Fragile Networks: Identifying Vulnerabilities and Synergies in an Uncertain Age

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3rd Yalta Optimization Conference August 2-4, 2010



Special thanks to the Organizers of this Conference for the opportunity to speak.

Дуже Вам Дякую!

Outline of Presentation

• Part I: Network Fundamentals, Efficiency Measurement, and Vulnerability Analysis

Part II: Applications and Extensions

• Part III: Mergers and Acquisitions, Network Integration, and Synergies

Part I

Why Study Fragile Networks?

Networks provide the foundations for transportation and logistics, for communication, energy provision, social interactions, financing, and economic trade.

Today, the subject has garnered great interest due to a spectrum of catastrophic events that have drawn attention to network vulnerability and fragility.

Since many networks that underlie our societies and economies are large-scale and complex in nature, they are liable to be faced with disruptions.

Recent disasters demonstrate the importance and the vulnerability of network systems

- 9/11 Terrorist Attacks, September 11, 2001;
- The biggest blackout in North America, August 14, 2003;
- Two significant power outages in September 2003 -- one in the UK and the other in Italy and Switzerland;
- The Indonesian tsunami (and earthquake), December 26, 2004;
- Hurricane Katrina, August 23, 2005;
- The Minneapolis I35 Bridge collapse, August 1, 2007;
- The Mediterranean cable destruction, January 30, 2008;
- The Sichuan earthquake on May 12, 2008;
- The Haiti earthquake that struck on January 12, 2010 and the Chilean one on February 27, 2010.

Earthquake Damage

Tsunami

letthesunshinein.wordpress.com

prcs.org.pk



Storm Damage

www.srh.noaa.gov





Infrastructure Collapse

www.10-7.com





The Haitian and Chilean Earthquakes



Disasters have brought an unprecedented impact on human lives in the 21st century and the number of disasters is growing.



Frequency of disasters [Source: Emergency Events Database (2008)]

Natural Disaster Trend and Number of People Affected (1975 – 2008)



EM-DAT: The OFDAXCRED International Disaster Database - www.emdatbe - Unitersité Catholique de Loruain, Brusseis - Belgiur

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Natural Disaster Trend Source: EM-DAT

Number of People Affected Source: EM-DAT

We are also in a New Era of Decision-Making Characterized by:

- complex interactions among decision-makers in organizations;
- alternative and at times conflicting criteria used in decision-making;
- constraints on resources: natural, human, financial, time, etc.;
- global reach of many decisions;
- high impact of many decisions;
- increasing risk and uncertainty, and
- the *importance of dynamics* and realizing a fast and sound response to evolving events.

This era is ideal for applying the tools of Fragile Networks.

Network problems are their own class of problems and they come in various forms and formulations, i.e., as optimization (linear or nonlinear) problems or as equilibrium problems and even dynamic network problems.

Network problems will be the focus of this talk with fragility as the major theme.

Interdisciplinary Impact of Networks Economics and odels and Algorithm OR/MS and Engineering Interregional Trade Energy General Equilibrium Manufacturing Industrial Organization **Telecommunications Networks** Portfolio Optimization Transportation Flow of Funds Supply Chains Accounting Biology Sociology **DNA Sequencing Computer Science** Social Networks **Targeted Cancer Routing Algorithms** Therapy Organizational Theory Price of Anarchy

Background and Network Fundamentals



Subway Network

Transportation, Communication, and Energy Networks



Railroad Network

Iridium Satellite Constellation Network

Satellite and Undersea Cable Networks

Ukrainian Gas Pipeline Network







Components of Common Physical Networks

Network System	Nodes	Links	Flows
Transportation	Intersections, Homes, Workplaces, Airports, Railyards	Roads, Airline Routes, Railroad Track	Automobiles, Trains, and Planes,
Manufacturing and logistics	Workstations, Distribution Points	Processing, Shipment	Components, Finished Goods
Communication	Computers, Satellites, Telephone Exchanges	Fiber Optic Cables Radio Links	Voice, Data, Video
Energy	Pumping Stations, Plants	Pipelines, Transmission Lines	Water, Gas, Oil, Electricity





Internet Traffic

Network



Systems





The study of the efficient operation of transportation networks dates to *ancient Rome* with a classical example being the publicly provided Roman road network and the *time of day chariot policy*, whereby chariots were banned from the ancient city of Rome at particular times of day.



The need to model and solve a spectrum of challenging network problems has given rise to new computational methodologies.



We need to capture not only network topology (how nodes are connected with the links) but also

the behavior of users of the networks and the induced flows!

Characteristics of Networks Today

- *large-scale nature* and complexity of network topology;
- congestion (leading to nonlinearities);
- alternative behavior of users of the network, which may lead to *paradoxical phenomena*;
- the *interactions among networks* themselves such as in transportation versus telecommunications;
- policies surrounding networks today may have a major impact not only economically but also socially, politically, and security-wise.

Networks in Action

- Some social network websites, such as facebook.com and myspace.com, have over 300 million users.
- Internet traffic is approximately *doubling* each year.
- In the US, the annual traveler delay per peak period (rush hour) has grown from 16 hours to 47 hours since 1982.
- The total amount of delay reached **3.7** *billion hours* in 2003.
- The wasted fuel amounted to 2.3 billion gallons due to engines idling in traffic jams (Texas Transportation Institute 2005 Urban Mobility Report).

Hence, many of the network problems today are flow-dependent and increasingly nonlinear, as opposed to linear.

Therefore, the underlying functions must capture, for example, congestion!

TRAFFIC

Capturing Link Congestion



For a typical user link travel time function, where the free flow travel time refers to the travel time to traverse a link when there is zero flow on the link (or zero vehicles). The importance of capturing user behavior on networks will now be illustrated through a famous paradox known as the *Braess paradox* in which travelers are assumed to behave in a *user-optimizing (U-O) manner*, as opposed to a *system-optimizing (S-O) one*.

Under U-O behavior, decision-makers act independently and selfishly with no concern of the impact of their travel choices on others.

Behavior on Congested Networks

Decision-makers select their cost-minimizing routes.



Flows are routed so as to minimize the total cost to society.

