An Integrated Disaster Relief Supply Chain Network Model With Time Targets and Demand Uncertainty

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The original model is based on the following paper:

Nagurney, A., Masoumi, A.H., and Yu, M. (2014) in *Regional Science Matters: Studies Dedicated to Walter Isard*, Nijkamp, P., Rose, A., and Kourtit, K., Editors, Springer, Heidelberg, Germany, in press.

Outlook

- Background and Introduction
- Integrated Network Model for Disaster Relief Supply Chains
- Numerical Example and Results
- Summary and Conclusion

Background: Natural Disasters

The number of natural disasters and the sizes of the populations affected by such events have been growing (Schultz, Koenig, and Noji (1996) and Nagurney and Qiang (2009)).

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Scientists are warning that we can expect more frequent extreme weather events in the future.

For instance, tropical cyclones which include hurricanes in the US are expected to be stronger as a result of global warming (Sheppard (2011) and Borenstein (2012)).

21st Century's Deadliest Natural Disasters

10. Great East Japan "Triple Disaster", 2011. Death toll: 18,400.

9. Gujarat earthquake, India and Pakistan, 2001. Death toll: 19,700.

8. European heat wave, 2003. Death toll: 40,000.

7. Bam earthquake, Iran, 2003. Death toll 26,000 – 43,000.

6. Russian heat wave, 2010. Death toll: 56,000.

5 .Sichuan earthquake, China, 2008. Death toll: 69,200.

4. Kashmir earthquake, Pakistan, India, and Afghanistan, 2005. Death toll: 75,000 – 86,000.

3. Cyclone Nargis, Myanmar, 2008. Death toll: 146,000.

2. Indian Ocean Tsunami, Indonesia, Sri Lanka, and India, 2004. Death toll: 230,000.

1. Haiti earthquake, 2010. Death toll: 316,000.

Natural Disasters with 100+ fatalities in the US in 21st Century

- Hurricane Rita: LA, TX - September 2005. Death toll: 120.
- Hurricane Ivan: TX, FL - September 2004. Death toll: 124.
- Hurricane Sandy: Eastern US - 2011. Death toll: 147.
- Tornado: MO - May 2011. Death toll: 160.
- Tornado: AL, TN, MS, GA, AR, VA - April 2011. Death toll: 346.
- Hurricane Katrina: FL, LA, MS, AZ - August 2005. Death toll: 1,836.

Background: Facts

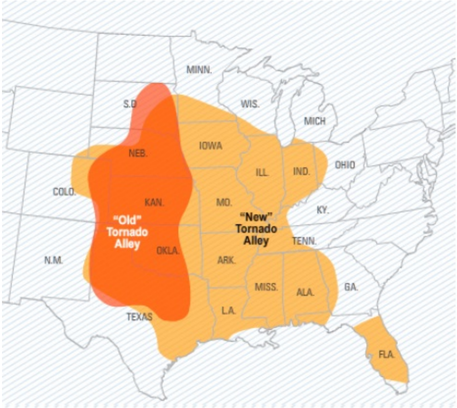

Figure 1: "Tornado Alley" grows wider (CoreLogic, Storm Prediction Center)

Background: Facts

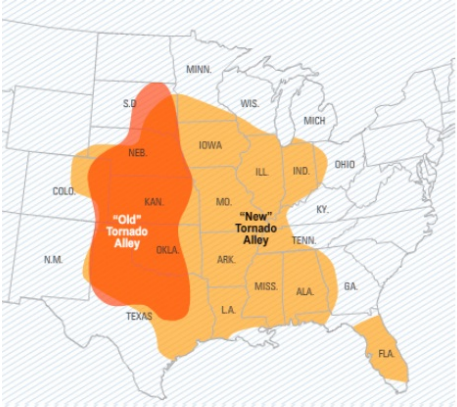

Figure 1: "Tornado Alley" grows wider (CoreLogic, Storm Prediction Center)

The traditional boundaries of Tornado Alley which has centered on the Plains states of Texas, Oklahoma, Kansas, Nebraska and South Dakota should be expanded to include much of the Midwest and Deep South, because the frequency and severity of tornadoes in those areas is much more widespread than commonly believed (Core Logic, April 2012).

