Competition for Medical Supplies Under Stochastic Demand in the Covid-19 Pandemic: A Generalized Nash Equilibrium Framework

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Background

With the Covid-19 pandemic, supply chains, including those for medical items, have been disrupted adding to the intense competition for such supplies.

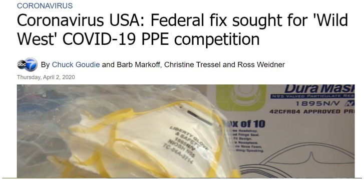
Op-Ed: Why a PPE shortage still plagues America and what we need to do about it

CNBC | 08-22

There Aren't Enough Medical Masks to Fight Coronavirus. Here's Why It's Not Going to Get Better Anytime Soon

Background

The great need for medical items from Personal Protective Equipment (PPEs) to ventilators and, now, even convalescent plasma, has led to intense competition for medical supplies among healthcare institutions and even regions, including states, as well as nations.



CORONAVIRUS

Coronavirus USA: Federal fix sought for 'Wild West' COVID-19 PPE competition

By Chuck Goudie and Barb Markoff, Christine Tressel and Ross Weidner

Thursday, April 2, 2020

Competition among state, local governments creates bidding war for medical equipment

Gov. Cuomo has called for the creation of a 'nationwide buying consortium.'

By ABC News

April 3, 2020, 12:33 PM • 8 min read

Global contest for medical equipment amidst the COVID19 pandemic

DHRITI KAMDAR

While the coronavirus pandemic continues to stay strong, the demand and supply for crucial medical equipment is highly unlikely to disappear.

Background

- China has historically produced half of the world's face masks, but with the coronavirus originating in Wuhan, China, the country dedicated the majority of the supply for their own citizens.
- Countries, such as Germany, even banned the export of PPEs.
- The intense competition for PPEs led to a dramatic increase in the price.
- The price of N95 masks grew from \$0.38 to \$5.75 each (a 1,413% increase) (Diaz, Sands, and Alesci (2020) and Berklan (2020)).
- Isolation protective gowns experienced a price increase from \$0.25 to \$5.00 (a 1900% increase).
- The price of reusable face shields going from \$0.50 to \$4.00 (a 700% increase).

Background

- We develop a **competitive game theory network model** for medical supplies inspired by the **Covid-19 pandemic**.
- It features salient characteristics of the realities of this pandemic in terms of **competition among organizations/institutions** for supplies under **limited capacities globally** as well as **uncertain demands**.
- Our model includes general transportation costs.
- Since organizations, notably, healthcare ones, compete with one another for the limited supplies, given the prices and their associated logistical costs as well as the expected loss due to possible shortages or surpluses, the model is a **Generalized Nash Equilibrium (GNE)** model.
- In the case of GNE models not only do the objective functions of the players in the game depend on the strategies of the other players but the feasible sets do as well.

Literature Review

- The first stochastic GNE model for disaster relief was constructed by Nagurney et al. (2020).
- The constructs that we utilize for handling the uncertain demands for medical items are based on results of Dong, Zhang, and Nagurney (2004), Nagurney, Yu, and Qiang (2011) and Nagurney, Masoumi, and Yu (2012, 2015).
- Mete and Zabinsky (2010) introduced a two-stage stochastic optimization model for storage and distribution of medical supplies but considered a single decision-maker.

The Generalized Nash Equilibrium Network Model for Medical Supplies under Stochastic Demand

The network consists of m supply locations for the medical supplies, with a typical supply point denoted by i, and n locations that are demand points, with a typical demand point denoted by j.