The Braess (1968) Paradox

Assume a network with a single O/D pair (1,4). There are 2 paths available to travelers: $p_1=(a,c)$ and $p_2=(b,d)$.

For a travel demand of **6**, the equilibrium path flows are $\mathbf{x}_{p_1}^*$

$$= x_{p_2}^* = 3$$
 and

The equilibrium path travel cost is $C_{p_1} = C_{p_2} = 83.$ c (

 $c_{a}(f_{a}) = 10 f_{a} c_{b}(f_{b}) = f_{b} + 50$ $c_{c}(f_{c}) = f_{c} + 50 c_{d}(f_{d}) = 10 f_{d}$

a

2

C

b

d

3

Adding a Link Increases Travel Cost for All!

- Adding a new link creates a new path $p_3=(a,e,d)$.
- The original flow distribution pattern is no longer an equilibrium pattern, since at this level of flow the cost on path p_3 ,

The new equilibrium flow pattern network is

 $\mathbf{x}_{p_1}^* = \mathbf{x}_{p_2}^* = \mathbf{x}_{p_3}^* = 2.$

The equilibrium path travel costs: $C_{p_4} =$

 $\begin{array}{c} 1 \\ a \\ c \\ c \\ 4 \end{array}$

 $c_{e}(f_{e}) = f_{e} + 10$

 $C_{p_2} = C_{p_3} = 92$

The Braess Paradox Around the World

1969 - Stuggart, Germany - Traffic worsened until a newly built road was closed.



1990 - Earth Day - New York City -42nd Street was closed and traffic flow improved.





2002 - Seoul, Korea - A 6 lane road built over the Cheonggyecheon River that carried 160,000 cars per day and was perpetually jammed was torn down to improve traffic flow.



Other Networks that Behave like Traffic Networks



The Internet

Supply Chain Networks



Electric Power Generation/Distribution Networks

Financial Networks



FRAGILE NETWORKS

Identifying Vulnerabilities and Synergies in an Uncertain World

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WILEY

This *paradox is relevant* not only to congested transportation networks but also to the Internet and electric power networks.

Hence, there are *huge implications* also for network design.

There are *two fundamental principles of travel behavior*, due to Wardrop (1952), which are referred to as user-optimal (U-O or network equilibrium) and system-optimal (S-O).

In a *user-optimized (network equilibrium) problem*, each user of a network system seeks to determine his/her costminimizing route of travel between an origin/destination pair, until an equilibrium is reached, in which no user can decrease his/her cost of travel by unilateral action.

In a system-optimized network problem, users are allocated among the routes so as to minimize the total cost in the system.

Both classes of problems, under certain imposed assumptions, possess optimization formulations.

Bureau of Public Roads (BPR) Link Cost Function

$$c_{a} = c_{a}^{0} \left[1 + \alpha \left(\frac{f_{a}}{t_{a}^{\prime}} \right)^{\beta} \right],$$

where, c_a and f_a are the travel time and link flow, respectively, on link a, c_a^0 is the free-flow travel time, and t'_a is the "practical capacity" of link a. The quantities α and β are model parameters, for which the values $\alpha = 0.15$ minutes and $\beta = 4$ are typical values. For example, these values imply that the practical capacity of a link is the flow at which the travel time is 15% greater than the free-flow travel time.

The User-Optimization (U-O) Problem Transportation Network Equilibrium

Consider a general network G = [N, L], where N denotes the set of nodes, and L the set of directed links. Let a denote a link of the network connecting a pair of nodes, and let p denote an acyclic path consisting of a sequence of links connecting an origin/destination (O/D) pair of nodes. P_w denotes the set of paths connecting the O/D pair of nodes w and P the set of all paths.

Let x_p represent the flow on path p and let f_a denote the flow on link a. The following conservation of flow equations must hold:

$$f_{a} = \sum_{p \in P} x_{p} \delta_{ap},$$

where $\delta_{ap} = 1$, if link *a* is contained in path *p*, and 0, otherwise. This expression states that the flow on a link *a* is equal to the sum of all the path flows on paths *p* that contain (traverse) link *a*.
Moreover, if we let d_w denote the demand associated with O/D pair w, then we must have that

$$d_w = \sum_{p \in P_w} x_p,$$

where $x_p \ge 0$, $\forall p$, that is, the sum of all the path flows between an origin/destination pair w must be equal to the given demand d_w .

Let c_a denote the user cost associated with traversing link a, and C_p the user cost associated with traversing the path p. Then

$$C_p = \sum_{a \in L} c_a \delta_{ap}.$$

In other words, the cost of a path is equal to the sum of the costs on the links comprising the path. In the classical model, $c_a = c_a(f_a), \forall a \in L$. In the most general case, $c_a = c_a(f), \forall a \in L$, where f is the vector of link flows.

Transportation Network Equilibrium Conditions

The network equilibrium conditions are then given by: For each path $p \in P_w$ and every O/D pair w:

$$C_p \left\{ \begin{array}{ll} = \lambda_w, & \text{if} \quad x_p^* > 0\\ \ge \lambda_w, & \text{if} \quad x_p^* = 0 \end{array} \right.$$

where λ_w is an indicator, whose value is not known a priori. The equilibrium conditions state that the user costs on all used paths connecting a given O/D pair will be minimal and equalized. This is Wardrop's first principle of travel behavior. As shown by Beckmann, McGuire, and Winsten (1956) and Dafermos and Sparrow (1969), if the user link cost functions satisfy the symmetry property that $\left[\frac{\partial c_a}{\partial c_b} = \frac{\partial c_b}{\partial c_a}\right]$, for all links a, b in the network then the solution to the above U-O problem can be reformulated as the solution to an associated optimization problem. For example, if we have that $c_a = c_a(f_a)$, for all links $a \in L$, then the solution to the U-O problem can be obtained by solving:

Minimize
$$\sum_{a \in L} \int_0^{f_a} c_a(y) dy$$

subject to:

$$egin{aligned} d_w &= \sum_{eta \in P_w} x_{m{
ho}}, & orall w \in W, \end{aligned} \ f_{m{a}} &= \sum_{eta \in P} x_{m{
ho}}, & orall a \in L, \cr x_{m{
ho}} &\geq 0, & orall p \in P. \end{aligned}$$

The System-Optimization (S-O) Problem

The above discussion focused on the user-optimized (U-O) problem. We now turn to the system-optimized (S-O) problem in which a central controller, say, seeks to minimize the total cost in the network system, where the total cost is expressed as

$$\sum_{a\in L} \hat{c}_a(f_a)$$

where it is assumed that the total cost function on a link *a* is defined as:

$$\hat{c}_a(f_a) \equiv c_a(f_a) \times f_a,$$

subject to the conservation of flow constraints, and the nonnegativity assumption on the path flows. Here separable link costs have been assumed, for simplicity, and other total cost expressions may be used, as mandated by the particular application.

