Supply Chain Networks, Wages, and Labor Productivity: Insights from Lagrange Analysis and Computations

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Acknowledgments

This talk is dedicated to essential workers, whose selflessness, expertise, and dedication have helped to sustain us in the Covid-19 pandemic.

I would also like to acknowledge all the freedom-loving people on the planet, including those fighting for their freedom in Ukraine.

It's All About People

A major research theme of ours in the COVID-19 pandemic is the inclusion of labor in supply chains, using optimization and game theory. The theme continues, as does its relevance, as the war on Ukraine continues to rage.

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January 29, 2021 in Supply Chain Networks

In the End, It's All About People

COVID-19 vaccine production reveals dependency on supply chains, labor workforce in the U.S.

By Anna Nagurney

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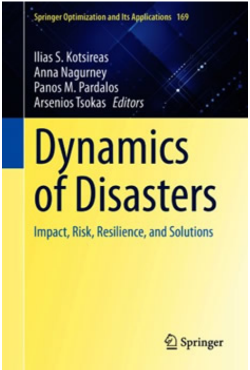
Research and Some Publications

"Perishable Food Supply Chain Networks with Labor in the Covid-19 Pandemic," A. Nagurney, in: *Dynamics of Disasters - Impact, Risk, Resilience, and Solutions*, **I.S. Kotsireas, A. Nagurney, P.M. Pardalos, and A. Tsokas, Editors, Springer International Publishing Switzerland, 2021, pp 173-193.**



Perishable Food Supply Chain Network Model with Labor

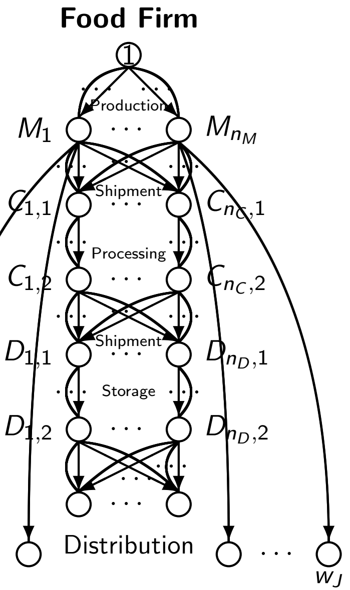
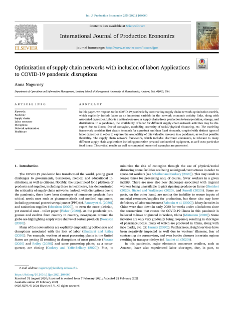


Figure: The Perishable Food Supply Chain Network Topology

In a series of papers we constructed supply chain network models with labor that included productivity factors and constraints on labor in order to identify the impacts of disruptions and to suggest possible mitigation procedures.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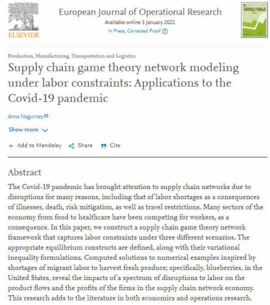
Optimization of supply chain networks with inclusion of labor: Applications to COVID-19 pandemic disruptions

Anna Nagurney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Supply chain game theory network modeling under labor constraints: Applications to the Covid-19 pandemic

Anna Nagurney

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Abstract

The Covid-19 pandemic has brought attention to supply chain networks due to disruptions for many reasons, including that of labor shortages as a consequences of illnesses, death, risk mitigation, as well as travel restrictions. Many sectors of the economy from food to healthcare have been competing for workers, as a consequence. In this paper, we construct a supply chain game theory network framework that captures labor constraints under three different scenarios. The appropriate equilibrium constructs are defined, along with their variational inequality formulations. Computed solutions to numerical examples inspired by shortages of migrant labor to harvest fresh produce; specifically, blueberries, in the United States, reveal the impacts of a spectrum of disruptions to labor on the product flows and the profits of the firms in the supply chain network economy. This research adds to the literature in both economics and operations research.