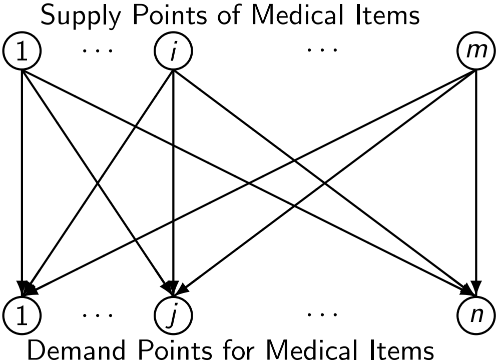
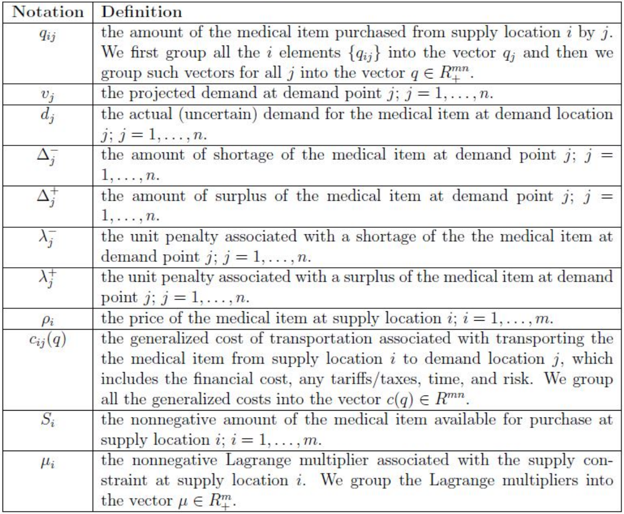


Figure: The Network Structure of the Competitive Game Theory Model for Medical Supplies

The Generalized Nash Equilibrium Network Model for Medical Supplies under Stochastic Demand

Table 1: Notation for the Medical Supply Generalized Nash Equilibrium Network Model



Notation	Definition
q_{ij}	the amount of the medical item purchased from supply location i by j. We first group all the i elements $\{q_{ij}\}$ into the vector q_j and then we group such vectors for all j into the vector $q \in R_+^{mn}$.
v_j	the projected demand at demand point j; $j = 1, \ldots, n$.
d_j	the actual (uncertain) demand for the medical item at demand location j; $j = 1, \ldots, n$.
Δ_j^-	the amount of shortage of the medical item at demand point j; $j = 1, \ldots, n$.
Δ_j^+	the amount of surplus of the medical item at demand point j; $j = 1, \ldots, n$.
λ_j^-	the unit penalty associated with a shortage of the the medical item at demand point j; $j = 1, \ldots, n$.
λ_j^+	the unit penalty associated with a surplus of the medical item at demand point j; $j = 1, \ldots, n$.
ρ_i	the price of the medical item at supply location i; $i = 1, \ldots, m$.
$c_{ij}(q)$	the generalized cost of transportation associated with transporting the the medical item from supply location i to demand location j, which includes the financial cost, any tariffs/taxes, time, and risk. We group all the generalized costs into the vector $c(q) \in R^{mn}$.
S_i	the nonnegative amount of the medical item available for purchase at supply location i; $i = 1, \ldots, m$.
μ_i	the nonnegative Lagrange multiplier associated with the supply constraint at supply location i. We group the Lagrange multipliers into the vector $\mu \in R_+^m$.

The Generalized Nash Equilibrium Network Model for Medical Supplies under Stochastic Demand

Stochastic Demand

Since d_j denotes the actual (uncertain) demand at destination point j, we have:

$$P_j(D_j) = P_j(d_j \leq D_j) = \int_0^{D_j} \mathcal{F}_j(t)dt, \qquad j = 1, \ldots, n, \tag{1}$$

where P_j and $\mathcal{F}_j$ denote the probability distribution function, and the probability density function of demand at point j, respectively. v_j is the "projected demand" for the medical item at demand point $j; j = 1, \ldots, n$.

The Generalized Nash Equilibrium Network Model for Medical Supplies under Stochastic Demand

Note that v_j is the "projected demand" for the medical item at demand point $j; j = 1, \ldots, n$.

