Supply Chain Network Design Under Profit Maximization and Oligopolistic Competition

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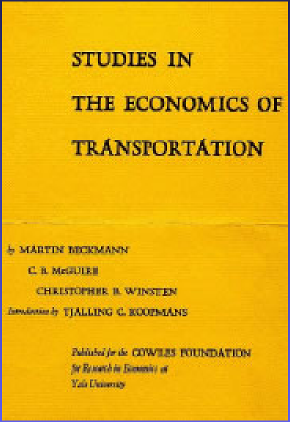
Outline

- Background and Motivation
- The Supply Chain Network Design Oligopoly Model
- Special Cases
- The Algorithm
- Numerical Examples
- Summary and Conclusions

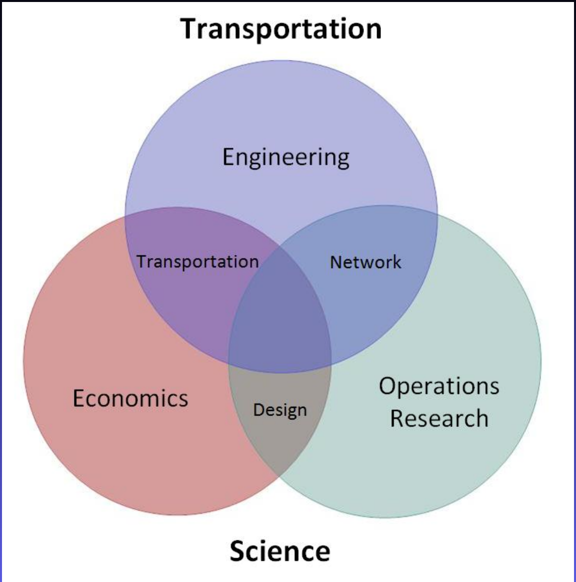
Network Design Must Capture the Behavior of Users.

This is something that we, in transportation science, are very aware of, going back to the seminal book by Beckmann, McGuire, and Winsten, *Studies in the Economics of Transportation* (1956); see also a retrospective on this book by Boyce, Mahmassani, and Nagurney (2005) in *Papers in Regional Science*.



There have been many researchers who have contributed significantly to this important topic.

I share with you some photos taken over the years and around the globe of some of them.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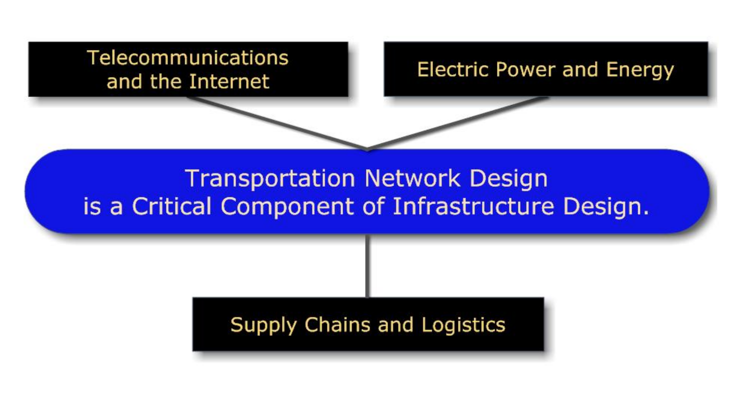
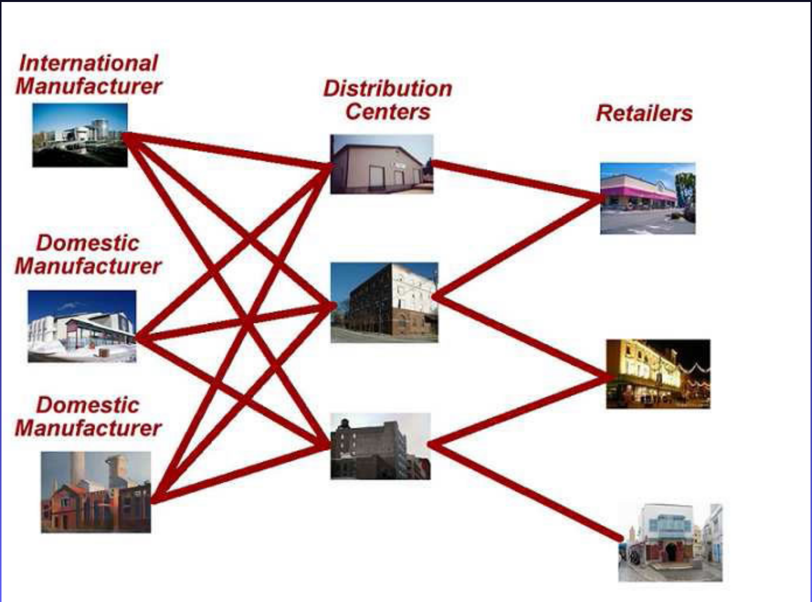

Telecommunications
and the Internet
Electric Power and Energy
Transportation Network Design
is a Critical Component of Infrastructure Design.
Supply Chains and Logistics

This presentation is based on the paper,

Supply Chain Network Design Under Profit Maximization and Oligopolistic Competition, Nagurney (2010),

Transportation Research E, *46*, 281-294, where additional background as well as references can be found.

Depiction of a Supply Chain



Some Examples of Oligopolies

- airlines
- freight carriers
- automobile manufacturers
- oil companies
- beer / beverage companies
- wireless communications
- certain financial institutions.