This presentation is based on the paper, of the same title, which is now in press in the *Journal of Global Optimization*:

Journal of Global Optimization
https://doi.org/10.1007/s10898-021-01122-y

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Supply chain networks, wages, and labor productivity: insights from Lagrange. analysis and computations

Anna Nagurney[1]

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Abstract
The COVID-19 pandemic has dramatically demonstrated the importance of labor to supply chain network activities from production to distribution with shortfalls in labor availability, for numerous reasons, resulting in product shortages and the reduction of profits of firms. Even as progress has been made through vaccinations, issues associated with labor are still arising. Increasing wages is a strategy to enhance labor productivity and, also to ameliorate, in part, labor shortages, but has not, until this work, been explored in a full supply chain network context. Specifically, in this paper, a game theory supply chain network model is constructed of firms competing in producing a substitutable, but differentiated, product, and seeking to determine their equilibrium product path flows, as well as hourly wages to pay their workers, under fixed labor amounts associated with links, and wage-responsive productivity factors. The theoretical and computational approach utilizes the theory of variational inequalities. We first introduce a model without wage bounds on links and then extend it to include wage bounds. Lagrange analysis is conducted for the latter model, which yields interesting insights, as well as an alternative variational inequality formulation. A series of numerical examples reveals that firms can gain in terms of profits by being willing to pay higher wages, resulting in benefits also for their workers, as well as consumers, who enjoy lower demand market prices for the products. However, sensitivity analysis should be conducted to determine the range of such wage bounds. Ultimately, we observed, that the profits may decrease and then stabilize. This work adds to the literature on the integration of concepts from economics and operations research for supply chain networks and also has policy implications.

Keywords Labor · Productivity · Wages · Supply chains · Networks · Game theory

1 Introduction

The COVID-19 pandemic has dramatically shown the importance of labor to global supply chains. Disruptions associated with lack of labor, due to reasons including illnesses and

Supply Chain Networks, Wages, and Productivity

In this paper, we explore the impacts of wage-responsive productivity of labor in supply chain networks on product consumer prices and profits of competing firms.

- Each link productivity factor is an increasing function of the wage on the link (and not fixed). Hence, the productivity factors are wage-responsive;
- The amount of labor available on each link is fixed;
- There is an upper bound on the wage on each link that a firm desires to pay. The previously noted work considered bounds on labor and not on wages;
- We conduct Lagrange analysis on the model, which yields an alternative variational inequality, amenable for elegant solution, plus managerial insights;
- We conduct sensitivity analysis on impacts of changes to wage-responsive productivity as well as bounds on wages that firms are willing to pay their employees.

The Nobel laureate Joseph Stiglitz (1982) also considered wage-responsive (dependent) productivity of labor but not in a supply chain network context as we do here, wherein different links of a firm associated with production, transportation, storage, and distribution and different sites can have distinct wage-responsive productivity factors and these can differ also across the supply chain links of the competing firms.

This paper adds to the recent literature on novel applications and extensions of supply chain networks using the rigorous methodology of the theory of variational inequalities to address challenges in the commercial sector inspired by the COVID-19 pandemic.

