Operational Research: The Transf**OR**mative Discipline for the 21st Century

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Many thanks to the Conference Chair of **OR60** – Professor Graham Rand, to the Conference Committee, and to the Operational Research Society for the invitation to speak to you.

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Outline

- Background and Inspiration
- Network Systems and the Braess Paradox
- Representation of Supply Chains as Networks
- Supply Chain Networks from Healthcare to Food
- Cybercrime and Cybersecurity
- Network Models and Disaster Relief
- OR100

Background and Inspiration

The Role of Great Britain

- **Operational Research (OR)** was first used in the late 1930s to describe the process of evaluation of radar as an essential tool in the air defense of Great Britain. The **ORigins** were in military applications.

- Lord Patrick Blackett was one of the four founders of **the OR Club** in 1948, which in 1953 became the **Operational Research (OR) Society**. He was the Director of Naval Operational Research at the Admiralty and a recipient of the Nobel Prize in physics in 1948.

The Role of Great Britain

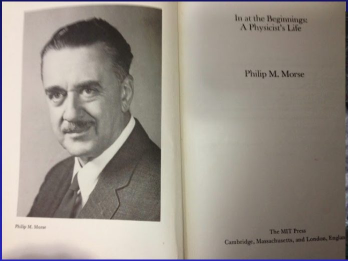
- **The first OR journal** - *Operational Research Quarterly* in 1950, which in 1978 became the *Journal of the Operational Research Society*.

- **OR1** takes place in 1958 in Harrogate.

- **The first dedicated Chair in OR** was established at Lancaster University in 1964, the same year as its founding.

Additional Acknowledgments

I would be remiss in also not acknowledging Philip M. Morse and George Dantzig on the other side of the Atlantic!



A Few Quotes

Philip M. Morse in his 1977 book, "In at the Beginnings" on page 318 writes:

The delights of research in O/R (he used the slash) are multiple. To me the pleasure coming from understanding how traffic behaves is as great as that coming from understanding how two atoms combine. In addition, the practical applications of O/R theory are often immediate and satisfying.

Morse ends his book with the following:

For those who like exploration, immersion in scientific research is not unsocial, is not dehumanizing; in fact, it is a lot of fun. And, in the end, if one is willing to grasp the opportunities, it can enable one to contribute something to human welfare.

OR - Our Profession

The founders of OR would be delighted to the see the growth of our discipline and profession with the discovery of and wide use of novel methodologies and innovative applications that they could not have envisioned in: industry, government, defense, healthcare, consultancies, and even, increasingly, in nonprofit organizations.

And, importantly, **students are drawn to Operational Research** because of its scope and great job opportunities!

Some of the Great **OR60 Streams**

Applications

Aviation
Behavioral OR; Community OR; Third Sector OR
Defense and Security
Energy
OR in Schools; OR in Sport; OR Consultancy and Case Studies
Risk
Sustainable Supply Chains
Timetabling and Scheduling: Transport and Logistics

Methodologies

Business Analytics, Data Mining, and Systems Thinking
Forecasting
Metaheuristics
Combinatorial, Network, and Robust Optimization
Simulation and Stochastic Processes.

For the Love of OR

When were you first captivated by OR?

From my first university course on the subject to my first projects in industry - working in the high tech defense sector on naval submarines in Newport, Rhode Island, I was drawn to the power of the subject, especially when combined with computing.

Network Systems and the Braess Paradox

I Work on the Modeling of Network Systems

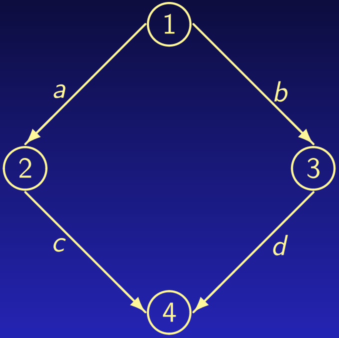
And utilize optimization, game theory, and also dynamical systems to gain insights as to the behavior of stakeholders.

Importance of Capturing Behavior on Networks - The Braess (1968) Paradox and User-Optimizing (U-O) Behavior

Assume a network with a single O/D pair (1,4). There are 2 paths available to travelers: $p_1 = (a, c)$ and $p_2 = (b, d)$.

For a travel demand of **6**, the equilibrium path flows are $x^*_{p_1} = x^*_{p_2} = 3$ and

The equilibrium path travel cost is $C_{p_1} = C_{p_2} = 83.$



$$c_a(f_a) = 10f_a, \quad c_b(f_b) = f_b + 50,$$

$$c_c(f_c) = f_c + 50, \quad c_d(f_d) = 10f_d.$$

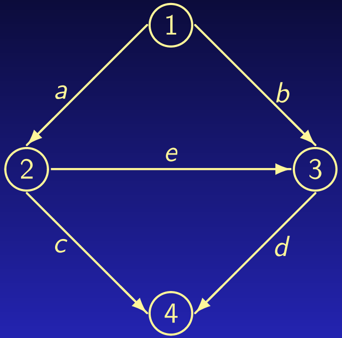
Adding a Link Increases Travel Cost for All!

Adding a new link creates a new path $p_3 = (a, e, d)$.

The original flow distribution pattern is no longer an equilibrium pattern, since at this level of flow the cost on path p_3, $C_{p_3} = 70$.

The new equilibrium flow pattern network is
$x^*_{p_1} = x^*_{p_2} = x^*_{p_3} = 2$.

The equilibrium path travel cost:
$C_{p_1} = C_{p_2} = C_{p_3} = 92$.



$c_e(f_e) = f_e + 10$

The 1968 Braess article has been translated from German to English:

"On a Paradox of Traffic Planning,"

D. Braess, A. Nagurney, and T. Wakolbinger, *Transportation Science* 39 (2005), pp 446-450.

Über ein Paradoxon aus der Verkehrsplanung

Von D. Braess, Münster [1])

Eingegangen am 28. März 1968

Zusammenfassung: Für die Straßenverkehrsplanung möchte man den Verkehrsfluß auf den einzelnen Straßen des Netzes abschätzen, wenn die Zahl der Fahrzeuge bekannt ist, die zwischen den einzelnen Punkten des Straßennetzes verkehren. Welche Wege am günstigsten sind, hängt nun nicht nur von der Beschaffenheit der Straße ab, sondern auch von der Verkehrsdichte. Es ergeben sich nicht immer optimale Fahrzeiten, wenn jeder Fahrer nur für sich den günstigsten Weg heraussucht. In einigen Fällen kann sich durch Erweiterung des Netzes der Verkehrsfluß sogar so umlagern, daß größere Fahrzeiten erforderlich werden.

Summary: For each point of a road network let be given the number of cars starting from it, and the destination of the cars. Under these conditions one wishes to estimate the distribution of the traffic flow. Whether a street is preferable to another one depends not only upon the quality of the road but also upon the density of the flow. If every driver takes that path which looks most favorable to him, the resultant running times need not be minimal. Furthermore it is indicated by an example that an extension of the road network may cause a redistribution of the traffic which results in longer individual running times.

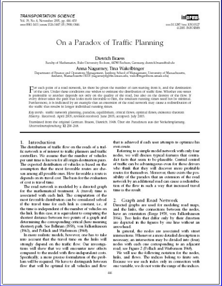
1. Einleitung

Für die Verkehrsplanung und Verkehrssteuerung interessiert, wie sich der Fahrzeugstrom auf die einzelnen Straßen des Verkehrsnetzes verteilt. Bekannt sei dabei die Anzahl der Fahrzeuge für alle Ausgangs- und Zielpunkte. Bei der Berechnung wird davon ausgegangen, daß von den möglichen Wegen jeweils der günstigste gewählt wird. Wie günstig ein Weg ist, richtet sich nach dem Aufwand, der zum Durchfahren nötig ist. Die Grundlage für die Bewertung des Aufwandes bildet die Fahrzeit.

Für die mathematische Behandlung wird das Straßennetz durch einen gerichteten Graphen beschrieben. Zur Charakterisierung der Bögen gehört die Angabe des Zeitaufwandes. Die Bestimmung der günstigen Stromverteilungen kann als gelöst betrachtet werden, wenn die Bewertung konstant ist, d. h., wenn die Fahrzeiten unabhängig von der Größe des Verkehrsflusses sind. Sie ist dann äquivalent mit der bekannten Aufgabe, den kürzesten Abstand zweier Punkte eines Graphen und den zugehörigen kritischen Pfad zu bestimmen [1], [5], [7].

Will man das Modell aber realistischer gestalten, ist zu berücksichtigen, daß die benötigte Zeit stark von der Stärke des Verkehrs abhängt. Wie die folgenden Untersuchungen zeigen, ergeben sich dann gegenüber dem Modell mit konstanter (belastungsunabhängiger) Bewertung z. T. völlig neue Aspekte. Dabei erweist sich schon eine Präzisierung der Problemstellung als notwendig; denn es ist zwischen dem Strom zu unterscheiden, der für alle am günstigsten ist,, und dem, der sich einstellt, wenn jeder Fahrer nur seinen eigenen Weg optimalisiert.