The Supply Chain Network Design Model

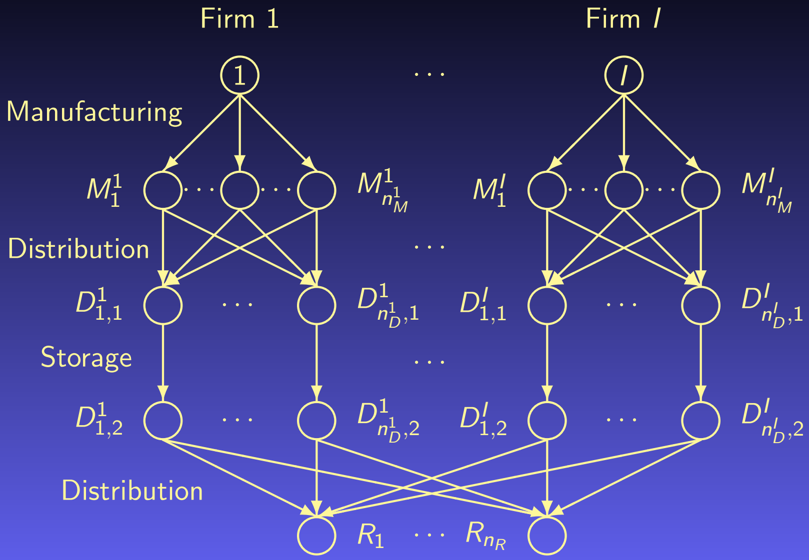


Figure 1: The Initial Supply Chain Network Topology of the Oligopoly

Some Notation

Firm i; $i = 1, \ldots, I$, is considering n^i_M manufacturing facilities / plants; n^i_D distribution centers, and serves the same n_R retail outlets / demand markets.

L^0_i denotes the set of directed links representing the economic activities associated with firm i; $i = 1, \ldots, I$ and $n_{L^0_i}$ denotes the number of links in L^0_i with n_{L^0} denoting the total number of links in the initial network L^0, where $L^0 \equiv \cup_{i=1,I} L^0_i$.

$G^0 = [N^0, L^0]$ denotes the graph consisting of the set of nodes N^0 and the set of links L^0 in Figure 1.

The Links

The possible manufacturing links from the top-tiered nodes i; $i = 1, \ldots, I$, are connected to the manufacturing nodes of the respective firm i, denoted by: $M_1^i, \ldots, M_{n_M^i}^i$.

The possible shipment links from the manufacturing nodes are connected to the distribution center nodes of each firm i; $i = 1, \ldots, I$, denoted by $D_{1,1}^i, \ldots, D_{n_D{}^i,1}^i$.

The possible links joining nodes $D_{1,1}^i, \ldots, D_{n_D^i,1}^i$ with nodes $D_{1,2}^i, \ldots, D_{n_D^i,2}^i$ for $i = 1, \ldots, I$ correspond to the storage links.

Finally, there are possible shipment links joining the nodes $D_{1,2}^i, \ldots, D_{n_D^i,2}^i$ for $i = 1, \ldots, I$ with the demand market nodes: $R_1, \ldots, R_{n_R}$.

The model can handle any prospective supply chain network topology provided that there is a top-tiered node to represent each firm and bottom-tiered nodes to represent the demand markets with a sequence of directed links, corresponding to at least one path, joining each top-tiered node with each bottom-tiered node.

The solution of the complete model will identify which links have positive capacities and, hence, should be retained in the final supply chain network design.

The Demand and Path Flow Notation

d_{R_k} denotes the demand for the product at demand market R_k; $k = 1, \ldots, n_R$. x_p denotes the nonnegative flow of the product on path p joining (origin) node i; $i = 1, \ldots, I$ with a (destination) demand market node.

The following conservation of flow equations must hold:

$$\sum_{p \in P^0_{R_k}} x_p = d_{R_k}, \quad k = 1, \ldots, n_R, \tag{1}$$

where $P^0_{R_k}$ is the set of paths connecting the (origin) nodes i; $i = 1, \ldots, I$ with (destination) demand market R_k. In particular, we have that $P^0_{R_k} = \cup_{i=1,\ldots,I} P^0_{R^i_k}$, where $P^0_{R^i_k}$ is the set of paths from origin node i to demand market k as in Figure 1.

The Demand Price Functions

We assume that there is a demand price function associated with the product at each demand market. We denote the demand price at demand market R_k by ρ_{R_k} and assume, as given, the demand price functions:

$$\rho_{R_k} = \rho_{R_k}(d), \quad k = 1, \ldots, n_R, \tag{2}$$

where d is the n_R-dimensional vector of demands at the demand markets.

The demand price functions are assumed to be continuous, continuously differentiable, and monotone decreasing. Note that the consumers at each demand market are indifferent as to which firm produced the homogeneous product.

The Path and Link Flows as Variables

f_a denotes the flow of the product on link a. The following conservation of flow equations must also be satisfied:

$$f_a = \sum_{p \in P^0} x_p \delta_{ap}, \quad \forall a \in L^0, \tag{3}$$

where $\delta_{ap} = 1$ if link a is contained in path p and $\delta_{ap} = 0$, otherwise. Here P^0 is the set of *all* paths in Figure 1, that is, $P^0 = \cup_{k=1,\ldots,n_R} P^0_{R_k}$. There are n_{P^0} paths in the network in Figure 1. P^0_i denotes the set of all paths from firm i to all the demand markets for $i = 1, \ldots, I$. There are $n_{P^0_i}$ paths from the firm i node to the demand markets.

The path flows must be nonnegative, that is,

$$x_p \geq 0, \quad \forall p \in P^0. \tag{4}$$

The Link Capacity Variables

$u_a, a \in L^0$ denotes the design capacity of link a, where we must have that

$$f_a \leq u_a, \quad \forall a \in L^0, \tag{5}$$

or, in view of (3)

$$\sum_{p \in P^0} x_p \delta_{ap} \leq u_a, \quad \forall a \in L^0. \tag{6}$$

The product flow on each link is bounded by the capacity (which is a strategic decision variable) on the link.

The Total Cost Function Expressions

The total operational cost on a link, $\hat{c}_a$, be it a manufacturing / production link, a shipment / distribution link, or a storage link a, is assumed, in general, to be a function of the flows of the product on all the links, that is,

$$\hat{c}_a = \hat{c}_a(f), \quad \forall a \in L^0, \tag{7}$$

where f is the vector of all the link flows.

