

# Competition for Blood Donations: A Nash Equilibrium Network Framework

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# Outline

- 1 Background and Motivation
- 2 Literature Review
- 3 The Competitive Network Model
- 4 The Algorithm and Case Study
- 5 Conclusions

# Background

- The blood banking industry is unique. The supply of the product is solely dependent on **donations from individuals**.
- On average, **13.6 million** whole blood and red blood cells are collected in the United States per year.
- The American Red Cross reports that the number of donors in the US in a year is approximately **6.8 million**.



# Motivation

- An estimated **38% of the US population** is eligible to donate blood at any given time. However, less than 10% of that eligible population actually donates blood each year. The percentage is lower in some countries such as Britain and New Zealand.
- The different blood service organizations have to **compete for this limited donor pool** in order to meet the demand.



# Motivation

- There has been a rise in the competition among the blood service organizations in recruiting and retaining donors.
- Donors in parts of the US have the option of donating to organizations such as the American Red Cross, America's Blood Center member organizations, or to local community blood banks and hospitals.
- Examples from industry: Blood Centers of the Pacific vs BloodSource in Sonoma County in 2011, Suncoast Communities Blood Bank vs Florida Blood Services in Sarasota, Florida in 2011.

# Motivation

## How to motivate donors?

- There are several operational aspects of the blood collection centers that can help motivate and retain donors.
- Factors: **satisfaction from the blood donation process, convenience, location of facilities, wait times, treatment by staff** of the organization collecting blood. These factors can be aggregated and termed as the **quality of services** offered by the blood service organizations.



- Literature emphasizes **donor satisfaction and service quality** as factors impacting donation decisions all over the world (Gillespie and Hillyer (2002), Schreiber et al. (2006), Cimarolli (2012), Al-Zubaidi and Al-Asousi (2012), Jain, Doshit, and Joshi (2015), Perera et al. (2015), Finck et al. (2016), Craig et al. (2016)).
- **Some optimization and game theory models** on the blood banking industry and healthcare include works by Cohen and Pierskalla (1975), Stewart (1992), Janssen and Mendys-Kamphorst (2004), Pierskalla (2005), Masoumi, Yu, and Nagurney (2012), Nagurney et al. (2013), Duan and Liao (2014), Osorio, Brailsford, and Smith (2015), Masoumi, Yu, and Nagurney (2017).

# The Competitive Network Model for Blood Donations

- There are  $m$  blood service organizations responsible for collection of blood, testing, processing, and distribution to hospitals and other medical facilities. A typical blood service organization is denoted by  $i$ .
- There are  $n$  regions in which blood collection can take place. A typical collection region is denoted by  $j$ .

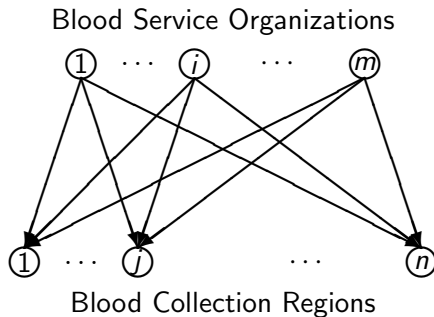


Figure: The Network Structure of the Game Theory Model for Blood Donations



# The Competitive Network Model for Blood Donations

- In this game theory model the blood service organizations compete for blood donations.
- The blood service organizations have, as their strategic variables, the **quality of services** that they provide donors at their collection sites in the regions.