Shortage and Surplus

The amounts of shortage and surplus at demand point j are calculated, respectively, according to:

$$\Delta_j^- \equiv \max\{0, d_j - v_j\}, \qquad j = 1, \ldots, n, \tag{2a}$$

$$\Delta_j^+ \equiv \max\{0, v_j - d_j\}, \qquad j = 1, \ldots, n. \tag{2b}$$

The expected values of shortage and surplus at each demand point are, hence:

$$E(\Delta_j^-) = \int_{v_j}^{\infty} (t - v_j)\mathcal{F}_j(t)dt, \qquad j = 1, \ldots, n, \tag{3a}$$

$$E(\Delta_j^+) = \int_0^{v_j} (v_j - t)\mathcal{F}_j(t)dt, \qquad j = 1, \ldots, n. \tag{3b}$$

The Generalized Nash Equilibrium Network Model for Medical Supplies under Stochastic Demand

Expected Penalties

The expected penalty incurred by demand point j due to the shortage and surplus of the medical item is equal to:

$$E(\lambda_j^- \Delta_j^- \; + \; \lambda_j^+ \Delta_j^+) = \lambda_j^- E(\Delta_j^-) + \lambda_j^+ E(\Delta_j^+), \qquad j = 1, \ldots, n. \tag{4}$$

Projected Demand

The projected demand at demand point j, v_j, is equal to the sum of flows of the medical item to j, that is:

$$v_j \equiv \sum_{i=1}^{m} q_{ij}, \qquad j = 1, \ldots, n. \tag{5}$$

The Generalized Nash Equilibrium Network Model for Medical Supplies under Stochastic Demand

Objective Function

The objective function of each demand point j is, hence, given by:

$$\text{Minimize} \quad \sum_{i=1}^{m} \rho_i q_{ij} + \sum_{i=1}^{m} c_{ij}(q) + \lambda_j^- E(\Delta_j^-) + \lambda_j^+ E(\Delta_j^+) \tag{6}$$

Constraints

$$\sum_{j=1}^{n} q_{ij} \leq S_i, \quad i = 1, \ldots, m, \tag{7}$$

$$q_{ij} \geq 0, \quad i = 1, \ldots, m. \tag{8}$$

The Generalized Nash Equilibrium Network Model for Medical Supplies under Stochastic Demand

- We assume that the total generalized transportation cost functions are continuously differentiable and convex.
- In our model, the transportation costs can, in general, depend upon the vector of medical item flows since there is competition for freight service provision in the pandemic.
- In the paper, we present some preliminaries that allow us to express the partial derivatives of the expected total shortage and discarding costs of the medical items at the demand points only in terms of the medical item flow variables.
- We prove that the third term in the Objective Function (6) is also convex.

The Generalized Nash Equilibrium Network Model for Medical Supplies under Stochastic Demand

Feasible Set

We define the feasible sets $K_j \equiv \{q_j \geq 0\}$; $j = 1, \ldots, n$. We define $K \equiv \prod_{i=1}^{I} K_i$. We also define the feasible set $\mathcal{S} \equiv \{q | q \, \text{satisfying} \, (7))\}$, which consists of the shared constraints.

Definition 1: Generalized Nash Equilibrium for Medical Items

A vector of medical items $q^* \in K \cap \mathcal{S}$ *is a Generalized Nash Equilibrium if for each demand point* j; $j = 1, \ldots, n$:

$$DU_j(q_j^*, \hat{q}_j^*) \leq DU_j(q_j, \hat{q}_j^*), \quad \forall q_j \in K_j \cap \mathcal{S}, \tag{9}$$

where $\hat{q}_j^* \equiv (q_1^*, \ldots, q_{j-1}^*, q_{j+1}^*, \ldots, q_n^*)$.

The Generalized Nash Equilibrium Network Model for Medical Supplies under Stochastic Demand

- According to (9), an equilibrium is established if no demand point has any incentive to unilaterally change its vector of medical item purchases/shipments.
- In our model not only does the objective function of a demand point depend not only on the vector of strategies of its own strategies and on those of the other demand points, but the feasible set does as well.
- This model is not a Nash (1950, 1951) model, but, rather, it is a Generalized Nash Equilibrium model.
- We define the feasible set $\mathcal{K} \equiv K \cap \mathcal{S}$.
- Our model captures the reality of the intense competitive landscape in the Covid-19 pandemic.

