Synergies and Vulnerabilities of Supply Chain Networks in a Global Economy: What We Can Learn from Half a Century of Advances in Transportation

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Thanks to Professors Pat Mokhtarian and Yueyue Fan for inviting me to UC Davis.
Interdisciplinary Impact of Networks

**Economics**
- Interregional Trade
- General Equilibrium
- Industrial Organization
- Portfolio Optimization
- Flow of Funds Accounting

**Mathematics**
- Networks

**Computer Science**
- Routing Algorithms

**Engineering**
- Energy
- Manufacturing
- Telecommunications
- Transportation

**Biology**
- DNA Sequencing
- Targeted Cancer Therapy

**Sociology**
- Social Networks
- Organizational Theory
Supply chain networks are the underpinning skeletons of the business world. These networks, more and more, are global in nature, with products consisting of parts manufactured in different regions of the world, assembled in yet other locations, and then shipped across continents and oceans to retailers and consumers.

Such complex networks consist of manufacturers (and their suppliers), shippers and carriers using various modes of transportation, distribution centers where the products are stored, and, ultimately, sent from to the customers.

Supply chains involve many decision-makers interacting with one another, sometimes competing, and at other times necessarily cooperating.
Depiction of a Supply Chain Network
Supply chain networks depend on infrastructure networks for their effective and efficient operations from: manufacturing and logistical networks, to transportation networks, to electric power networks, financial networks, and telecommunication networks, most, notably, the Internet.

No supply chain, logistics system, or infrastructure system is immune to disruptions and as long as there have been supply chains there have been disruptions.

However, in the past decade there have been vivid high-profile examples of supply chain disruptions and their impacts. Supply chain disruptions and the associated risk are major topics now in theoretical and applied research, as well as in practice, since risk in the context of supply chains may be associated with the production/procurement processes, the transportation/shipment of the goods, and/or the demand markets.
Transportation, Communication, and Energy Networks

Bus Network

Railroad Network

Iridium Satellite Constellation Network

Satellite and Undersea Cable Networks

Duke Energy Gas Pipeline Network
## Components of Common Networks

<table>
<thead>
<tr>
<th>Network System</th>
<th>Nodes</th>
<th>Links</th>
<th>Flows</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transportation</td>
<td>Intersections, Homes, Workplaces, Airports, Railyards</td>
<td>Roads, Airline Routes, Railroad Track</td>
<td>Automobiles, Trains, and Planes,</td>
</tr>
<tr>
<td>Manufacturing and logistics</td>
<td>Workstations, Distribution Points</td>
<td>Processing, Shipment</td>
<td>Components, Finished Goods</td>
</tr>
<tr>
<td>Communication</td>
<td>Computers, Satellites, Telephone Exchanges</td>
<td>Fiber Optic Cables, Radio Links</td>
<td>Voice, Data, Video</td>
</tr>
<tr>
<td>Energy</td>
<td>Pumping Stations, Plants</td>
<td>Pipelines, Transmission Lines</td>
<td>Water, Gas, Oil, Electricity</td>
</tr>
</tbody>
</table>
US Railroad Freight Flows

Railroad Freight Density
(million gross tons)

- Blue: Under 10 mgt
- Green: 10 to 20 mgt
- Yellow: 20 to 40 mgt
- Orange: 40 to 60 mgt
- Light Orange: 60 to 100 mgt
- Red: Over 100 mgt

Natural Gas Pipeline Network in the US

Interstate Natural Gas Flow Summary, 2003
World Oil Trading Network
Moreover, in recent decades, the focus has been on lean supply chains and, although such supply chains may work well when the environment is predictable and steady, they may be very sensitive to disruptions since they lack redundancy and slack in their systems.

Furthermore, firms today may be much less vertically integrated (and, clearly, more global). Decades ago, Ford Motor Company and other automobile manufacturers and even IBM produced their products, essentially in their entirety.

Lynn (2006) has argued that globalization has led to extremely fragile supply chains. Suppliers today may be in parts of the world that are unstable and subject to natural disasters, political instability, and strife.
In fact, Craighead, Blackhurst, Rungtusanatham, and Handfield (2007) have argued that supply chain disruptions and the associated operational and financial risks are the most pressing issue faced by firms in today’s competitive global environment.

Notably, the focus of research has been on “demand-side” risk, which is related to fluctuations in the demand for products, as opposed to the “supply-side” risk, which deals with uncertain conditions that affect the production and transportation processes of the supply chain.
Major Recent Supply Chain Disruptions

Several recent major disruptions:

► In March 2000, a lightning bolt struck a Philips Semiconductor plant in Albuquerque, New Mexico, and created a 10-minute fire that resulted in the contamination of millions of computer chips and subsequent delaying of deliveries to its two largest customers: Finland’s Nokia and Sweden’s Ericsson.

► Ericsson used the Philips plant as its sole source and reported a $400 million loss because it did not receive the chip deliveries in a timely manner whereas Nokia moved quickly to tie up spare capacity at other Philips plants and refitted some of its phones so that it could use chips from other US suppliers and from Japanese suppliers.

► Nokia managed to arrange alternative supplies and, therefore, mitigated the impact of the disruption.

► Ericsson learned a painful lesson from this disaster.
The West Coast port lockout in 2002, which resulted in a 10 day shutdown of ports in early October, typically, the busiest month. 42% of the US trade products and 52% of the imported apparel go through these ports, including Los Angeles. Estimated losses were one billion dollars per day.
The economic and financial troubles of the automobile companies in the United States among the “Big Three” are creating a domino effect throughout the supply chain and the vast network of auto supplier firms. For example, GM alone has approximately 2,000 suppliers, whereas Ford has about 1,600 suppliers, and Chrysler about 900 suppliers. Although Ford is in better shape in terms of the cash the company has, it shares most of the same big parts suppliers, so a disruption in the supply chain that a bankruptcy would invariably cause would hurt Ford too, and even halt production temporarily.
As summarized by Sheffi (2005), one of the main characteristics of disruptions in supply networks is “the seemingly unrelated consequences and vulnerabilities stemming from global connectivity.”