The S-O Optimality Conditions

Under the assumption of strictly increasing user link cost functions, the optimality conditions are: For each path $p \in P_w$, and every O/D pair w:

$$\hat{C}'_{p} \left\{ \begin{array}{ll} = \mu_{w}, & \text{if} \quad x_{p} > 0\\ \geq \mu_{w}, & \text{if} \quad x_{p} = 0, \end{array} \right.$$

where \hat{C}'_p denotes the marginal total cost on path p, given by:

$$\hat{C}'_{p} = \sum_{a \in L} \frac{\partial \hat{c}_{a}(f_{a})}{\partial f_{a}} \delta_{ap}.$$

The above conditions correspond to Wardrop's second principle of travel behavior. What is the S-O solution for the two Braess networks (before and after the addition of a new link e)?

Before the addition of the link e, we may write:

$$\hat{c}_a'=20f_a,\quad \hat{c}_b'=2f_b+50,$$

$$\hat{c}_{c}' = 2f_{c} + 50, \quad \hat{c}_{d}' = 20f_{d}.$$

It is easy to see that, in this case, the S-O solution is identical to the U-O solution with $x_{p_1} = x_{p_2} = 3$ and $\hat{C}'_{p_1} = \hat{C}'_{p_2} = 116$. Furthermore, after the addition of link e, we have that $\hat{c}'_e = 2f_e + 10$. The new path p_3 is not used in the S-O solution, since with zero flow on path p_3 , we have that $\hat{C}'_{p_3} = 170$ and $\hat{C}'_{p_1} = \hat{C}'_{p_2}$ remains at 116.

Another Example

b

 $c_{b}(f_{b}) = f_{b} + 1$

a

Assume a network with a single O/D pair (1,2). There are 2 paths available to travelers: $p_1 = a$ and $p_2 = b$.

For a travel demand of 1, the U-O path flows are: $x_{p_1}^* = 1; x_{p_2}^* = 0$ and the total cost under U O behavior is TO = 1 $C_a(f_a) = f_a$

the total cost under U-O behavior is $TC_{u_0} = 1$.

The S-O path flows are: $\mathbf{x}_{p_1} = \frac{3}{4}$; $\mathbf{x}_{p_2} = \frac{1}{4}$ and the total cost under S-O behavior is $TC_{S-0} = \frac{7}{8}$.

The Price of Anarchy

The price of anarchy is defined as the ratio of the TC under U-O behavior to the TC under S-O behavior:

 $\rho = TC_{U-O} / TC_{S-O}$

See Roughgarden (2005), *Selfish Routing and the Price of Anarchy*.

Question: When does the U-O solution coincide with the S-O solution?

Answer: In a general network, with user link cost functions given by: $c_a(f_a) = c_a^0 f_a^\beta$, for all links, with $c_a^0 \ge 0$ and $\beta \ge 0$.

Note that for $c_a(f_a)=c_a^o$, that is, in the case of uncongested networks, this result always holds.

Recall again the Braess Network where we add the link e.



What happens if the demand varies over time?

The U-O Solution of the Braess Network with Added Link (Path) and Time-Varying Demands



In Demand Regime I, only the new path is used. In Demand Regime II, the Addition of a New Link (Path) Makes Everyone Worse Off! In Demand Regime III, only the original paths are used.



Network 1 is the Original Braess Network - Network 2 has the added link.

The new link is NEVER used after a certain demand is reached even if the demand approaches infinity.

Hence, in general, except for a limited range of demand, building the new link is a complete waste!

If the symmetry assumption does not hold for the user link costs functions, which is always satisfied by separable user link cost functions, then the (U-O) equilibrium conditions can no longer be reformulated as an associated optimization problem and the equilibrium conditions are formulated and solved as a *variational inequality (VI)* problem!

Smith (1979), Dafermos (1980)

VI Formulation of Transportation Network Equilibrium (Dafermos (1980), Smith (1979))

A traffic path flow pattern satisfies the above equilibrium conditions if and only if it satisfies the variational inequility problem: determine $x^* \in K$, such that

$$\sum_{p} C_p(x^*) \times (x_p - x_p^*) \ge 0, \quad \forall x \in K.$$

Finite-dimensional variational inequality theory has been applied to-date to the wide range of equilibrium problems noted above.

In particular, the finite-dimensional variational inequality problem is to determine $x^* \in K \subset R^n$ such that

$$\langle F(x^*), x - x^* \rangle \ge 0, \quad \forall x \in K,$$

where $\langle \cdot, \cdot \rangle$ denoted the inner product in \mathbb{R}^n and K is closed and convex.

A Geometric Interpretation of a Variational Inequality



The variational inequality problem, contains, as special cases, such classical problems as:

- systems of equations
- optimization problems
- complementarity problems and is also closely related to fixed point problems.

Hence, it is a unifying mathematical formulation for a variety of mathematical programming problems.

Transportation and Other Network Systems

The TNE Paradigm is the Unifying Paradigm for a Variety of Network Systems:

Transportation Networks

- the Internet
- Financial Networks
- Supply Chains

Electric Power Networks

Other Related Applications

- Telecommuting/Commuting Decision-Making
- Teleshopping/Shopping Decision-Making
- Supply Chain Networks with Electronic Commerce
- Financial Networks with Electronic Transactions
- Reverse Supply Chains with E-Cycling
- Knowledge Networks
- Social Networks integrated with Economic Networks (Supply Chains and Financial Networks)

The Equivalence of Supply Chains and Transportation Networks



Nagurney, Transportation Research E 42 (2006), pp 293-316.