The Supply Chain Network Game Theory Models with Wage-Responsive Productivity

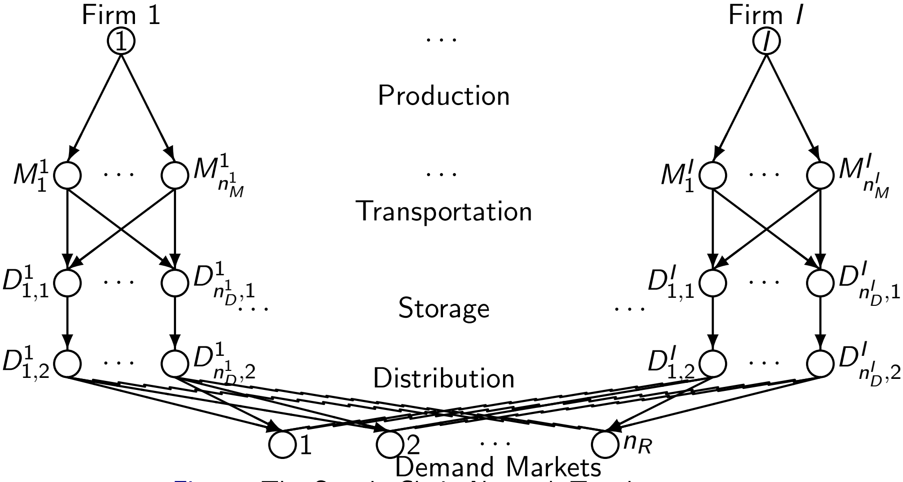


Figure: The Supply Chain Network Topology

Table: Notation for the Models with Wage-Dependent Labor Productivity

Notation	Definition
x_p; $p \in P_k^i$	nonnegative product flow on path p beginning at firm node i and ending at demand market k; $i = 1, \ldots, I$; $k = 1, \ldots, n_R$. Firm i's product path flows are grouped into the vector $x^i \in R_+^{n_{P^i}}$. All the firms' product path flows are grouped into the vector $x \in R_+^{n_P}$.
d_{ik}	demand for the product of firm i at demand market k; $i = 1, \ldots, I$; $k = 1, \ldots, n_R$. We group the $\{d_{ik}\}$ elements for firm i into the vector $d^i \in R_+^{n_R}$. All the demands are grouped into the vector $d \in R_+^{I \times n_R}$.
f_a	nonnegative flow of the product on link a, $\forall a \in L$. All the link flows are grouped into the vector $f \in R_+^{n_L}$.
l_a^{fixed}	fixed amount of labor on link a (typically denoted in person hours).
w_a	wage for a unit of labor on link a per hour the cognizant firm is willing to pay, on links $a \in L^i$ for $i = 1, \ldots, I$.
$\bar{w}_a$	upper bound on wage on link a that the firm responsible for the link is willing to pay, for $a \in L^i$ for $i = 1, \ldots, I$.
$\alpha_a w_a$	productivity factor relating input of labor to output of product flow on link a, where α_a is given $\forall a \in L$ and is positive and is refered to as the *wage-responsiveness productivity factor*.
$\hat{c}_a(f)$	total operational cost associated with link a, $\forall a \in L$.
$\rho_{ik}(d)$	demand price function for firm i's product at demand market k; $i = 1, \ldots, I$; $k = 1, \ldots, n_R$.

The Model Without Wage Bounds

All the product paths flows must be nonnegative:

$$x_p \geq 0, \quad \forall p \in P^i, \quad \forall i. \tag{1}$$

The demand for each product must be satisfied at each demand market, that is, for each firm i: $i = 1, \ldots, I$:

$$\sum_{p \in P_k^i} x_p = d_{ik}, \quad k = 1, \ldots, n_R. \tag{2}$$

The link flows of each firm i; $i = 1, \ldots, I$, are related to the product path flows thus:

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L^i, \tag{3}$$

where $\delta_{ap} = 1$, if link a is contained in path p, and 0, otherwise. A novel features of our model is the use of the following equations:

$$f_a = \alpha_a w_a l_a^{fixed}, \quad \forall a \in L^i, \quad i = 1, \ldots, I. \tag{4}$$

The Utility Functions of the Competing Firms

The utility function of each firm i, U^i; $i = 1, \dots, I$, represents the profit, which is the difference between its revenue, $\sum_{k=1}^{n_R} \rho_{ik}(d)d_{ik}$, and its total operational costs and all the wages paid for labor, $\sum_{a\in L^i} \hat{c}_a(f) - \sum_{a\in L^i} w_a l_a^{fixed}$:

$$U^i = \sum_{k=1}^{n_R} \rho_{ik}(d)d_{ik} - \sum_{a\in L^i} \hat{c}_a(f) - \sum_{a\in L^i} w_a l_a^{fixed}. \quad (5)$$

Due to (2), we can define demand price functions $\tilde{\rho}_{ik}(x) \equiv \rho_{ik}(d)$, $\forall i$, $\forall k$, and, due to (3), we can define the total operational link cost functions $\tilde{c}_a(x) \equiv \hat{c}_a(f)$, $\forall a \in L$. Also, using (4), and, subsequently, (3), we conclude that

$$w_a l_a^{fixed} = \frac{f_a}{\alpha_a l_a^{fixed}} l_a^{fixed} = \frac{(\sum_{p\in P} x_p \delta_{ap})}{\alpha_a}, \quad \forall a \in L. \quad (6)$$

The Utility Functions of the Competing Firms

The Utility Functions

We define $\tilde{U}_i(x) \equiv U_i$; $i = 1, \ldots, I$ and, by also making use of (2):

$$\tilde{U}^i(x) = \sum_{k=1}^{n_R} \sum_{p \in P_k^i} \tilde{\rho}_{ik}(x) x_p - \sum_{a \in L^i} \tilde{c}_a(x) - \sum_{a \in L^i} \frac{\left(\sum_{p \in P} x_p \delta_{ap}\right)}{\alpha_a}, \forall i. \quad (7)$$

The feasible set K_i for firm i is defined as: $K_i \equiv \{x^i | x^i \in R_+^{n_{P^i}}$, for $i = 1, \ldots, I\}$. Also, $K \equiv \prod_{i=1}^{I} K_i$.