Indeed, supply chain disruptions may have impacts that propagate not only locally but globally and, hence, a holistic, system-wide approach to supply chain network modeling and analysis is essential in order to be able to capture the complex interactions among decision-makers.
Disasters in Transportation Networks

www.salem-news.com

www.boston.com
Disasters in Electric Power Networks

www.cellar.org

media.collegepublisher.com

www.crh.ncaa.gov
Disasters in Communication Networks

www.tx.mb21.co.uk

www.w5jgv.com

www.wirelessestimator.com
Motivation for Our Research

Hence, the rigorous modeling and analysis of supply chain networks, in the presence of possible disruptions is imperative since disruptions may have lasting major financial consequences.

Hendricks and Singhal (2005) analyzed 800 instances of supply chain disruptions experienced by firms whose stocks are publicly traded. They found that the companies that suffered supply chain disruptions experienced share price returns 33 percent to 40 percent lower than the industry and the general market benchmarks. Furthermore, share price volatility was 13.5 percent higher in these companies in the year following a disruption than in the prior year.

A company that experiences a supply chain disruption can expect to experience significant decreases in sales growth, stock return, and shareholder wealth for two years or more following the incident (Hendricks and Singhal (2003, 2005)). It is evident that only well-prepared companies can effectively cope with supply chain disruptions.
The Supply Chain's Impact on Stock Price

<table>
<thead>
<tr>
<th>% Increase in Stock Price</th>
<th>1% Revenue Increase</th>
<th>1% Operational Expenses Decrease</th>
<th>20% Inventory Reduction</th>
<th>5% Fixed-Asset Utilization Increase</th>
<th>5% Days Sales Decrease</th>
</tr>
</thead>
</table>
| Source: S&P 500 Survey 2002

Supply Chain Management Review • March 2005
Motivation for Our Research

The goal of supply chain risk management is to alleviate the consequences of disruptions and risks or, simply put, to increase the *robustness* of a supply chain. However, there are very few quantitative models for measuring supply chain robustness.

Snyder and Daskin (2005) examined supply chain disruptions in the context of facility location. The objective of their model was to select locations for warehouses and other facilities that minimize the transportation costs to customers and, at the same time, account for possible closures of facilities that would result in re-routing of the product. However, as commented in Snyder and Shen (2006), “Although these are multi-location models, they focus primarily on the local effects of disruptions.”
To-date, most supply disruption studies have focused on a local point of view, in the form of a single-supplier problem (see, e. g., Gupta (1996) and Parlar (1997)) or a two-supplier problem (see, e. g., Parlar and Perry (1996)).

Very few studies/papers have examined supply chain risk management in an environment with multiple decision-makers and in the case of uncertain demands (cf. Tomlin (2006)).
Characteristics of Networks Today Including Supply Chains

- *Large-scale nature* and complexity of network topology;

- *Congestion*; in the US we are experiencing a freight capacity crisis;

- the *interactions among networks* themselves such as in transportation versus telecommunications;

- *Dynamics* and global reach; increasing risk and uncertainty.
Traffic Congestion
There are *two fundamental principles of travel behavior* (Wardrop (1952)):

- User-optimization (or network equilibrium)
- System-optimization

(Beckmann, McGuire, and Winsten (1956), Dafermos and Sparrow (1969)).

These concepts correspond to decentralized versus centralized decision-making and are extremely relevant in today's networked economies and societies.
In a user-optimized (network equilibrium) problem, each user of a network system seeks to determine his/her cost-minimizing route of travel between an origin/destination pair, until an equilibrium is reached, in which no user can decrease his/her cost of travel by unilateral action.

In a system-optimized network problem, users are allocated among the routes so as to minimize the total cost in the system.

Both classes of problems, under certain imposed assumptions, possess convex optimization formulations.
Capturing Link Congestion

For a typical user link travel time function, where the free flow travel time refers to the travel time to traverse a link when there is zero flow on the link (or zero vehicles).
BPR Link Cost Function

A common link performance function is the Bureau of Public Roads (BPR) cost function developed in 1964. This equation is given by

\[ c_a = c_a^0 \left[ 1 + \alpha \left( \frac{f_a}{t'_a} \right)^\beta \right], \]

where, \( c_a \) and \( f_a \) are the travel time and link flow, respectively, on link \( a \), \( c_a^0 \) is the free-flow travel time, and \( t'_a \) is the “practical capacity” of link \( a \). The quantities \( \alpha \) and \( \beta \) are model parameters, for which the values \( \alpha = 0.15 \) minutes and \( \beta = 4 \) are typical values. For example, these values imply that the practical capacity of a link is the flow at which the travel time is 15% greater than the free-flow travel time.
The Transportation Network Equilibrium (TNE) Problem and Methodological Tools
Transportation applications have motivated the development of methodological tools in different disciplines, many of which have been motivated and derived from the book, *Studies in the Economics of Transportation*, Beckmann, McGuire, and Winsten (1956); see Boyce, Mahmassani, and Nagurney, *Papers in Regional Science* 84 (2005), 85-103.
The Transportation Social - Knowledge Network

On the Beach in Mallacoota, Australia

Professors Beckmann and Dafermos at Anna Nagurney’s Post-Ph.D. Defense Party in Barus Holley

INFORMS Honoring the 50th Anniversary of the Publication of Studies in the Economics of Transportation

Professor Beckmann with Professor Michael Florian of Montreal

Professors Beckmann and McGuire
• *alternative behaviors of the users of the network*

– System-optimized (S-O) (centralized supply chain) versus

– user-optimized (U-O) (decentralized supply chain),

which may lead to paradoxical phenomena (Braess Paradox and the Merger Paradox).
Network Equilibrium Problem
Derived from Transportation

(U-O Problem)
Consider a general network $G = [N, L]$, where $N$ denotes the set of nodes, and $L$ the set of directed links. Let $a$ denote a link of the network connecting a pair of nodes, and let $p$ denote a path consisting of a sequence of links connecting an O/D pair. $P_w$ denotes the set of paths, assumed to be acyclic, connecting the O/D pair of nodes $w$ and $P$ the set of all paths.