The Generalized Nash Equilibrium Network Model for Medical Supplies under Stochastic Demand

Definition 2: Variational Equilibrium

A vector of medical items $q^* \in \mathcal{K}$ is a Variational Equilibrium of the above Generalized Nash Equilibrium problem if it is a solution to the following variational inequality:

$$\sum_{j=1}^{n} \sum_{i=1}^{m} \frac{\partial DU_j(q^*)}{q_{ij}} \times (q_{ij} - q_{ij}^*) \geq 0, \quad \forall q \in \mathcal{K}, \tag{10}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in mn-dimensional Euclidean space.

In expanded form, the variational inequality in (10) is: determine $q^* \in \mathcal{K}$ such that

$$\sum_{j=1}^{n} \sum_{i=1}^{m} \left[\rho_i + \sum_{l=1}^{m} \frac{\partial c_{lj}(q^*)}{\partial q_{ij}} + \lambda_j^+ P_j(\sum_{l=1}^{m} q_{lj}^*) - \lambda_j^- (1 - P_j(\sum_{l=1}^{m} q_{lj}^*)) \right] \times \left[q_{ij} - q_{ij}^* \right] \geq 0, \tag{11}$$

The Generalized Nash Equilibrium Network Model for Medical Supplies under Stochastic Demand

Standard Form

From Nagurney (1999) we know that finite-dimensional variational inequality problem, VI$(F, \mathcal{K})$, is to determine a vector $X^* \in \mathcal{K} \subset R^N$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{12}$$

where F is a given continuous function from $\mathcal{K}$ to R^N, and $\mathcal{K}$ is a given closed, convex set.

We let $X \equiv q$ and $F(X)$ be the vector with elements: $\{\frac{\partial DU_j(q^*)}{q_{ij}}\}$, $\forall j, i$ with $\mathcal{K}$ as originally defined and $N = mn$. Then, clearly, variational inequality (11) can be put into standard form (12), under our assumptions.

The Generalized Nash Equilibrium Network Model for Medical Supplies under Stochastic Demand

We associate a nonnegative Lagrange multiplier μ_i with constraint (7), for each supply location $i = 1, \ldots, m$. We group all the Lagrange multipliers into the vector $\mu \in R^m_+$. We define the feasible set $\mathcal{K}^2 \equiv \{(q, \mu) | q \geq 0, \mu \geq 0\}$.

Then, using arguments as in Nagurney, Salarpour, and Daniele (2019), an alternative variational inequality for (11) is: determine $(q^*, \mu^*) \in \mathcal{K}^2$ such that

$$\sum_{j=1}^{n}\sum_{i=1}^{m}\left[\rho_i + \sum_{l=1}^{m}\frac{\partial c_{lj}(q^*)}{\partial q_{ij}} + \lambda_j^+ P_j(\sum_{l=1}^{m} q_{lj}^*) - \lambda_j^-(1 - P_j(\sum_{l=1}^{m} q_{lj}^*) + \mu_i^*\right] \times [q_{ij} - q_{ij}^*]$$

$$+ \sum_{i=1}^{m}\left[S_i - \sum_{j=1}^{n} q_{ij}^*\right] \times [\mu_i - \mu_i^*] \geq 0, \quad \forall (q, \mu) \in \mathcal{K}^2. \tag{13}$$

The Generalized Nash Equilibrium Network Model for Medical Supplies under Stochastic Demand

Variational inequality (13) can also be put into standard form (12) if we define $X \equiv (q, \mu)$ and $F(X) \equiv (F^1(X), F^2(X))$ where $F^1(X)$ has as its (i, j)-th component:

$\rho_i + \sum_{l=1}^{m} \frac{\partial c_{lj}(q)}{\partial q_{ij}} + \lambda_j^+ P_j(\sum\limits_{l=1}^{m} q_{lj}) - \lambda_j^-(1 - P_j(\sum\limits_{l=1}^{m} q_{lj}) + \mu_i$; $i = 1, \ldots, m$; $j = 1, \ldots, n$, and the i-th component of $F^2(X)$ is $S_i - \sum_{j=1}^{n} q_{ij}$, for $i = 1, \ldots, m$. Furthermore, $\mathcal{K} \equiv \mathcal{K}^2$ and $N = mn + m$.

Illustrative Examples

- The illustrative examples that are inspired by the Covid-19 pandemic and associated challenges in procuring N95 face masks.
- The supply point sells 20-pack N95 masks in the form of large bulks of 1000 packs each; therefore, one unit of item flow from the supply point to a demand point, q_{ij}, represents 1000 of 20-pack N95 masks.
- The demand at the demand point is uniformly distributed between 100 and 1,000 of large bulks.

(PBS)

Illustrative Examples

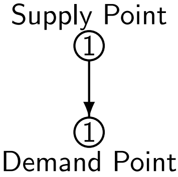


Figure: Network Topology for Illustrative Example 1

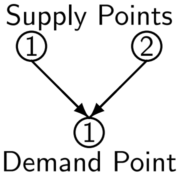


Figure: Network Topology for Illustrative Example 2

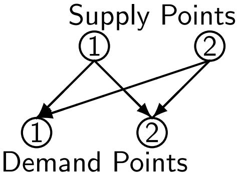


Figure: Network Topology for Illustrative Example 3

Illustrative Examples

- We assume that the price of each 20-pack N95 mask during the pandemic is \$25, so that the purchase price of each large bulk is $\rho_1 = 25,000$.
- We assume that, for every 2,000 people who do not use the face mask, one person would die due to the disease.
- Although it is not easy to value people's lives, we assume a \$200,000 equivalent for each loss. As a result, the penalty, λ_1^-, on the shortage of one item flow, which is equivalent to 20,000 N95 masks, is set at \$2,000,000.
- We also consider a penalty of $\lambda_1^+ = 100,000$ on surplus item flows to avoid overloading.

Illustrative Examples

Data for Illustrative Example 1

$$\rho_1 = 25,000, \quad S_1 = 1,000, \quad c_{11}(q) = q_{11}^2 + 3q_{11},$$

$$\lambda_1^- = 2,000,000, \quad \lambda_1^+ = 100,000.$$

We can rewrite variational inequality (13) for this example as: determine $(q^*, \mu^*) \in \mathcal{K}^2$ such that:

$$\left[25000 + 2q_{11}^* + 3 + 100000\left(\frac{q_{11}^* - 100}{900}\right) - 2000000\left(\frac{1000 - q_{11}^*}{900}\right) + \mu_1^*\right] \times [q_{11} - q_{11}^*]$$

$$+ [1000 - q_{11}^*] \times [\mu_1 - \mu_1^*] \geq 0, \quad \forall (q, \mu) \in \mathcal{K}^2$$

The solution to the above variational inequality, which we obtained analytically, is:

$$q_{11}^* = 945.62, \quad \mu_1^* = 0.00.$$

Illustrative Examples

Additional Data for Illustrative Example 2

$$\rho_2 = 10,000, \quad S_2 = 500, \quad c_{21}(q) = 2q_{21}^2 + 4q_{21}.$$

Additional Data for Illustrative Example 3

The demand for the new demand point is uniformly distributed between 100 and 500. The generalized transportation cost functions and the penalty coefficients associated with the second demand point are:

$$c_{12}(q) = 2q_{12}^2 + 3q_{12}, \quad c_{22}(q) = 3q_{22}^2 + 4q_{22},$$

$$\lambda_2^- = 2,000,000, \quad \lambda_2^+ = 100,000.$$

Illustrative Examples

- The projected demand value $v_1 = 945.62$ which is very close to the upper bound.
- The disutility of the organization in this logistical operation is equal to 67,543,534.04.
- With the addition of a new supply point that offers lower price,the decision-makers purchase more items from supply point 2.
- The supply capacity of the new supply point is half that of the first supply point, and we see that all its capacity has been used. Therefore, the associated equilibrium Lagrange multiplier is positive.
- Now, with greater flexibility in the supply chain due to the addition of a new supply point, the disutility of the organization at the demand point has declined, dropping to 59,860,548.75.