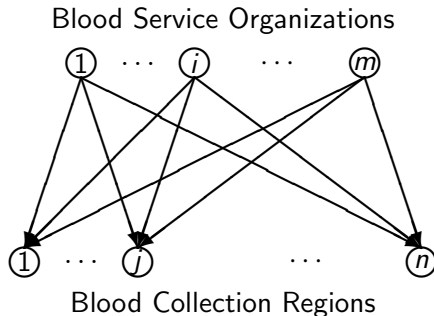


Figure: The Network Structure of the Game Theory Model for Blood Donations

# The Competitive Network Model for Blood Donations

## Quality constraint

There is a non-negative lower bound and a positive upper bound on the quality of service,  $Q_{ij}$ , that  $i$  provides in region  $j$  such that:

$$\underline{Q}_{ij} \leq Q_{ij} \leq \bar{Q}_{ij}, \quad j = 1, \dots, n. \quad (1)$$

## Cost of collection

Each blood service organization  $i$  incurs a total cost  $\hat{c}_{ij}$  associated with collecting blood in region  $j$  given by:

$$\hat{c}_{ij} = \hat{c}_{ij}(Q), \quad j = 1, \dots, n, \quad (2)$$

where  $\hat{c}_{ij}$  is assumed to be convex and continuously differentiable for all  $i, j$ .

## Monetized utility

Each blood organization,  $i$ , enjoys a utility associated with the service given by:

$$\omega_i \sum_{j=1}^n \gamma_{ij} Q_{ij}, \quad (3)$$

where the  $\omega_i$  and the  $\gamma_{ij}$ s;  $j = 1, \dots, n$ , take on positive values.

# The Competitive Network Model for Blood Donations

## Blood donations

Each blood service organization,  $i$ , receives a volume of blood donations in region  $j$ , denoted by  $P_{ij}$ ;  $j = 1, \dots, n$ , where

$$P_{ij} = P_{ij}(Q), \quad (4)$$

where each  $P_{ij}$  is assumed to be concave and continuously differentiable.

# The Competitive Network Model for Blood Donations

## Revenue

Each blood service organization,  $i$ , achieves revenue that is associated with its blood collection activities over the time horizon, given by:

$$\pi_i \sum_{j=1}^n P_{ij}(Q), \quad (5)$$

where  $\pi_i$  is an average price for blood (typically, measured in pints) for blood service organization  $i$ ;  $i = 1, \dots, m$ .

## Optimization Problem

Each blood service organization,  $i$ , seeks to maximize its transaction utility,  $U_i$ . Hence, the optimization problem is as follows:

$$\text{Maximize } U_i = \pi_i \sum_{j=1}^n P_{ij}(Q) + \omega_i \sum_{j=1}^n \gamma_{ij} Q_{ij} - \sum_{j=1}^n \hat{c}_{ij}(Q) \quad (6)$$

subject to (1).

# The Competitive Network Model for Blood Donations

## Definition 1: Nash Equilibrium for Blood Donations

A quality service level pattern  $Q^* \in K$  is said to constitute a Nash Equilibrium in blood donations if for each blood service organization  $i; i = 1, \dots, m$ ,

$$U_i(Q_i^*, \hat{Q}_i^*) \geq U_i(Q_i, \hat{Q}_i^*), \quad \forall Q_i \in K^i, \quad (7)$$

where

$$\hat{Q}_i^* \equiv (Q_1^*, \dots, Q_{i-1}^*, Q_{i+1}^*, \dots, Q_m^*). \quad (8)$$

- According to (7), a Nash Equilibrium is established if no blood service organization can improve upon its transaction utility by altering its quality service levels, given that the other organizations have decided on their quality service levels.

## Theorem 1: Variational Inequality Formulation of the Nash Equilibrium for Blood Donations

A quality service level pattern  $Q^* \in K$  is a Nash Equilibrium according to Definition 1 if and only if it satisfies the variational inequality problem:

$$-\sum_{i=1}^m \sum_{j=1}^n \frac{\partial U_i(Q^*)}{\partial Q_{ij}} \times (Q_{ij} - Q_{ij}^*) \geq 0, \quad \forall Q \in K \quad (9)$$

or, equivalently, the variational inequality:

$$\sum_{i=1}^m \sum_{j=1}^n \left[ \sum_{k=1}^n \frac{\partial \hat{c}_{ik}(Q^*)}{\partial Q_{ij}} - \omega_i \gamma_{ij} - \pi_i \sum_{k=1}^n \frac{\partial P_{ik}(Q^*)}{\partial Q_{ij}} \right] \times [Q_{ij} - Q_{ij}^*] \geq 0, \quad \forall Q \in K. \quad (10)$$