In addition, the total design cost, $\hat{\pi}_a$, associated with each link a, is a function of the design capacity on the link, that is,

$$\hat{\pi}_a = \hat{\pi}_a(u_a), \quad \forall a \in L^0. \tag{8}$$

The Vectors of Strategy Variables and the Profit Functions of the Firms

X_i denotes the vector of strategy variables associated with firm i; $i = 1, \ldots, I$, where X_i is the vector of path flows associated with firm i, and the vector of link capacities, $X_i \equiv \{\{\{x_p\}|p \in P_i^0\}; \{\{u_a\}|a \in L_i^0\}\} \in R_+^{n_{P_i^0}+n_{L_i^0}}$. X is then the vector of all the firms' strategies, $X \equiv \{\{X_i\}|i = 1, \ldots, I\}$.

The profit function U_i of firm i; $i = 1, \ldots, I$, is the difference between the firm's revenue and its total costs:

$$U_i = \sum_{k=1}^{n_R} \rho_{R_k}(d) \sum_{p \in P^0_{R^i_k}} x_p - \sum_{a \in L_i^0} \hat{c}_a(f) - \sum_{a \in L_i^0} \hat{\pi}_a(u_a). \qquad (9)$$

In view of (1) – (9):

Supply Chain Design Network Equilibrium

We consider the *oligopolistic market mechanism in which the firms select their supply chain network link capacities and product path flows in a noncooperative manner*, each one trying to maximize its own profit.

Definition 1: Supply Chain Network Design Cournot-Nash Equilibrium

A path flow and design capacity pattern $X^* \in \mathcal{K}^0 = \prod_{i=1}^{I} \mathcal{K}_i^0$ *is said to constitute a supply chain network design Cournot-Nash equilibrium if for each firm* i*;* $i = 1, \ldots, I$*:*

$$U_i(X_i^*, \hat{X}_i^*) \geq U_i(X_i, \hat{X}_i^*), \quad \forall X_i \in \mathcal{K}_i^0, \tag{11}$$

where $\hat{X}_i^* \equiv (X_1^*, \ldots, X_{i-1}^*, X_{i+1}^*, \ldots, X_I^*)$ *and*
$\mathcal{K}_i^0 \equiv \{X_i | X_i \in R_+^{n_{P_i^0}+n_{L_i^0}}$*, and (6) is satisfied*$\}$.

The Variational Inequality Formulation

Theorem 1

Assume that for each firm i; $i = 1, \dots, I$, the profit function $U_i(X)$ is concave with respect to the variables in X_i, and is continuously differentiable. Then $X^* \in \mathcal{K}^0$ is a supply chain network design Cournot-Nash equilibrium according to Definition 1 if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^{I} \langle \nabla_{X_i} U_i(X^*)^T, X_i - X_i^* \rangle \geq 0, \quad \forall X \in \mathcal{K}^0, \tag{12}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in the corresponding Euclidean space and $\nabla_{X_i} U_i(X)$ denotes the gradient of $U_i(X)$ with respect to X_i.

The solution of variational inequality (12) is equivalent to the solution of variational inequality: determine $(x^*, u^*, \lambda^*) \in \mathcal{K}^1$ *satisfying:*

$$\sum_{i=1}^{I} \sum_{k=1}^{n_R} \sum_{p \in P^0_{R^i_k}} \left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} + \sum_{a \in L^0} \lambda^*_a \delta_{ap} - \rho_{R_k}(x^*) - \sum_{l=1}^{n_R} \frac{\partial \rho_{R_l}(x^*)}{\partial d_{R_k}} \sum_{p \in P^0_{R^i_k}} x^*_p \right]$$

$$\times [x_p - x^*_p] + \sum_{a \in L^0} \left[\frac{\partial \hat{\pi}_a(u^*_a)}{\partial u_a} - \lambda^*_a \right] \times [u_a - u^*_a]$$

$$+ \sum_{a \in L^0} \left[u^*_a - \sum_{p \in P^0} x^*_p \delta_{ap} \right] \times [\lambda_a - \lambda^*_a] \geq 0, \quad \forall (x, u, \lambda) \in \mathcal{K}^1, \quad (13)$$

where $\mathcal{K}^1 \equiv \{(x, u, \lambda) | x \in R^{n_{P^0}}_+; u \in R^{n_{L^0}}_+; \lambda \in R^{n_{L^0}}_+\}$ *and* $\frac{\partial \hat{C}_p(x)}{\partial x_p} \equiv \sum_{b \in L^0_i} \sum_{a \in L^0_i} \frac{\partial \hat{c}_b(f)}{\partial f_a} \delta_{ap}$ *for paths* $p \in P^0_i$; $i = 1, \ldots, I$.

Standard Variational Inequality Form

Variational inequality (13) can be put into standard form as below: determine $X^* \in \mathcal{K}$ such that

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{14}$$

where $\mathcal{K}$ is closed and convex and $F(X)$ is a continuous function from $\mathcal{K}$ to R^n. Indeed, we can define $\mathcal{K} \equiv \mathcal{K}^1$ and let $F(X)$ be the vector with $n_{P^0} + 2n_{L^0}$ components given by the specific terms preceding the first multiplication sign in (13), the second in (13), and the third in (13).

Relationship to a Spatial Oligopoly Problem

Corollary 1

Assume that there are I firms in the supply chain network design oligopoly model and that each firm is considering a single manufacturing plant and a single distribution center. Assume also that the distribution costs from each manufacturing plant to the distribution center and the storage costs are all equal to zero. Then the resulting model is a generalization of the spatial oligopoly model of Dafermos and Nagurney (1987) with the inclusion of design capacities as strategic variables and whose underlying initial network structure is given in Figure 2.

Proof: Follows from Dafermos and Nagurney (1987) and Nagurney (1993).