Background: Facts

Tornado risk extends across much of the eastern half of the nation, and at least 26 states have some area facing extreme tornado risk. Kansas, Florida, Iowa, Louisiana and Mississippi were the top five states for the most tornadoes from 1980 to 2009.

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The worst outbreak was in the Southeast in April 2011, when 321 people died and $7.3 billion in insured losses occurred; the most expensive tornado outbreak ever recorded (USA Today, April 2012).

Background: Facts

In the US alone, the average number of disasters per year that exceeded a cost of 1 billion dollars in damages increased from 3.6 in the 2001-2005 period to 5.8 in 2006-2010 (NOAA).

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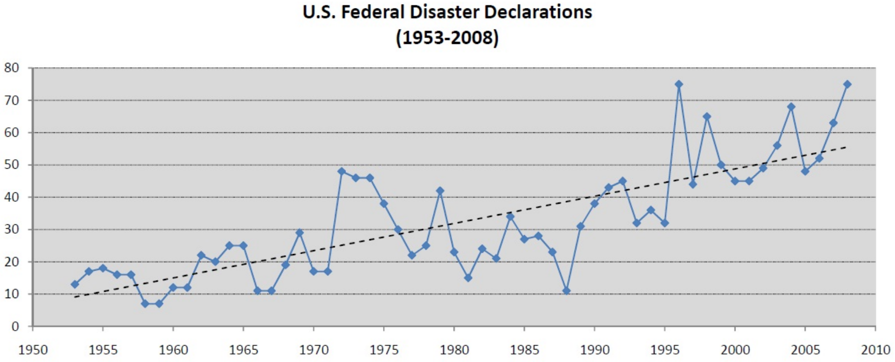

U.S. Federal Disaster Declarations, 1953-2008 with trend line (Source Data: FEMA)

Background: Natural Disasters

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Hence, being prepared against potential disasters leads to reduced vulnerability and a lower number of fatalities.

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Hence, being prepared against potential disasters leads to reduced vulnerability and a lower number of fatalities.

"During a natural disaster, one has only two options: **to become a victim, or to become a responder**" (Alvendia-Quero (2012)).

Background: Disaster Relief Supply Chains

The *complexity* of disaster relief supply chains originates from *several inherent factors*:

- **Large demands** for relief products,
- Level of **uncertainty**,
- **Irregularities** in the size, the timing, and the location of relief product demand patterns,
- Disaster-driven supply chains are typically **incident-responsive**, and
- Develop new networks of relationships within **days** or even **hours**, and have very **short life-cycles**.

Background: Criticality of *Time*

TIME plays a substantial role in the construction and operation of disaster relief networks.

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FEMA's key benchmarks in response and recovery:

- To meet the survivors initial demands within 72 hours,
- To restore basic community functionality within 60 days, and
- To return to as normal of a situation within 5 years.

Relevant Literature

- Okuyama, Y. (2003). Economics of natural disasters: A critical review. Paper presented at the 50th North American Meeting, Regional Science Association International Philadelphia, Pennsylvania.
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Mathematical Model

We propose an integrated supply chain network model for disaster relief.

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Our mathematical framework is of system-optimization type where the organization aims to satisfy the uncertain demands subject to the minimization of total operational costs while the sequences of activities leading to the ultimate delivery of the relief good are targeted to be completed within a certain time.