[1]) Priv.-Doz. Dr. Dietrich Braess, Institut für numerische und instrumentelle Mathematik, 44 Münster, Hüfferstr. 3 a.



TRANSPORTATION SCIENCE
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On a Paradox of Traffic Planning

Dietrich Braess
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For each point of a road network, let there be given the number of cars starting from it, and the destination of the cars. Under these conditions one wishes to estimate the distribution of traffic flow. Whether one street is preferable to another depends not only on the quality of the road, but also on the density of the flow. If every driver takes the path that looks most favorable to him, the resultant running times need not be minimal. Furthermore, it is indicated by an example that an extension of the road network may cause a redistribution of the traffic that results in longer individual running times.

Key words: traffic network planning; paradox; equilibrium; critical flows; optimal flows; existence theorem
History: Received: April 2005; revision received: June 2005; accepted: July 2005.

Translated from the original German: Braess, Dietrich. 1968. Über ein Paradoxon aus der Verkehrsplanung. *Unternehmensforschung* **12** 258–268.

1. Introduction

The distribution of traffic flow on the roads of a traffic network is of interest to traffic planners and traffic controllers. We assume that the number of vehicles per unit time is known for all origin-destination pairs. The expected distribution of vehicles is based on the assumption that the most favorable routes are chosen among all possible ones. How favorable a route is depends on its travel cost. The basis for the evaluation of cost is travel time.

The road network is modeled by a directed graph for the mathematical treatment. A (travel) time is associated with each link. The computation of the most favorable distribution can be considered solved if the travel time for each link is constant, i.e., if the time is independent of the number of vehicles on the link. In this case, it is equivalent to computing the shortest distance between two points of a graph and determining the corresponding critical (here meaning shortest) path. See Bellman (1958), von Falkenhausen (1963), and Pollack and Wiebenson (1960).

In more realistic models, however, one has to take into account that the travel time on the links will strongly depend on the traffic flow. Our investigations will show that we will encounter new effects compared to the model with flow-independent costs. Specifically, a more precise formulation of the problem will be required. We have to distinguish between flow that will be optimal for all vehicles and flow that is achieved if each user attempts to optimize his own route.

Referring to a simple model network with only four nodes, we will discuss typical features that contradict facts that seem to be plausible. Central control of traffic can be advantageous even for those drivers who think that they will discover more profitable routes for themselves. Moreover, there exists the possibility of the paradox that an extension of the road network by an additional road can cause a redistribution of the flow in such a way that increased travel time is the result.

2. Graph and Road Network

Directed graphs are used for modeling road maps, and the links, the connections between the nodes, have an orientation (Berge 1958, von Falkenhausen 1966). Two links that differ only by their direction are depicted in the figures by one line without an arrowhead.

In general, the nodes are associated with street intersections. Whenever a more detailed description is necessary, an intersection may be divided into (four) nodes with each one corresponding to an adjacent road; see Figure 2 (Pollack and Wiebenson 1960).

We will use the following notation for the nodes, links, and flows. The indices belong to finite sets. Because we use each index only in connection with one variable, we do not write the range of the indices.

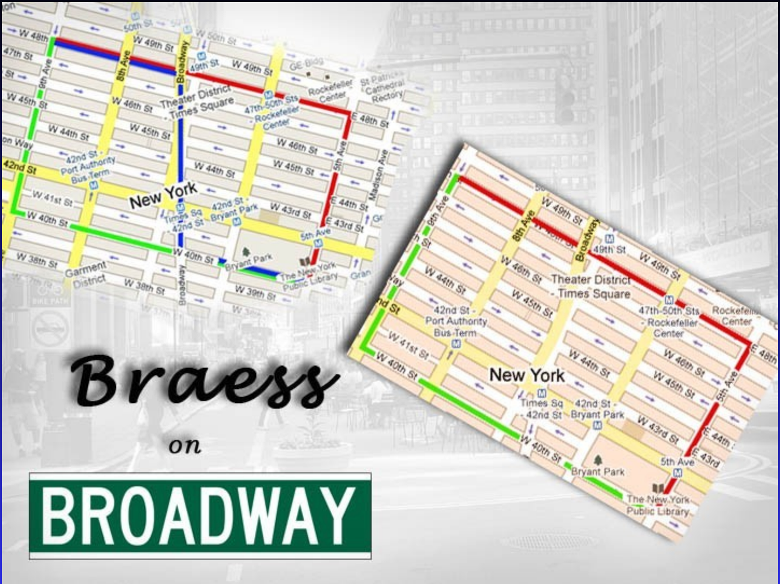
446

The Braess Paradox Around the World

1969 - Stuttgart, Germany - The traffic worsened until a newly built road was closed.

1990 - Earth Day - New York City - 42^{nd} Street was closed and traffic flow improved.

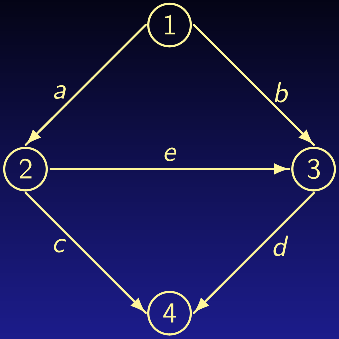
2002 - Seoul, Korea - A 6 lane road built over the Cheonggyecheon River that carried 160,000 cars per day and was perpetually jammed was torn down to improve traffic flow.


Braess
on
BROADWAY
New York
Theater District
- Times Square
47th-50th Sts
- Rockefeller
Center
Rockefeller
Center
42nd St -
Port Authority
Bus Term
Times Sq
- 42nd St
42nd St -
Bryant Park
Bryant Park
5th Ave
The New York
Public Library
W 49th St
W 46th St
W 45th St
W 44th St
W 43rd St
W 41st St
W 40th St
W 38th St
W 39th St
E 48th St
E 44th St
E 43rd St
8th Ave
9th Ave
5th Ave
Broadway
Madison Ave
Garment
District

Interview on Broadway for *America Revealed* on March 15, 2011

Under S-O behavior, the total cost in the network is minimized, and the new route p_3, under the same demand, would not be used.

The Braess paradox never occurs in S-O networks.



Recall the Braess network with the added link e.

What happens as the demand increases?

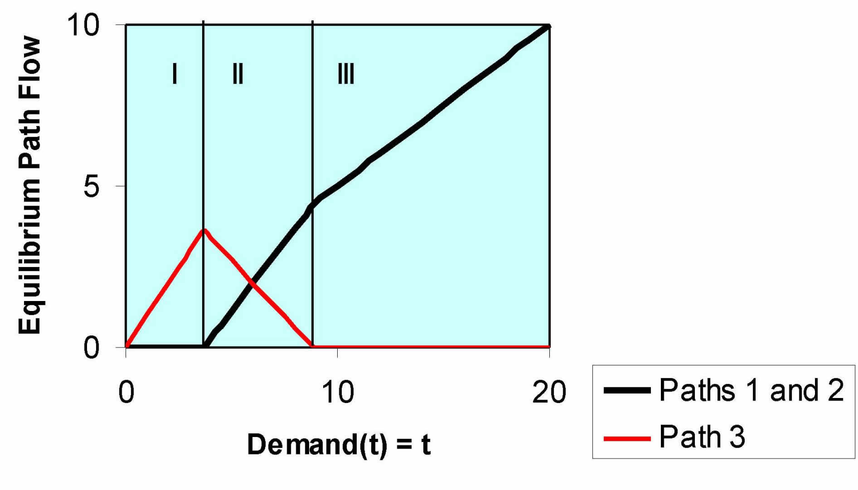
For Networks with Time-Dependent Demands

We Use Evolutionary Variational Inequalities

Radcliffe Institute for Advanced Study – Harvard University 2005-2006

Research with Professor David Parkes of Harvard University and Professor Patrizia Daniele of the University of Catania, Italy

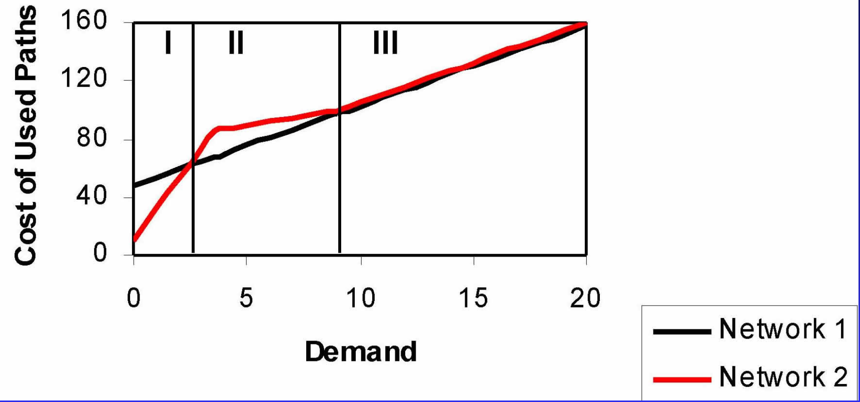
The U-O Solution of the Braess Network with Added Link (Path) and Time-Varying Demands Solved as an *Evolutionary Variational Inequality* (Nagurney, Daniele, and Parkes, *Computational Management Science* **4** (2007), pp 355-375).