Supply Chain - Transportation Supernetwork Representation



Nagurney, Ke, Cruz, Hancock, Southworth, Environment and Planning B 29 (2002), 795-818.

The fifth chapter of Beckmann, McGuire, and Winsten's book, *Studies in the Economics of Transportation* (1956) describes some *unsolved problems* including a single commodity network equilibrium problem that the authors imply could be generalized to capture electric power networks.

Specifically, they asked whether electric power generation and distribution networks can be reformulated as transportation network equilibrium problems.

Electric Power Supply Chains



The Electric Power Supply Chain Network



Nagurney and Matsypura, Proceedings of the CCCT (2004).

The Transportation Network Equilibrium Reformulation of Electric Power Supply Chain Networks



Electric Power Supply Network

Transportation Chain Network

Nagurney, Liu, Cojocaru, and Daniele, Transportation Research E 43 (2007).

In 1952, Copeland wondered whether money flows like water or electricity.

The Transportation Network Equilibrium Reformulation of the Financial Network Equilibrium Model with Intermediation



Liu and Nagurney, Computational Management Science 4 (2007), pp 243-281.

We have shown that *money* as well as *electricity* flow like *transportation* and have answered questions posed fifty years ago by Copeland and by Beckmann, McGuire, and Winsten!

Recent Literature on Network Vulnerability

- Latora and Marchiori (2001, 2002, 2004)
- Holme, Kim, Yoon and Han (2002)
- Taylor and D'este (2004)
- Murray-Tuite and Mahmassani (2004)
- Chassin and Posse (2005)
- Barrat, Barthélemy and Vespignani (2005)
- Sheffi (2005)
- Dall'Asta, Barrat, Barthélemy and Vespignani (2006)
- Jenelius, Petersen and Mattson (2006)
- Taylor and D'Este (2007)

Network Centrality Measures

- Barrat et al. (2004, pp. 3748), *The identification of the most central nodes in the system is a major issue in network characterization.*
- Centrality Measures for Non-weighted Networks
 - Degree, betweenness (node and edge), closeness (Freeman (1979), Girvan and Newman (2002))
 - Eigenvector centrality (Bonacich (1972))
 - Flow centrality (Freeman, Borgatti and White (1991))
 - Betweenness centrality using flow (Izquierdo and Hanneman (2006))
 - Random-work betweenness, Current-flow betweenness (Newman and Girvan (2004))
- Centrality Measures for Weighted Networks (Very Few)
 Weighted betweenness centrality (Dall'Asta et al. (2006))
 Network efficiency measure (Latora-Marchiori (2001))

Some of Our Research on Network Efficiency, Vulnerability, and Robustness

A Network Efficiency Measure for Congested Networks, Nagurney and Qiang, *Europhysics Letters* **79**, August (2007), p1-p5.

A Transportation Network Efficiency Measure that Captures Flows, Behavior, and Costs with Applications to Network Component Importance Identification and Vulnerability, Nagurney and Qiang, *Proceedings of the POMS 18th Annual Conference*, Dallas, Texas (2007).

A Network Efficiency Measure with Application to Critical Infrastructure Networks, Nagurney and Qiang, *Journal of Global Optimization* **40** (2008), pp 261-275.

Robustness of Transportation Networks Subject to Degradable Links, Nagurney and Qiang, *Europhysics Letters* **80**, December (2007).

A Unified Network Performance Measure with Importance Identification and the Ranking of Network Components, Qiang and Nagurney, *Optimization Letters* **2** (2008), pp 127-142.

Which Nodes and Links Really Matter?

A New Network Performance/Efficiency Measure with Applications to a Variety of Network Systems

The Nagurney and Qiang (N-Q) Network Efficiency Measure

The network performance/efficiency measure $\mathcal{E}(G,d)$, for a given network topology G and demand vector d, is defined as

$$\mathcal{E}(G,d) = \frac{\sum_{w \in W} \frac{d_w}{\lambda_w}}{n_W},$$

where n_w is the number of O/D pairs in the network and λ_w is the equilibrium disutility/price for O/D pair *w*.

Nagurney and Qiang, Europhysics Letters 79 (2007).

Importance of a Network Component

Definition:

The importance, I(g), of a network component $g \in G$ is measured by the relative network efficiency drop after g is removed from the network:

$$I(g) = \frac{\triangle \mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}(G, d) - \mathcal{E}(G - g, d)}{\mathcal{E}(G, d)},$$

where *G*-*g* is the resulting network after component g is removed.
The Latora and Marchiori (L-M) Network Efficiency Measure

Definition:

The network performance/efficiency measure, E(G) for a given network topology, G, is defined as:

$$E(G) = \frac{1}{n(n-1)} \sum_{i \neq j \in G} \frac{1}{d_{ij}}$$

where *n* is the number of nodes in the network and d_{ij} is the shortest path length between node *i* and node *j*.

The L-M Measure vs. the N-Q Measure

Theorem: Equivalence in a Special Case

If positive demands exist for all pairs of nodes in the network, *G*, and each of demands is equal to 1, and if d_{ij} is set equal to λ_w , where w=(i,j), for all $w \in W$, then the N-Q and L-M network efficiency measures are one and the same.

The Approach to Identifying the Importance of Network Components

The elimination of a link is treated in the N-Q network efficiency measure by removing that link while the removal of a node is managed by removing the links entering and exiting that node.

In the case that the removal results in no path connecting an O/D pair, we simply assign the demand for that O/D pair to an abstract path with a cost of infinity.

The N-Q measure is well-defined even in the case of disconnected networks.

According to the European Environment Agency (2004), since 1990, the *annual number of extreme weather and climate related events has doubled*, in comparison to the previous decade. These events account for approximately 80% of all economic losses caused by catastrophic events. In the course of climate change, catastrophic events are projected to occur more frequently (see Schulz (2007)).

Schulz (2007) applied the Nagurney and Qiang (2007) network efficiency measure to a German highway system in order to identify the critical road elements and found that this measure provided more reasonable results than the measure of Taylor and D'Este (2007).