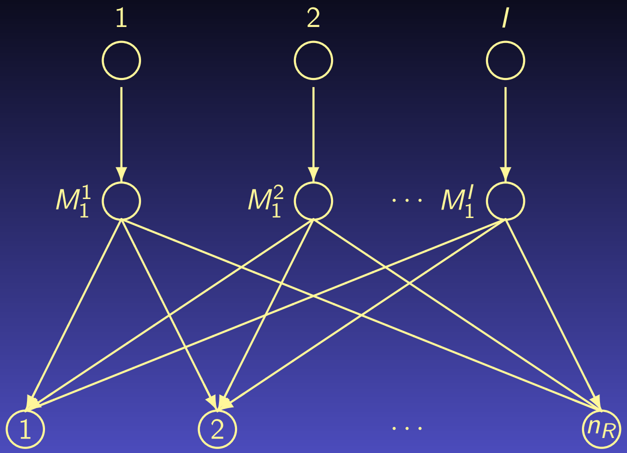


Figure 2: The Initial Topology of the Spatial Oligopoly Design Problem

Relationship to the Classical Oligopoly

It is also interesting to relate the supply chain network oligopoly model to the classical Cournot (1838) oligopoly model.

Corollary 2

Assume that there is a single manufacturing plant associated with each firm in the above supply chain network design model and a single distribution center. Assume also that there is a single demand market. Assume that the manufacturing cost of each firm depends only upon its own output. Then, if the storage and distribution cost functions are all identically equal to zero the above design model collapses to an extension of the classical oligopoly model in quantity variables and with capacity design variables. Furthermore, if $I = 2$, one then obtains a generalization of the classical duopoly model.

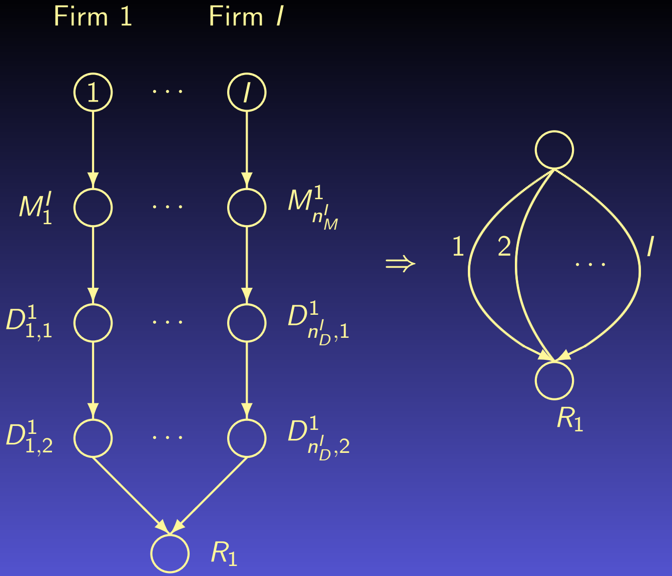


Figure 3: The Initial Topology of the "Classical" Oligopoly Design Problem

With the variational inequality formulation of the competitive supply chain network design problem we can:

- consider problems in which there are nonlinearities as well as asymmetries in the underlying functions, for which an optimization formulation would no longer suffice and
- obtain further insights into competitive oligopolistic equilibrium problems that characterize a variety of industries with the generalization of the inclusion of explicit design variables.

As is well-known, intuition in the case of equilibrium problems, as opposed to optimization problems, may be easily confounded (as in the Braess paradox; see Braess (1968), Nagurney and Boyce (2005), and Braess, Nagurney, and Wakolbinger (2005)).

An oligopolistic supply chain network design framework that allows for the computation of solutions can enable firms to investigate their optimal strategies in the presence of competition.

In the *special case where there is only a single firm and the demands at the demand markets are fixed and known*, then this model collapses to a system-optimization supply chain network design model developed earlier by Nagurney (2009).

Other related research of ours includes multicriteria decision-making in supply chain networks design, as in the case of *sustainability and environmental concerns*, and supply chain network design in the case of critical needs products (medicines, vaccines, etc.), *uncertainty on the demand side*, as well as *redesign issues*.

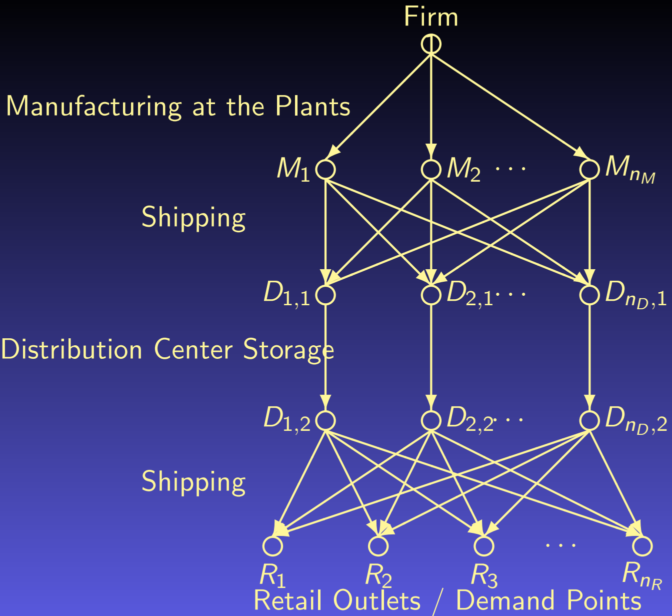
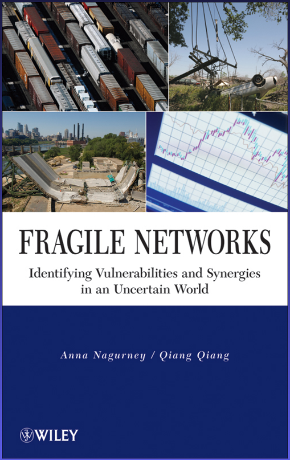


Figure 4: Initial Topology in the Case of System-Optimization

Some other issues related to network design are explored in our *Fragile Networks* book.