# The Competitive Network Model for Blood Donations

We can put the variational inequality formulations of the Nash Equilibrium problem into standard variational inequality form (see Nagurney (1999)), that is: determine  $X^* \in \mathcal{K} \subset R^N$ , such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \quad (11)$$

where  $F$  is a given continuous function from  $\mathcal{K}$  to  $R^N$  and  $\mathcal{K}$  is a closed and convex set.

## Existence

Existence of a solution  $Q^*$  to variational inequality (9) and also (10) is guaranteed from the standard theory of variational inequalities (cf. Nagurney (1999)) since the function  $F(X)$  that enters the variational inequality is continuous and the feasible set  $K$  is compact.

## Uniqueness

If  $F(X)$  is strictly monotone, that is:

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle > 0, \quad \forall X^1, X^2 \in \mathcal{K}, X^1 \neq X^2, \quad (12)$$

then the equilibrium solution  $X^*$  and, hence,  $Q^*$  is unique.

# The Algorithm

An iteration  $\tau + 1$  of the Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993), is:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau}F(X^{\tau})), \quad (13)$$

The Euler method, the sequence  $\{a_{\tau}\}$  must satisfy:  $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$ ,  $a_{\tau} > 0$ ,  $a_{\tau} \rightarrow 0$ , as  $\tau \rightarrow \infty$ .

# Explicit Formula for the Euler Method Applied to the Blood Donation Game Theory Model

Closed form expression for the quality service levels  $i = 1, \dots, m; j = 1, \dots, n$ , at iteration  $\tau + 1$ :

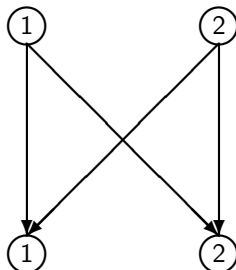
$$Q_{ij}^{\tau+1} = \max\{\underline{Q}_{ij}, \min\{\bar{Q}_{ij}, Q_{ij}^{\tau} + a_{\tau}(\pi_i \sum_{k=1}^n \frac{\partial P_{ik}(Q^{\tau})}{\partial Q_{ij}} + \omega_i \gamma_{ij} - \sum_{k=1}^n \frac{\partial \hat{c}_{ik}(Q^{\tau})}{\partial Q_{ij}})\}\} \quad (14)$$

# Case Study: American Red Cross and United Blood Services

- The American Red Cross (cf. Arizona Blood Services Region (2016)) issued a call for donations.
- Low supply of blood due to seasonal colds and flu and the devastating impact of Hurricane Matthew.
- On October 8, 2016, Hurricane Matthew made landfall that affected such states as Florida, Georgia, and the Carolinas, and disrupted blood donations in many locations in the Southeast of the US.
- We focus on Tucson, Arizona, where the American Red Cross has held recent blood drives at multiple locations and where there are also competitors for blood, including the United Blood Services.

# Case Study: American Red Cross and United Blood Services Example 1

Blood Service Organizations



Blood Collection Regions

We consider a month of collection of whole blood cells. According to Meyer (2017), Executive Vice President of the American Red Cross, productive Red Cross sites collect, on the average, 700-840 whole blood units a month.

# Case Study: American Red Cross and United Blood Services Example 1

The blood donation functions for the American Red Cross (organization 1) are:

$$P_{11}(Q) = 10Q_{11} - Q_{21} - Q_{22} + 20 + 10 + 100$$

$$P_{12}(Q) = 12Q_{12} - Q_{21} - 2Q_{22} + 20 + 15 + 100.$$

The blood donation functions for the United Blood Services (organization 2) are:

$$P_{21}(Q) = 11Q_{21} - Q_{11} - Q_{12} + 28 + 15 + 80$$

$$P_{22}(Q) = 12Q_{22} - Q_{11} - Q_{12} + 28 + 27 + 80.$$

The utility function components of the transaction utilities of these blood service organizations are:

$$\omega_1 = 9, \quad \gamma_{11} = 8, \quad \gamma_{12} = 9, \quad \omega_2 = 10, \quad \gamma_{21} = 9, \quad \gamma_{22} = 10.$$