The N-Q measure can also be used to asses which links should be added to improve efficiency. It was used for the evaluation of the proposed North Dublin (Ireland) Metro system (October 2009 Issue of ERCIM News).

Example 1

Assume a network with two O/D pairs: $w_1 = (1,2)$ and $w_2 = (1,3)$ with demands: $d_{w_1} = 100$ and $d_{w_2} = 20$.

The paths are: for w_1 , $p_1 = a$; for w_2 , $p_2 = b$.

The U-O equilibrium path flows are: $x_{p_1}^* = 100, x_{p_2}^* = 20.$



 $c_a(f_a) = 0.01 f_a + 19$ $c_b(f_b) = 0.05 f_b + 19$

The (U-O) equilibrium path costs are: $C_{p_1} = C_{p_2} = 20$.

The Importance and Ranking of Links for Example 1

Link	Importance Value from Our Measure	Importance Ranking from Our Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
а	0.8333	1	0.5000	1
b	0.1667	2	0.5000	1

The Importance and Ranking of Nodes for Example 1

Node	Importance Value from Our Measure	Importance Ranking from Our Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
1	1.0000	1	1.0000	1
2	0.8333	2	0.5000	2
3	0.1667	3	0.5000	2

Example 2 – The Braess Network



The Importance and Ranking of Links for Example 2

Link	Importance Value from Our Measure	Importance Ranking from Our Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
а	0.2096	1	0.1056	3
b	0.1794	2	0.2153	2
С	0.1794	2	0.2153	2
d	0.2069	1	0.1056	3
е	-0.1084	3	0.3616	1

The Importance and Ranking of Nodes for Example 2

Node	Importance Value from Our Measure	Importance Ranking from Our Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
1	1.0000	1	N/A	N/A
2	0.2096	2	0.7635	1
3	0.2096	2	0.7635	1
4	1.0000	1	N/A	N/A

Example 3

The network :



 $w_1 = (1, 20)$ $w_2 = (1, 19)$ $d_{w_1} = 100$ $d_{w_2} = 100$

Nagurney, Transportation Research B (1984)

Example 3: Link Cost Functions

Link a	Link Cost Function $c_a(f_a)$	Link a	Link Cost Function $c_a(f_a)$
1	$.00005f_1^4 + 5f_1 + 500$	15	$.00003f_{15}^4 + 9f_{15} + 200$
2	$.00003f_2^4 + 4f_2 + 200$	16	$8f_{16} + 300$
3	$.00005f_3^4 + 3f_3 + 350$	17	$.00003f_{17}^4 + 7f_{17} + 450$
4	$.00003f_4^4 + 6f_4 + 400$	18	$5f_{18} + 300$
5	$.00006f_5^4 + 6f_5 + 600$	19	$8f_{19} + 600$
6	$7f_6 + 500$	20	$.00003f_{20}^4 + 6f_{20} + 300$
7	$.00008f_7^4 + 8f_7 + 400$	21	$.00004f_{21}^4 + 4f_{21} + 400$
8	$.00004f_8^4 + 5f_8 + 650$	22	$.00002f_{22}^4 + 6f_{22} + 500$
9	$.00001f_9^4 + 6f_9 + 700$	23	$.00003f_{23}^4 + 9f_{23} + 350$
10	$4f_{10} + 800$	24	$.00002f_{24}^4 + 8f_{24} + 400$
11	$.00007f_{11}^4 + 7f_{11} + 650$	25	$.00003f_{25}^4 + 9f_{25} + 450$
12	$8f_{12} + 700$	26	$.00006f_{26}^4 + 7f_{26} + 300$
13	$.00001f_{13}^4 + 7f_{13} + 600$	27	$.00003f_{27}^4 + 8f_{27} + 500$
14	$8f_{14} + 500$	28	$.00003f_{28}^4 + 7f_{28} + 650$

Algorithms for Solution

The projection method (cf. Dafermos (1980) and Nagurney (1999)) embedded with the equilibration algorithm of Dafermos and Sparrow (1969) was used for the computations.

In addition, the column generation method of Leventhal, Nemhauser, and Trotter (1973) was implemented to generate paths, as needed, in the case of the largescale Sioux Falls network example.

Example 3: The Importance and Ranking of Links

Link a	Importance Value	Importance Ranking	Link a	Importance Value	Importance Ranking
1	0.9086	3	15	0.0000	22
2	0.8984	4	16	0.0001	21
3	0.8791	6	17	0.0000	22
4	0.8672	7	18	0.0175	18
5	0.8430	9	19	0.0362	17
6	0.8226	11	20	0.6641	14
7	0.7750	12	21	0.7537	13
8	0.5483	15	22	0.8333	10
9	0.0362	17	23	0.8598	8
10	0.6641	14	24	0.8939	5
11	0.0000	22	25	0.4162	16
12	0.0006	20	26	0.9203	2
13	0.0000	22	27	0.9213	1
14	0.0000	22	28	0.0155	19

Example 3: Link Importance Rankings



Example 4 - Sioux Falls Network

The network data are from LeBlanc, Morlok, and Pierskalla (1975).

The network has 528 O/D pairs, 24 nodes, and 76 links.

The user link cost functions are of BPR form.



Example 4 - Sioux Falls Network Link Importance Rankings



Comparative Importance of Links for the Network of Baden-Württemberg using the N-Q Measure

(29)



Modelling and analysis of transportation networks in earthquake prone areas, S. Tyagunov, C. Schulz, F. Wenzel, L. Stempniewski, and M. Kostenko

The Network Efficiency Measure for Dynamic Networks

A network like the Internet is volatile. Its traffic patterns can change quickly and dramatically... The assumption of a static model is therefore particularly suspect in such networks. (Roughgarden (2005)).

An Efficiency Measure for Dynamic Networks with Application to the Internet and Vulnerability Analysis (Nagurney and Qiang), *Netnomics* 9 (2008), pp 1-20.

