Synergies and Vulnerabilities of Supply Chain Networks in a Global Economy

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Outline

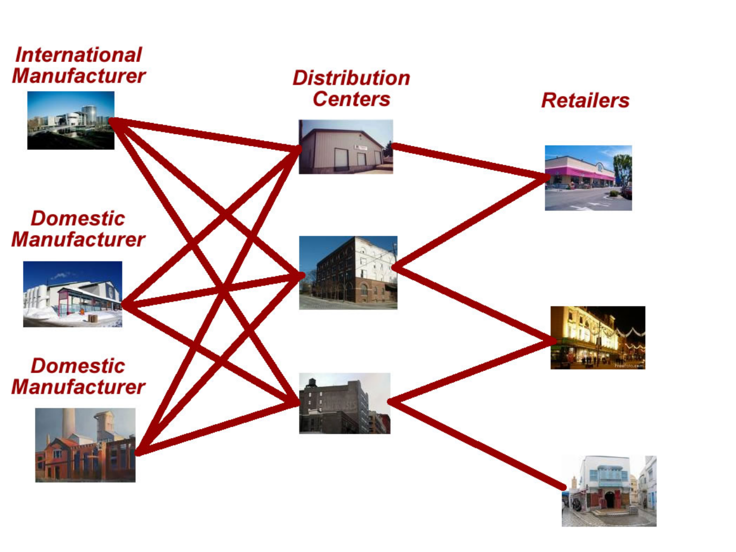
- Today's Supply Chain Networks
- Major Recent Supply Chain Disruptions
- Motivation for Our Research
- Network Vulnerabilities and Supply Chain Networks
- Mergers and Acquisitions and Supply Chain Network Synergies
- Summary and Conclusions

Today's Supply Chain Networks

Supply chain networks are the underpinning skeletons of the business world. These networks, more and more, are global in nature, with products consisting of parts manufactured in different regions of the world, assembled in yet other locations, and then shipped across continents and oceans to retailers and consumers.

Such complex networks consist of manufacturers (and their suppliers), shippers and carriers using various modes of transportation, distribution centers where the products are stored, and, ultimately, sent from to the customers.

Supply chains involve many decision-makers interacting with one another, sometimes competing, and at other times necessarily cooperating.



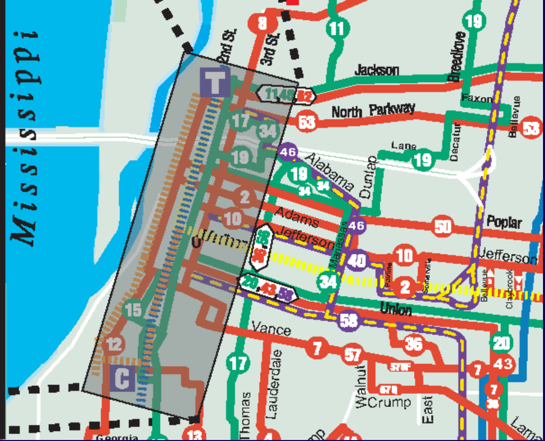
Depiction of a Supply Chain Network

Supply chain networks depend on infrastructure networks for their effective and efficient operations from: manufacturing and logistical networks, to transportation networks, to electric power networks, financial networks, and telecommunication networks, most, notably, the Internet.

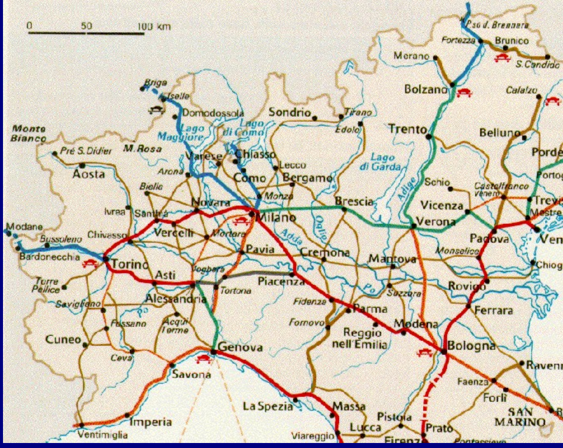
No supply chain, logistics system, or infrastructure system is immune to disruptions and as long as there have been supply chains there have been disruptions.