# Case Study: American Red Cross and United Blood Services Example 1

The total costs of operating the blood collection sites over the time horizon, which must cover costs of employees, supplies, and energy, and providing the level of quality service, are:

$$\begin{aligned}\hat{c}_{11}(Q) &= 5Q_{11}^2 + 10,000, & \hat{c}_{12}(Q) &= 18Q_{12}^2 + 12,000. \\ \hat{c}_{21}(Q) &= 4.5Q_{21}^2 + 12,000, & \hat{c}_{22}(Q) &= 5Q_{22}^2 + 14,000.\end{aligned}$$

The bounds on the quality levels are:

$$\begin{aligned}\underline{Q}_{11} &= 50, \bar{Q}_{11} = 80, & \underline{Q}_{12} &= 40, \bar{Q}_{12} = 70, \\ \underline{Q}_{21} &= 60, \bar{Q}_{21} = 90, & \underline{Q}_{22} &= 70, \bar{Q}_{22} = 90.\end{aligned}$$

The prices, which correspond to the collection component of the blood supply chain, are:  $\pi_1 = 70$  and  $\pi_2 = 60$ .



# Case Study: American Red Cross and United Blood Services Example 1

Solutions:  $Q_{11}^* = 77.2$ ,  $Q_{12}^* = 40.$ ,  $Q_{21}^* = 83.3$ ,  $Q_{22}^* = 82$ .

According to the solution, the Red Cross stands to collect **736.7 units of blood at region 1**, since  $P_{11}(Q^*) = 736.7$  and **367.7 units of whole blood at region 2**. United Blood Services, on the other hand, stand to collect, since  $P_{21}(Q^*) = 922.1$ , **that number of units per month at region 1**, and **1001.80 units in region 2** (since  $P_{22}(Q^*) = 1001.8$ ).

**Hence, the United Blood Services collect a larger number of units of blood in the two regions.**

# Case Study: American Red Cross and United Blood Services Example 1

According to the Lagrangean analysis (as in our paper) only  $Q_{12}^*$  is at its lower bound and no quality service levels are at their upper bounds:  $\bar{\lambda}_{11}^1 = 0$ ,  $\bar{\lambda}_{21}^1 = 0$ ,  $\bar{\lambda}_{22}^1 = 0$ , and  $\bar{\lambda}_{11}^2 = 0$ ,  $\bar{\lambda}_{12}^2 = 0$ ,  $\bar{\lambda}_{21}^2 = 0$ ,  $\bar{\lambda}_{22}^2 = 0$ .

Also, since  $Q_{12}^* = \underline{Q}_{12}$ , we compute

$$\bar{\lambda}_{12}^1 = \sum_{k=1}^2 \frac{\partial \hat{c}_{1k}(Q^*)}{\partial Q_{12}} - \omega_1 \gamma_{12} - \pi_1 \sum_{k=1}^2 \frac{\partial P_{1k}(Q^*, \beta_1, t_{12})}{\partial Q_{12}} = 1359.$$

The American Red Cross suffers a marginal loss given by  $\bar{\lambda}_{12}^1$ . The transaction utilities at the equilibrium quality levels are:

$$U_1(Q^*) = 5,507.20, \quad U_2(Q^*) = 38,485.99.$$

**In this illustrative example, the United Blood Services organization provides a higher level of quality services at each of its locations in Tucson and garners a higher transaction utility than the American Red Cross.**

## Case Study: Example 2

**Example 2 has the same network topology as Example 1. The data are also identical to those in Example 1.**

New  $P_{ij}$  functions:  $\alpha_{ij}\sqrt{P_{ij}}$  for  $i = 1, 2; j = 1, 2$  with  $\alpha_{11} = 50$ ,  $\alpha_{12} = 30$ ,  $\alpha_{21} = 40$ , and  $\alpha_{22} = 20$ .