The Network Efficiency Measure for Dynamic Networks – Continuous Time

The network efficiency for the network G with time-varying demand d for $t \in [0, T]$, denoted by $\mathcal{E}(G, d, T)$, is defined as follows:

$$\mathcal{E}(G,d,T) = \frac{\int_0^T \left[\sum_{w \in W} \frac{d_w(t)}{\lambda_w(t)}\right]/n_W dt}{T}$$

Note that the above measure is the average network performance over time of the dynamic network.

The Network Efficiency Measure for Dynamic Networks – Discrete Time

Let d_w^1 , d_w^2 , ..., d_w^H denote demands for O/D pair w in H discrete time intervals, given, respectively, by: $[t_0, t_1], (t_1, t_2], ..., (t_{H-1}, t_H]$, where $t_H \equiv T$. Assume that the demand is constant in each such time interval for each O/D pair. Denote the corresponding minimal costs for each O/D pair w at the H different time intervals by: $\lambda_w^1, \lambda_w^2, ..., \lambda_w^H$. The demand vector d, in this special discrete case, is a vector in $R^{n_W \times H}$.

Dynamic Network Efficiency: Discrete Time Version

The network efficiency for the network (G, d) over H discrete time intervals: $[t_0, t_1], (t_1, t_2], ..., (t_{H-1}, t_H]$, where $t_H \equiv T$, and with the respective constant demands: $d_w^1, d_w^2, ..., d_w^H$ for all $w \in W$ is defined as follows:

$$\mathcal{E}(G, d, t_{H} = T) = \frac{\sum_{i=1}^{H} \left[\left(\sum_{w \in W} \frac{d_{w}^{i}}{\lambda_{w}^{i}} \right) (t_{i} - t_{i-1}) / n_{W} \right]}{t_{H}}$$

Importance of Nodes and Links in the Dynamic Braess Network Using the N-Q Measure when T=10

Link	Importance Value	Importance Ranking
а	0.2604	1
Ь	0.1784	2
С	0.1784	2
d	0.2604	1
е	-0.1341	3

Link *e* is never used after t = 8.89 and in the range $t \in [2.58, 8.89]$, it increases the cost, so the fact that link e has a negative importance value makes sense; over ne, its removal ould, on the erage, improve e network iciency!

Node	Importance Value	Importance Ranking	tir
1	1.0000	1	W
2	0.2604	2	av
3	0.2604	2	th
4	1.0000	1	ef

The Advantages of the N-Q Network Efficiency Measure

- The measure captures demands, flows, costs, and behavior of users, in addition to network topology.
- The resulting importance definition of network components is applicable and well-defined even in the case of disconnected networks.
- It can be used to identify the importance (and ranking) of either nodes, or links, or both.
- It can be applied to assess the efficiency/performance of a wide range of network systems.
- It is applicable also to elastic demand networks (Qiang and Nagurney, *Optimization Letters* (2008)).
- It is applicab to dynamic networks (Nagurney and Qiang, Netnomics (2008)).

Part II

Robustness in Engineering and Computer Science

IEEE (1990) defined robustness as the degree to which a system of component can function correctly in the presence of invalid inputs or stressful environmental conditions.

Gribble (2001) defined system robustness as the ability of a system to continue to operate correctly across a wide range of operational conditions, and to fail gracefully outside of that range.

Schillo et al. (2001) argued that robustness has to be studied *in relation to some definition of the performance measure*.

Motivation for Research on Transportation Network Robustness

According to the American Society of Civil Engineering:

Poor maintenance, natural disasters, deterioration over time, as well as unforeseen attacks now lead to estimates of **\$94** *billion in the US* in terms of needed repairs for roads alone.

Poor road conditions in the United States cost US motorists *\$54 billion in repairs and operating costs annually*.

Transportation Network Robustness

The focus of the robustness of networks (and complex networks) has been on the impact of different network measures when facing the removal of nodes on networks.

We focus on the *degradation of links through reductions in their capacities* and the effects on the induced travel costs in the presence of known travel demands and different functional forms for the links.

"Robustness" in Transportation

Sakakibara et al. (2004) proposed a topological index. The authors considered a transportation network to be robust if it is "dispersed" in terms of the number of links connected to each node.

Scott et al. (2005) examined transportation network robustness by analyzing the increase in the total network cost after removal of certain network components. A New Approach to Transportation Network Robustness

The Importance of Studying Transportation Network Robustness

The US is experiencing a *freight capacity crisis* that threatens the strength and productivity of the US economy. According to the American Road & Transportation Builders Association (see Jeanneret (2006)), nearly 75% of US freight is carried in the US on highways, and bottlenecks are causing truckers **243** *million hours* of delay annually with an estimated associated cost of **\$8** *billion*.

The number of motor vehicles in the US has risen by **157** *million* (or 212.16%) since 1960 while the population of licensed drivers grew by 109 million (or 125.28%) (US Department of Transportation (2004)).

The Transportation Network Robustness Measure

Nagurney and Qiang, Europhysics Letters, 80, December (2007)

The robustness measure \mathcal{R}^{γ} for a transportation network G with the vector of demands d, the vector of user link cost functions c, and the vector of link capacities u is defined as the relative performance retained under a given uniform capacity retention ratio γ ($\gamma \in (0, 1]$) so that the new capacities are given by γu . Its mathematical definition is given as:

$$\mathcal{R}^{\gamma} = \mathcal{R}(G, c, d, \gamma, u) = \frac{\mathcal{E}^{\gamma}}{\mathcal{E}} \times 100\%$$

where \mathcal{E} and \mathcal{E}^{γ} are the network performance measures with the original capacities and the remaining capacities, respectively.

We utilize BPR functions user link cost functions c for the robustness analysis.

A Simple Example

Assume a network with one O/D pair: $w_1 = (1,2)$ with demand given by $d_{w_1} = 10$.

The paths are: $p_1 = a$ and $p_2 = b$. In the BPR link cost function, k=1 and $\beta=4$; $c_a^{\ o}=10$ and $c_a^{\ o}=1$.

Assume that there are two sets of capacities:

Capacity Set A, where $u_a = u_b = 50$; Capacity Set B, where $u_a = 50$ and

Capacity Set B, where u_a =50 and u_b =10.