In Demand Regime I, Only the New Path is Used.

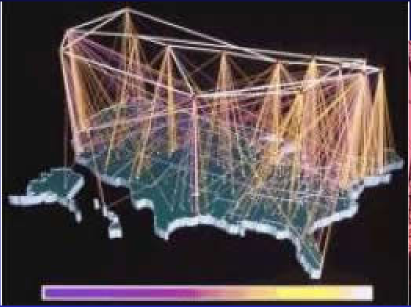
In Demand Regime II, the travel demand lies in the range [2.58, 8.89], and *the Addition of a New Link (Path) Makes Everyone Worse Off*!

In Demand Regime III, when the travel demand exceeds 8.89, *Only the Original Paths are Used!*



The new path is never used, under U-O behavior, when the demand exceeds 8.89, even when the demand goes out to infinity!

Other Networks that Behave like Traffic Networks



The Internet and electric power networks and even supply chains!

Representation of Supply Chains as Networks

Much of My Recent Research Had Been on Supply Chains

Characteristics of Supply Chains and Networks Today

- *large-scale nature* and complexity of network topology;
- *congestion*, which leads to nonlinearities;
- *alternative behavior of users of the networks*, which may lead to paradoxical phenomena;
- *possibly conflicting criteria associated with optimization*;
- *interactions among the underlying networks themselves*, such as the Internet with electric power networks, financial networks, and transportation and logistical networks;
- recognition of *their fragility and vulnerability*;
- policies surrounding networks today may have major impacts not only economically, but also *socially, politically, and security-wise*.

Representation of Supply Chains as Networks

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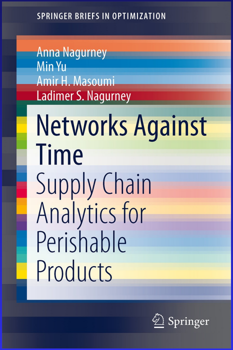
• build meaningful extensions using the graphical/network conceptualization.

Supply Chain Networks from Healthcare to Food

A Multidisciplinary Approach

In our research on perishable and time-sensitive product supply chains, we utilize results from physics, chemistry, biology, and medicine in order to capture the perishability of various products over time from healthcare products such as blood, medical nucleotides, and pharmaceuticals to food.



Supply Chain Networks – Optimization Models

Blood Supply Chains for the Red Cross

A. Nagurney, A. H. Masoumi, and M. Yu, "Supply Chain Network Operations Management of a Blood Banking System with Cost and Risk Minimization," *Computational Management Science* **9(2)** (2012), pp 205-231.

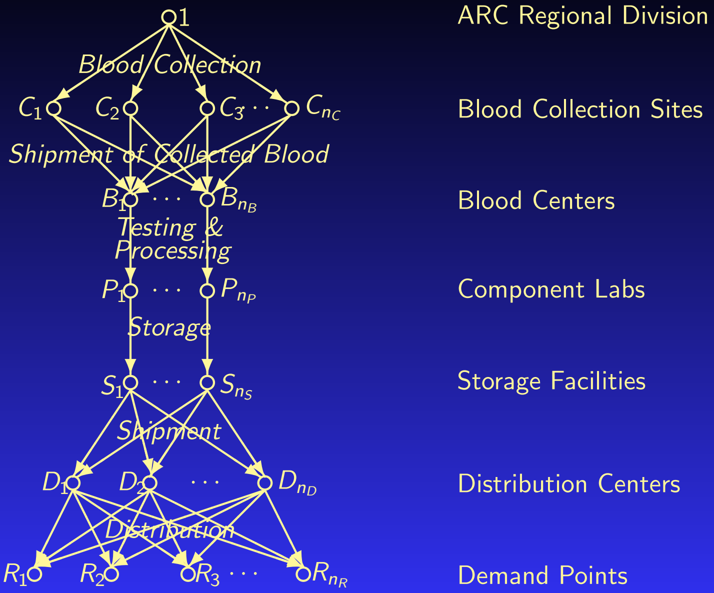
Blood Supply Chains for the Red Cross

The American Red Cross is the major supplier of blood products to hospitals and medical centers satisfying about 40% of the demand for blood components nationally.

Blood Supply Chains for the Red Cross

- The shelf life of platelets is 5 days and of red blood cells is 42.
- Over 39,000 donations are needed everyday in the US.
- Blood is a perishable product that cannot be manufactured but must be donated.
- As of February 1, 2018, the American Red Cross was facing a critical emergency need for blood and platelet donors. Severe winter weather forced the cancellation of hundreds of blood drives, resulting in nearly tens of thousands donations uncollected. In addition, flu in the US was close to epidemic levels.
- There is increasing competition among blood service organizations for donors and, overall, there has been a decrease in demand because of improved medical procedures.

Supply Chain Network Topology for a Regionalized Blood Bank



Blood Supply Chains for the Red Cross

Our blood supply chain network optimization model for the management of the procurement, testing and processing, and distribution has such novel features as:

- It captures *perishability of this life-saving product* through the use of arc multipliers;
- It contains *discarding costs* associated with waste/disposal;
- It handles *uncertainty* associated with demand points;
- It assesses *costs associated with shortages/surpluses at the demand points*, and
- It quantifies the *supply-side risk* associated with procurement.

Medical Nuclear Supply Chains

We developed a medical nuclear supply chain network design model which captures the decay of the radioisotope molybdenum.

"Medical Nuclear Supply Chain Design: A Tractable Network Model and Computational Approach," A. Nagurney and L. S. Nagurney, *International Journal of Production Economics* **140(2)** (2012), pp 865-874.

Medical Nuclear Supply Chains

Medical nuclear supply chains are essential supply chains in healthcare and provide the conduits for products used in nuclear medical imaging, which is routinely utilized by physicians for diagnostic analysis for both cancer and cardiac problems.

Such supply chains have unique features and characteristics due to the products' time-sensitivity, along with their hazardous nature.

Salient Features:

- complexity
- economic aspects
- underlying physics of radioactive decay
- importance of considering both waste management and risk management.

Medical Nuclear Supply Chains

Over **100,000** hospitals in the world use radioisotopes (World Nuclear Association (2011)).

Technetium, ^{99m}Tc, which is a decay product of Molybdenum-99, ^{99}Mo, is the most commonly used medical radioisotope, used in more than 80% of the radioisotope injections, with more than 30 million procedures worldwide each year.

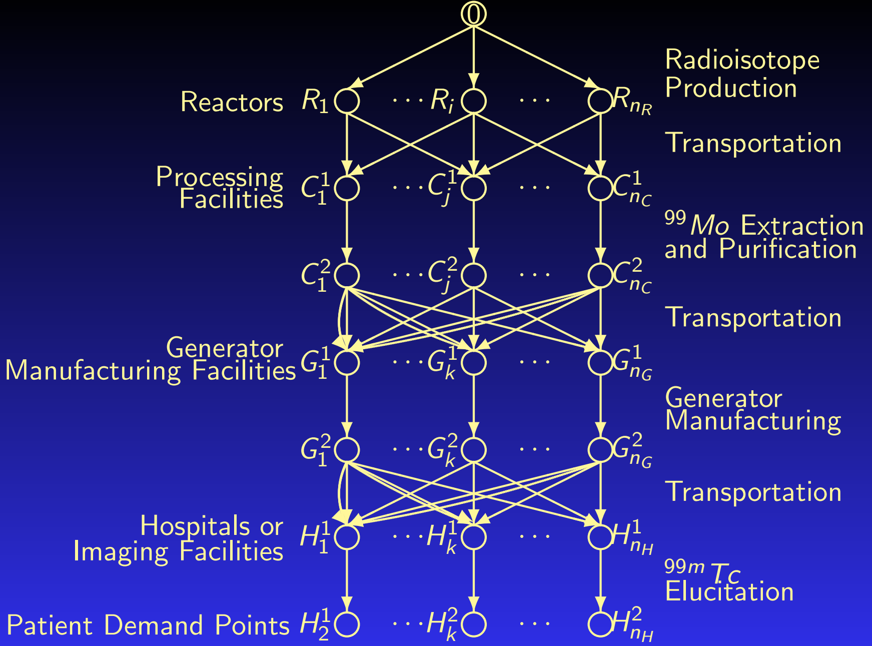
The half-life of Molybdenum-99 is 66 hours.

Each day, **41,000** nuclear medical procedures are performed in the United States using Technetium-99m.

Medical Nuclear Supply Chains

For over two decades, all of the Molybdenum necessary for US-based nuclear medical diagnostic procedures has come from **foreign** sources.

The majority of the reactors are between **40 and 50 years old**. Several of the reactors currently used are due to be retired by the end of this decade (Seeverens (2010) and OECD Nuclear Energy Agency (2010a))



The Medical Nuclear Supply Chain Network Topology

Arc Multipliers

Because of the exponential decay of molybdenum, we have that the quantity of the radioisotope:

$$N(t) = N_0 e^{-\lambda t}$$

so that an arc multiplier on a link a that takes t_a hours of time corresponds to:

$$\alpha_a = e^{-\frac{ln2}{66.7} t_a}.$$

Supply Chain Networks – Game Theory Models

Game Theory

There are many game theory problems and tools for solving them. There is noncooperative game theory, in which the players or decision-makers compete with one anther, and cooperative game theory, in which players cooperate with one another.