Computed equilibrium quality service levels are:

$$Q_{11}^* = 72.43, \quad Q_{12}^* = 40.00, \quad Q_{21}^* = 64.61, \quad Q_{22}^* = 70.00.$$

The Euler method requires 34 iterations to converge to this solution.

## Case Study: Example 2

$Q_{12}^*$  and  $Q_{22}^*$  are at their lower bounds. Lagrange analysis shows blood service organization 1 suffers a marginal loss of 737.03 associated with its services in region 2 and blood service organization 2 suffers a marginal loss of 354.85 associated with its services in region 2.

The values of the transaction utilities of the blood service organizations at these equilibrium values are:  $U_1 = 67,860.92$ ,  $U_2 = 43,229.16$ .

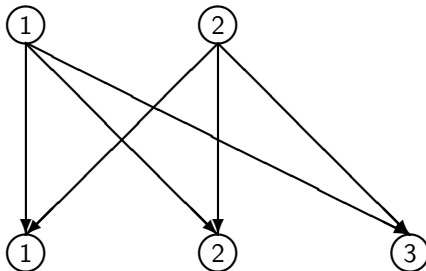
Blood collected by organization 1:  $P_{11} = 1341.37$  units in region 1 and  $P_{12} = 607.74$  units of blood in region 2. Blood collected by organization 2:  $P_{21} = 1074.27$  units in region 1 and  $P_{22} = 587.39$  units of blood in region 2.

# Case Study: Example 3

**Example 3 has the network topology in Figure 3.**

In this example, the lower bounds associated with the blood service organizations servicing region 3 in terms of collections are set to 0.

Blood Service Organizations



Blood Collection Regions

## Case Study: Example 3

The data are as follows:  $\alpha_{13} = 40$ ,  $\alpha_{23} = 30$ , and

$$P_{13}(Q) = 40\sqrt{10Q_{13} - Q_{23} + 50}, \quad P_{23}(Q) = 30\sqrt{11Q_{23} - Q_{13} + 50},$$

$$\gamma_{13} = 9, \quad \gamma_{23} = 10,$$

and

$$\hat{c}_{13}(Q) = 10Q_{13}^2 + 15,000, \quad \hat{c}_{23}(Q) = 9Q_{23}^2 + 13,000.$$

The lower and upper bounds on the new links, in turn, are:

$$\underline{Q}_{13} = 0, \quad \underline{Q}_{23} = 0,$$

$$\bar{Q}_{13} = 60, \quad \bar{Q}_{23} = 70.$$

## Case Study: Example 3

The Euler method converges in 34 iterations to the following equilibrium quality level pattern:

$$Q_{11}^* = 72.43, \quad Q_{12}^* = 40.00, \quad Q_{13}^* = 38.84,$$

$$Q_{21}^* = 64.61, \quad Q_{22}^* = 70.00, \quad Q_{23}^* = 33.70.$$

$$\bar{\lambda}_{13}^1 = \bar{\lambda}_{13}^2 = \bar{\lambda}_{23}^1 = \bar{\lambda}_{23}^2 = 0.00.$$

The transaction utility for blood service organization 1,  $U_1 = 41,057.70$ , and the transaction utility for blood service organization 2,  $U_2 = 23,469.59$ .

## Case Study: Example 3

Blood collection by organization 1 in region 3 is **804.73** units of blood.  
Collection by organization 2 in region 3 is **586.24** units of blood.