However, in the past decade there have been vivid high-profile examples of supply chain disruptions and their impacts. Supply chain disruptions and the associated risk are major topics now in theoretical and applied research, as well as in practice, since risk in the context of supply chains may be associated with the production/procurement processes, the transportation/shipment of the goods, and/or the demand markets.

Transportation, Communication, and Energy Networks



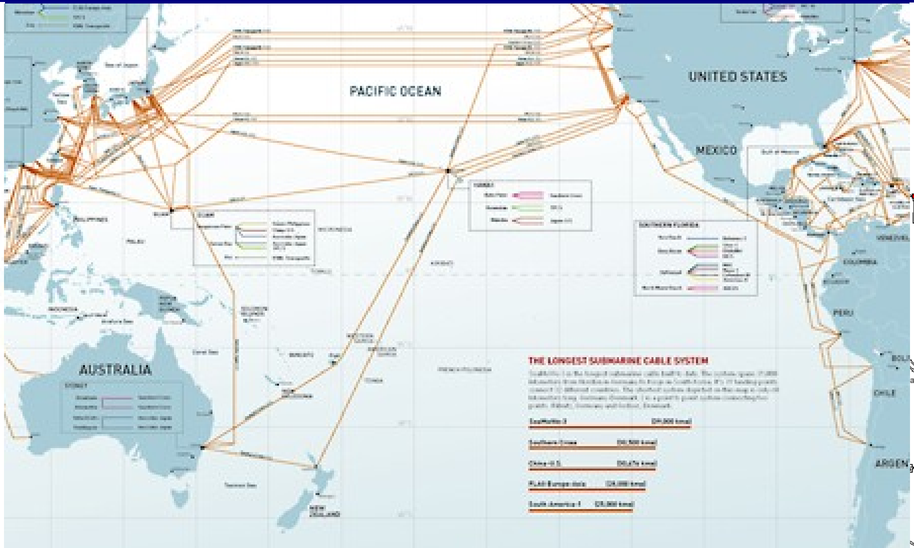
Bus Network



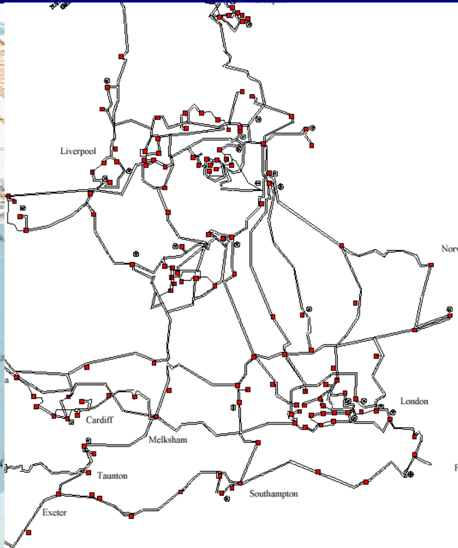
Rail Network

Iridium Satellite Constellation Network

Satellite and Undersea Cable Networks



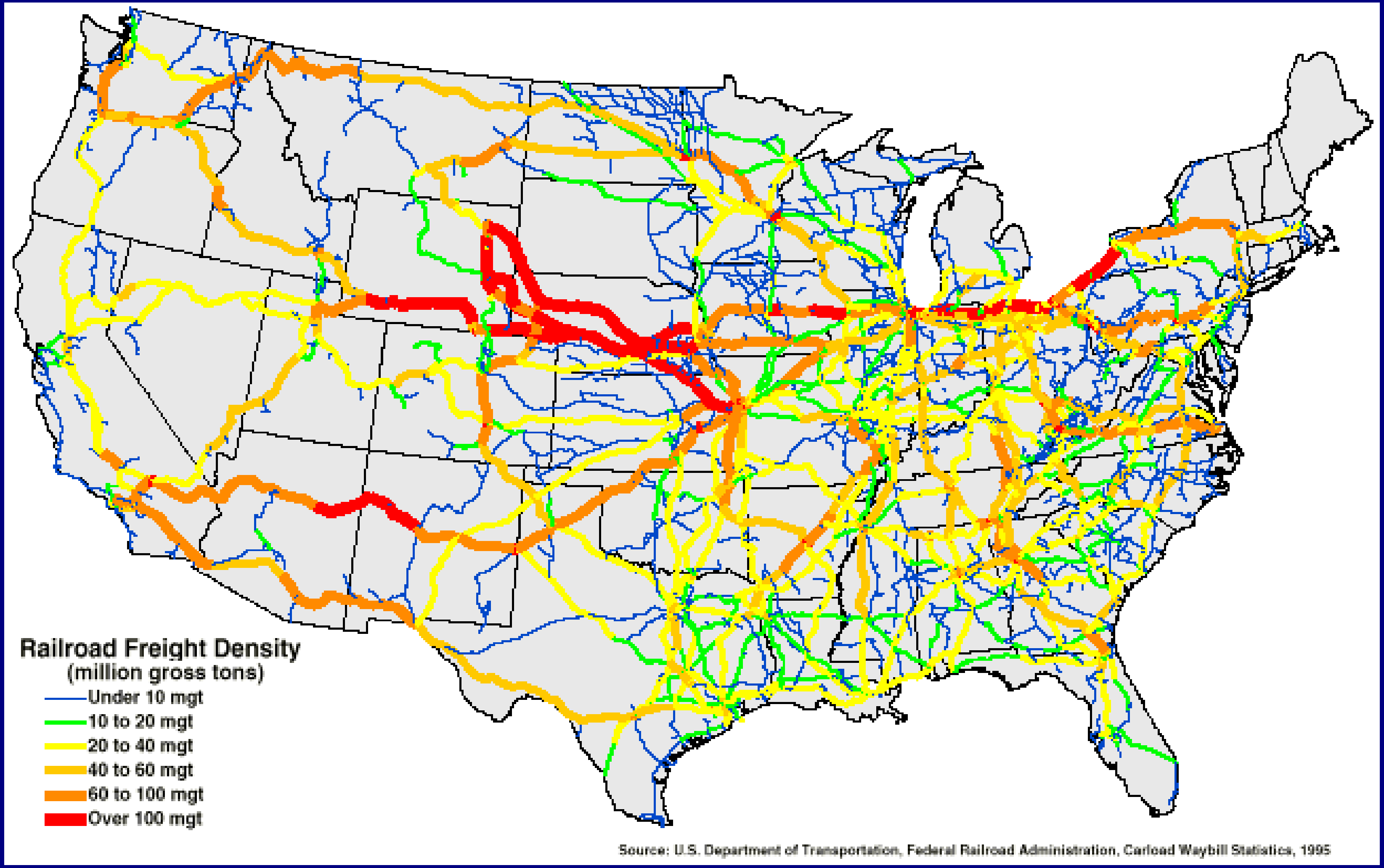
British Electricity Grid



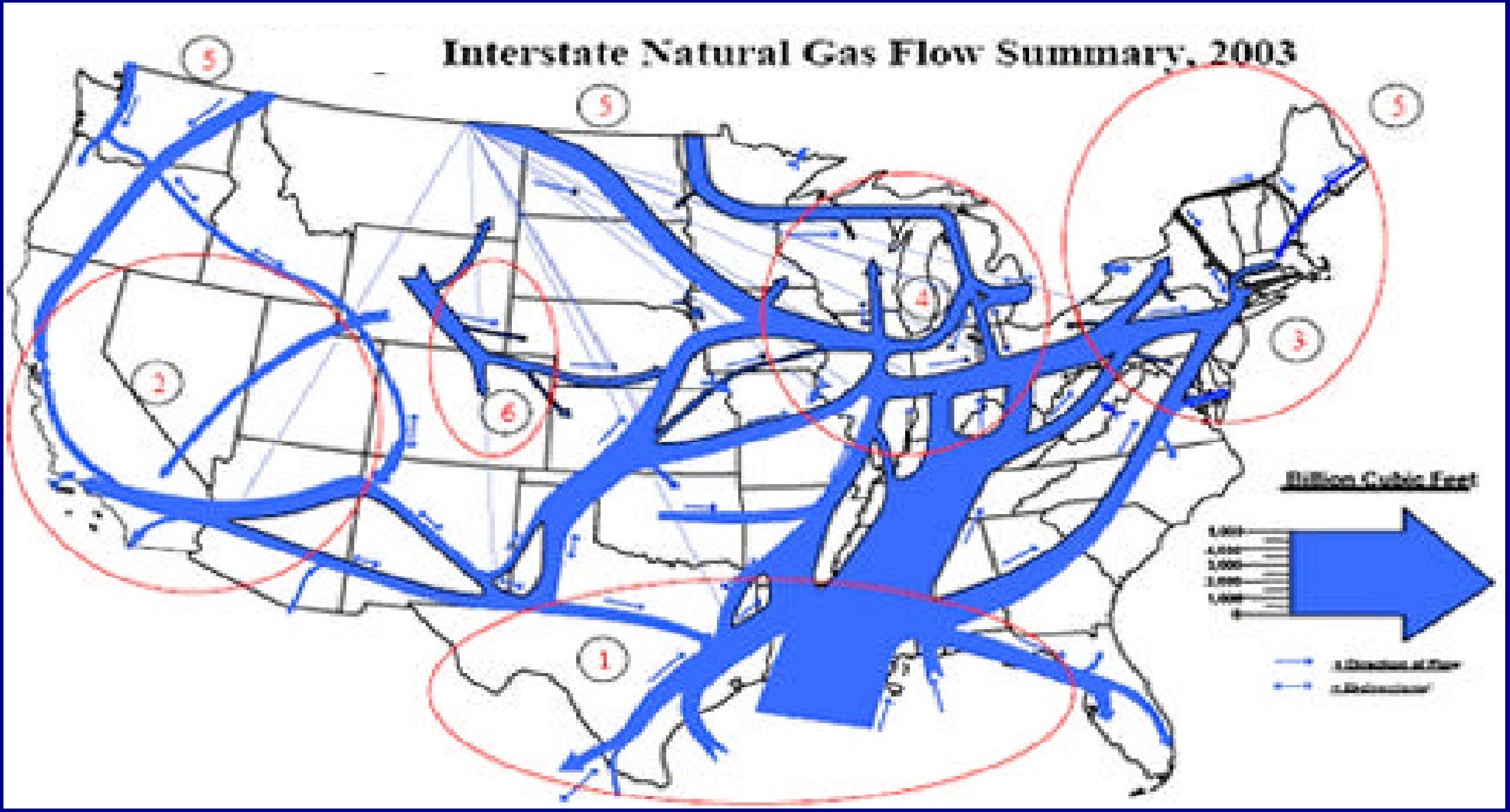
Components of Common Networks

Network System	Nodes	Links	Flows
Transportation	Intersections, Homes, Workplaces, Airports, Railyards	Roads, Airline Routes, Railroad Track	Automobiles, Trains, and Planes,
Manufacturing and logistics	Workstations, Distribution Points	Processing, Shipment	Components, Finished Goods
Communication	Computers, Satellites, Telephone Exchanges	Fiber Optic Cables Radio Links	Voice, Data, Video
Energy	Pumping Stations, Plants	Pipelines, Transmission Lines	Water, Gas, Oil, Electricity