The Supply Chain Nash Equilibrium

Each firm i; $i = 1, \ldots, I$, seeks to determine its vector of strategies consisting of its product path flows $x^i \in R_+^{n_{P^i}}$ that maximizes its profits, $\tilde{U}^i(x)$, satisfying the Nash (1950,1951) equilibrium conditions in the definition below. We assume that the utility function of each firm is concave wrt its strategies and is continuously differentiable.

Definition 1: Supply Chain Network Nash Equilibrium for the Game Theory Model Without Wage Bounds

A path flow pattern $x^* \in K$ is a supply chain network Nash Equilibrium if for each firm i; $i = 1, \ldots, I$:

$$\tilde{U}^i(x^{i*}, \hat{x}^{i*}) \geq \tilde{U}^i(x^i, \hat{x}^{i*}), \quad \forall x^i \in K_i, \tag{8}$$

where $\hat{x}^{i*} \equiv (x^{1*}, \ldots, x^{i-1*}, x^{i+1*}, \ldots, x^{I*})$.

According to (8), a Nash equilibrium is achieved when no firm, acting unilaterally, can improve upon its profits.

Variational Inequality Formulations

Using the classical theory of Nash equilibria and variational inequalities, it follows that the solution to the above Nash Equilibrium problem (see Nash (1950, 1951)) coincides with the solution of the variational inequality problem: determine $x^* \in K$, such that

$$-\sum_{i=1}^{I}\sum_{k=1}^{n_R}\sum_{p\in P_k^i}\frac{\partial \tilde{U}^i(x^*)}{\partial x_p}\times(x_p-x_p^*)\geq 0, \quad \forall x\in K, \tag{9}$$

or: determine $x^* \in K$, such that

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\sum_{p\in P_k^i}\left[\frac{\partial \tilde{C}_p(x^*)}{\partial x_p}+\sum_{a\in L^i}\frac{1}{\alpha_a}\delta_{ap}-\tilde{\rho}_{ik}(x^*)-\sum_{l=1}^{n_R}\frac{\partial \tilde{\rho}_{il}(x^*)}{\partial x_p}\sum_{q\in P_l^i}x_q^*\right]\times[x_p-x_p^*] \tag{10}$$

where

$$\frac{\partial \tilde{C}_p(x)}{\partial x_p}\equiv\sum_{a\in L^i}\sum_{b\in L^i}\frac{\partial \hat{c}_b(f)}{\partial f_a}\delta_{ap},\ \forall p\in P^i,\forall i;\ \frac{\partial \tilde{\rho}_{il}(x)}{\partial x_p}\equiv\frac{\partial \rho_{il}(d)}{\partial d_{ik}},\forall p\in P_k^i,\forall i,k. \tag{11}$$

The Model with Wage Bounds plus Lagrange Analysis

We now extend the above model to introduce upper bounds on wages that the firms are willing to pay their workers per hour. We allow for distinct upper limits on different links. Specifically, the model remains as above except for the addition of the following constraints:

$$w_a \leq \bar{w}_a, \quad \forall a \in L. \tag{12}$$

Making use of (3) and (4), (12) can be reexpressed as:

$$\sum_{p \in P} x_p \delta_{ap} \leq \bar{w}_a \alpha_a l_a^{fixed}, \quad \forall a \in L. \tag{13}$$

We define the feasible set
$K_1^i \equiv \{x^i \geq 0$, and (13) holds for all $a \in L^i\}$, with $K_1 \equiv \prod_{i=1}^{I} K_1^i$.
With the wage link upper bounds the statement of the Nash equilibrium according to Definition 1 is still relevant but over the feasible set K_1. The variational inequality (10) also holds but with the new feasible set K_1.