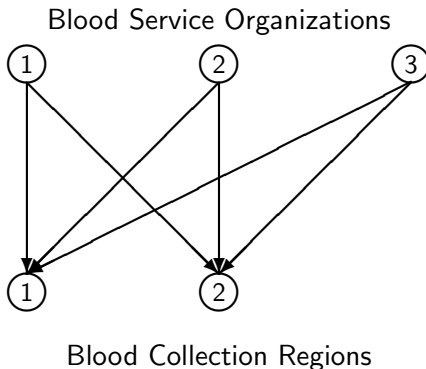
Since now both organizations operate a facility in an additional region the costs for organization 1 are equal to **167,283.03** and for organization 2 the costs are **127,589.64**.

Revenue of organization 1 is now **196,739.25** and that of organization 2 is **134,874.17**.



# Case Study: Example 4

**Example 4 is constructed from Example 2 but there is a new competitor, blood service organization 3.**



## Case Study: Example 4

The data for blood service organization 3 are:

$$P_{31}(Q) = 50\sqrt{11Q_{31} - Q_{21} + 50}, \quad P_{32}(Q) = 40\sqrt{10Q_{32} - Q_{12} + 2000},$$
$$\omega_3 = 10, \quad \gamma_{31} = 10, \quad \gamma_{32} = 11,$$

Total cost functions given by:

$$\hat{c}_{31}(Q) = 6q_{31}^2 + 10,000 \quad \hat{c}_{32}(Q) = 5Q_{32}^2 + 12,000,$$

Lower and upper bounds are as follows:

$$\underline{Q}_{31} = 50, \quad \bar{Q}_{31} = 90,$$
$$\underline{Q}_{32} = 40, \quad \bar{Q}_{32} = 80.$$

## Case Study: Example 4

The price  $\pi_3 = 80$ .

The Euler method requires 40 iterations for convergence and yields the following equilibrium quality service level pattern:

$$Q_{11}^* = 72.43, \quad Q_{12}^* = 40, \quad Q_{21}^* = 64.61, \quad Q_{22}^* = 70$$

$$Q_{31}^* = 70.73, \quad Q_{32}^* = 66.65.$$

Blood service organization 3 has a transaction utility  $U_3 = 104,706.44$ . Its quality levels do not lie at the bounds so that:  $\bar{\lambda}_{31}^1 = \bar{\lambda}_{31}^2 = \bar{\lambda}_{32}^1 = \bar{\lambda}_{32}^2 = 0$ .

The amounts of blood donations received by organization 3 are:  $P_{31} = 1,381.47$  and  $P_{32} = 2,049.99$ . Revenue is: **274,516.72** with its cost equal to **184,922.09**.

## Case Study: Example 5

**Example 5 is constructed from Example 4.**

We assume that some time has transpired and now both blood service organizations 1 and 2 realize that there is more competition from blood service organization 3.

For blood service organization 1:

$$P_{11}(Q) = 50\sqrt{10Q_{1,1} - Q_{2,1} - Q_{22} - .5Q_{3,1} + 130},$$

$$P_{12}(Q) = 30\sqrt{12Q_{1,2} - Q_{2,1} - 2Q_{2,2} - .3Q_{3,2} + 135},$$

and for blood service organization 2:

$$P_{21}(Q) = 40\sqrt{11Q_{2,1} - Q_{1,1} - Q_{1,2} - .2Q_{2,1} + 113},$$

$$P_{22}(Q) = 20\sqrt{12Q_{2,2} - Q_{1,1} - Q_{1,2} - .3Q_{3,2} + 135}.$$

## Case Study: Example 5

Quality service level pattern:

$$Q_{11}^* = 73.57, \quad Q_{12}^* = 40, \quad Q_{21}^* = 64.99, \quad Q_{22}^* = 70$$

$$Q_{31}^* = 70.73, \quad Q_{32}^* = 66.65.$$

Utilities

$$U_1 = 64,439.25, \quad U_2 = 42,572.30, \quad \text{and} \quad U_3 = 104,222.39.$$