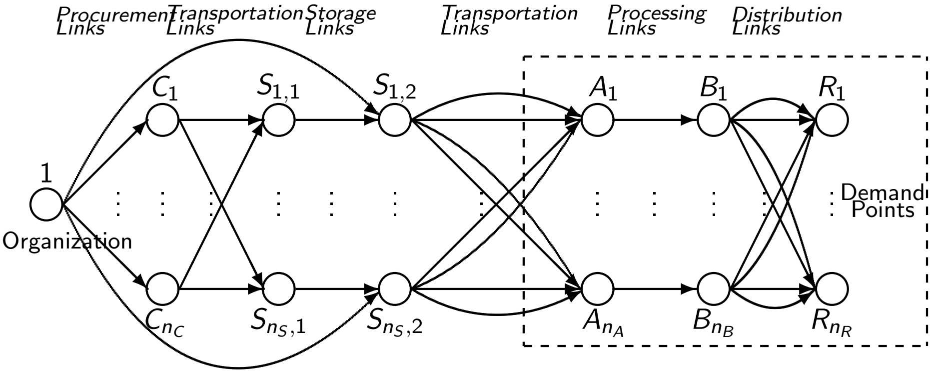

Figure 2: Network Topology of the Integrated Disaster Relief Supply Chain

$G = [N, L]$: supply chain network graph,

N: set of nodes,

L: set of links (arcs).

Mathematical Model

Notations:

$\mathcal{P}_k$: set of paths connecting the origin (node 1) to demand point k,
$\mathcal{P}$: set of all paths joining the origin node to the destination nodes, and
n_p: total number of paths in the supply chain.

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Total Operational Cost Function

$$\hat{c}_a(f_a) = f_a \times c_a(f_a), \qquad \forall a \in L, \tag{1}$$

where:

f_a: flow of the disaster relief product on link a, and
$c_a(f_a)$: unit operational cost function on link a.

Probability Distribution of Demand

$$P_k(D_k) = P_k(d_k \leq D_k) = \int_0^{D_k} \mathcal{F}_k(t)dt, \qquad k = 1, \ldots, n_R, \qquad (2)$$

where:

d_k: actual value of demand at point k,

P_k: probability distribution function of demand at point k,

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Demand Shortage and Surplus

$$\Delta_k^- \equiv \max\{0, d_k - v_k\}, \qquad k = 1, \ldots, n_R, \qquad (3a)$$

$$\Delta_k^+ \equiv \max\{0, v_k - d_k\}, \qquad k = 1, \ldots, n_R, \qquad (3b)$$

where:
v_k: "projected demand" for the disaster relief item at point k.

Expected Values of Shortage and Surplus

$$E(\Delta_k^-) = \int_{v_k}^{\infty} (t - v_k)\mathcal{F}_k(t)dt, \qquad k = 1, \ldots, n_R, \tag{4a}$$

$$E(\Delta_k^+) = \int_{0}^{v_k} (v_k - t)\mathcal{F}_k(t)dt, \qquad k = 1, \ldots, n_R. \tag{4b}$$

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$$E(\Delta_k^+) = \int_{0}^{v_k} (v_k - t)\mathcal{F}_k(t)dt, \qquad k = 1, \ldots, n_R. \qquad (4b)$$

Expected Penalty due to Shortage and Surplus

$$E(\lambda_k^- \Delta_k^- \; + \; \lambda_k^+ \Delta_k^+) = \lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+), \qquad k = 1, \ldots, n_R, \quad (5)$$

where:

λ_k^-: unit penalty associated with the shortage of the relief item at point k,

λ_k^+: unit penalty associated with the surplus of the relief item at point k.

Path Flows

$$x_p \geq 0, \qquad \forall p \in \mathcal{P}. \tag{6}$$

x_p: flow of the disaster relief goods on path p joining node 1 with a demand node.

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Relationship between Path Flows and Projected Demand

$$v_k \equiv \sum_{p \in \mathcal{P}_k} x_p, \qquad k = 1, \ldots, n_R. \tag{7}$$

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Relationship between Link Flows and Path Flows

$$f_a = \sum_{p \in \mathcal{P}} x_p \; \delta_{ap}, \qquad \forall a \in L. \tag{8}$$

δ_{ap} is equal to 1 if link a is contained in path p and is 0, otherwise.

Capturing Time Aspect in Formulation

Completion Time

$$\tau_a(f_a) = g_a f_a + h_a, \qquad \forall a \in L, \tag{9}$$

where:

τ_a: completion time of the activity on link a. Note: $h_a \geq 0$, and $g_a \geq 0$.