Let $x_p$ represent the flow on path $p$ and $f_a$ the flow on link $a$. The following conservation of flow equation must hold:

$$f_a = \sum_{p \in P} x_p \delta_{ap},$$

where $\delta_{ap} = 1$, if link $a$ is contained in path $p$, and 0, otherwise. This expression states that the load on a link $a$ is equal to the sum of all the path flows on paths $p$ that contain (traverse) link $a$. 
Moreover, if we let $d_w$ denote the demand associated with O/D pair $w$, then we must have that

$$d_w = \sum_{p \in P_w} x_p,$$

where $x_p \geq 0$, $\forall p$, that is, the sum of all the path flows between an origin/destination pair $w$ must be equal to the given demand $d_w$.

Let $c_a$ denote the user cost associated with traversing link $a$, which is assumed to be continuous, and $C_p$ the user cost associated with traversing the path $p$. Then

$$C_p = \sum_{a \in L} c_a \delta_{ap}.$$
Network Equilibrium

The network equilibrium conditions are then given by: For each path \( p \in P_w \) and every O/D pair \( w \):

\[
C_p \begin{cases}
\equiv \lambda_w, & \text{if} \quad x^*_p > 0 \\
\geq \lambda_w, & \text{if} \quad x^*_p = 0
\end{cases}
\]

where \( \lambda_w \) is an indicator, whose value is not known a priori. These equilibrium conditions state that the user costs on all used paths connecting a given O/D pair will be minimal and equalized.
The Braess (1968) Paradox

Assume a network with a single O/D pair (1,4). There are 2 paths available to travelers: \( p_1=(a,c) \) and \( p_2=(b,d) \).

For a travel demand of 6, the equilibrium path flows are \( x_{p_1}^* = x_{p_2}^* = 3 \) and

The equilibrium path travel cost is

\[
C_{p_1} = C_{p_2} = 83.
\]

\[
c_a(f_a) = 10 f_a \quad c_b(f_b) = f_b + 50 \\
c_c(f_c) = f_c + 50 \quad c_d(f_d) = 10 f_d
\]
Adding a new link creates a new path $p_3=(a,e,d)$.

The original flow distribution pattern is no longer an equilibrium pattern, since at this level of flow the cost on path $p_3$, $C_{p_3}=70$.

The new equilibrium flow pattern network is

$x_{p_1}^* = x_{p_2}^* = x_{p_3}^* = 2$.

The equilibrium path travel costs:

$C_{p_1} = C_{p_2} = C_{p_3} = 92$.
The 1968 Braess article has been translated from German to English and appears as

On a Paradox of Traffic Planning

by Braess, Nagurney, Wakolbinger

in the November 2005 issue of Transportation
The NE Paradigm is the Unifying Paradigm:

- Transportation Networks
- The Internet
- Financial Networks
- Decentralized Supply Chains.
The System-Optimization (S-O) Problem

The above discussion focused on the user-optimized (U-O) problem. We now turn to the system-optimized (S-O) problem in which a central controller, say, seeks to minimize the total cost in the network system, where the total cost is expressed as

$$\sum_{a \in L} \hat{c}_a(f_a)$$

where it is assumed that the total cost function on a link $a$ is defined as:

$$\hat{c}_a(f_a) \equiv c_a(f_a) \times f_a,$$

subject to the conservation of flow constraints, and the nonnegativity assumption on the path flows. Here separable link costs have been assumed, for simplicity, and other total cost expressions may be used, as mandated by the particular application.
The S-O Optimality Conditions

Under the assumption of strictly increasing user link cost functions, the optimality conditions are: For each path $p \in P_w$, and every O/D pair $w$:

$$\hat{C}_p' \begin{cases} = \mu_w, & \text{if } x_p > 0 \\ \geq \mu_w, & \text{if } x_p = 0, \end{cases}$$

where $\hat{C}_p'$ denotes the marginal total cost on path $p$, given by:

$$\hat{C}_p' = \sum_{a \in L} \frac{\partial \hat{c}_a(f_a)}{\partial f_a} \delta_{ap}.$$

The above conditions correspond to Wardrop’s second principle of travel behavior.
What is the S-O solution for the two Braess networks (before and after the addition of a new link e)?

Before the addition of the link e, we may write:

\[ \hat{c}'_a = 20f_a, \quad \hat{c}'_b = 2f_b + 50, \]
\[ \hat{c}'_c = 2f_c + 50, \quad \hat{c}'_d = 20f_d. \]

It is easy to see that, in this case, the S-O solution is identical to the U-O solution with \( x_{p_1} = x_{p_2} = 3 \) and \( \hat{C}'_{p_1} = \hat{C}'_{p_2} = 116. \)

Furthermore, after the addition of link e, we have that \( \hat{c}'_e = 2f_e + 10. \) The new path \( p_3 \) is not used in the S-O solution, since with zero flow on path \( p_3, \) we have that \( \hat{C}'_{p_3} = 170 \) and \( \hat{C}'_{p_1} = \hat{C}'_{p_2} \) remains at 116.
If the symmetry assumption does not hold for the user link costs functions, then the transportation network equilibrium conditions can no longer be reformulated as an associated optimization problem and the equilibrium conditions are formulated and solved as a variational inequality problem!
VI Formulation of TNE
Dafermos (1980), Smith (1979)

A traffic path flow pattern satisfies the above equilibrium conditions if and only if it satisfies the variational inequality problem: determine $x^* \in K$, such that

$$\sum_p C_p(x^*) \times (x_p - x_p^*) \geq 0, \quad \forall x \in K.$$ 

Finite-dimensional variational inequality theory has been applied to-date to the wide range of equilibrium problems noted above.