Robustness of the Simple Network



Capacity Retention Ratio y

Another Example: Braess Network with BPR Functions

Instead of using the original cost functions, we construct a set of BPR functions as below under which the Braess Paradox still occurs. The new demand is 110.

$$c_a(f_a) = 1 + (\frac{f_a}{20})^{\beta}, \quad c_b(f_b) = 50(1 + (\frac{f_b}{50})^{\beta}),$$
$$c_c(f_c) = 50(1 + (\frac{f_b}{50})^{\beta}), \quad c_d(f_d) = 1 + (\frac{f_d}{20})^{\beta},$$
$$c_e(f_e) = 10(1 + (\frac{f_e}{100})^{\beta}).$$



 $\beta = 1$





β= 2





 $\beta = 4$

 $\beta = 3$

Example: The Anaheim, California Network

There are 461 nodes, 914 links, and 1, 406 O/D pairs in the Anaheim network.


Robustness vs. Capacity Retention Ratio for the Anaheim Network



Different Perspectives on Transportation Network Robustness: Relative Total Cost Indices

- The index is based on the two behavioral solution concepts, namely, the total cost evaluated under the U-O flow pattern, denoted by TC_{U-O}, and the S-O flow pattern, denoted by TC_{S-O}, respectively.
- The relative total cost index for a transportation network *G* with the vector of demands *d*, the vector of user link cost functions *c*, and the vector of link capacities *u* is defined as the relative total cost increase under a given uniform capacity retention ratio *γ* (*γ* ∈ (0, 1]) so that the new capacities are given by *γu*. Let *c* denote the vector of BPR user link cost functions and let *d* denote the vector of O/D pair travel demands.

We still utilize BPR functions user link cost functions c for the robustness analysis.

$$\mathcal{I}_{U-O}^{\gamma} = \mathcal{I}_{U-O}(G, c, d, \gamma, u) = \frac{TC_{U-O}^{\gamma} - TC_{U-O}}{TC_{U-O}} \times 100\%,$$

where TC_{U-O} and TC_{U-O}^{γ} are the total network costs evaluated under the U-O flow pattern with the original capacities and the remaining capacities (i.e., γu), respectively.

$$\mathcal{I}_{S-O}^{\gamma} = \mathcal{I}_{S-O}(G, c, d, \gamma, u) = \frac{TC_{S-O}^{\gamma} - TC_{S-O}}{TC_{S-O}} \times 100\%,$$

where TC_{s-o} and TC_{s-o}^{γ} are the total network costs evaluated under the S-O flow pattern with the original capacities and the remaining capacities (i.e., γu), respectively.

Example: The Sioux Falls Network



From the above figure, we can see that the Sioux-Falls network is always more robust under U-O behavior except when β is equal to 2 and the capacity retention ratio is between 0.5 and 0.9.

Example: The Anaheim Network



Ratio of I_{u-o} to I_{s-o} for the Anaheim Network under the Capacity Retention Ratio γ

Relationship Between the Price of Anarchy and the Relative Total Cost Indices

- p captures the relationship between total cost across distinct behavioral principles.
- The two relative total cost indices are focused on the degradation of network performance *within* U-O or S-O behavior.
- The relationship between the ratio of the two indices and the price of anarchy:

$$\frac{I_{S-O}^{\gamma}}{I_{U-O}^{\gamma}} = \frac{\left[TC_{S-O}^{\gamma} - TC_{S-O}\right]}{\left[TC_{U-O}^{\gamma} - TC_{U-O}\right]} \times \rho.$$

The result from the above ratio can be less than 1, greater than 1, or equal to 1, depending on the network and data.



Depiction of a Global Supply Chain



Supply Chain Disruptions

- In March 2000, a lightning bolt struck a Philips Semiconductor plant in Albuquerque, New Mexico, and created a 10-minute fire that resulted in the contamination of millions of computer chips and subsequent delaying of deliveries to its two largest customers: Finland's Nokia and Sweden's Ericsson.
- Ericsson used the Philips plant as its sole source and reported a \$400 million loss because it did not receive the chip deliveries in a timely manner whereas Nokia moved quickly to tie up spare capacity at other Philips plants and refitted some of its phones so that it could use chips from other US suppliers and from Japanese suppliers.
- Nokia managed to arrange alternative supplies and, therefore, mitigated the impact of the disruption.
- Ericsson learned a painful lesson from this disaster.



The West Coast port lockout in 2002, which resulted in a 10 day shutdown of ports in early October, typically, the busiest month. 42% of the US trade products and 52% of the imported apparel go through these ports, including Los Angeles. Estimated losses were one billion dollars per day.



SUPPLY CHAIN MANAGEMENT REVIEW · MARCH 2005

As summarized by Sheffi (2005), one of the main characteristics of disruptions in supply networks is *the seemingly unrelated consequences and vulnerabilities stemming from global connectivity.*

Supply chain disruptions may have impacts that propagate not only locally but globally and, hence, a holistic, system-wide approach to supply chain network modeling and analysis is essential in order to be able to capture the complex interactions among decision-makers.

The Multitiered Network Structure of a Supply Chain



Assumptions

- Manufacturers and retailers are multicriteria decisionmakers
- Manufacturers and retailers try to:
 - Maximize profit
 - Minimize risk
 - Individual weight is assigned to the risk level according to decision-maker's attitude towards risk.
- Nash Equilibrium is the underlying behavioral principle.

A Supply Chain Network Performance Measure

The supply chain network performance measure, \mathcal{E}^{SCN} , for a given supply chain, and expected demands: \hat{d}_k ; k = 1, 2, ..., o, is defined as follows:

$$\mathcal{E}^{SCN} \equiv \frac{\sum_{k=1}^{o} \frac{\hat{d}_{k}}{\rho_{3k}}}{o},$$

where o is the number of demand markets in the supply chain network, and \hat{d}_k and ρ_{3k} denote, respectively, the expected equilibrium demand and the equilibrium price at demand market k.

Supply Chain Robustness Measurement

Let \mathcal{E}_w denote the supply chain performance measure with random parameters fixed at a certain level as described above. Then, the supply chain network robustness measure, \mathcal{R} , is given by the following:

$$\mathcal{R}^{SCN} = \mathcal{E}^0_{SCN} - \mathcal{E}_w,$$

where \mathcal{E}_{SCN}^{0} gauges the supply chain performance based on the supply chain model, but with weights related to risks being zero.

