}$	192.38	192.38	109.35
$x^*_{p_5}$	142.85	142.62	131.97
$x^*_{p_6}$	53.08	52.86	42.63
$x^*_{p_7}$	143.81	143.12	132.36
$x^*_{p_8}$	54.04	53.36	43.02

Equilibrium Link Labor Values

Table: Equilibrium Link Labor Values for Examples 1 Through 3

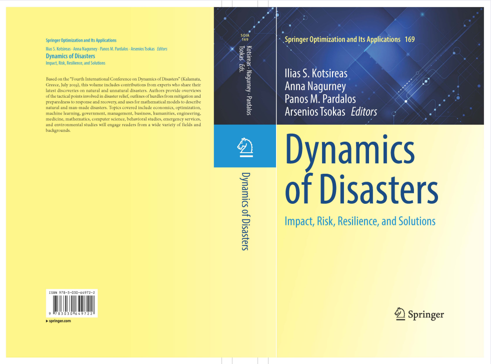
Equilibrium Link Labor Values	Ex. 1	Ex. 2	Ex. 3
l_a^*	10.00	10.00	5.26
l_b^*	11.65	11.65	4.95
l_c^*	2.40	2.40	1.26
l_d^*	2.91	2.91	1.24
l_e^*	10.62	10.62	5.00
l_f^*	1.72	1.72	0.30
l_g^*	3.59	3.59	2.20
l_h^*	12.46	12.42	11.49
l_i^*	4.46	4.43	3.57
l_j^*	2.87	2.86	2.64
l_k^*	1.07	1.06	0.86
l_l^*	5.63	5.60	5.00
l_m^*	1.96	1.95	1.75
l_n^*	1.98	1.96	1.75

Farmers should do everything possible to secure the health of workers at their production/harvesting and other facilities, so that the blueberries can be harvested in a timely manner and so that profits do not suffer. Keeping workers healthy, through appropriate measures, impacts the bottom line!

Some Additional Research

Some Additional Research

A. Nagurney, “Perishable Food Supply Chain Networks with Labor in the Covid-19 Pandemic,” in: *Dynamics of Disasters - Impact, Risk, Resilience, and Solutions*, **I.S. Kotsireas, A. Nagurney, P.M. Pardalos, and A. Tsokas, Editors, Springer Nature Switzerland AG, 2021, pp 173-193.**



Some Additional Research

A. Nagurney, "Attracting International Migrant Labor: Investment Optimization to Alleviate Supply Chain Labor Shortages," *Operations Research Perspectives* **9 (2022), 100233.**

A. Nagurney, "Optimization of Investments in Labor Productivity in Supply Chain Networks," *International Transactions in Operational Research* **29(4), (2022), pp 2116-2144. This article was recognized with an Editor's Choice Award.**

A. Nagurney, "Supply Chain Networks, Wages, and Labor Productivity: Insights from Lagrange Analysis and Computations," *Journal of Global Optimization* **83, (2022), pp 615-638.**

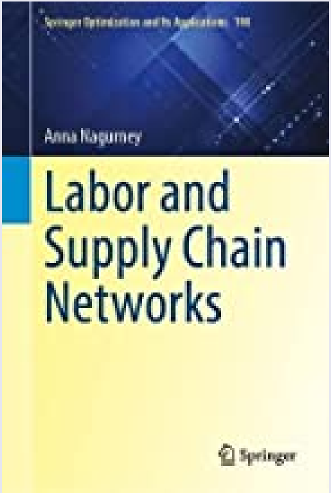
Some Additional Research

The numerical results in our papers clearly reveal the importance of a holistic approach to supply chain network modeling since decisions made by a specific firm can have unexpected impacts on other competing firms in the supply chain network economy.

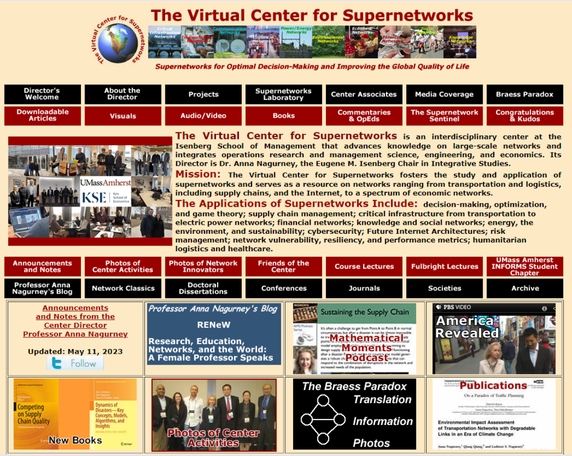
Our results also strongly suggest that having wages and labor equilibrate without any wage ceilings can be beneficial for an individual firm and also for firms engaged in competition.

And, most importantly, taking care of workers is critical in times of peace and war!

New Book


Springer Optimization and Its Applications 198
Anna Nagurney
Labor and Supply Chain Networks
Springer

Thank You Very Much!



More information on our work can be found on the Supernetwork Center site:

https://supernet.isenberg.umass.edu/