In particular, the finite-dimensional variational inequality problem is to determine $x^* \in K \subset \mathbb{R}^n$ such that

$$\langle F(x^*), x - x^* \rangle \geq 0, \quad \forall x \in K,$$

where $\langle \cdot, \cdot \rangle$ denoted the inner product in $\mathbb{R}^n$ and $K$ is closed and convex.
A Geometric Interpretation of a Variational Inequality and a Projected Dynamical System

Dupuis and Nagurney (1993)
Nagurney and Zhang (1996)
The variational inequality problem, contains, as special cases, such classical problems as:

- systems of equations
- optimization problems
- complementarity problems

and is also closely related to fixed point problems.

*Hence, it is a unifying mathematical formulation for a variety of mathematical programming problems.*
The Equivalence of Decentralized Supply Chains and Transportation Networks

Supply Chain -Transportation Supernetwork Representation

Electric Power Supply Chains
The Electric Power Supply Chain Network

The Transportation Network Equilibrium Reformulation of Electric Power Supply Chain Networks

Electric Power Supply Chain Network with Fuel Suppliers

In 1952, Copeland wondered whether money flows like water or electricity.
The Transportation Network Equilibrium Reformulation of the Financial Network Equilibrium Model with Intermediation

We have shown that *money* as well as *electricity* flow like *transportation* and have answered questions posed fifty years ago by Copeland and by Beckmann, McGuire, and Winsten!
Additional disasters that have demonstrated the importance and the vulnerability of network systems.

Examples:
• 9/11 Terrorist Attacks, September 11, 2001;
• The biggest blackout in North America, August 14, 2003;
• Two significant power outages in September 2003 -- one in the UK and the other in Italy and Switzerland;
• Hurricane Katrina, August 23, 2005;
• The Minneapolis I35 Bridge Collapse, August 1, 2007
• The severance of the Mediterranean cable in 2008.
Our Research on Network Efficiency, Vulnerability, and Robustness


Robustness of Transportation Networks Subject to Degradable Links, Nagurney and Qiang, *Europhysics Letters*, 80, December (2007).

A New Network Performance/Efficiency Measure with Applications to Infrastructure Networks including Supply Chains
The network performance/efficiency measure $\mathcal{E}(G,d)$, for a given network topology $G$ and fixed demand vector $d$, is defined as

$$\mathcal{E}(G,d) = \frac{\sum_{w \in W} \frac{d_w}{\lambda_w}}{n_W},$$

where $n_w$ is the number of O/D pairs in the network and $\lambda_w$ is the equilibrium disutility for O/D pair $w$.

Definition: Importance of a Network Component

The importance, \( I(g) \), of a network component \( g \in G \) is measured by the relative network efficiency drop after \( g \) is removed from the network:

\[
I(g) = \frac{\Delta \mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}(G, d) - \mathcal{E}(G - g, d)}{\mathcal{E}(G, d)},
\]

where \( G-g \) is the resulting network after component \( g \) is removed.
The Approach to Study the Importance of Network Components

The elimination of a link is treated in the N-Q network efficiency measure by removing that link while the removal of a node is managed by removing the links entering and exiting that node.

In the case that the removal results in no path connecting an O/D pair, we simply assign the demand for that O/D pair to an abstract path with a cost of infinity. Hence, our measure is well-defined even in the case of disconnected networks.

The measure generalizes the Latora and Marchiori network measure for complex networks.
The Latora and Marchiori (L-M) Network Efficiency Measure

Definition: The L-M Measure

The network performance/efficiency measure, $E(G)$ for a given network topology, $G$, is defined as:

$$E(G) = \frac{1}{n(n-1)} \sum_{i \neq j \in G} \frac{1}{d_{ij}}$$

where $n$ is the number of nodes in the network and $d_{ij}$ is the shortest path length between node $i$ and node $j$. 
Assume a network with two O/D pairs: 
\( w_1 = (1,2) \) and \( w_2 = (1,3) \) with demands: 
\( d_{w_1} = 100 \) and \( d_{w_2} = 20 \).

The paths are:
for \( w_1 \), \( p_1 = a \); for \( w_2 \), \( p_2 = b \).

The equilibrium path flows are:
\( x_{p_1}^* = 100 \), \( x_{p_2}^* = 20 \).

The equilibrium path travel costs are:
\( c_{p_1} = c_{p_2} = 20 \).
### Importance and Ranking of Links and Nodes

<table>
<thead>
<tr>
<th>Link</th>
<th>Importance Value from Our Measure</th>
<th>Importance Ranking from Our Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.8333</td>
<td>1</td>
</tr>
<tr>
<td>b</td>
<td>0.1667</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Node</th>
<th>Importance Value from Our Measure</th>
<th>Importance Ranking from Our Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.8333</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>0.1667</td>
<td>3</td>
</tr>
</tbody>
</table>
The network data are from LeBlanc, Morlok, and Pierskalla (1975).

The network has 528 O/D pairs, 24 nodes, and 76 links.

The user link cost functions are of Bureau of Public Roads (BPR) form.
The Bureau of Public Roads (BPR) link cost functional form is:

\[ c_a(f_a) = t_a^0 \left[ 1 + k \left( \frac{f_a}{u_a} \right)^\beta \right] \quad \forall a \in L \]

where \( k \) and \( \beta \) are greater than zero and the \( u \)’s are the practical capacities on the links.
Example - Sioux Falls Network Link Importance Rankings
The Advantages of the N-Q Network Efficiency Measure

- The measure captures demands, flows, costs, and behavior of users, in addition to network topology;
- The resulting importance definition of network components is applicable and well-defined even in the case of disconnected networks;
- It can be used to identify the importance (and ranking) of either nodes, or links, or both; and
- It can be applied to assess the efficiency/performance of a wide range of network systems.
- It is applicable also to elastic demand networks (Qiang and Nagurney, Optimization Letters (2008)).
- It has been extended to dynamic networks (Nagurney and Qiang, Netnomics, in press).
The focus of the robustness of networks (and complex networks) has been on the impact of different network measures when facing the removal of nodes on networks.