John F. Nash

In noncooperative games, the governing concept is that of Nash equilibrium. In cooperative games, we can apply Nash bargaining theory.

Electric Power Supply Chains

We developed *an empirical, large-scale electric supply chain network equilibrium model*, formulated it as a VI problem, and were able to solve it by *exploiting the connection between electric power supply chain networks and transportation networks* using our proof of a hypothesis posed in the classic book, *Studies in the Economics of Transportation*, by Beckmann, McGuire, and Winsten (1956).

The paper, "An Integrated Electric Power Supply Chain and Fuel Market Network Framework: Theoretical Modeling with Empirical Analysis for New England," by Zugang Liu and Anna Nagurney was published in *Naval Research Logistics* **56** (2009), pp 600-624.

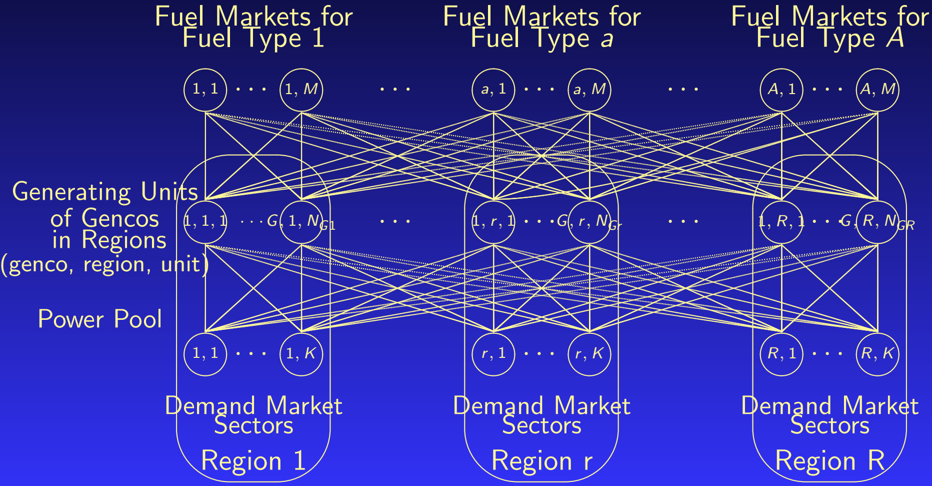
An Empirical Example of an Electric Power Supply Chain for New England

There are 82 generating companies who own and operate 573 generating units. We considered 5 types of fuels: natural gas, residual fuel oil, distillate fuel oil, jet fuel, and coal. The whole area was divided into 10 regions:

1. Maine,
2. New Hampshire,
3. Vermont,
4. Connecticut (excluding Southwest Connecticut),
5. Southwestern Connecticut (excluding the Norwalk-Stamford area),
6. Norwalk-Stamford area,
7. Rhode Island,
8. Southeastern Massachusetts,
9. Western and Central Massachusetts,
10. Boston/Northeast Massachusetts.

Graphic of New England

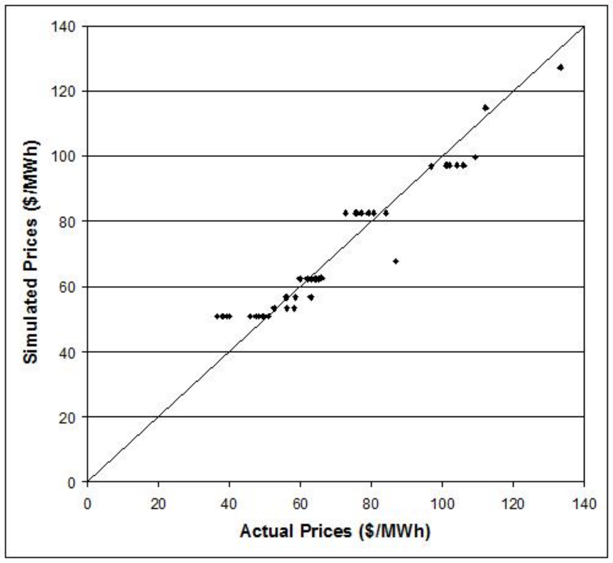
The Electric Power Supply Chain Network with Fuel Supply Markets



We tested the model on the data of July 2006 which included $24 \times 31 = 744$ hourly demand/price scenarios. We sorted the scenarios based on the total hourly demand, and constructed the load duration curve. We divided the duration curve into 6 blocks ($L_1 = 94$ hours, and $L_w = 130$ hours; $w = 2, ..., 6$) and calculated the average regional demands and the average weighted regional prices for each block.

The empirical model had on the order of $20,000$ *variables.*

Actual Prices Vs. Simulated Prices ($/Mwh)



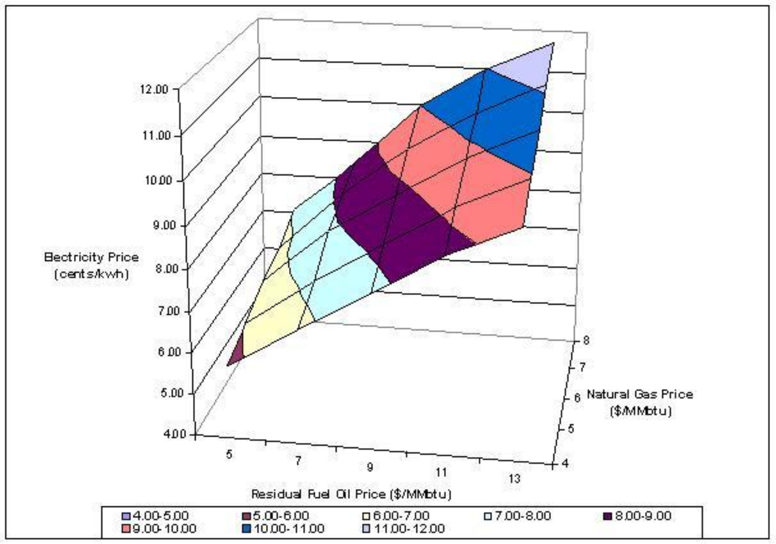
Sensitivity Analysis

We used the same demand data, and then varied the prices of natural gas and residual fuel oil. We assumed that the percentage change of distillate fuel oil and jet fuel prices were the same as that of the residual fuel oil price.

The next figure presents the average electricity price for the two peak blocks under oil/gas price variations.

The surface in the figure represents the average peak electricity prices under different natural gas and oil price combinations.

Sensitivity Analysis



Food Supply Chains

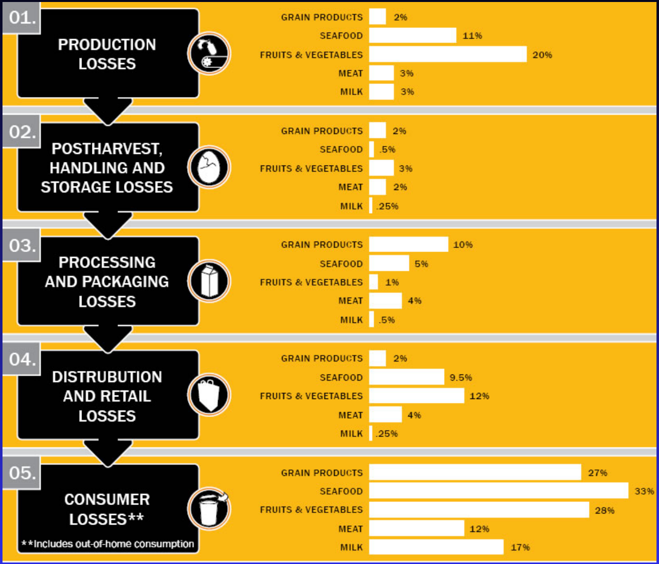
Food is something anyone can relate to.