The Model with Wage Bounds plus Lagrange Analysis

We define $V(x)$ as

$$V(x) \equiv \sum_{i=1}^{I} \sum_{k=1}^{n_R} \sum_{p \in P_k^i} \left[\frac{\partial \tilde{C}_p(x^*)}{\partial x_p} + \sum_{a \in L^i} \frac{1}{\alpha_a} \delta_{ap} - \tilde{\rho}_{ik}(x^*) - \sum_{l=1}^{n_R} \frac{\partial \tilde{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} x_q^* \right] \times [x_p - x_p^*] \quad (14)$$

and observe that the variational inequality with wage bounds can be rewritten as the following minimization problem:

$$\min_{K_1} V(x) = V(x^*) = 0. \quad (15)$$

In order to construct the Lagrange function, we reformulate the constraints as below, with the associated Lagrange multiplier next to the corresponding constraint:

$$e_a = \sum_{p \in P} x_p \delta_{ap} - \bar{w}_a \alpha_a l_a^{fixed} \leq 0, \quad \lambda_a, \forall a,$$

$$g_p = -x_p \leq 0, \quad \epsilon_p, \forall p, \quad (16)$$

and

$$\Gamma(x) = (e_a, g_p)_{a \in L; p \in P}. \quad (17)$$

We now construct the Lagrange function $\mathcal{L}(x, \lambda, \epsilon)$

$$= \sum_{i=1}^{I} \sum_{k=1}^{n_R} \sum_{p \in P_k^i} \left[\frac{\partial \tilde{C}_p(x^*)}{\partial x_p} + \sum_{a \in L^i} \frac{1}{\alpha_a} \delta_{ap} - \tilde{\rho}_{ik}(x^*) - \sum_{l=1}^{n_R} \frac{\partial \tilde{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} x_q^* \right]$$

$$\times [x_p - x_p^*] + \sum_{a \in L} e_a \lambda_a + \sum_{p \in P} g_p \epsilon_p, \forall x \in R_+^{n_P}, \forall \lambda \in R_+^{n_L}, \forall \epsilon \in R_+^{n_P}, \quad (18)$$

where λ is the vector of all λ_as and ϵ is the vector of all ϵ_ps.
The feasible set K_1 is convex and the Slater condition is satisfied. Indeed, we know that $\Gamma(x)$ is convex and $\exists \bar{x} \in R_+^{n_P}$: $\Gamma(\bar{x}) < 0$, since we can always construct a small enough path flow pattern. Hence, if x^* is a minimal solution to problem (15), there exist $\lambda^* \in R_+^{n_L}$ and $\epsilon^* \in R_+^{n_P}$ such that the vector $(x^*, \lambda^*, \epsilon^*)$ is a saddle point of the Lagrange function (18):

$$\mathcal{L}(x^*, \epsilon, \lambda) \leq \mathcal{L}(x^*, \epsilon^*, \lambda^*) \leq \mathcal{L}(x, \epsilon^*, \lambda^*) \quad (19)$$

and

$$e_a^* \lambda_a^*, \quad \forall a,$$
$$g_p^* \epsilon_p^* = 0, \quad \forall p. \quad (20)$$

The Model with Wage Bounds plus Lagrange Analysis

From the right-hand side of (19) it follows that $x^* \in R_+^{n_P}$ is a minimal point of the function $\mathcal{L}(x, \epsilon^*, \lambda^*)$ in the whole space R^{n_P} and, therefore, we have that for all $p \in P_k^i$, $\forall i, k$:

$$\frac{\partial \mathcal{L}(x^*, \epsilon^*, \lambda^*)}{\partial x_p} = \left[\frac{\partial \tilde{C}_p(x^*)}{\partial x_p} + \sum_{a \in L^i} \frac{1}{\alpha_a} \delta_{ap} - \tilde{\rho}_{ik}(x^*) - \sum_{l=1}^{n_R} \frac{\partial \tilde{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} x_q^* \right]$$

$$+ \sum_{a \in L} \lambda_a^* \delta_{ap} - \epsilon_p^* = 0, \qquad (21)$$

together with conditions (20).