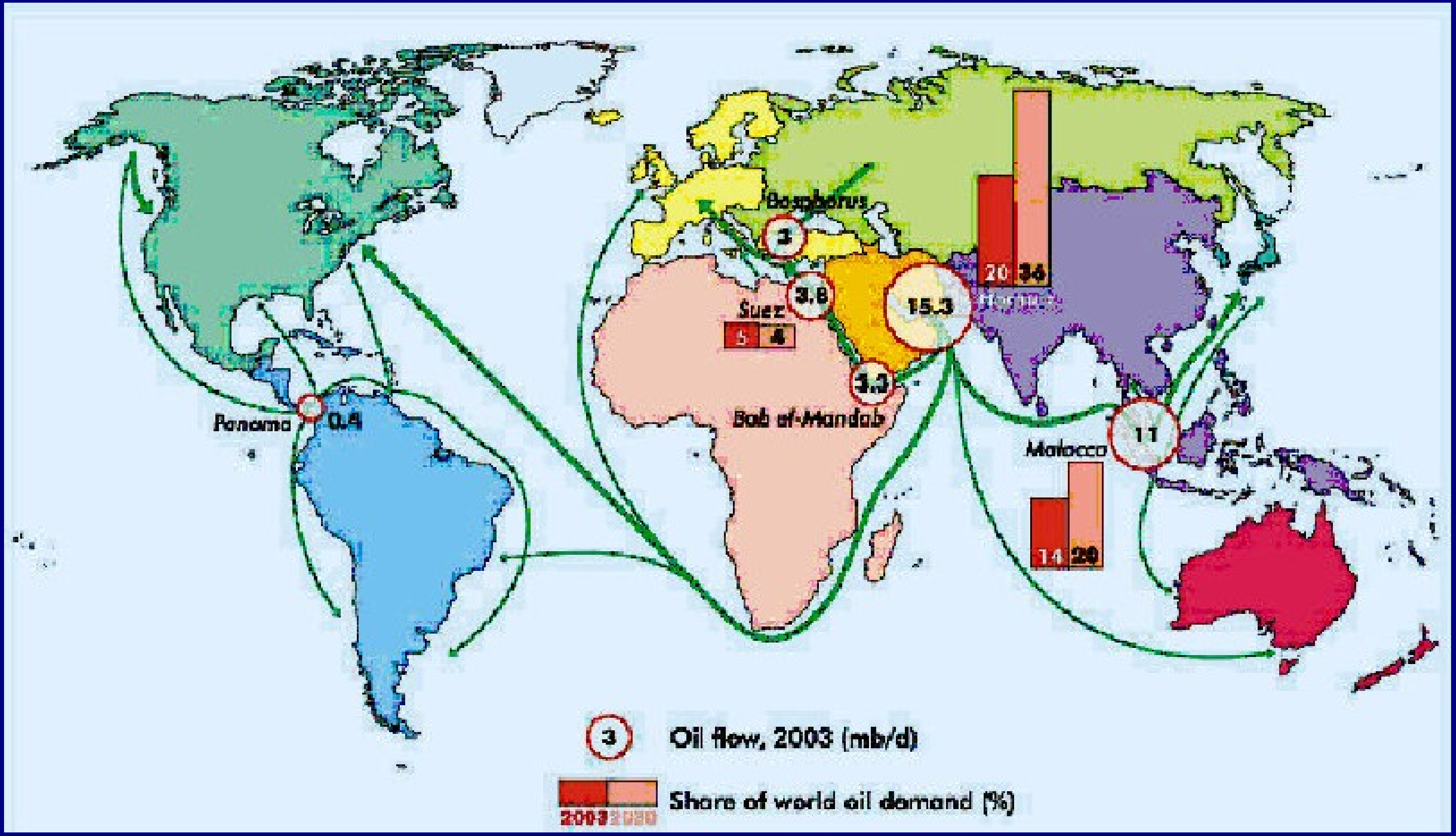
US Railroad Freight Flows



Natural Gas Pipeline Network in the US



World Oil Trading Network



Moreover, in recent decades, the focus has been on lean supply chains and, although such supply chains may work well when the environment is predictable and steady, they may be very sensitive to disruptions since they lack redundancy and slack in their systems.