Fascinating Facts About Food Perishability

THE SHELF LIFE OF FOOD

Foods unopened, uncut or uncooked unless stated otherwise	COUNTER/PANTRY (1 DAY ◄—► 1 MONTH)	REFRIGERATOR (1 DAY ◄—► 3 MONTHS)	FREEZER (1 MONTH ◄—► 1 YEAR)
APPLES	2-4 weeks	1-2 months	8-12 months
BANANAS	2-7 days	5-9 days	2-3 months
CANTALOUPE	Until ripe	1 week	8-12 months
CARROTS	Up to 4 days	4-5 weeks	8-12 months
CUCUMBERS	1-3 days	1 week	8-12 months
EGGS	Few hours	3-4 weeks	Do not freeze
MILK	Few hours	5-7 days	1 month
YOGURT	Few hours	2-3 weeks	1-2 months

Fascinating Facts About Food Perishability



Source: Food and Agriculture Organization 2011

Fascinating Facts About Food Perishability

Source: Food and Agriculture Organization 2011

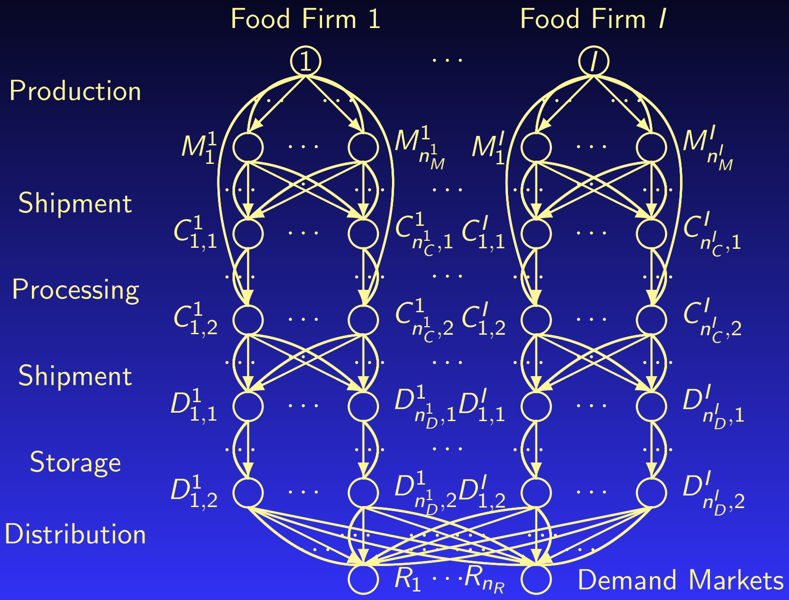
Fresh Produce Food Supply Chains

Our fresh produce supply chain network oligopoly model:

1. captures the deterioration of fresh food along the entire supply chain from a network perspective;
2. handles the time decay through the introduction of arc multipliers;
3. formulates oligopolistic competition with product differentiation;
4. includes the disposal of the spoiled food products, along with the associated costs;
5. allows for the assessment of alternative technologies involved in each supply chain activity.

Reference: "Competitive Food Supply Chain Networks with Application to Fresh Produce," Min Yu and Anna Nagurney, *European Journal of Operational Research* **224(2)** (2013), pp 273-282.

Fresh Produce Food Supply Chains



Fresh Produce Food Supply Chains

Food products also deteriorate over time and especially fresh produce. According to Nahmias (1982), each unit has a probability of $e^{\lambda t_a}$ of surviving t_a units of time where λ is the decay rate. Hence, arc multipliers can be constructed in a similar manner as those for the medical nuclear supply chain:

$$\alpha_a = e^{-\lambda t_a}$$

where λ is the decay rate for the food.

In rare cases, food deterioration follows the zero order reactions with linear decay (see Tijskens and Polderdijk (1996) and Rong, Akkerman, and Grunow (2011)). Then,

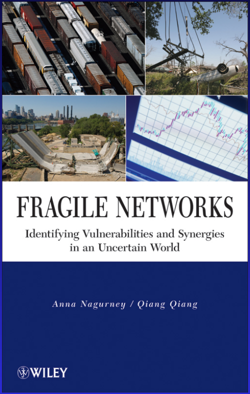
$$\alpha_a = 1 - \lambda t_a$$

for a post-production link.

Cybercrime and Cybersecurity

How I Became Interested in Cybersecurity

One of my books, written with a UMass Amherst PhD alum, was "hacked" and digital copies of it posted on websites around the globe.



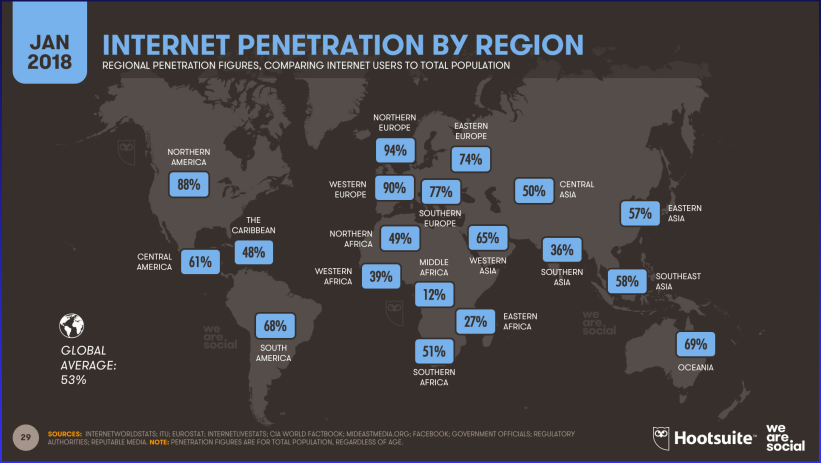
In a sense, this may be viewed as a compliment since clearly someone had determined that it has some sort of *value*.

The publisher John Wiley & Sons was notified and lawyers got involved but how do you contact and then influence those responsible for postings on rather anonymous websites?

About the same time news about cyberattacks was getting prominent attention in the media and there were those interested in working with us on related research on cybersecurity.

The Internet has transformed the ways in which individuals, groups, organizations communicate, obtain information, access entertainment, and conduct their economic and social activities.

In 2012, there were over 2.4 billion users. In 2018, the number of Internet users has surpassed 4 billion users, more than half of the world's population.



Some Recent Major Cyberattacks

- **Equifax**: In September 2017, it was revealed that names, SSNs, birthdates, drivers' license information, and credit card numbers on about 143 million U.S. consumers was compromised in a cybersecurity breach that began in mid-May and was discovered only on July 29, 2017 (Bloomberg (2017)). In late February 2018, Equifax disclosed that it had discovered that an additional 2.4 million U.S. consumers were affected by the cyberattack (Reuters (2018)).

- **"WannaCry" ransomware**: Began in mid-May 2017. It crippled National Health Services (NHS) hospitals in the UK, hobbling emergency rooms, delaying vital medical procedures, and creating chaos (WIRED (2017)).

Some Recent Major Cyberattacks

• **Banks:** The Carbanak group, also known as Anunak, was exposed in 2015 after supposedly stealing upwards of $1 billion from more than 100 banks across 30 countries (The New York Times (2015)).

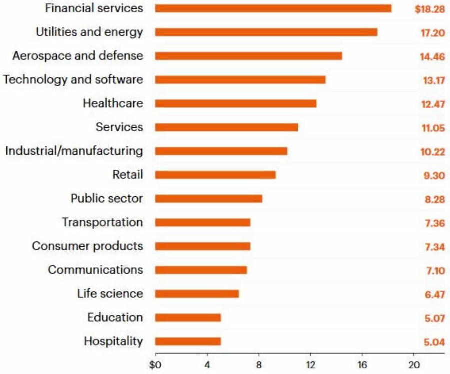
• **US Office of Personnel Management:** In June 2015, OPM discovered that sensitive information, including SSNs of 21.5 million federal employees was stolen (WIRED (2016)).

• **Sony Pictures Entertainment** The attack on Sony in 2014 destroyed data on more than 3,000 computers and disclosed prerelease films and embarrassing emails of executives (Fortune (2015)).

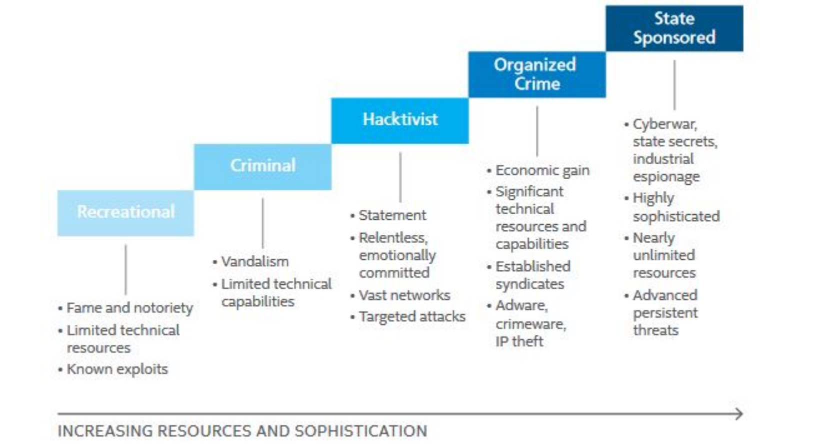
• **Target, Home Depot, Michaels Stores, Staples, and eBay:** These were breached in 2014 - card data and personal information of millions of customers were stolen (The New York Times (2015)).

Cost of Cybercrime

- **Cybercrimes are costly for organizations.** According to Forbes (2017), cybercrime will cost the world about $6 trillion per year on average through 2021. All industries fall victim to cybercrime, but to different degrees. **Average annual costs per company caused by global cybercrime as of 2017 by sector (in million US$) (Ponemon(2017))**



Changing Attacker Profiles



McAfee Labs Threats Report, August 2015

Cybercrime

Clearly, hackers go where there is money.

A Predictive Network Economic Model of Cybercrime

Network Economics of Cybercrime

We lay the foundation for the development of network economics based models for cyberccrime in financial services.

Financial services firms as well as hackers are economic agents.

Our view is that financial firms produce/possess commodities (or products) that hackers (criminals) seek to obtain.