The Model with Wage Bounds plus Lagrange Analysis

Theorem

Conditions (20) and (21) correspond to an equivalent variational inequality to the one in (10), but over the feasible set K_1, given by: determine $(x^*, \epsilon^*, \lambda^*) \in R_+^{2n_P+n_L}$ such that

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\sum_{p\in P_k^i}\left[\frac{\partial \tilde{C}_p(x^*)}{\partial x_p}+\sum_{a\in L^i}\frac{1}{\alpha_a}\delta_{ap}-\tilde{\rho}_{ik}(x^*)-\sum_{l=1}^{n_R}\frac{\partial\tilde{\rho}_{il}(x^*)}{\partial x_p}\sum_{q\in P_l^i}x_q^*+\sum_{a\in L^i}\lambda_a^*\delta_{ap}-\epsilon_p^*\right]\times[x_p-x_p^*]$$

$$+\sum_{p\in P}x_p^*\times\left[\epsilon_p-\epsilon_p^*\right]+\sum_{a\in L}\left[\bar{w}_a\alpha_a l_a^{fixed}-\sum_{p\in P}x_p^*\delta_{ap}\right]\times[\lambda_a-\lambda_a^*]\geq 0,\quad \forall(x,\epsilon,\lambda)\in R_+^{2n_P+n_L}, \quad (22)$$

or simplified as: determine $(x^*, \lambda^*) \in R_+^{n_P+n_L}$ such that:

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\sum_{p\in P_k^i}\left[\frac{\partial \tilde{C}_p(x^*)}{\partial x_p}+\sum_{a\in L^i}\frac{1}{\alpha_a}\delta_{ap}+\sum_{a\in L^i}\lambda_a^*\delta_{ap}-\tilde{\rho}_{ik}(x^*)-\sum_{l=1}^{n_R}\frac{\partial\tilde{\rho}_{il}(x^*)}{\partial x_p}\sum_{q\in P_l^i}x_q^*\right]\times[x_p-x_p^*]$$

$$+\sum_{a\in L}\left[\bar{w}_a\alpha_a l_a^{fixed}-\sum_{p\in P}x_p^*\delta_{ap}\right]\times[\lambda_a-\lambda_a^*]\geq 0,\quad \forall(x,\lambda)\in R_+^{n_P+n_L}. \quad (23)$$

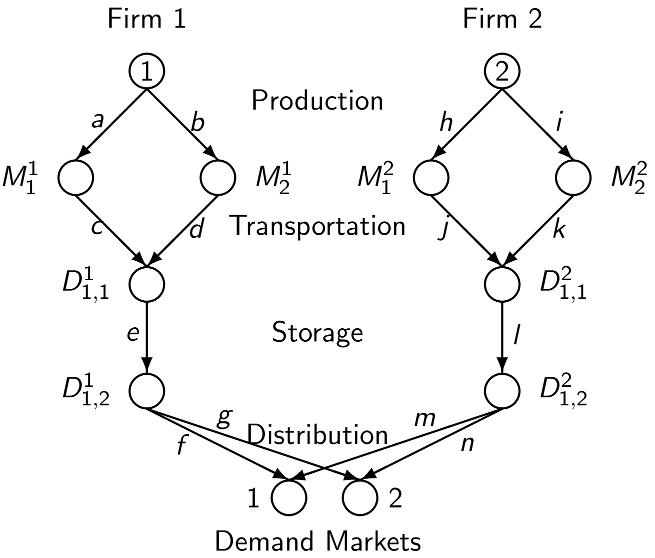


Figure: The Supply Chain Network Topology for the Numerical Examples

Numerical Examples

The total operational link cost functions are:

$$\hat{c}_a(f) = 2f_a^2, \quad \hat{c}_b(f) = 2f_b^2, \quad \hat{c}_c(f) = .5f_c^2, \quad \hat{c}_d = .5f_d^2,$$

$$\hat{c}_e(f) = f_e^2 + 2f_e, \quad \hat{c}_f(f) = .5f_f^2, \quad \hat{c}_g(f) = .5f_g^2,$$

$$\hat{c}_h(f) = 1.5f_h^2, \quad \hat{c}_i(f) = 1.5f_i^2 + f_i, \quad \hat{c}_j(f) = f_j^2 + 2f_j, \quad \hat{c}_k = f_k^2,$$

$$\hat{c}_l(f) = .5f_l^2, \quad \hat{c}_m(f) = .5f_m^2 + f_m, \quad \hat{c}_n(f) = f_n^2 + 2f_n.$$

The demand price functions are:

$$\rho_{11}(d) = -5d_{11} - 2d_{21} + 800, \quad \rho_{12}(d) = -5d_{12} - d_{22} + 850,$$

$$\rho_{21}(d) = -3d_{21} - d_{11} + 700, \quad \rho_{22}(d) = -5d_{22} - .5d_{12} + 750.$$

The operational cost functions and demand price functions are constructed to reflect a fairly high value product that is not that expensive to produce, transport, store, and distribute.