Illustrative Examples

- In Example 3, it can be seen that the full capacity of supply point 2 has not been assigned to demand point 1, since the organization at demand point 1 now competed with the organization at demand point 2.
- The major part of the demand point 1's procurement of the N95 masks is from supply point 1 that has a larger capacity as compared to supply point 2.
- The addition of a new demand point to the competition has changed the strategies of the organization at demand point 1, and we can see the impact on its disutility. Its disutility has now increased to 62,580,546.57. The disutility of the second demand point is 28,457,845.74.

Qualitative Properties

Theorem 2: Monotonicity

The function $F(X)$ is monotone, for all $X \in \mathcal{K}$, if all the generalized transportation cost functions c_{ij}, $i = 1, \ldots, m$; $j = 1, \ldots, n$, are convex.

Theorem 3: Uniqueness

The function $F(X)$ is strictly monotone for all $X \in \mathcal{K}$, if all the generalized transportation cost functions c_{ij}; $i = 1, \ldots, m$; $j = 1, \ldots, n$, are strictly convex. Then the variational inequality (13) has a unique solution in $\mathcal{K}$.

Theorem 4: Lipschitz Continuity

If the generalized transportation cost functions c_{ij}, for all i and j, have bounded second order partial derivatives, then the function $F(X)$ that enters the variational inequality problem (13) is Lipschitz continuous; that is, there exists a constant $L > 0$, known as the Lipschitz constant, such that

$$\|F(X^1) - F(X^2)\| \leq L\|X^1 - X^2\|, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{14}$$

Algorithm

Modified Projection Method (Korpelevich (1977))

Step 0: Initialization
Initialize with $X^0 \in \mathcal{K}$. Set the iteration counter $\tau := 1$ and let β be a scalar such that $0 < \beta \leq \frac{1}{L}$, where L is the Lipschitz constant.

Step 1: Computation
Compute $\bar{X}^\tau$ by solving the variational inequality subproblem:

$$\langle \bar{X}^\tau + \beta F(X^{\tau-1}) - X^{\tau-1}, X - \bar{X}^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{15}$$

Step 2: Adaptation
Compute X^τ by solving the variational inequality subproblem:

$$\langle X^\tau + \beta F(\bar{X}^\tau) - X^{\tau-1}, X - X^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{16}$$

Step 3: Convergence Verification
If $|X^\tau - X^{\tau-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\tau := \tau + 1$ and go to Step 1.

Algorithm

Explicit Formula for the Medical Item Flow

Determine $\bar{q}_{ij}^{\tau}$ for each i, j at Step 1 iteration τ according to:

$$\bar{q}_{ij}^{\tau} = \max\{0, q_{ij}^{\tau-1} + \beta(-\rho_i - \sum_{l=1}^{m} \frac{\partial c_{lj}(q^{\tau-1})}{\partial q_{ij}} - \lambda_j^{+} P_j(\sum_{l=1}^{m} q_{lj}^{\tau-1}) +$$

$$\lambda_j^{-}(1 - P_j(\sum_{l=1}^{m} q_{lj}^{\tau-1})) - \mu_i^{\tau-1})\}. \tag{17}$$

Explicit Formula for the Lagrange Multiplier

Determine $\bar{\mu}_i^{\tau}$ for each i at Step 1 iteration τ according to:

$$\bar{\mu}_i^{\tau} = \max\{0, \mu_i^{\tau-1} + \beta(-S_i + \sum_{j=1}^{n} q_{ij}^{\tau-1})\}. \tag{18}$$