The Algorithm – the Euler Method

At an iteration τ of the Euler method (see Dupuis and Nagurney (1993), Nagurney and Zhang (1996)) one computes:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau} F(X^{\tau})), \tag{15}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem: determine $X^* \in \mathcal{K}$ such that

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{16}$$

where $\langle \cdot, \cdot \rangle$ is the inner product in n-dimensional Euclidean space, $X \in R^n$, and $F(X)$ is an n-dimensional function from $\mathcal{K}$ to R^n, with $F(X)$ being continuous (see also (14)).

The sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \to 0$, as $\tau \to \infty$.

Explicit Formulae for (15) to the Supply Chain Network Design Variational Inequality (13)

$$x_p^{\tau+1} = \max\{0, x_p^\tau + a_\tau(\rho_{R_k}(x^\tau) - \sum_{l=1}^{n_R} \frac{\partial \rho_l(x^\tau)}{\partial d_{R_k}} \sum_{p \in P^0_{R_k^i}} x_p^\tau - \frac{\partial \hat{C}_p(x^\tau)}{\partial x_p}$$

$$- \sum_{a \in L^0} \lambda_a^\tau \delta_{ap})\}, \forall i, \forall k, \forall p \in P^0_{R_k^i}; \quad (17)$$

$$u_a^{\tau+1} = \max\{0, u_a^\tau + a_\tau(\lambda_a^\tau - \frac{\partial \hat{\pi}_a(u_a^\tau)}{\partial u_a})\}, \quad \forall a \in L^0; \quad (18)$$

$$\lambda_a^{\tau+1} = \max\{0, \lambda_a^\tau + a_\tau(\sum_{p \in P^0} x_p^\tau \delta_{ap} - u_a^\tau)\}, \quad \forall a \in L^0. \quad (19)$$

Numerical Examples

The Firms

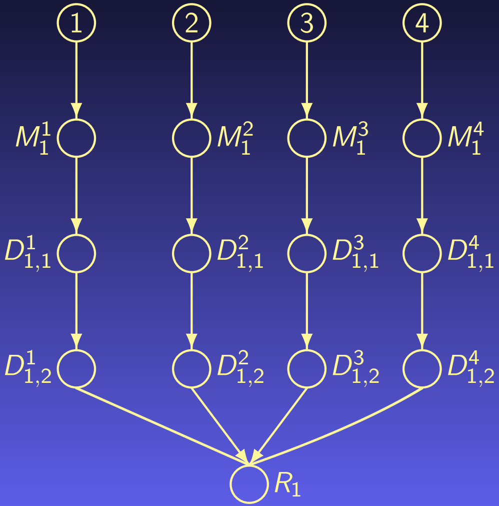


Figure 5: Initial Topology for Examples 1 and 2

Example 1

For simplicity, we let all the total operational cost functions on the links be equal and given by:

$$\hat{c}_a(f) = 2f_a^2 + f_a, \quad \forall a \in L_i^0; \; i = 1, 2, 3, 4. \tag{20}$$

The total design cost functions on the links were:

$$\hat{\pi}_a(u_a) = 5u_a^2 + u_a, \quad \forall a \in L_i^0; \; i = 1, 2, 3, 4. \tag{21}$$

The demand price function at the single demand market was:

$$\rho_{R_1}(d) = -d_{R_1} + 200. \tag{22}$$

The paths were: p_1, p_2, p_3, and p_4 corresponding to firm 1 through firm 4, respectively, with each path originating in its top-most firm node and ending in the demand market node as in Figure 5.

The Euler method converged to the equilibrium solution:

$$x^*_{p_1} = x^*_{p_2} = x^*_{p_3} = x^*_{p_4} = 3.15,$$

$$u^*_a = 3.15, \quad \forall a \in L^0, \quad f^*_a = 3.15, \quad \forall a \in L^0,$$
$$\lambda^*_a = 32.46, \quad \forall a \in L^0.$$

The demand was 12.60 and the demand market price was $\rho_{R_1} = 187.40$. Each firm earned a profit of 287.88.

The supply chain network design, since all links had positive capacities (as well as positive flows), had the topology in Figure 5.

Example 2

Example 2 was constructed from Example 1 and had the same data except that the demand price at the demand market was greatly reduced to:

$$\rho_{R_1}(d) = -d_{R_1} + 5. \tag{23}$$

The Euler method converged to the solution with all path flows, link flows, and design capacities equal to 0.00 with the Lagrange multipliers all equal to 1.05. Hence, not one of the firms enters into this market and none of these firms produces the product.

Therefore, the final supply chain network design is, in this case, the null set!

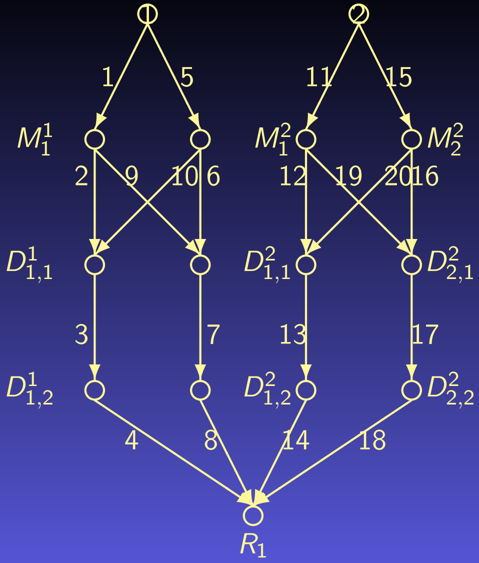


Figure 6: Initial Supply Chain Topology for Examples 3 and 4

Example 3

Example 3 had the initial supply chain network topology as in Figure 6, where we also define the links. Specifically, there were two firms considering two manufacturing plants each, two distribution centers each, and considering serving a single demand market.

This example represents the following scenario.

Let the first firm be located in the US, for example, where there are higher manufacturing costs and also higher costs associated with both constructing the manufacturing plants and the distribution centers; see: $\hat{c}_1$, $\hat{c}_5$, and $\hat{\pi}_1$, $\hat{\pi}_5$.