We focus on the *degradation of links through reductions in their capacities* and the effects on the induced costs in the presence of known demands and different functional forms for the links.
The Time-Dependent (Demand-Varying) Braess Paradox and Evolutionary Variational Inequalities
Recall the Braess Network where we add the link e.
The Solution of an Evolutionary (Time-Dependent) Variational Inequality for the Braess Network with Added Link (Path)
In Demand Regime I, only the new path is used.  
In Demand Regime II, the Addition of a New Link (Path) Makes Everyone Worse Off!  
In Demand Regime III, only the original paths are used.

Network 1 is the Original Braess Network - Network 2 has the added link.
The new link is NEVER used after a certain demand is reached even if the demand approaches infinity.

Hence, in general, except for a limited range of demand, building the new link is a complete waste!
In the paper:


we have extended the performance measure of Nagurney and Qiang to handle disruptions in supply chains under risk and uncertainty.
Contributions of This Research

- Developed a multi-tiered, multi transportation modal supply chain network with interactions among various decision-makers.

- The model captures the supply-side risks together with uncertain demand.

- The mean-variance approach is used to model individual’s attitude towards risks.

- Developed a weighted measure to study the supply chain network performance.
Figure: The Multitiered Network Structure of the Supply Chain
Assumptions

- Manufacturers and retailers are multicriteria decision-makers
- Manufacturers and retailers try to
  - Maximize profit
  - Minimize risk
  - Individual weight assigned to the risk level according to decision maker’s attitude towards risk
- Nash Equilibrium
For each manufacturer $i$, there is a random parameter $\alpha_i$ that reflects the impact of disruption to his production cost function. The expected production cost function is given by:

$$\hat{F}_i(Q^1) \equiv \int f_i(Q^1, \alpha_i) d\mathcal{F}_i(\alpha_i), \quad i = 1, \ldots, m.$$ 

The variance of the above production cost function is denoted by $VF_i(Q^1)$ where $i = 1, \ldots, m$.

We assume that each manufacturer has $g$ types of transportation modes available to ship the product to the retailers, the cost of which is also subject to disruption impacts. The expected transportation cost function is given by:

$$\hat{C}_{ij}^u(q_{ij}^u) \equiv \int c_{ij}^u(q_{ij}^u, \beta_{ij}^u) d\mathcal{F}_{ij}^u(\beta_{ij}^u), \quad i = 1, \ldots, m; \quad j = 1, \ldots, n; \quad u = 1, \ldots, g.$$ 

We further denote the variance of the above transportation cost function as $VC_{ij}^u(Q^1)$ where $i = 1, \ldots, m; \quad j = 1, \ldots, n; \quad u = 1, \ldots, g$. 
Manufacturer’s Maximization Problem

Maximize

\[ \sum_{j=1}^{n} \sum_{u=1}^{g} \rho_{1ij}^{u} q_{ij}^{u} - \hat{F}_i(Q^1) - \sum_{j=1}^{n} \sum_{u=1}^{g} \hat{C}_{ij}^{u}(q_{ij}^{u}) \]

\[ -\theta_i \left[ \sum_{i=1}^{m} VF_i(Q^1) + \sum_{j=1}^{n} \sum_{u=1}^{g} VC_{ij}^{u}(q_{ij}^{u}) \right] \]

Nonnegative weight \( \theta_i \) is assigned to the variance of the cost functions for each manufacturer to reflect his attitude towards disruption risks.

Anna Nagurney
Synergies and Vulnerabilities of Supply Chain Networks
We assume that for each manufacturer, the production cost function and the transaction cost function without disruptions are continuously differentiable and convex. Hence, the optimality conditions for all manufacturers simultaneously (cf. Bazaraa, Sherali, and Shetty (1993) and Nagurney (1999)) can be expressed as the following VI:

**The Optimal Conditions for All Manufacturers**

Determine $Q^{1*} \in R^{mng}_+$ satisfying:

$$
\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{u=1}^{g} \left[ \frac{\partial \hat{F}_i(Q^{1*})}{\partial q_{ij}^u} + \frac{\partial \hat{C}_{ij}^u(q_{ij}^{u*})}{\partial q_{ij}^u} + \theta_i \left( \frac{\partial VF_i(Q^{1*})}{\partial q_{ij}^{u*}} \right) 
\right. \\
\left. + \frac{\partial VC_{ij}^u(q_{ij}^{u*})}{\partial q_{ij}^{u*}} \right] - \rho_{1ij}^{u*} \times [q_{ij}^u - q_{ij}^{u*}] \geq 0, \quad \forall Q^1 \in R^{mng}_+.
$$
A random risk/disruption related random parameter \( \eta_j \) is associated with the handling cost of retailer \( j \). The expected handling cost is:

\[
\hat{C}_j^1(Q^1, Q^2) \equiv \int c_j(Q^1, Q^2, \eta_j) dF_j(\eta_j), \quad j = 1, \ldots, n
\]

The variance of the handling cost function is denoted by \( VC_j^1(Q^1, Q^2) \) where \( j = 1, \ldots, n \).

**Retailer’s Maximization Problem**

The objective function for distributor \( j; j = 1, \ldots, n \) can be expressed as follows:

Maximize

\[
\sum_o \sum_h \rho_{2jk} q_{jk} - \hat{C}_j^1(Q^1, Q^2) - \sum_i \sum_u \rho_{1ij} q_{ij}^u - \omega_j VC_j^1(Q^1, Q^2)
\]

subject to:

\[
\sum_o \sum_h q_{jk} \leq \sum_i \sum_u q_{ij}^u
\]

and the nonnegativity constraints: \( q_{ij}^u \geq 0 \) for all \( i, j, \) and \( u; q_{jk}^v \geq 0 \) for all \( j, k, \) and \( v \).
We assume that, for each retailer, the handling cost without disruptions is continuously differentiable and convex.