Numerical Examples: Example 1

- The network consists of a single supply point and a single demand point as in the network in Figure 2.
- The q_{ij}s are in units since these medical practices are small relative to hospitals, etc.
- We assumed a uniform probability distribution in the range $[100, 1000]$ at the demand point.
- The additional data for this example are:

$$\rho_1 = 2, \quad S_1 = 1,000, \quad c_{11}(q) = .005q_{11}^2 + .01q_{11},$$

$$\lambda_1^- = 1,000, \quad \lambda_1^+ = 10.$$

- The computed equilibrium solution is:

$$q_{11}^* = 980.56, \quad \mu_1^* = 0.00.$$

- The projected demand of 980.56 is close to the upper bound of the probability distribution at the demand point.

Numerical Examples: Example 2

- There is one supply point and two demand points.
- The network topology is as in Figure 5.
- This example has the same data as Numerical Example 1 except for the following additional data for the new demand point:

$$c_{12}(q) = .01q_{12}^2 + .02, \quad \lambda_2^- = 1000, \quad \lambda_2^+ = 10.$$

- The modified projection method converged to the following equilibrium solution:

$$q_{11}^* = 502.20, \quad q_{12}^* = 497.80, \quad \mu_1^* = 541.61.$$

- The available supply of 1,000 N95 masks is exhausted between the two demand points, and, hence, the associated Lagrange multiplier μ_1^* is positive.

Numerical Examples: Example 3

- There are two supply points and two demand points.
- The topology is as in Figure 4.
- The data are same as that of Numerical Example 2 with the following additions:

$$S_2 = 500, \quad \rho_2 = 3, \quad c_{21}(q) = .015q_{21}^2 + .03, \quad c_{22}(q) = .02q_{22}^2 + .04q_{22}.$$

- The modified projection method yielded the following equilibrium solution:

$$q_{11}^* = 526.31, \quad q_{12}^* = 473.69, \quad q_{21}^* = 225.57,$$

$$q_{22}^* = 274.43, \quad \mu_1^* = 261.17, \quad \mu_2^* = 258.65.$$

Numerical Examples: Example 4

The network consists of two supply points and three demand points.

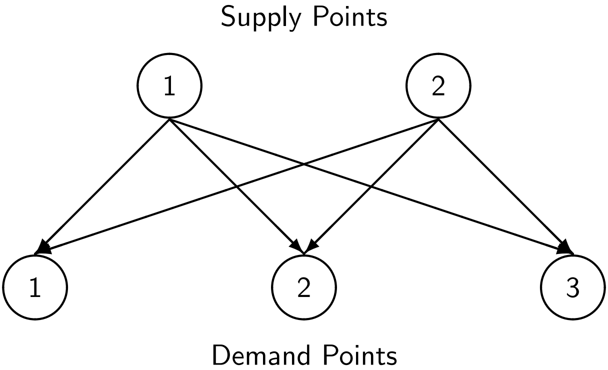


Figure: Network Topology for Numerical Example 4

Numerical Examples: Example 4

- Numerical Example 4 has the same data as Numerical Example 3 but with the addition of data for demand point 3 as follows:

$$c_{13}(q) = .01q_{13}^2 + .02q_{13}, \quad c_{23}(q) = .015q_{23}^2 + .03q_{23},$$

$$\lambda_3^- = 1000, \quad \lambda_3^+ = 10.$$

- The probability distribution for the N95 masks associated with demand point 3 is uniform with a lower bound of 200 and an upper bound of 1000.
- The modified projection method yielded the following equilibrium solution:

$$q_{11}^* = 360.11, \quad q_{12}^* = 318.83, \quad q_{13}^* = 321.06,$$

$$q_{21}^* = 122.29, \quad q_{22}^* = 161.10, \quad q_{23}^* = 216.62,$$

$$\mu_1^* = 565.25, \quad \mu_2^* = 564.16.$$

Numerical Examples: Example 5

There are two supply points and four demand points.