Lagrangian analysis

$$\bar{\lambda}_{11}^1 = \bar{\lambda}_{11}^2 = 0, \quad \bar{\lambda}_{21}^1 = \bar{\lambda}_{21}^2 = 0, \quad \bar{\lambda}_{31}^1 = \bar{\lambda}_{31}^2 = 0 \quad \text{and also} \quad \bar{\lambda}_{32}^1 = \bar{\lambda}_{32}^2 = 0.$$
$$\bar{\lambda}_{12}^1 = 720.98, \quad \bar{\lambda}_{12}^2 = 0, \quad \text{and} \quad \bar{\lambda}_{22}^1 = 351.79, \quad \bar{\lambda}_{22}^2 = 0.$$

## Blood donations

The volumes of blood donations are now as follows: For organization 1:  $P_{11} = 1,318.43$ ,  $P_{12} = 592.46$ ; for organization 2:  $P_{21} = 1,059.31$ ,  $P_{22} = 580.15$ , and for organization 3:  $P_{31} = 1,381.22$ ,  $P_{32} = 2,049.99$ .

## Costs

For organization 1 cost is equal to **77,860.27**. Organization 2 encumbers costs equal to **68,644.69**. Organization 3 incurs costs of **185,38.53**.

## Revenue

Organization 1 has a revenue of **133,762.72**. Organization 2 gains a revenue of **98,367.77**. Organization 3 obtains a revenue of **274,497.03**.

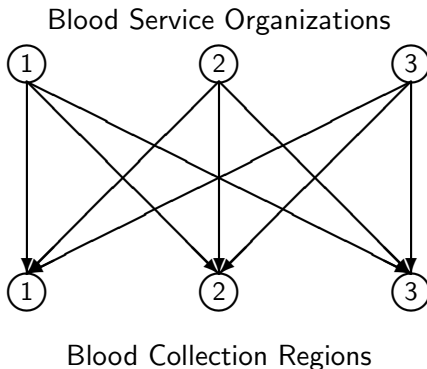
## Case Study: Example 5

- Blood service organization 1 suffers a marginal loss of 720.98 associated with its services in region 2.
- Blood service organization 2 suffers a marginal loss of 351.79 associated with its services in region 2.
- With increased competition blood donors benefit in that the quality service levels provided are now as high as in Example 4.
- Both blood service organizations 1 and 2 provide a higher quality service in region 1 than in Example 4 but, at a higher cost so their transaction utilities are lower now than in Example 4.
- Blood collections from both regions decrease for organizations 1 and 2. However, **due to the presence of a competing organization the overall blood collection increases**. This finding is consistent with the empirical findings in Bose (2014).

# Case Study: Example 6

**Example 6 is constructed from Example 5.**

There is an additional collection region.





## Case Study: Example 6

The data remain as in Example 5 with the addition of the new data below:

$$\alpha_{13} = 40, \quad \alpha_{23} = 30, \quad \alpha_{33} = 50,$$

$$P_{13}(Q) = 40\sqrt{10Q_{13} - Q_{23} - .2Q_{33} + 150},$$

$$P_{23}(Q) = 30\sqrt{11Q_{23} - Q_{13} - .2Q_{33} + 150},$$

$$P_{33}(Q) = 50\sqrt{10Q_{33} - Q_{23} - .3Q_{13} + 100},$$

$$\hat{c}_{13}(Q) = 100Q_{13}^2 + 15,000, \quad \hat{c}_{23}(Q) = 9Q_{23}^2 + 13000$$

$$\hat{c}_{33}(Q) = 8Q_{33}^2 + 10000$$

Lower and upper bounds on the new links to region 3 are given by:

$$\underline{Q}_{13} = 0, \quad \underline{Q}_{23} = 0, \quad \underline{Q}_{33} = 40,$$

$$\bar{Q}_{13} = 60, \quad \bar{Q}_{23} = 70, \quad \bar{Q}_{33} = 90.$$

## Case Study: Example 6

The Euler method, again, converges in 40 iterations to the following equilibrium pattern:

$$\begin{aligned} Q_{11}^* &= 73.57, & Q_{12}^* &= 40, & Q_{13}^* &= 36.32, \\ Q_{21}^* &= 64.99, & Q_{22}^* &= 70, & Q_{23}^* &= 31.51, \\ Q_{31}^* &= 70.73, & Q_{32}^* &= 66.65, & Q_{33}^* &= 56.39. \end{aligned}$$

The transaction utilities are now:  $U_1 = 129,918.82$ ,  $U_2 = 58,877.95$ , and  $U_3 = 168,602.63$ .