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Completion Time on Paths

$$\tau_p = \sum_{a \in L} \tau_a(f_a)\delta_{ap} = \sum_{a \in L} (g_a f_a + h_a)\delta_{ap}, \qquad \forall p \in \mathcal{P}. \tag{10}$$

Where:

τ_p: completion time of the sequence of activities on path p.

$$h_p = \sum_{a \in L} h_a \delta_{ap}, \qquad \forall p \in \mathcal{P}. \tag{11}$$

Hence,

$$\tau_p = h_p + \sum_{a \in L} g_a f_a \delta_{ap}, \qquad \forall p \in \mathcal{P}. \tag{12}$$

h_p: sum of the uncongested terms h_as on path p.

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Time Targets

$$\tau_p \leq T_k, \qquad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R, \tag{13}$$

T_k: target for the completion time of the activities on paths corresponding to demand point k determined by the organization's decision-maker.

$$\sum_{a \in L} g_a f_a \delta_{ap} \leq T_k - h_p, \qquad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R. \tag{14}$$

$$T_{kp} = T_k - h_p, \qquad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R. \tag{15}$$

T_{kp}: target time for demand point k with respect to path p.

$$\sum_{a \in L} g_a f_a \delta_{ap} \leq T_{kp}, \qquad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R. \tag{16}$$

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$$\sum_{a \in L} g_a f_a \delta_{ap} \leq T_{kp}, \qquad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R. \tag{16}$$

Late Delivery (Deviations) with Respect to Target Times

$$\sum_{a \in L} g_a f_a \delta_{ap} - z_p \leq T_{kp}, \qquad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R. \tag{17}$$

z_p: amount of deviation with respect to target time T_{kp} corresponding to the "late" delivery of product to point k on path p.

Non-negativity

$$z_p \geq 0, \qquad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R. \tag{18}$$

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Time Constraint

$$\sum_{q \in \mathcal{P}} \sum_{a \in L} g_a x_q \delta_{aq} \delta_{ap} - z_p \leq T_{kp}, \qquad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R. \tag{19}$$

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$$\hat{C}_p(x) = x_p \times C_p(x) = x_p \times \sum_{a \in L} c_a(f_a) \delta_{ap}, \qquad \forall p \in \mathcal{P}, \tag{20}$$

$\hat{C}_p(x)$: total operational cost function on path p.
C_p: unit operational cost on path p.

$\gamma_k(z)$: **tardiness penalty function** corresponding to demand point k.

Optimization Formulation

$$\text{Minimize} \sum_{p \in \mathcal{P}} \hat{C}_p(x) \; + \; \sum_{k=1}^{n_R} (\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)) + \sum_{k=1}^{n_R} \gamma_k(z), \quad (21)$$

subject to: constraints (6), (18), and (19).

$\gamma_k(z)$: **tardiness penalty function** corresponding to demand point k.

Optimization Formulation

$$\text{Minimize} \sum_{p\in\mathcal{P}} \hat{C}_p(x) + \sum_{k=1}^{n_R}(\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)) + \sum_{k=1}^{n_R} \gamma_k(z), \quad (21)$$

subject to: constraints (6), (18), and (19).

Feasible Set

$$K = \{(x, z, \omega) | x \in R_+^{n_p}, z \in R_+^{n_p}, \text{ and } \omega \in R_+^{n_p}\}, \quad (23)$$

where x is the vector of path flows, z is the vector of time deviations on paths, and ω is the vector of Lagrange multipliers corresponding to (19).

Variational Inequality Formulation

The optimization problem (21), subject to its constraints (6), (18), and (19), is equivalent to the variational inequality problem: determine the vector of optimal path flows, the vector of optimal path time deviations, and the vector of optimal Lagrange multipliers $(x^*, z^*, \omega^*) \in K$*, such that:*

$$\sum_{k=1}^{n_R} \sum_{p \in \mathcal{P}_k} \left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} + \lambda_k^+ P_k(\sum_{q \in \mathcal{P}_k} x_q^*) - \lambda_k^- (1 - P_k(\sum_{q \in \mathcal{P}_k} x_q^*)) \right.$$