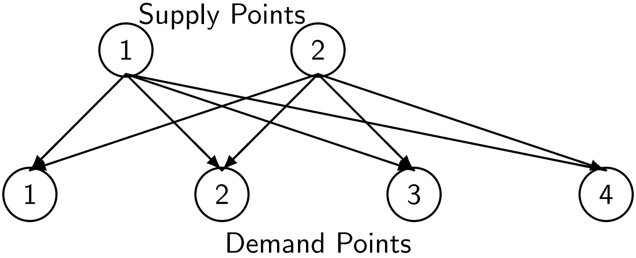


Figure: Network Topology for Numerical Example 5

Numerical Examples: Example 5

- Additional data for the new demand point 4:

$$c_{14}(q) = .015q_{14}^2 + .03q_{14}, \quad c_{24}(q) = .025q_{24}^2 + .05q_{24},$$

$$\lambda_4^- = 1000, \quad \lambda_4^+ = 10.$$

- The modified projection method now yielded the following equilibrium solution:

$$q_{11}^* = 260.73, \quad q_{12}^* = 229.36, \quad q_{13}^* = 251.22, \quad q_{14}^* = 258.69,$$

$$q_{21}^* = 79.57, \quad q_{22}^* = 109.17, \quad q_{23}^* = 160.46,$$

$$q_{24}^* = 150.81, \quad \mu_1^* = 725.71, \quad \mu_2^* = 724.91.$$

Results

- In Numerical Example 2 we see that with increased competition for N95 mask supplies from the second demand point, the first demand point has a large reduction in procured supplies, as compared to the volume received in Numerical Example 1.
- With the addition of a new supply point in Numerical Example 3, both demand points gain significantly in terms of the volume of N95 that each procures and the supplies at each supply point are fully sold out.
- In Numerical Example 4 with increasing competition for the N95 masks with another demand point, both demand points 1 and 2 experience decreases in procurement of supplies. The two supply points again fully sell out of their N95 masks.
- In Numerical Example 5 the suppliers of the N95 sell out their supplies. However, the demand points lose in term of supply procurement for their organizations with the increased demand and competition from and yet another demand point

Summary and Conclusions

- Medical supplies are essential in the battle against the coronavirus that causes Covid-19.
- The demand for medical supplies globally from PPEs to ventilators has created an intense competition.
- We developed a **Generalized Nash Equilibrium** model that consists of **multiple supply points** for the **medical items** and **multiple demand points** with the demand at the latter being **stochastic**.
- Using some recently introduced machinery we were able to provide alternative **variational inequality formulations** of the equilibrium conditions.

Summary and Conclusions

- We utilized the **variational inequality** with not only medical item product flows as variables but also the **Lagrange multipliers** associated with the supply capacities of the medical items at the supply point.
- We studied the model **quantitatively** through illustrative examples that we were able to solve analytically as well as via numerical examples for which we utilized an algorithm that we proposed.
- The findings from the numerical examples confirm that more supply points with sufficient supplies are needed to ensure that organizations are not deprived of critical supplies due to competition.
- As a result of this competition and limited local availability; in particular in the case of supplies such as masks and even coronavirus test kits, we are seeing several countries now setting up local production sites.
- This model can be applied to study the network economics of a spectrum of medical items, both in the near term, and in the longer term, as when vaccines as well as medicines for Covid-19 become available.

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This work is dedicated to all essential workers, including: healthcare workers, first responders, freight service providers, grocery store workers, farmers, and educators, who sacrificed so much in the Covid-19 pandemic.

The Virtual Center for Supernetworks is an interdisciplinary center at the Isenberg School of Management that advances knowledge on large-scale networks and integrates operations research and management science, engineering, and economics. Its Director is Dr. Anna Nagurney, the John F. Smith Memorial Professor of Operations Management.

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Thank you!