The second firm, on the other hand, is located outside the US where there are lower manufacturing costs and storage costs as well as associated design costs for the facilities; see $\hat{c}_{11}$, $\hat{c}_{15}$, and $\hat{\pi}_{11}$, $\hat{\pi}_{15}$. However, the demand market is in the US and, hence, the second firm faces higher transportation costs to deliver the product to the demand market, as can be seen from the function data in Tables 1 and 2; see $\hat{c}_{14}$ and $\hat{c}_{18}$ versus $\hat{c}_4$ and $\hat{c}_8$.

The total cost data, along with the computed link flows, capacities, and Lagrange multipliers, are reported in Tables 1 and 2.

The demand price function was:

$$\rho_{R_1}(d) = -d_{R_1} + 300. \tag{24}$$

Table 1: Total Cost Functions and Solution for Example 3

Link a	$\hat{c}_a(f)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	λ_a^*
1	$2f_1^2 + 2f_1$	$5u_1^2 + u_1$	7.28	7.28	74.05
2	$f_2^2 + 2f_2$	$.5u_2^2 + u_2$	3.98	3.98	4.98
3	$2.5f_3^2 + f_3$	$5u_3^2 + 2u_3$	7.54	7.54	76.23
4	$f_4^2 + f_4$	$.5u_4^2 + u_4$	7.54	7.54	8.54
5	$3f_5^2 + 2f_5$	$6u_5^2 + u_5$	5.73	5.73	70.05
6	$.5f_6^2 + f_6$	$.5u_6^2 + u_6$	2.17	2.17	3.17
7	$1.5f_7^2 + f_7$	$10u_7^2 + u_7$	5.47	5.47	110.07
8	$f_8^2 + f_8$	$.5u_8^2 + u_8$	5.47	5.47	6.48
9	$.5f_9^2 + f_9$	$.5u_9^2 + u_9$	3.30	3.30	4.30
10	$f_{10}^2 + f_{10}$	$.5u_{10}^2 + u_{10}$	3.56	3.56	4.56
11	$.5f_{11}^2 + f_{11}$	$4u_{11}^2 + u_{11}$	7.15	7.15	58.23
12	$f_{12}^2 + f_{12}$	$.5u_{12}^2 + u_{12}$	2.98	2.98	3.98
13	$.5f_{13}^2 + f_{13}$	$2.5u_{13}^2 + u_{13}$	7.14	7.14	36.72
14	$4f_{14}^2 + f_{14}$	$5u_{14}^2 + 5u_{14}$	7.14	7.14	76.19
15	$f_{15}^2 + f_{15}$	$3u_{15}^2 + u_{15}$	8.12	8.12	49.76

Table 2: Total Cost Functions and Solution for Example 3 (continued)

Link a	$\hat{c}_a(f)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	λ_a^*
16	$f_{16}^2 + f_{16}$	$.5u_{16}^2 + u_{16}$	3.96	3.96	4.96
17	$.5f_{17}^2 + f_{17}$	$2.5u_{17}^2 + u_{17}$	8.13	8.13	41.66
18	$3.5f_{18}^2 + f_{18}$	$4u_{18}^2 + 2u_{18}$	8.13	8.13	66.89
19	$f_{19}^2 + f_{19}$	$.5u_{19}^2 + u_{19}$	4.17	4.17	5.17
20	$.5f_{20}^2 + f_{20}$	$.5u_{20}^2 + u_{20}$	4.16	4.16	5.16

We also provide the computed equilibrium path flows. There were four paths for each firm and we label the paths as follows (please refer to Figure 6): for firm 1:

$$p_1 = (1, 2, 3, 4),\ p_2 = (1, 9, 7, 8),\ p_3 = (5, 6, 7, 8),\ p_4 = (5, 10, 3, 4),$$

for firm 2:

$$p_5 = (11, 12, 13, 14),\ p_6 = (11, 19, 17, 18),\ p_7 = (15, 16, 17, 18),$$
$$p_8 = (15, 20, 13, 14).$$

The computed equilibrium path flow pattern was:

$$x^*_{p_1} = 3.98,\ x^*_{p_2} = 3.30,\ x^*_{p_3} = 2.17,\ x^*_{p_4} = 3.56,$$
$$x^*_{p_5} = 2.98,\ x^*_{p_6} = 4.17,\ x^*_{p_7} = 3.96,\ x^*_{p_8} = 4.16.$$

Example 4

Example 4 also had the initial supply chain network topology as in Figure 6. Example 4 had the same data as Example 3 except we reduced the capacity design costs associated with the shipment links from the second firm to the demand market as in Tables 3 and 4; see $\hat{\pi}_{14}$ and $\hat{\pi}_{18}$.

The total cost data, along with the computed link flows, capacities, and Lagrange multipliers, are reported in Tables 3 and 4.

Table 3: Total Cost Functions and Solution for Example 4

Link a	$\hat{c}_a(f)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	λ_a^*
1	$2f_1^2 + 2f_1$	$5u_1^2 + u_1$	7.17	7.17	72.87
2	$f_2^2 + 2f_2$	$.5u_2^2 + u_2$	3.92	3.92	4.92
3	$2.5f_3^2 + f_3$	$5u_3^2 + 2u_3$	7.42	7.42	75.01
4	$f_4^2 + f_4$	$.5u_4^2 + u_4$	7.42	7.42	8.42
5	$3f_5^2 + 2f_5$	$6u_5^2 + u_5$	5.64	5.64	68.93
6	$.5f_6^2 + f_6$	$.5u_6^2 + u_6$	2.13	2.13	3.14
7	$1.5f_7^2 + f_7$	$10u_7^2 + u_7$	5.39	5.39	108.30
8	$f_8^2 + f_8$	$.5u_8^2 + u_8$	5.39	5.39	6.39
9	$.5f_9^2 + f_9$	$.5u_9^2 + u_9$	3.25	3.25	4.25
10	$f_{10}^2 + f_{10}$	$.5u_{10}^2 + u_{10}$	3.51	3.51	4.50
11	$.5f_{11}^2 + f_{11}$	$4u_{11}^2 + u_{11}$	9.23	9.23	74.74
12	$f_{12}^2 + f_{12}$	$.5u_{12}^2 + u_{12}$	4.04	4.04	5.04
13	$.5f_{13}^2 + f_{13}$	$2.5u_{13}^2 + u_{13}$	9.64	9.64	49.24
14	$4f_{14}^2 + f_{14}$	$.5u_{14}^2 + u_{14}$	9.64	9.64	10.64
15	$f_{15}^2 + f_{15}$	$3u_{15}^2 + u_{15}$	10.49	10.49	63.90