$$\left. + \sum_{q \in \mathcal{P}} \sum_{a \in L} \omega_q^* g_a \delta_{aq} \delta_{ap} \right] \times [x_p - x_p^*] + \sum_{k=1}^{n_R} \sum_{p \in \mathcal{P}_k} \left[\frac{\partial \gamma_k(z^*)}{\partial z_p} - \omega_p^* \right] \times [z_p - z_p^*]$$

$$+ \sum_{k=1}^{n_R} \sum_{p \in \mathcal{P}_k} \left[T_{kp} + z_p^* - \sum_{q \in \mathcal{P}} \sum_{a \in L} g_a x_q^* \delta_{aq} \delta_{ap} \right] \times [\omega_p - \omega_p^*] \geq 0, \qquad \forall (x, z, \omega) \in K, \tag{24}$$

where

$$\frac{\partial \hat{C}_p(x)}{\partial x_p} \equiv \sum_{a \in L} \frac{\partial \hat{c}_a(f_a)}{\partial f_a} \delta_{ap}, \quad \forall p \in \mathcal{P}_k; \;\; k = 1, \ldots, n_R. \tag{25}$$

Solution Method and Numerical Examples

Solution Algorithm

Explicit Formulae for the Euler Method Applied to the Variational Inequality (24)

At iteration $\tau + 1$:

$$x_p^{\tau+1} = \max\{0, x_p^\tau + a_\tau(\lambda_k^-(1 - P_k(\sum_{q\in\mathcal{P}_k} x_q^\tau)) - \lambda_k^+ P_k(\sum_{q\in\mathcal{P}_k} x_q^\tau) - \frac{\partial \hat{C}_p(x^\tau)}{\partial x_p}$$

$$-\sum_{q\in\mathcal{P}}\sum_{a\in L}\omega_q^\tau g_a \delta_{aq}\delta_{ap})\}, \qquad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R, \tag{31}$$

$$z_p^{\tau+1} = \max\{0, z_p^\tau + a_\tau(\omega_p^\tau - \frac{\partial \gamma_k(z^\tau)}{\partial z_p})\}, \qquad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R, \text{ and} \tag{32}$$

$$\omega_p^{\tau+1} = \max\{0, \omega_p^\tau + a_\tau(\sum_{q\in\mathcal{P}}\sum_{a\in L} g_a x_q^\tau \delta_{aq}\delta_{ap} - T_{kp} - z_p^\tau)\},$$

$$\forall p \in \mathcal{P}_k; \qquad k = 1, \ldots, n_R. \tag{33}$$

A Large Scale Numerical Example

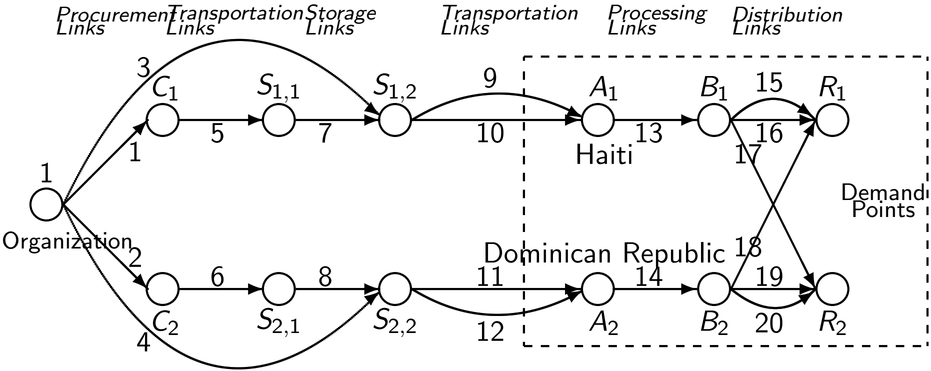

Figure 3: Network Topology of the Larger Disaster Relief Supply Chain Numerical Example

Numerical Example Data

R_1 is assumed to have a higher demand for relief goods due to a larger population and its potential higher vulnerability to the disasters as compared to R_2.