We assume that the firms (as well as the hackers) can be located in different regions of a country or in different countries. Financial service firms may also be interpreted as prey and the hackers as predators.

Commodities or products that the hackers seek to acquire may include: credit card numbers, password information, specific documents, etc.

The financial firms are the producers of these commodities whereas the hackers act as agents and "sell" these products, if they acquire them, at the "going" market prices.

There is a "price" at which the hackers acquire the financial commodity from a financial institution and a price at which they sell the hacked product in the demand markets. The former we refer to as the supply price and the latter is the demand price.

In addition, we assume that there is **a transaction cost associated between each pair of financial and demand markets for each commodity**. These transaction costs can be generalized costs that also capture risk.

Indeed, if the cyber criminals do not find demand markets for their acquired financial commodities (since there are no consumers willing to pay the price) then there is no economic incentive for them to acquire the financial commodities.

To present another criminal network analogue – consider the market for illegal drugs, with the U.S. market being one of the largest, if not the largest one. If there is no demand for the drugs then the suppliers of illegal drugs cannot recover their costs of production and transaction and the flows of drugs will go to zero.

According to a recent Rand report, for many, the cyber black market can be more profitable than the illegal drug trade.

Network Economics of Cybercrime

• After the major Target breach, **some credit cards obtained thus initially sold for $135 each on the black market, but, within weeks, as banks started to cancel the cards, the price dropped to $8** and, seven months after Target learned about the breach, the cards had essentially no value.

• In addition, different “brands of credit cards can be viewed as different products since they command different prices on the black market. For example, according to Leinwand Leger (2014) credit cards with the highest credit limits, such as an American Express Platinum card, command the highest prices.

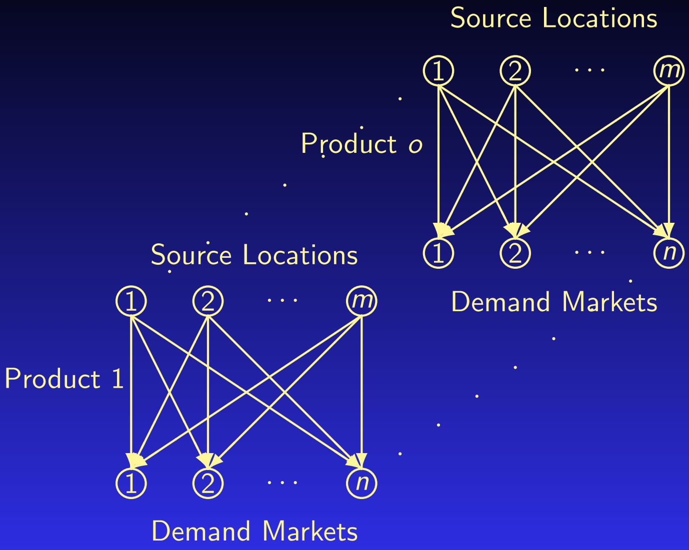
• A card number with a low limit might sell for $1 or $2, while a high limit card number can sell for $15 or considerably more, as noted above. **Hacked credit card numbers of European credit cards can command prices five times higher than U.S. cards** (see Peterson (2013)).

Perishability and Cybercrime in Financial Products

There is a short time window during which the value of a financial product acquired through cybercrime is positive but it decreases during the time window. Hence, financial products such as credit cards that are hacked can be treated as perishable products such as fruits, vegetables, etc.

Perishability and Cybercrime in Financial Products

This part of the presentation is based on the paper, "A Multiproduct Network Economic Model of Cybercrime in Financial Services," Anna Nagurney, *Service Science* 7(1) (2015) pp 70-81.



Structure of the Network Economic Problem

Some Notation - Variables

Let Q_{ij}^k denote the nonnegative amount of financial product k obtained from i and shipped to j. Q is the vector of Q_{ij}^ks.

Let s_i^k denote the nonnegative supply of financial product k at i and let d_j^k be the demand for k and j. s is the vector of s_i^ks and d is the vector of d_j^ks.

T_{ij}^k is the time between the acquisition of product k from source location i and its sale at j.

$T_{ave,j}^k$ is the average time for delivery of product k at demand market j, where $T_{ave,j}^k = \frac{\sum_{i=1}^m T_{ij}^k Q_{ij}^k}{d_j^k}$. T_{ave} is the vector of $T_{ave,j}^k$s.

Some Notation - Functions

Let $\pi_i^k(s)$ denote the price of acquiring product k at source location i.

Let $\rho_j^k(d, T_{ave})$ denote the demand price of financial product k at demand market j.

Let $\hat{c}_{ij}^k(Q)$ denote the unit transaction cost associated with transacting product k between i and j.

Conservation of Flow Equations

The conservation of flow equations are:

$$s_i^k = \sum_{j=1}^{n} Q_{ij}^k, \quad k = 1, \ldots, o; i = 1, \ldots, m,$$

$$d_j^k = \sum_{i=1}^{m} Q_{ij}^k, \quad k = 1, \ldots, o; i = 1, \ldots, n,$$

$$Q_{ij}^k \geq 0, \quad k = 1, \ldots, o; i = 1, \ldots, m; j = 1, \ldots, n.$$

In addition, we introduce the following expression, which captures time:

$$t_{ij}^k Q_{ij}^k + h_{ij}^k = T_{ij}^k, \quad k = 1, \ldots, o; i = 1, \ldots, m; j = 1, \ldots, n.$$

In view of the conservation of flow equations, we can define new demand price functions $\hat{\rho}_j^k$, $\forall k, \forall j$ as follows:

$$\hat{\rho}_j^k(Q) \equiv \rho_j^k(d, T_{ave}), \quad k = 1, \ldots, o; j = 1, \ldots, n.$$

If the demand at a demand market for a product is equal to zero, we remove that demand market from the network for that product since the corresponding time average would not be defined.

Also, we can define new supply price functions $\hat{\pi}_i^k$, $\forall k, \forall i$ as:

$$\hat{\pi}_i^k(Q) \equiv \pi_i^k(s), \quad k = 1, \ldots, o; j = 1, \ldots, n,$$

which allow us to construct a variational inequality formulation governing the equilibrium conditions below with nice features for computations. We assume that all the functions in the model are continuous.

The Network Economic Equilibrium Conditions

The network economic equilibrium conditions for cybercrime have been achieved if for all products k; $k = 1, \dots, o$, and for all pairs of markets (i, j); $i = 1, \dots, m$; $j = 1, \dots, n$, the following conditions hold:

$$\hat{\pi}_i^k(Q^*) + c_{ij}^k(Q^*) \begin{cases} = \hat{\rho}_j^k(Q^*), & \text{if} \quad {Q_{ij}^k}^* > 0 \\ \geq \rho_j^k(Q^*), & \text{if} \quad {Q_{ij}^k}^* = 0, \end{cases}$$

where recall that $\hat{\pi}_i^k$ denotes the price of product k at source location i, c_{ij}^k denotes the unit transaction cost associated with k between (i, j), and $\hat{\rho}_j^k$ is the demand price of k at demand market j. ${Q_{ij}^k}^*$ is the equilibrium flow of product k between i and j with Q^* being the vector of all such flows.

We define the feasible set $K \equiv \{Q | Q \in R_+^{omn}\}$.

VI Formulation of the Equilibrium Conditions

Theorem: Variational Inequality Formulation

A product flow pattern $Q^* \in K$ is a cybercrime network economic equilibrium if and only if it satisfies the variational inequality problem:

$$\sum_{k=1}^{o}\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\hat{\pi}_i^k(Q^*) + c_{ij}^k(Q^*) - \hat{\rho}_j^k(Q^*)\right]\times(Q_{ij}^k - Q_{ij}^{k^*}) \geq 0, \quad \forall Q \in K.$$

The above variational inequality problem can be put into standard form (see Nagurney (1999)): determine $X^* \in \mathcal{K}$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}.$$

We define $\mathcal{K} \equiv K$, $X \equiv Q$, and $F(X) \equiv (F_{kij}(X))$; $k = 1, \ldots, o$; $i = 1, \ldots, m$; $j = 1, \ldots, n$, where $F_{kij} = \hat{\pi}_i^k(Q) + c_{ij}^k(Q) - \hat{\rho}_j^k(Q)$.

Methodology - The Variational Inequality Problem

We utilize the theory of variational inequalities for the formulation, analysis, and solution of cybercrime as well as cybersecurity investment problems.

Definition: The Variational Inequality Problem
The finite-dimensional variational inequality problem, $\mathrm{VI}(F, \mathcal{K})$*, is to determine a vector* $X^* \in \mathcal{K}$*, such that:*

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

where F *is a given continuous function from* $\mathcal{K}$ *to* R^N*,* $\mathcal{K}$ *is a given closed convex set, and* $\langle \cdot, \cdot \rangle$ *denotes the inner product in* R^N*.*

Methodology - The Variational Inequality Problem

The vector X consists of the decision variables – typically, the flows (products, prices, investments, security levels, etc.).

$\mathcal{K}$ is the feasible set representing how the decision variables are constrained – for example, the flows may have to be nonnegative; budget constraints may have to be satisfied; similarly, quality and/or time constraints may have to be satisfied.