### The Optimal Conditions for All Retailers

Determine \((Q_1^*, Q_2^*, \gamma^*) \in R_+^{mng+noh+n}\) satisfying:

\[
\begin{align*}
\sum_{i=1}^m \sum_{j=1}^n \sum_{u=1}^g \left[ \frac{\partial \hat{C}_j^1(Q_1^*, Q_2^*)}{\partial q_{ij}^u} + \rho_{1ij}^u + \omega_j \frac{\partial VC_j^1(Q_1^*, Q_2^*)}{\partial q_{ij}^u} - \gamma_j^* \right] \\
\times \left[ q_{ij}^u - q_{ij}^{u*} \right] + \sum_{j=1}^n \sum_{k=1}^o \sum_{v=1}^h \left[ -\rho_{2jk}^v + \gamma_j^* + \frac{\partial \hat{C}_j^1(Q_1^*, Q_2^*)}{\partial q_{jk}^v} \right] \\
+ \omega_j \frac{\partial VC_j^1(Q_1^*, Q_2^*)}{\partial q_{jk}^v} \right] \times \left[ q_{jk}^v - q_{jk}^{v*} \right] \\
+ \sum_{j=1}^n \left[ \sum_{i=1}^m \sum_{u=1}^g q_{ij}^{u*} - \sum_{k=1}^o \sum_{v=1}^h q_{jk}^{v*} \right] \times \left[ \gamma_j - \gamma_j^* \right] \geq 0, \ \forall (Q_1, Q_2, \gamma) \in R_+^{mng+noh+n}
\end{align*}
\]
The Market Stochastic Economic Equilibrium Conditions

For any retailer with associated demand market $k; k = 1, \ldots, o$:

\[ \hat{d}_k(\rho^*_3) \begin{cases} 
\leq \sum_{j=1}^{o} \sum_{v=1}^{h} q^v_{jk}, & \text{if } \rho^*_3 = 0, \\
= \sum_{j=1}^{o} \sum_{v=1}^{h} q^v_{jk}, & \text{if } \rho^*_3 > 0,
\end{cases} \]

\[ \rho^v_{2jk} + c^v_{jk}(Q^{2*}) \begin{cases} 
\geq \rho^*_3, & \text{if } q^v_{jk} = 0, \\
= \rho^*_3, & \text{if } q^v_{jk} > 0.
\end{cases} \]
The above market equilibrium conditions are equivalent to the following VI problem, after taking the expected value and summing over all retailers/demand markets $k$:

**Equivalent VI Problem**

Determine $(Q^{2*}, \rho_3^*) \in R^{noh+o}_+$ satisfying:

$$
\sum_{k=1}^{o} \left( \sum_{j=1}^{n} \sum_{v=1}^{h} q_{jk}^{v*} - \hat{d}_k(\rho_3^*) \right) \times [\rho_{3k} - \rho_{3k}^{*}] 
$$

$$
+ \sum_{k=1}^{o} \sum_{j=1}^{n} \sum_{v=1}^{h} (\rho_{2jk}^{v} + c_{jk}^{v}(Q^{2*}) - \rho_{3k}^{*}) \times [q_{jk}^{v} - q_{jk}^{v*}] \geq 0, \ \forall \rho_3 \in R^{o}_+, \forall Q^{2} \in R^{noh}_+, 
$$

where $\rho_3$ is the $o$-dimensional vector with components: $\rho_{31}, \ldots, \rho_{3o}$ and $Q^{2}$ is the $noh$-dimensional vector.
Remark:

We are interested in the cases where the expected demands are positive, that is, \( \hat{d}_k(\rho_3) > 0, \forall \rho_3 \in R_+^o \) for \( k = 1, \ldots, o \). Furthermore, we assume that the unit transaction costs: \( c_{jk}^v(Q^2) > 0, \forall j, k, \forall Q^2 \neq 0 \). Under the above assumptions, we can show that \( \rho_{3k}^* > 0 \) and \( \hat{d}_k(\rho_{3}^*) = \sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{v=1}^{h} q_{jk}^v, \forall k \).

Definition: Supply Chain Network Equilibrium with Uncertainty and Expected Demands

The equilibrium state of the supply chain network with disruption risks and expected demands is one where the flows of the product between the tiers of the decision-makers coincide and the flows and prices satisfy the sum of conditions of manufacturers, distributors, and demand markets.
Theorem: VI Formulation of the Supply Chain Network Equilibrium with Uncertainty and Expected Demands

**Determine** \((Q_1^*, Q_2^*, \gamma^*, \rho_3^*) \in R^{mng+n+noh+n+o}_+\) satisfying:

\[
\sum_{i=1}^m \sum_{j=1}^n \sum_{u=1}^g \left[ \frac{\partial \hat{F}_i(Q_1^{1*})}{\partial q_{ij}^u} + \frac{\partial \hat{C}_ij(Q_{1j}^{u*})}{\partial q_{ij}^u} + \theta_i \left( \frac{\partial V_F_i(Q_1^{1*})}{\partial q_{ij}^u} + \frac{\partial V_{C_j}(q_{ij}^{u*})}{\partial q_{ij}^u} \right) \right] \\
+ \frac{\partial \hat{C}_j(Q_1^{1*}, Q_2^{2*})}{\partial q_{ij}^u} + \varpi_j \frac{\partial V_{C_j}(Q_1^{1*}, Q_2^{2*})}{\partial q_{ij}^u} - \gamma_j^* \times [q_{ij}^u - q_{ij}^{u*}] \\
+ \sum_{j=1}^n \sum_{k=1}^o \sum_{v=1}^h \left[ \frac{\partial \hat{C}_j(Q_1^{1*}, Q_2^{2*})}{\partial q_{jk}^v} + \varpi_j \frac{\partial V_{C_j}(Q_1^{1*}, Q_2^{2*})}{\partial q_{jk}^v} \right] \\
+ \gamma_j^* + c_{jk}^v(Q_2^{2*}) - \rho_{3k}^* \times [q_{jk}^v - q_{jk}^{v*}] \\
+ \sum_{j=1}^n \left[ \sum_{l=1}^m \sum_{u=1}^g q_{lj}^{u*} - \sum_{k=1}^o \sum_{v=1}^h q_{jk}^{v*} \right] \times \left[ \gamma_j - \gamma_j^* \right] + \sum_{j=1}^o \sum_{k=1}^n \sum_{v=1}^h q_{jk}^{v*} - d_k(\rho_3^*) \times [\rho_{3k} - \rho_{3k}^*] \geq 0,
\]

\[
\forall (Q_1^1, Q_2^2, \gamma, \rho_3) \in R^{mng+n+noh+n+o}_+.
\]

where \(\mathcal{K} \equiv \{(Q_1^1, Q_2^2, \gamma, \rho_3) \parallel (Q_1^1, Q_2^2, \gamma, \rho_3) \in R^{mng+n+noh+n+o}_+\}.\)
A Supply Chain Network Performance Measure