The demand for the relief item at R_1 and R_2 is assumed to follow a **uniform distribution** on the intervals [25,40] and [10,20], respectively.

Unit shortage and surplus penalties at demand points R_1 and R_2:

$$\lambda^-_{R_1} = 10,000, \ \lambda^+_{R_1} = 100,$$

$$\lambda^-_{R_2} = 7,500, \ \lambda^+_{R_2} = 150.$$

Target times of delivery at demand points:

$$T_{R_1} = 72, \ T_{R_2} = 70.$$

Tardiness penalty functions at demand points:

$$\gamma_{R_1}(z) = 3(\sum_{p \in \mathcal{P}_{R_1}} z_p^2), \quad \gamma_{R_2}(z) = 3(\sum_{p \in \mathcal{P}_{R_2}} z_p^2).$$

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The Euler method (cf.(31)–(33)) for the solution of variational inequality (24) was implemented in FORTRAN on a PC. We set the sequence as $\{a^\tau\} = .1(1, \frac{1}{2}, \frac{1}{2}, \ldots)$, and the convergence tolerance was 10^{-6}.

Table 2: Functions and the Optimal Flows on Links in the Numerical Example

Link	$\hat{c}_a(f_a)$	$\tau_a(f_a)$	f_a^*
1	$3f_1^2 + 2f_1$	0	19.22
2	$2f_2^2 + 2.5f_2$	0	20.02
3	$5f_3^2 + 4f_3$	$3f_3 + 3$	0.00
4	$4.5f_4^2 + 3f_4$	$4f_4 + 2$	0.00
5	$f_5^2 + 2f_5$	0	19.22
6	$f_6^2 + .5f_6$	0	20.02
7	$2.5f_7^2 + 3f_7$	0	19.22
8	$3.5f_8^2 + 2f_8$	0	20.02
9	$7f_9^2 + 5f_9$	$2f_9 + 2$	19.22
10	$4f_{10}^2 + 6f_{10}$	$10f_{10} + 6$	0.00
11	$2.5f_{11}^2 + 4f_{11}$	$7.5f_{11} + 5$	0.23
12	$4.5f_{12}^2 + 5f_{12}$	$1.5f_{12} + 1.5$	19.79
13	$2f_{13}^2 + 4f_{13}$	$2f_{13} + 2$	19.22
14	$f_{14}^2 + 3f_{14}$	$1.5f_{14} + 1$	20.02
15	$4f_{15}^2 + 5f_{15}$	$3f_{15} + 3$	13.95
16	$2.5f_{16}^2 + 2f_{16}$	$5f_{16} + 4$	5.28
17	$3f_{17}^2 + 4f_{17}$	$6.5f_{17} + 3$	0.00
18	$4f_{18}^2 + 4f_{18}$	$7f_{18} + 5$	6.85
19	$3f_{19}^2 + 3f_{19}$	$4f_{19} + 5$	5.68
20	$3.5f_{20}^2 + 5f_{20}$	$3.5f_{20} + 4$	7.49

Table 3: Path Definitions, Target Times, Optimal Path Flows, Time Deviations, and Lagrange Multipliers for the Numerical Example