Table 4: Total Cost Functions and Solution for Example 4 (continued)

Link a	$\hat{c}_a(f)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	λ_a^*
16	$f_{16}^2 + f_{16}$	$.5u_{16}^2 + u_{16}$	4.89	4.89	5.89
17	$.5f_{17}^2 + f_{17}$	$2.5u_{17}^2 + u_{17}$	10.08	10.08	51.44
18	$3.5f_{18}^2 + f_{18}$	$.5u_{18}^2 + u_{18}$	10.08	10.08	11.08
19	$f_{19}^2 + f_{19}$	$.5u_{19}^2 + u_{19}$	5.19	5.19	6.19
20	$.5f_{20}^2 + f_{20}$	$.5u_{20}^2 + u_{20}$	5.60	5.60	6.60

The demand price was: 267.47 and the total profit earned by both firms was: 4,493.29.

The computed equilibrium path flows were:

$$x_{p_1}^* = 3.92, \quad x_{p_2}^* = 3.25, \quad x_{p_3}^* = 2.13, \quad x_{p_4}^* = 3.51,$$

$$x_{p_5}^* = 4.04, \quad x_{p_6}^* = 5.19, \quad x_{p_7}^* = 4.89, \quad x_{p_8}^* = 5.60.$$

One can see that the second firm manufactured a greater volume of the product than the first firm and provided more of the product to the consumers at the demand market. In this example the firm that had lower overseas costs was more competitive once the shipment costs were further reduced.

The final supply chain network topology under the optimal design for both Examples 3 and 4 remained as in Figure 6.

Example 5

Example 5 had the same data as Example 4 but now we added a second demand market with the initial supply chain network topology being as depicted in Figure 7. The links are labeled on that figure. We assumed that the second demand market was located in the US with the shipment costs set to reflect this scenario.

The demand price function for the first demand market remained as in Examples 3 and 4. The demand price function for the new demand market was:

$$\rho_{R_2}(d) = -2d_{R_2} + 500. \tag{25}$$

The remainder of the input data and the computed solution are given in Tables 5 and 6.

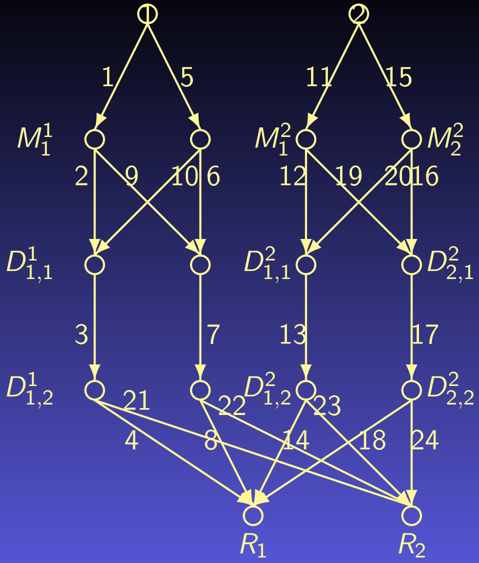


Figure 7: Initial Supply Chain Topology for Example 5

Table 5: Total Cost Functions and Solution for Example 5

Link a	$\hat{c}_a(f)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	λ_a^*
1	$2f_1^2 + 2f_1$	$5u_1^2 + u_1$	10.29	10.29	104.12
2	$f_2^2 + 2f_2$	$.5u_2^2 + u_2$	5.75	5.75	6.75
3	$2.5f_3^2 + f_3$	$5u_3^2 + 2u_3$	10.83	10.83	109.09
4	$f_4^2 + f_4$	$.5u_4^2 + u_4$	0.00	0.00	0.04
5	$3f_5^2 + 2f_5$	$6u_5^2 + u_5$	8.12	8.12	98.65
6	$.5f_6^2 + f_6$	$.5u_6^2 + u_6$	3.04	3.04	4.04
7	$1.5f_7^2 + f_7$	$10u_7^2 + u_7$	7.58	7.58	151.81
8	$f_8^2 + f_8$	$.5u_8^2 + u_8$	0.00	0.00	0.56
9	$.5f_9^2 + f_9$	$.5u_9^2 + u_9$	4.54	4.54	5.54
10	$f_{10}^2 + f_{10}$	$.5u_{10}^2 + u_{10}$	5.08	5.08	6.08
11	$.5f_{11}^2 + f_{11}$	$4u_{11}^2 + u_{11}$	15.84	15.84	127.61
12	$f_{12}^2 + f_{12}$	$.5u_{12}^2 + u_{12}$	6.98	6.98	7.98
13	$.5f_{13}^2 + f_{13}$	$2.5u_{13}^2 + u_{13}$	16.66	16.66	84.34
14	$4f_{14}^2 + f_{14}$	$.5u_{14}^2 + u_{14}$	2.33	2.33	3.33
15	$f_{15}^2 + f_{15}$	$3u_{15}^2 + u_{15}$	18.02	18.02	109.00

Table 6: Total Cost Functions and Solution for Example 5 (continued)