The supply chain network performance measure, $\mathcal{E}$, for a given supply chain, and expected demands: $\hat{d}_k$; $k = 1, 2, \ldots, o$, is defined as follows:

$$
\mathcal{E} \equiv \frac{\sum_{k=1}^{o} \frac{\hat{d}_k}{\rho_{3k}}}{o},
$$

where $o$ is the number of demand markets in the supply chain network, and $\hat{d}_k$ and $\rho_{3k}$ denote, respectively, the expected equilibrium demand and the equilibrium price at demand market $k$. 
Assume that all the random parameters take on a given threshold probability value; say, for example, 95%. Moreover, assume that all the cumulative distribution functions for random parameters have inverse functions. Hence, we have that: \( \alpha_i = \mathcal{F}_{i}^{-1}(0.95) \), for \( i = 1, \ldots, m \); 
\( \beta_{ij}^u = \mathcal{F}_{ij}^{u^{-1}}(0.95) \), for \( i = 1, \ldots, m; j = 1, \ldots, n \), and so on.

Supply Chain Robustness Measurement

Let \( \mathcal{E}_w \) denote the supply chain performance measure with random parameters fixed at a certain level as described above. Then, the supply chain network robustness measure, \( \mathcal{R} \), is given by the following:

\[
\mathcal{R} = \mathcal{E}^0 - \mathcal{E}_w,
\]

where \( \mathcal{E}^0 \) gauges the supply chain performance based on the supply chain model, but with weights related to risks being zero.

\( \mathcal{E}^0 \) examines the “base” supply chain performance while \( \mathcal{E}_w \) assesses the supply chain performance measure at some prespecified uncertainty level. If their difference is small, a supply chain maintains its functionality well and we consider the supply chain to be robust.
Where Are We Now?

Empirical Case Study

• New England electric power market and fuel markets
• 82 generators who own and operate 573 power plants
• 5 types of fuels: natural gas, residual fuel oil, distillate fuel oil, jet fuel, and coal
• Hourly demand/price data of July 2006 (24 × 31 = 744 scenarios)
• 6 blocks (L1 = 94 hours, and Lw = 130 hours; w = 2, ..., 6)
The New England Electric Power Supply Chain Network with Fuel Suppliers
Predicted Prices vs. Actual Prices ($/Mwh)
Fragile Networks: Identifying Vulnerabilities and Synergies in an Uncertain World

Anna Nagurney and Qiang (Patrick) Qiang

Wiley & Sons

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Today, supply chains are more extended and complex than ever before. At the same time, the current competitive economic environment requires that firms operate efficiently, which has spurred research to determine how to utilize supply chains more effectively.

There is also a pronounced amount of merger activity. According to Thomson Financial, in the first nine months of 2007 alone, worldwide merger activity hit $3.6 trillion, surpassing the total from all of 2006 combined.

Notable examples: K-Mart and Sears in the retail industry in 2004 and Federated and May in 2005, Coors and Molson in the beverage industry in 2005, and the recently proposed merger between Anheuser Busch and InBev.
According to Kusstatscher and Cooper (2005) there were five major waves of Merger & Acquisition (M&A) activity:

The First Wave: 1898-1902: an increase in horizontal mergers that resulted in many US industrial groups;

The Second Wave: 1926-1939: mainly public utilities;

The Third Wave: 1969-1969: diversification was the driving force;

The Fourth Wave: 1983-1986: the goal was efficiency;

The Fifth Wave: 1997 until the early years of the 21st century: globalization was the motto.

In 1998, M&As reached $2.1 trillion worldwide; in 1999, the activity exceeded $3.3 trillion, and in 2000, almost $3.5 was reached.
A survey of 600 executives involved in their companies’ mergers and acquisitions (M&A) conducted by Accenture and the Economist Unit (see Byrne (2007)) found that less than half (45%) achieved expected cost-saving synergies.

Langabeer and Seifert (2003) determined a direct correlation between how effectively supply chains of merged firms are integrated and how successful the merger is. They concluded, based on the empirical findings of Langabeer (2003), who analyzed hundreds of mergers over the preceding decade, that

**Improving Supply Chain Integration between Merging Companies is the Key to Improving the Likelihood of Post-Merger Success!**
Recently, we introduced a system-optimization perspective for supply chains in which firms are engaged in multiple activities of production, storage, and distribution to the demand markets and proposed a cost synergy measure associated with evaluating proposed mergers:


In that paper, the merger of two firms was modeled and the demands for the product at the markets, which were distinct for each firm prior to the merger, were assumed to be fixed.
Figure 1: Case 0: Firms A and B Prior to Horizontal Merger (Nagurney (2009))
Figure 2: Case 1: Firms $A$ and $B$ Merge (Nagurney (2009))
Figure 3: Case 2: Firms A and B Merge (Nagurney (2009))
Figure 4: Case 3: Firms A and B Merge (Nagurney (2009))
Synergy Measure

The measure that we utilized in Nagurney (2009) to capture the gains, if any, associated with a horizontal merger Case $i; i = 1, 2, 3$ is as follows:

$$S^i = \left[ \frac{TC^0 - TC^i}{TC^0} \right] \times 100\%,$$

where $TC^i$ is the total cost associated with the value of the objective function $\sum_{a \in L_i} \hat{c}_a(f_a)$ for $i = 0, 1, 2, 3$ evaluated at the optimal solution for Case $i$. Note that $S^i; i = 1, 2, 3$ may also be interpreted as synergy.
This framework can also be applied to teaming of humanitarian organizations in the case of humanitarian logistics operations.
The Supply Chain Network Oligopoly Model (Nagurney (2008b))

Figure 5: Supply Chain Network Structure of the Oligopoly
Assume that the profit functions are concave and continuously differentiable.