Numerical Examples

The α_a and the l_a^{fixed} parameters (cf. (4)) are, for $a \in L$ as follows:

$$\alpha_a = .5,\ \alpha_b = .5,\ \alpha_c = .3,\ \alpha_d = .3,\ \alpha_e = .4,\ \alpha_f = .5,\ \alpha_g = .3,$$

$$\alpha_h = .3,\ \alpha_i = .4, \alpha_j = .3, \alpha_k = .3, \alpha_l = .5,\ \alpha_m = .3,\ \alpha_n = .3,$$

$$l_a^{fixed} = 10,\ l_b^{fixed} = 10,\ l_c^{fixed} = 9,\ l_d^{fixed} = 7,\ l_e^{fixed} = 8,\ l_f^{fixed} = 6,$$

$$l_g^{fixed} = 8,\ l_h^{fixed} = 3,\ l_i^{fixed} = 3,\ l_j^{fixed} = 9,\ l_k^{fixed} = 9,\ l_l^{fixed} = 8,$$

$$l_m^{fixed} = 7,\ l_n^{fixed} = 8.$$

The paths are defined as: path $p_1 = (a, c, e, f)$, path $p_2 = (b, d, e, f)$, path $p_3 = (a, c, e, g)$, path $p_4 = (b, d, e, g)$, path $p_5 = (h, j, l, m)$, path $p_6 = (i, k, l, m)$, path $p_7 = (h, j, l, n)$, and path $p_8 = (i, k, l, n,)$.

Series 1: Examples 1 Through 4

Example 1 has all link wage bounds $\bar{w}_a = 10$; Example 2 has all wage bounds $\bar{w}_a = 15$; Example 3 has all wage bounds $\bar{w}_a = 20$; and Example 4 has all wage bounds $\bar{w}_a = 25$. We use the modified projection method for the computations.

Example 1 Results

The modified projection method converges to the following equilibrium path flow pattern:

$$x^*_{p_1} = 7.19, \quad x^*_{p_2} = 7.19, \quad x^*_{p_3} = 8.81, \quad x^*_{p_4} = 8.81,$$

$$x^*_{p_5} = 0.59, \quad x^*_{p_6} = 2.09, \quad x^*_{p_7} = 8.41, \quad x^*_{p_8} = 9.91.$$

The demand market prices are:

$$\rho_{11} = 722.75, \quad \rho_{12} = 743.56, \quad \rho_{21} = 677.59, \quad \rho_{22} = 722.86.$$

The profit for Firm 1 is: 20,530.75 and the profit for Firm 2 is: 13,628.85.

Example 2 Results

The modified projection method converges to the following equilibrium path flow pattern:

$$x^*_{p_1} = 11.19, \quad x^*_{p_2} = 11.19, \quad x^*_{p_3} = 12.81, \quad x^*_{p_4} = 12.81,$$

$$x^*_{p_5} = 1.94, \quad x^*_{p_6} = 4.19, \quad x^*_{p_7} = 11.56, \quad x^*_{p_8} = 13.81.$$

The demand market prices are now:

$$\rho_{11} = 675.81, \quad \rho_{12} = 696.54, \quad \rho_{21} = 659.22, \quad \rho_{22} = 711.82.$$

The profit for Firm 1 is now: 26,605.95, whereas the profit for Firm 2 is: 19,213.26.

With the wage bounds raised, reflecting that the firms are willing to pay their workers more for their labor, the profit for each firm increases, while the demand market prices that consumers pay decrease, signaling a win-win situation.

Example 3 Results

The modified projection method for Example 3, with all wage bounds set to 20, converges to the following equilibrium path flow pattern:

$$x^*_{p_1} = 15.20, \quad x^*_{p_2} = 15.20, \quad x^*_{p_3} = 16.81, \quad x^*_{p_4} = 16.81,$$

$$x^*_{p_5} = 3.29, \quad x^*_{p_6} = 6.29, \quad x^*_{p_7} = 14.71, \quad x^*_{p_8} = 17.71.$$

The demand market prices are now:

$$\rho_{11} = 628.88, \quad \rho_{12} = 649.53, \quad \rho_{21} = 640.86, \quad \rho_{22} = 700.78.$$

The profit for Firm 1 is now: 29,897.83 and the profit for Firm 2 is: 24,012.63.