All of the Lagrange multipliers are equal to 0 except for the following:  
 $\bar{\lambda}_{12}^1 = 720.98$ ,  $\bar{\lambda}_{22}^1 = 351.79$ .

## Blood donations

The volumes of blood donations are now: for organization 1:

$P_{11} = 1,318.43$ ,  $P_{12} = 592.46$ ,  $P_{13} = 867.59$ ; for organization 2:

$P_{21} = 1,059.31$ ,  $P_{22} = 580.15$ ,  $P_{23} = 635.70$ , and for organization 3:

$P_{31} = 1,381.22$ ,  $P_{32} = 2,049.99$ , and  $P_{33} = 1,246.49$ .

## Net Revenue

The net revenue of organization 1 is equal to **118,439.83**; that of organization 2 is: **42,877.63**, and that of organization 3: **147,850.66**.

**All blood service organizations gain by servicing another region even in the case of competition.**

# Conclusions

- In this paper, we develop a game theory model for blood donations that focuses on blood service organizations.
- The blood service organizations **compete for blood donations** in different regions.
- Donors respond to the **quality of service** that the blood service organizations provide in blood collection.
- We formulate the governing equilibrium conditions as a **variational inequality problem** and prove that the solution is guaranteed to exist.
- We established (in the paper) additional theoretical results based on **Lagrange theory** associated with the **lower and upper bounds on the quality service levels**.

# Conclusions

- The results demonstrate how **increased competition** can yield **benefits** for blood donors in terms of **quality level of service**.
- Increased competition also **increases the total blood collection** although **collections by individual organizations decrease**.
- Blood service organizations who do “good,” can also be **financially sustainable** even in the face of competition.
- This research adds to the literature on **game theory and healthcare** and, specifically, to game theory and blood supply chains, which has been very limited, to-date.

# Thank you !

▶ <https://supernet.isenberg.umass.edu/visuals.html>



## The Virtual Center for Supernetworks



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**The Virtual Center for Supernetworks** is an interdisciplinary center at the Isenberg School of Management that advances knowledge on large-scale networks and integrates operations research and management science, engineering, and economics. Its Director is Dr. Anna Nagurney, the John F. Smith Memorial Professor of Operations Management.

**Mission:** The Virtual Center for Supernetworks fosters the study and application of supernetworks and serves as a resource on networks ranging from transportation and logistics, including supply chains, and the Internet, to a spectrum of economic networks.

**The Applications of Supernetworks Include:** decision-making, optimization, and game theory; supply chain management; critical infrastructure from transportation to electric power networks; financial networks; knowledge and social networks; energy, the environment, and sustainability; cybersecurity; Future Internet Architectures; risk management; network vulnerability, resiliency, and performance metrics; humanitarian logistics and healthcare.

Announcements and Notes	Photos of Center Activities	Photos of Network Innovators	Friends of the Center	Course Lectures	Fulbright Lectures	UMass Amherst INFORMS Student Chapter
Professor Anna Nagurney's Blog	Network Classics	Doctoral Dissertations	Conferences	Journals	Societies	Archive

[Announcements and Notes from the Center Director](#)  
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[Professor Anna Nagurney's Blog](#)

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Research, Education, Networks, and the World:  
A Female Professor Speaks

Sustaining the Supply Chain

45 often a challenge to get from Point A to Point B in normal circumstances, but after a disaster it can be even more difficult to do so. In this episode, we explore how the mathematical modeling of supply chains can help us understand the challenges of managing after a disaster. In this episode, we explore how the combination of disruptions in the network and increased needs of the population.

▶ PBS VIDEO

America Revealed