Furthermore, firms today may be much less vertically integrated (and, clearly, more global). Decades ago, Ford Motor Company and other automobile manufacturers and even IBM produced their products, essentially in their entirety.

Lynn (2006) has argued that globalization has led to extremely fragile supply chains. Suppliers today may be in parts of the world that are unstable and subject to natural disasters, political instability, and strife.

In fact, Craighead, Blackhurst, Rungtusanatham, and Handfield (2007) have argued that supply chain disruptions and the associated operational and financial risks are the most pressing issue faced by firms in today's competitive global environment.

Notably, the focus of research has been on "demand-side" risk, which is related to fluctuations in the demand for products, as opposed to the "supply-side" risk, which deals with uncertain conditions that affect the production and transportation processes of the supply chain.

Major Recent Supply Chain Disruptions

Several recent major disruptions:

- In March 2000, a lightning bolt struck a Philips Semiconductor plant in Albuquerque, New Mexico, and created a 10-minute fire that resulted in the contamination of millions of computer chips and subsequent delaying of deliveries to its two largest customers: Finland's Nokia and Sweden's Ericsson.
- Ericsson used the Philips plant as its sole source and reported a $400 million loss because it did not receive the chip deliveries in a timely manner whereas Nokia moved quickly to tie up spare capacity at other Philips plants and refitted some of its phones so that it could use chips from other US suppliers and from Japanese suppliers.
- Nokia managed to arrange alternative supplies and, therefore, mitigated the impact of the disruption.
- Ericsson learned a painful lesson from this disaster.

- The West Coast port lockout in 2002, which resulted in a 10 day shutdown of ports in early October, typically, the busiest month. 42% of the US trade products and 52% of the imported apparel go through these ports, including Los Angeles. Estimated losses were one billion dollars per day.

The economic and financial troubles of the automobile companies in the United States among the “Big Three” are creating a domino effect throughout the supply chain and the vast network of auto supplier firms. For example, GM alone has approximately 2,000 suppliers, whereas Ford has about 1,600 suppliers, and Chrysler about 900 suppliers. Although Ford is in better shape in terms of the cash the company has, it shares most of the same big parts suppliers, so a disruption in the supply chain that a bankruptcy would invariably cause would hurt Ford too, and even halt production temporarily.

As summarized by Sheffi (2005), one of the main characteristics of disruptions in supply networks is "the seemingly unrelated consequences and vulnerabilities stemming from global connectivity."

Indeed, supply chain disruptions may have impacts that propagate not only locally but globally and, hence, a *holistic, system-wide approach* to supply chain network modeling and analysis is essential in order to be able to capture the complex interactions among decision-makers.