The function F that enters the variational inequality represents functions that capture the behavior in the form of the functions such as costs, profits, risk, etc.

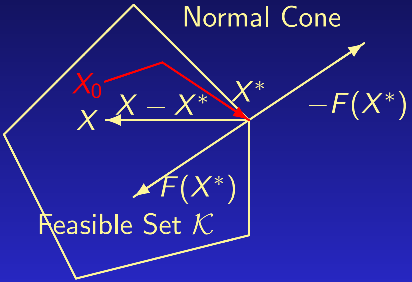
The variational inequality problem contains, as special cases, such mathematical programming problems as:

- systems of equations,
- optimization problems,
- complementarity problems,
- game theory problems, operating under Nash equilibrium,
- and is related to the fixed point problem.

Hence, it is a natural methodology for a spectrum of supply chain network problems from centralized to decentralized ones as well as to design problems.

Geometric Interpretation of VI($F, \mathcal{K}$) and a Projected Dynamical System (Dupuis and Nagurney, Nagurney and Zhang)

In particular, $F(X^*)$ is "orthogonal" to the feasible set $\mathcal{K}$ at the point X^*.



Associated with a VI is a Projected Dynamical System, which provides natural underlying dynamics associated with travel (and other) behavior to the equilibrium.

To model the *dynamic behavior of complex networks*, we utilize *projected dynamical systems* (PDSs) advanced by Dupuis and Nagurney (1993) in *Annals of Operations Research* and by Nagurney and Zhang (1996) in our book *Projected Dynamical Systems and Variational Inequalities with Applications*.

Such nonclassical dynamical systems are now being used in

evolutionary games (Sandholm (2005, 2011)),

ecological predator-prey networks (Nagurney and Nagurney (2011a, b)), and

even *neuroscience* (Girard et al. (2008)

dynamic spectrum model for cognitive radio networks (Setoodeh, Haykin, and Moghadam (2012)).

The Euler Method

At each iteration τ one solves the following problem:

$$X^{\tau+1} = P_{\mathcal{K}}(X^\tau - a_\tau F(X^\tau)),$$

where $P_{\mathcal{K}}$ is the projection operator.

As shown in Dupuis and Nagurney (1993) and Nagurney and Zhang (1996), for convergence of the general iterative scheme, which induces the Euler method, among other methods, the sequence $\{a_\tau\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$.

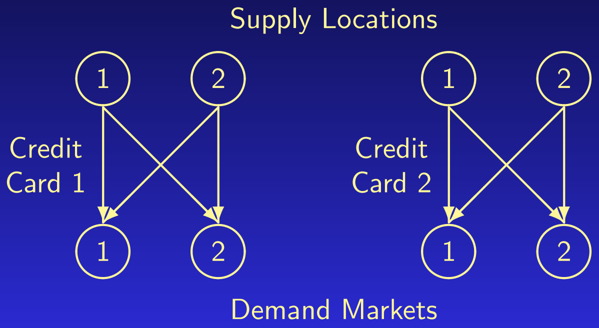
Explicit Formulae

In particular, we have the following closed form expression for the product flows $k = 1, \ldots, m$; $i = 1, \ldots, m$; $j = 1, \ldots, n$:

$$Q_{ij}^{k\,\tau+1} = \max\{0, Q_{ij}^{k\,\tau} + a_\tau(\hat{\rho}_j^k(Q^\tau) - c_{ij}^k(Q^\tau) - \hat{\pi}_i^k(Q^\tau)\}.$$

Numerical Examples: 2 Financial Products, 2 Supply Markets, and 2 Demand Markets

The network topology of the examples is in the figure below.



Topology of Examples

Numerical Examples: 2 Financial Products, 2 Supply Markets, and 2 Demand Markets

Example 1

The supply price functions are:

$$\pi_1^1(s) = 5s_1^1 + s_2^1 + 2, \quad \pi_2^1(s) = 2s_2^1 + s_1^1 + 1,$$

$$\pi_1^2(s) = 2s_1^2 + s_1^1 + 1, \quad \pi_2^2(s) = s_2^2 + .5s_2^1 + 1.$$

The unit transaction cost functions are:

$$c_{11}^1(Q) = .03{Q_{11}^1}^2 + 3Q_{11}^1 + 1, \quad c_{21}^1(Q) = .02{Q_{21}^1}^2 + 2Q_{21}^1 + 2,$$

$$c_{11}^2(Q) = .01{Q_{11}^2}^2 + Q_{11}^2 + 1, \quad c_{21}^2(Q) = .001{Q_{21}^2}^2 + .1Q_{21}^2 + 1,$$

$$c_{12}^1(Q) = .01{Q_{12}^1}^2 + Q_{12}^1 + 1, \quad c_{22}^1(Q) = .01{Q_{22}^1}^2 + Q_{22}^1 + 1,$$

$$c_{12}^2(Q) = .01{Q_{12}^2}^2 + Q_{12}^2 + 1, \quad c_{22}^2(Q) = .02{Q_{22}^2}^2 + 2Q_{22}^2 + 2.$$

Numerical Examples: 2 Financial Products, 2 Supply Markets, and 2 Demand Markets

Example 1

The demand price functions are:

$$\rho_1^1(d, T_{ave}) = -2d_1^1 - d_1^2 - .5\,T_{ave,1}^1 + 500,$$

$$\rho_1^2(d) = -3d_1^2 - d_1^1 - .1\,T_{ave,1}^2 + 300,$$

$$\rho_2^1(d, T_{ave}) = -d_2^1 - .5d_2^2 - .2\,T_{ave,2}^1 + 200,$$

$$\rho_2^2(d, T_{ave}) = -2d_2^2 - d_2^1 - .1\,T_{ave,2}^2 + 100.$$

Numerical Examples: 2 Financial Products, 2 Supply Markets, and 2 Demand Markets

Example 1 The time expressions are:

$$T^1_{11} = .1Q^1_{11} + 10, \quad T^1_{21} = .5Q^1_{21} + 5,$$

$$T^2_{11} = .1Q^2_{11} + 20, \quad T^2_{21} = .5Q^2_{21} + 15,$$

$$T^1_{12} = .1Q^1_{12} + 10, \quad T^1_{22} = .1Q^1_{22} + 10,$$

$$T^2_{12} = .5Q^2_{12} + 5, \quad T^2_{22} = .5Q^2_{22} + 10,$$

so that

$$T^1_{ave,1} = \frac{T^1_{11}Q^1_{11} + T^1_{21}Q^1_{21}}{d^1_1}, \quad T^2_{ave,1} = \frac{T^2_{11}Q^2_{11} + T^2_{21}Q^2_{21}}{d^2_1}.$$

$$T^1_{ave,2} = \frac{T^1_{12}Q^1_{12} + T^1_{22}Q^1_{22}}{d^1_2}, \quad T^2_{ave,2} = \frac{T^2_{12}Q^2_{12} + T^2_{22}Q^2_{22}}{d^2_2}.$$

The Euler method converged to the solution reported in Tables 1 and 2.

Example 2

Example 2 has the same data as Example 1 except that now we have a modification in the demand price function associated with the second product at demand market 2 so that:

$$\rho_2^2(d, T_{ave}) = -2d_2^2 - d_2^1 - .1T_{ave,2}^2 + 200.$$

Such a change might represent that the value of this financial product has increased at that demand market.

Example 3

Example 3 was constructed from Example 2 and had the same data except that we increased the fixed terms in all the transaction cost functions so that:

$$c_1^1(Q) = .03{Q_{11}^1}^2 + 3Q_{11}^1 + 10, \quad c_{21}^1(Q) = .02{Q_{21}^1}^2 + 2Q_{21}^1 + 20,$$

$$c_{11}^2(Q) = .01{Q_{11}^2}^2 + Q_{11}^2 + 10, \quad c_{21}^1(Q) = .001{Q_{21}^2}^2 + .1Q_{21}^2 + 10,$$

$$c_{12}^1(Q) = .01{Q_{12}^1}^2 + Q_{12}^1 + 10, \quad c_{22}^1(Q) = .01{Q_{22}^1}^2 + Q_{22}^1 + 10,$$

$$c_{12}^2(Q) = .01{Q_{12}^2}^2 + Q_{12}^2 + 10, \quad c_{22}^2(Q) = .02{Q_{22}^2}^2 + 2Q_{22}^2 + 20.$$

This could represent the situation that the cybercriminals have a harder time fencing all the products at all the demand markets. The results are reported in Tables 1 and 2.