Link a	$\hat{c}_a(f)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	λ_a^*
16	$f_{16}^2 + f_{16}$	$.5u_{16}^2 + u_{16}$	8.34	8.34	9.34
17	$.5f_{17}^2 + f_{17}$	$2.5u_{17}^2 + u_{17}$	17.20	17.20	87.04
18	$3.5f_{18}^2 + f_{18}$	$.5u_{18}^2 + u_{18}$	1.51	1.51	2.51
19	$f_{19}^2 + f_{19}$	$.5u_{19}^2 + u_{19}$	8.86	8.86	9.86
20	$.5f_{20}^2 + f_{20}$	$.5u_{20}^2 + u_{20}$	9.68	9.68	10.68
21	$.5f_{21}^2 + f_{21}$	$2.5u_{21}^2 + f_{21}$	10.83	10.83	23.66
22	$f_{22}^2 + f_{22}$	$.5u_{22}^2 + f_{22}$	7.58	7.58	17.16
23	$2f_{23}^2 + f_{23}$	$.5u_{23}^2 + f_{23}$	14.33	14.33	15.33
24	$1.5f_{24}^2 + f_{22}$	$.5u_{24}^2 + f_{24}$	15.70	15.70	16.69

For completeness, we also provide the computed path flows. We retained the numbering and the definitions of the first eight paths for the first demand market as in Examples 3 and 4 (but associated now with Figure 7). The new paths for the first firm to the second demand market are:

$$p_9 = (1, 2, 3, 21),\ p_{10} = (1, 9, 7, 22),\ p_{11} = (5, 6, 7, 22),$$

$$p_{12} = (5, 10, 3, 21),$$

and for the second firm to the second demand market:

$$p_{13} = (11, 12, 13, 23),\ p_{14} = (11, 19, 17, 24),\ p_{15} = (15, 16, 17, 24),$$

$$p_{16} = (15, 20, 13, 23).$$

The computed equilibrium path flow solution:

$$x_{p_1}^* = 0.00,\ x_{p_2}^* = 0.00,\ x_{p_3}^* = 0.00,\ x_{p_4}^* = 0.00,$$

$$x_{p_5}^* = 0.49,\ x_{p_6}^* = 0.72,\ x_{p_7}^* = 0.79,\ x_{p_8}^* = 1.84.$$

$$x_{p_9}^* = 5.75,\ x_{p_{10}}^* = 4.54,\ x_{p_{11}}^* = 3.04,\ x_{p_{12}}^* = 5.08,$$

$$x_{p_{13}}^* = 6.49,\ x_{p_{14}}^* = 8.15,\ x_{p_{15}}^* = 7.55,\ x_{p_{16}}^* = 7.84.$$

It is very interesting to note that the first firm no longer provides any of the product to the first demand market and, in fact, all its associated path flows to the first demand market are now equal to zero as are the link flows on links 4 and 8. In addition, the associated design capacities on links 4 and 8 are also equal to zero.

For Example 5, the optimal supply chain network design is as in Figure 8.

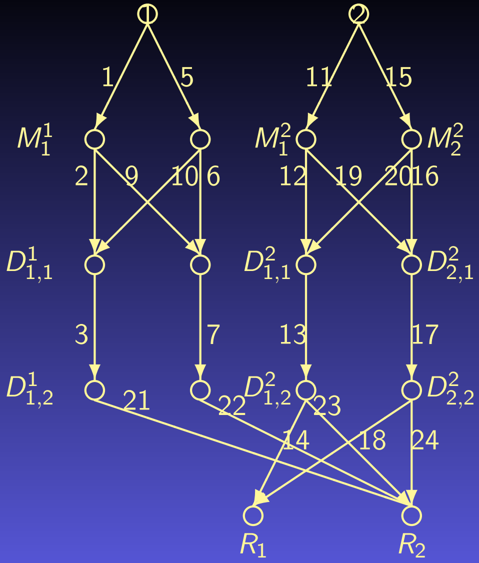


Figure 8: The Optimal Supply Chain Network Topology for Example 5

Summary and Conclusions

- We developed a multimarket supply chain network design model in an oligopolistic setting.
- The firms select not only their optimal product flows but also the capacities associated with the various supply chain activities of production / manufacturing, storage, and distribution / shipment.
- We formulated the supply chain network design problem as a variational inequality problem and then proposed an algorithm, which fully exploits the underlying structure of these network problems, and yields closed form expressions at each iterative step.
- The network formalism proposed here, which captures competition on the production, distribution, as well as demand market dimensions, enables the investigation of economic issues surrounding supply chain network design.

Summary and Conclusions

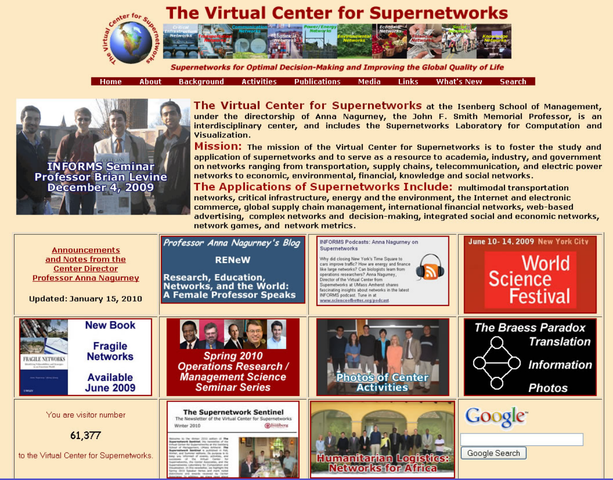
- It allows for the identification (and generalization) of special cases of oligopolistic market equilibrium problems, spatial, as well as aspatial, through the underlying network structure, that have appeared in the literature.
- The network structure allows one to visualize graphically the proposed supply chain network topology and the final optimal / equilibrium design.
- This paper illustrates the power of computational methodologies to explore issues regarding competing firms and network design.
- We demonstrate that such problems can be formulated and solved without using discrete variables but, rather only continuous variables.

Future Research

The research can be extended in several directions:

- One can construct multiproduct versions of the oligopolistic supply chain network design model developed here, and one can also consider more explicitly international/global issues with the incorporation of exchange rates and risk.
- It would also be worthwhile to formulate supply chain network redesign oligopolistic models.
- Further computational experimentation as well as theoretical developments and empirical applications would also be of value.

Thank You!



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