We consider the usual oligopolistic market mechanism in which the \( I \) firms produce and distribute the product in a noncooperative manner, each one trying to maximize its own profit. We seek to determine a nonnegative path flow pattern \( x \) for which the \( I \) firms will be in a state of equilibrium as defined below.

**Definition: Supply Chain Network Cournot-Nash Equilibrium**

A product flow pattern \( x^* \in R_+^{nP_0} \) is said to constitute a supply chain network Cournot-Nash equilibrium if for each firm \( i; \ i = 1, \ldots, I \):

\[
 u_i(x^*_i, \hat{x}^*_i) \geq u_i(x_i^*, \hat{x}^*_i), \quad \forall x_i \in R_+^{nP_0_i},
\]

where \( x_i \equiv \{x_p|p \in P_0^i\} \) and \( \hat{x}^*_i \equiv (x_1^*, \ldots, x_{i-1}^*, x_{i+1}^*, \ldots, x_I^*) \).
Theorem: Variational Inequality Formulation

Assume that for each firm \( i; i = 1, \ldots, I \), the profit function \( u_i(x) \) is concave with respect to the variables \( x_p; p \in P_i^0 \), and is continuously differentiable. Then \( x^* \in R_{+}^{nP0} \) is a supply chain network Cournot-Nash equilibrium if and only if it satisfies the variational inequality:

\[
- \sum_{i=1}^{I} \sum_{p \in P_i^0} \frac{\partial u_i(x^*)}{\partial x_p} \times (x_p - x_p^*) \geq 0, \quad \forall x \in R_{+}^{nP0},
\]
or, equivalently: determine \( x^* \in \mathcal{K}^0 \) satisfying:

\[
\sum_{i=1}^{I} \sum_{k=1}^{n_{R}} \sum_{p \in P_{R_k}^{0}} \left[ \frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \rho_{R_k}(x^*) - \sum_{l=1}^{n_{R}} \frac{\partial \rho_{R_l}(x^*)}{\partial d_{R_k}} \sum_{p \in P_{R_k}^{0}} x^*_p \right] \\
\times [x_p - x^*_p] \geq 0, \; \forall x \in \mathcal{K}^0,
\]

where \( \mathcal{K}^0 \equiv \{x | x \in \mathbb{R}_+^{nP_0} \} \) and \( \frac{\partial \hat{C}_p(x)}{\partial x_p} \equiv \sum_{b \in L_i^0} \sum_{a \in L_i^0} \frac{\partial \hat{c}_b(f)}{\partial f_a} \delta_{a p} \) for paths \( p \in P_i^0 \).

Proof: Follows directly from Gabay and Moulin (1982) and Dafermos and Nagurney (1987). Here we have also utilized the fact that the demand price functions can be reexpressed directly as a function of path flows.
It is interesting to relate this supply chain network oligopoly model to the spatial oligopoly model proposed by Dafermos and Nagurney (1987), which is done in the following corollary.

**Corollary: Relationship to the Spatial Oligopoly Model**

Assume that there are $I$ firms in the supply chain network oligopoly model and that each firm has a single manufacturing plant and a single distribution center. Assume also that the distribution costs from each manufacturing plant to the distribution center and the storage costs are all equal to zero. Then the resulting model is isomorphic to the spatial oligopoly model of Dafermos and Nagurney (1987) whose underlying network structure is given in Figure 6.

Figure 6: Network Structure of the Spatial Oligopoly

Anna Nagurney
Synergies and Vulnerabilities of Supply Chain Networks
The relationship between the supply chain network oligopoly model to the classical Cournot (1838) oligopoly model is now given (see also Gabay and Moulin (1982) and Nagurney (1993)).

Corollary: Relationship to Classical Oligopoly Model

Assume that there is a single manufacturing plant associated with each firm in the above model, and a single distribution center. Assume also that there is a single demand market. Assume also that the manufacturing cost of each manufacturing firm depends only upon its own output. Then, if the storage and distribution cost functions are all identically equal to zero the above model collapses to the classical oligopoly model in quantity variables. Furthermore, if \( I = 2 \), one then obtains the classical duopoly model.
Figure 7: Network Structure of the Classical Oligopoly
Mergers Through Coalition Formation

Figure 6: Mergers of the First $n_{1'}$ Firms and the Next $n_{2'}$ Firms
In this talk we have demonstrated the richness of network concepts for quantifying synergies as well as vulnerabilities associated with supply chain networks in the global economy.

The need for performance metrics as well as analytics has never been more profound nor more feasible.

By focusing on interdisciplinary research and practice we can better identify which nodes and links in supply chain networks truly matter and whether or not to participate in any merger based on potential and predetermined synergies.
Thank You!

The Virtual Center for Supernetworks

The Virtual Center for Supernetworks at the Isenberg School of Management, under the directorship of Anna Nagurney, the John F. Smith Memorial Professor, is an interdisciplinary center, and includes the Supernetworks Laboratory for Computation and Visualization.

Mission: The mission of the Virtual Center for Supernetworks is to foster the study and application of supernetworks and to serve as a resource to academia, industry, and government on networks ranging from transportation, supply chains, telecommunication, and electric power networks to economic, environmental, financial, knowledge and social networks.

The Applications of Supernetworks Include: multimodal transportation networks, critical infrastructure, energy and the environment, the Internet and electronic commerce, global supply chain management, international financial networks, web-based advertising, complete networks and decision-making, integrated social and economic networks, network games, and network metrics.

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