Some of the Relevant Papers for Part II

Environmental Impact Assessment of Transportation Networks with Degradable Links in an Era of Climate Change, Nagurney, Qiang, and Nagurney, *International Journal of Sustainable Transportation* 4: (2010), pp 154-171.

Modeling of Supply Chain Risk Under Disruptions with Performance Measurement and Robustness Analysis, Qiang, Nagurney, and Dong, invited chapter in: **Managing Supply Chain Risk and Vulnerability: Tools and Methods for Supply Chain Decision Makers**, T. Wu, and J. Blackhurst, Editors (2009), Springer, 91-111. Part III

We have been focusing on network vulnerability and robustness analysis. We also have results in terms of *synergy* in the case of network integration as would occur in mergers and acquisitions.

In this framework we model the economic activities of each firm as a S-O problem on a network.

Some of the References

A System-Optimization Perspective for Supply Chain Network Integration: The Horizontal Merger Case, Nagurney, *Transportation Research E* 45: (2009), pp 1-19.

Multiproduct Supply Chain Horizontal Network Integration: Models, Theory, and Computational Results, Nagurney, Woolley, and Qiang, *International Transactions in Operational Research* 17: (2010), pp 333-349.

Mergers and Acquisitions and Supply Chain Network Synergies

Today, supply chains are more extended and complex than ever before. At the same time, the current competitive economic environment requires that firms operate efficiently, which has spurred research to determine how to utilize supply chains more effectively.

There is also a pronounced amount of merger activity. According to Thomson Financial, in the first nine months of 2007 alone, worldwide merger activity hit \$3.6 trillion, surpassing the total from all of 2006 combined.

Notable examples: KMart and Sears in the retail industry in 2004 and Federated and May in 2005, Coors and Molson in the beverage industry in 2005, and the recently proposed merger between Anheuser Busch and InBev. According to Kusstatscher and Cooper (2005) there were five major waves of of Merger & Acquisition (M &A) activity:

The First Wave: 1898-1902: an increase in horizontal mergers that resulted in many US industrial groups; The Second Wave: 1926-1939: mainly public utilities; The Third Wave: 1969-1973: diversification was the driving force; The Fourth Wave: 1983-1986: the goal was efficiency;

The Fifth Wave: 1997 until the early years of the 21st century: globalization was the motto.

In 1998, M&As reached \$2.1 trillion worldwide; in 1999, the activity exceeded \$3.3 trillion, and in 2000, almost \$3.5 was reached.

A survey of 600 executives involved in their companies' mergers and acquisitions (M&A) conducted by Accenture and the Economist Unit (see Byrne (2007)) found that less than half (45%) achieved expected cost-saving synergies.

Langabeer and Seifert (2003) determined a direct correlation between how effectively supply chains of merged firms are integrated and how successful the merger is. They concluded, based on the empirical findings of Langabeer (2003), who analyzed hundreds of mergers over the preceding decade, that

Improving Supply Chain Integration between Merging Companies is the Key to Improving the Likelihood of Post-Merger Success!

Supply Chain Prior to the Merger



Supply Chain Post-Merger



Quantifying the Synergy of the Merger

The synergy associated with the total generalized costs which captures the total costs is defined as:

$$\mathcal{S}^{\mathcal{T}\mathcal{C}} \equiv [rac{\mathcal{T}\mathcal{C}^0 - \mathcal{T}\mathcal{C}^1}{\mathcal{T}\mathcal{C}^0}] imes 100\%.$$

This framework can also be applied to teaming of humanitarian organizations in the case of humanitarian logistics operations; http://hlogistics.som.umass.edu

Humanitarian Logistics: Networks for Africa



Rockefeller Foundation Bellagio Center Conference, Bellagio, Lake Como, Italy

May 5-9, 2008

Conference Organizer: Anna Nagurney, John F. Smith Memorial Professor University of Massachusetts at Amherst



Famine in Southern Africa

Ethiopia's Food Crisis

Flooding in Kenya





Vulnerability of Humanitarian Supply Chains

Extremely poor logistic infrastructures: Modes of transportation include trucks, barges, donkeys in Afganistan, and elephants in Cambodia (Shister (2004)).

To ship the humanitarian goods to the affected area in the first 72 hours after disasters is crucial. The successful execution is not just a question of money but a difference between life and death (Van Wassenhove (2006)).

Corporations' expertise with logistics could help public response efforts for nonprofit organizations (Sheffi (2002), Samii et al.(2002)).

In the humanitarian sector, organizations are 15 to 20 years behind, as compared to the commercial arena, regarding supply chain network development (Van Wassenhove (2006)).

Supply Chains of Humanitarian Organizations A and B Prior to the Integration



Supply Chain Network after Humanitarian Organizations A and B Integrate their Chains



References - for Further Reading

Link to Network Economics course materials as well as several other related courses conducted by Nagurney on her Fulbright in Austria: http://supernet.som.umass.edu/austria_lectures/fulmain.html Overview article on Network Economics by Nagurney: http://supernet.som.umass.edu/articles/NetworkEconomics.pdf Background article on the importance of the Beckmann, McGuire, and Winsten book, *Studies in the Economics of Transportation*: http://tsap.civil.northwestern.edu/boyce_pubs/retrospective_on_beckmann.pdf

Preface to the translation of the Braess (1968) article and the translation: http://tsap.civil.northwestern.edu/bouce_pubs/preface_to.pdf http://homepage.rub.de/Dietrich.Braess/Paradox-BNW.pdf

Link to numerous articles on network modeling and applications, vulnerability and robustness analysis, as well as network synergy: http://supernet.som.umass.edu/dart.html

Link to books of interest: http://supernet.som.umass.edu/bookser.html



Supernetworks for Optimal Decision-making and Improving the Global Quality of Life

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The Virtual Center for Supernetworks at the Isenberg School of Management, under the directorship of Anna Nagurney, the John F. Smith Memorial Professor, is an interdisciplinary center, and includes the Supernetworks Laboratory for Computation and Visualization.

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