	Path Definition	T_{kp}	x_p^*	z_p^*	ω_p^*
$\mathcal{P}_{R_1}$: Set of Paths Corresponding to Demand Point R_1	$p_1 = (1,5,7,9,13,15)$	65	13.95	53.66	321.99
	$p_2 = (1,5,7,9,13,16)$	64	5.28	39.23	235.39
	$p_3 = (1,5,7,10,13,15)$	61	0.00	19.32	115.90
	$p_4 = (1,5,7,10,13,16)$	60	0.00	4.83	28.99
	$p_5 = (2,6,8,11,14,18)$	61	0.06	18.67	112.03
	$p_6 = (2,6,8,12,14,18)$	64.5	6.79	43.12	258.75
	$p_7 = (3,9,13,15)$	62	0.00	56.66	339.99
	$p_8 = (3,9,13,16)$	61	0.00	42.23	253.39
	$p_9 = (3,10,13,15)$	58	0.00	22.34	134.05
	$p_{10} = (3,10,13,16)$	57	0.00	7.84	47.03
	$p_{11} = (4,11,14,18)$	59	0.00	20.71	124.24
	$p_{12} = (4,12,14,18)$	62.5	0.00	45.24	271.46
$\mathcal{P}_{R_2}$: Set of Paths Corresponding to Demand Point R_2	$p_{13} = (1,5,7,9,13,17)$	63	0.00	13.87	83.25
	$p_{14} = (1,5,7,10,13,17)$	59	0.00	0.00	0.00
	$p_{15} = (2,6,8,11,14,19)$	59	0.13	0.00	0.00
	$p_{16} = (2,6,8,11,14,20)$	60	0.04	0.00	0.00
	$p_{17} = (2,6,8,12,14,19)$	62.5	5.55	19.91	119.44
	$p_{18} = (2,6,8,12,14,20)$	63.5	7.45	22.40	134.43
	$p_{19} = (3,9,13,17)$	60	0.00	16.90	101.41
	$p_{20} = (3,10,13,17)$	56	0.00	0.00	0.00
	$p_{21} = (4,11,14,19)$	57	0.00	0.00	0.00
	$p_{22} = (4,11,14,20)$	58	0.00	0.00	0.00
	$p_{23} = (4,12,14,19)$	60.5	0.00	21.96	131.77
	$p_{24} = (4,12,14,20)$	61.5	0.00	24.48	146.85

Numerical Example: A Variant

We assumed that the organization will now procure the items locally and, hence, the time functions associated with the direct procurement links 3 and 4 are now greatly reduced. The remainder of the input data remains as in the previous example.

As in Table 4, now both the storage links for pre-positioning (links 7 and 8) and for post-disaster procurement (links 3 and 4) have positive flows.

Table 4: Numerical Example Variant - Optimal Link Flows

Link	$\hat{c}_a(f_a)$	$\tau_a(f_a)$	f_a^*
1	$3f_1^2 + 2f_1$	0	12.02
2	$2f_2^2 + 2.5f_2$	0	11.21
3	$5f_3^2 + 4f_3$	$.1f_3 + 1$	7.35
4	$4.5f_4^2 + 3f_4$	$.1f_4 + 1$	8.88
5	$f_5^2 + 2f_5$	0	12.02
6	$f_6^2 + .5f_6$	0	11.21
7	$2.5f_7^2 + 3f_7$	0	12.02
8	$3.5f_8^2 + 2f_8$	0	11.21
9	$7f_9^2 + 5f_9$	$2f_9 + 2$	19.37
10	$4f_{10}^2 + 6f_{10}$	$10f_{10} + 6$	0.00
11	$2.5f_{11}^2 + 4f_{11}$	$7.5f_{11} + 5$	0.24
12	$4.5f_{12}^2 + 5f_{12}$	$1.5f_{12} + 1.5$	19.86
13	$2f_{13}^2 + 4f_{13}$	$2f_{13} + 2$	19.37
14	$f_{14}^2 + 3f_{14}$	$1.5f_{14} + 1$	20.10
15	$4f_{15}^2 + 5f_{15}$	$3f_{15} + 3$	14.04
16	$2.5f_{16}^2 + 2f_{16}$	$5f_{16} + 4$	5.33
17	$3f_{17}^2 + 4f_{17}$	$6.5f_{17} + 3$	0.00
18	$4f_{18}^2 + 4f_{18}$	$7f_{18} + 5$	6.84
19	$3f_{19}^2 + 3f_{19}$	$4f_{19} + 5$	5.72
20	$3.5f_{20}^2 + 5f_{20}$	$3.5f_{20} + 4$	7.53

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A network optimization model was developed for the supply chain management of a disaster relief (humanitarian) organization in charge of procurement and distribution of relief items to a geographic region prone to natural disasters. The model:

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- includes penalties associated with shortages/surpluses at the demand points with respect to the uncertain demand, and
- enables prioritizing the demand points based on the population, geographic location, etc., by assigning different time targets.

Thank You!

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