With the wage bounds further increased that the profits of both firms increase (as compared to their respective values in Examples 1 and 2) and, again, the demand market prices decrease at all demand markets under more generous upper bounds on wages.

Example 4 Results

The algorithm for Example 4, with all wage bounds now set to 25, converges to the following equilibrium path flow pattern:

$$x^*_{p_1} = 18.69, \quad x^*_{p_2} = 18.69, \quad x^*_{p_3} = 20.30, \quad x^*_{p_4} = 20.30,$$

$$x^*_{p_5} = 4.67, \quad x^*_{p_6} = 8.42, \quad x^*_{p_7} = 17.83, \quad x^*_{p_8} = 21.58.$$

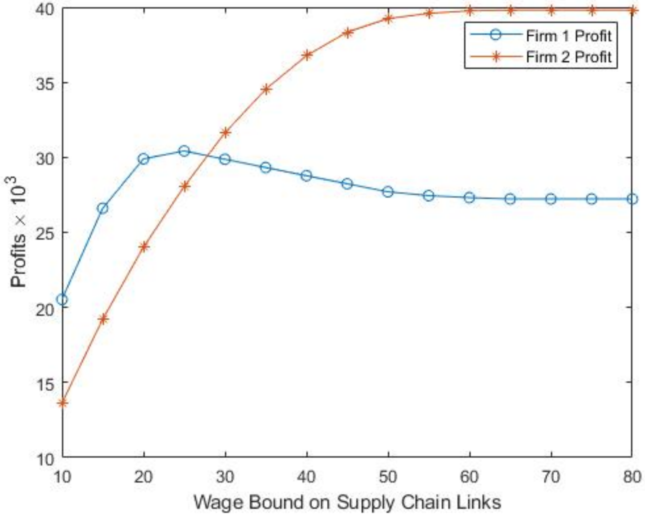
The demand market prices are:

$$\rho_{11} = 586.95, \quad \rho_{12} = 607.61, \quad \rho_{21} = 623.37, \quad \rho_{22} = 690.28.$$

The profit for Firm 1 is now: 30,425.58 and the profit for Firm 2 is: 28,060.79.

We see that the results are quite robust and reveal that raising wages can benefit both firms as well as consumers.

We now proceed to conduct additional sensitivity analysis.



We see something interesting happening, and this further emphasizes the importance of having a rigorous theoretical and computational framework for conducting such exercises.

At a wage bound of 30, Firm 1 now has a lower profit than it had at a wage bound of 25, whereas the profit of Firm 2 continues to increase, but at a decreasing rate. Plus, Firm 2 now has a profit exceeding that of Firm 1. And, for link wage bounds of 65 or higher, the profit of Firm 1 stabilizes at 27,225.99 and that of Firm 2 at 39,800.45.

Sensitivity Analysis

It is important to emphasize that the model has, in effect, link production functions that relate labor, which is fixed, and the wage-responsiveness productivity factor and wage to the product output on each link.

However, the product flows are associated with paths, since the product requires multiple supply chain links, beginning from production to ultimate distribution, and the latter, for each firm and demand market pair sum up to the demand.

All these are intricately related. Of course, the solution of the supply chain network model without bounds for this dataset would yield the same profits (and equilibrium pattern) as obtained for wages on the supply chain links of 65 or above.

Numerical Results and Insights

Our paper has many additional numerical examples as well as results of sensitivity analysis exercises.

• **Firms that are willing to pay their workers higher hourly wages can enjoy higher profits**, and can, hence, beat their competitors, in terms of lower prices for consumers at the demand markets, higher wages for the workers, and, of course, higher profits.

• **Our framework also has relevance for addressing, in part, the labor shortage in various industrial sectors and even in freight services** since we show that, even with fixed labor amounts associated with various supply chain network economic activities, having more productive labor, that is wage-sensitive, can yield financial gains for firms.

Numerical Results and Insights

• However, it is important to conduct sensitivity analysis, since, after a point that the wage bounds are raised, a firm may experience a decline in profits. Furthermore, ultimately, the profits of competing firms may stabilize and there will be no change with increases in the wage bounds. This is also interesting, since it suggests a natural type of wage "cap."

• **This work adds to the literature on the integration of principles from economics and operations research perspectives for supply chains.**

Thank You!

For more information: https://supernet.isenberg.umass.edu/