Results

Equilibrium Solutions for the Examples

Financial Flows	Example 1	Example 2	Example 3
$Q_{11}^{1\ *}$	25.93	26.31	26.21
$Q_{12}^{1\ *}$	0.00	0.00	0.00
$Q_{21}^{1\ *}$	46.73	48.28	46.45
$Q_{22}^{1\ *}$	16.77	12.50	11.61
$Q_{11}^{2\ *}$	11.69	4.81	3.47
$Q_{12}^{2\ *}$	6.09	23.46	23.59
$Q_{21}^{2\ *}$	37.56	39.27	39.57
$Q_{22}^{2\ *}$	0.00	12.67	9.69

Results

Incurred Equilibrium Prices and Average Times

Prices	**Example 1**	**Example 2**	**Example 3**
$\rho_1^1(d^*, T^*_{ave})$	294.07	295.07	300.35
$\rho_1^2(d^*, T^*_{ave})$	76.52	89.85	94.87
$\rho_2^1(d^*, T^*_{ave})$	175.51	164.94	167.28
$\rho_2^2(d^*, T^*_{ave})$	69.98	113.86	120.52
Average Times	**Example 1**	**Example 2**	**Example 3**
$T^1_{ave,1}$	22.74	23.32	22.59
$T^2_{ave,1}$	30.78	33.09	33.62
$T^1_{ave,2}$	23.35	22.50	22.32
$T^2_{ave,2}$	10.61	13.75	13.08

Managerial Insights

• The examples show the quantified impacts of changes in the data on the equilibrium financial product flows, and on the incurred demand prices and average times for product delivery.

• The results are consistent with existing data on hacked credit cards. For example, Goncharov (2012) reports that the cost, that is, the supply price, of hacking into various accounts can range anywhere from \$16 to over \$325. Also, as reported in Ablon, Libicki, and Golay (2014), following an initial breach, the markets may get flooded with cybercrime products leading to a decrease in prices, which the structure of our demand price functions capture.

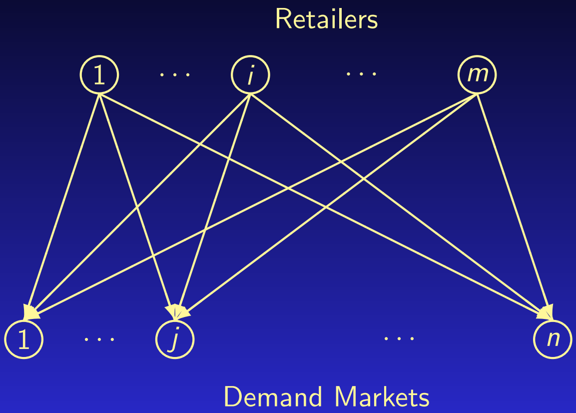
Managerial Insights

- Credit cards acquired in the Target breach initially fetched from $20 to $135 depending on the type of card, expiration date as well as limit (cf. Ablon, Libicki, and Golay (2014)). Although our numerical study did not focus on a specific historical data breach, the results are not inconsistent with results obtained in practice.

- Finally, the model captures the crucial time element in the demand market pricing of products obtained through cybercrime with a focus on financial services.

Cybersecurity and Supply Chains

Figure 1: Supply chains are also vulnerable to cyberattacks and can serve as entre points



The Structure of the Supply Chain Network Game Theory Model

Some Other Examples of Our Recent Cybersecurity Work

"Multifirm Models of Cybersecurity Investment Competition vs. Cooperation and Network Vulnerability," Anna Nagurney and Shivani Shukla, *European Journal of Operational Research* 260(2) (2017) pp 588-600.

"A Supply Chain Network Game Theory Model of Cybersecurity Investments with Nonlinear Budget Constraints," Anna Nagurney, Patrizia Daniele, and Shivani Shukla, *Annals of Operations Research* 248(1) (2017) pp 405-427.

"Cybersecurity Investments with Nonlinear Budget Constraints: Analysis of the Marginal Expected Utilities," Patrizia Daniele, Antonio Maugeri, and Anna Nagurney, in: *Operations Research, Engineering, and Cyber Security*, Th.M. Rassias and N.J. Daras (Eds.), Springer International Publishing Switzerland (2017) pp 117-134.

Network Models and Disaster Relief

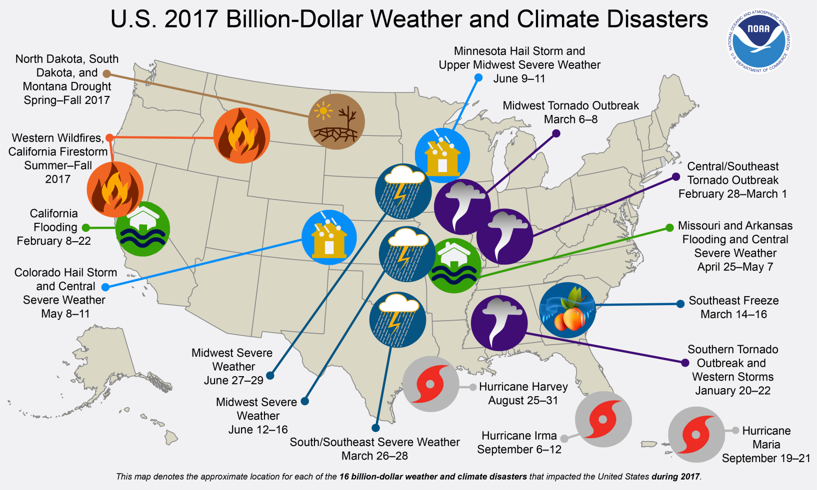
Network Models Are Also Very Useful in Disaster Relief

Examples of Some Disasters

- The biggest blackout in North America, August 14, 2003;
- Two significant power outages in September 2003 – one in the UK and the other in Italy and Switzerland;
- The Indonesian tsunami (and earthquake), December 26, 2004;
- Hurricane Katrina, August 23, 2005;
- The Sichuan earthquake on May 12, 2008;
- The Haiti earthquake that struck on January 12, 2010 and the Chilean one on February 27, 2010;
- The triple disaster in Japan on March 11, 2011;
- Superstorm Sandy, October 29, 2012;
- Hurricanes Harvey, Irma, and Maria that struck in 2017.

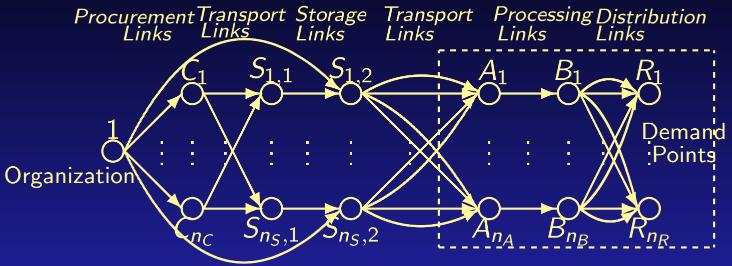
Hurricane Katrina, Fukushima, and Superstorm Sandy

Billion Dollar Disasters in the United States in 2017



2017 Set a Record for Losses in the US from Natural Disasters

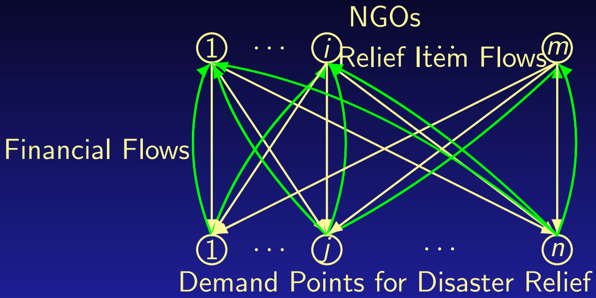
Disasters have a catastrophic effect on human lives and a region's or even a nation's resources.



Network Topology of the Integrated Disaster Relief Supply Chain

A. Nagurney, A. H. Masoumi, and M. Yu, "An Integrated Disaster Relief Supply Chain Network Model with Time Targets and Demand Uncertainty." In: *Regional Science Matters: Studies Dedicated to Walter Isard*, P. Nijkamp, A. Rose, and K. Kourtit, Editors, Springer International Publishing Switzerland (2015), pp 287-318.

A Game Theory Disaster Relief Model



The Network Structure of an Integrated Logistics and Financial Game Theory Model for Disaster Relief

A. Nagurney et al., "A Variational Equilibrium Network Framework for Humanitarian Organizations in Disaster Relief: Effective Product Delivery Under Competition for Financial Funds." In press in: *Dynamics of Disasters: Algorithmic Approaches and Applications*, Springer International Publishers Switzerland (2018).

OR100

OR100

OR100 will take place at Lancaster University.

Face to face time will still be essential for scientific exchanges but delegates will have traveled via a hyperloop and/or autonomous vehicles, among other means, to the conference.

Current students and postdocs in the **OR60** audience will be the senior statesmen and stateswomen and will look forward to another half century of contributions to OR due to health and medical advances.

Advanced computing, not even conceivable today, will be pervasive, but with a human touch.

OR will be a household name because of its sustained and significant impact.

OR100

OR models and techniques will have helped to revolutionize:

- transport
- logistics
- manufacturing
- healthcare
- energy technologies and pollution reduction
- the mitigation of climate change and the response to disasters

and played pivotal roles in global peace, the alleviation of hunger, enhanced security, and prosperity.

OR will continue to forge scientific friendships and collaborations across national boundaries.

OR100

And if anyone in the audience was at **OR1** at Harrogate in 1958, or your advisor was, or a collaborator of yours was, we applaud you.

The wonderful OR network with strong linkages to its rich past continues to grow and evolve now and into the future.

THANK YOU!

For more information, see: http://supernet.isenberg.umass.edu