Disasters in Transportation Networks

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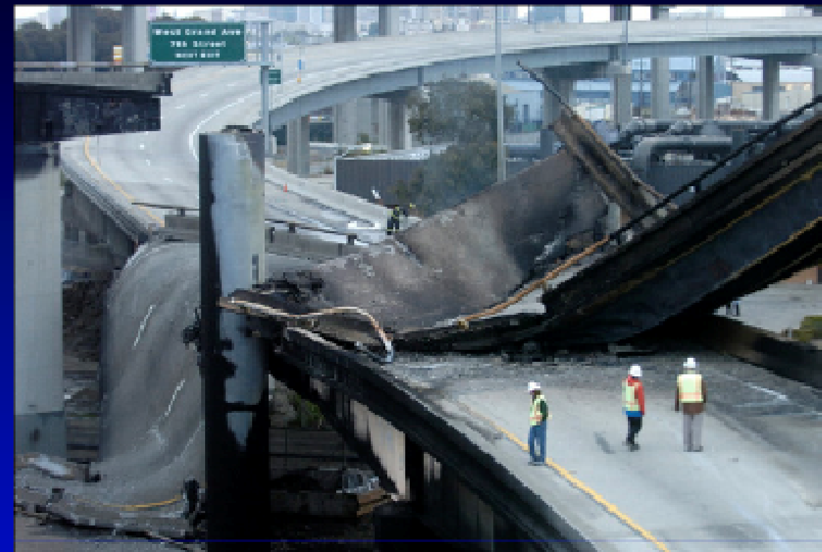

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Disasters in Electric Power Networks

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Disasters in Communication Networks

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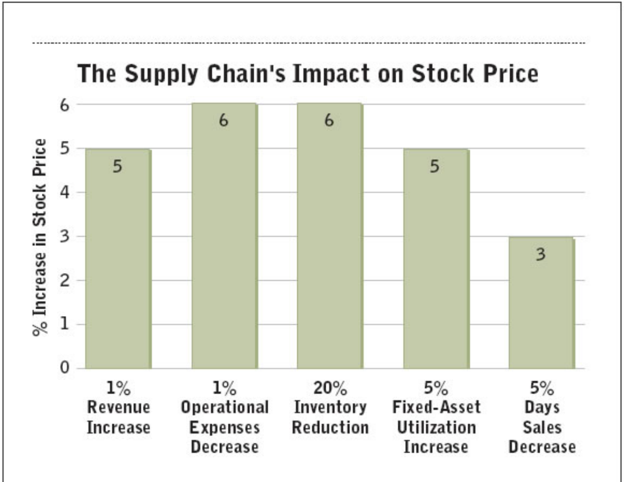
Motivation for Our Research

Hence, the rigorous modeling and analysis of supply chain networks, in the presence of possible disruptions is imperative since disruptions may have lasting major financial consequences.

Hendricks and Singhal (2005) analyzed 800 instances of supply chain disruptions experienced by firms whose stocks are publicly traded. They found that the companies that suffered supply chain disruptions experienced share price returns 33 percent to 40 percent lower than the industry and the general market benchmarks. Furthermore, share price volatility was 13.5 percent higher in these companies in the year following a disruption than in the prior year.

A company that experiences a supply chain disruption can expect to experience significant decreases in sales growth, stock return, and shareholder wealth for two years or more following the incident (Hendricks and Singhal (2003, 2005)). It is evident that only well-prepared companies can effectively cope with supply chain disruptions.



Source: S&P 500 Survey 2002

SUPPLY CHAIN MANAGEMENT REVIEW · MARCH 2005

Motivation for Our Research

The goal of supply chain risk management is to alleviate the consequences of disruptions and risks or, simply put, to increase the *robustness* of a supply chain. However, there are very few quantitative models for measuring supply chain robustness.

Snyder and Daskin (2005) examined supply chain disruptions in the context of facility location. The objective of their model was to select locations for warehouses and other facilities that minimize the transportation costs to customers and, at the same time, account for possible closures of facilities that would result in re-routing of the product. However, as commented in Snyder and Shen (2006), "Although these are multi-location models, they focus primarily on the local effects of disruptions."

To-date, most supply disruption studies have focused on a local point of view, in the form of a single-supplier problem (see, e. g., Gupta (1996) and Parlar (1997)) or a two-supplier problem (see, e. g., Parlar and Perry (1996)).

Very few studies/papers have examined supply chain risk management in an environment with multiple decision-makers and in the case of uncertain demands (cf. Tomlin (2006)).

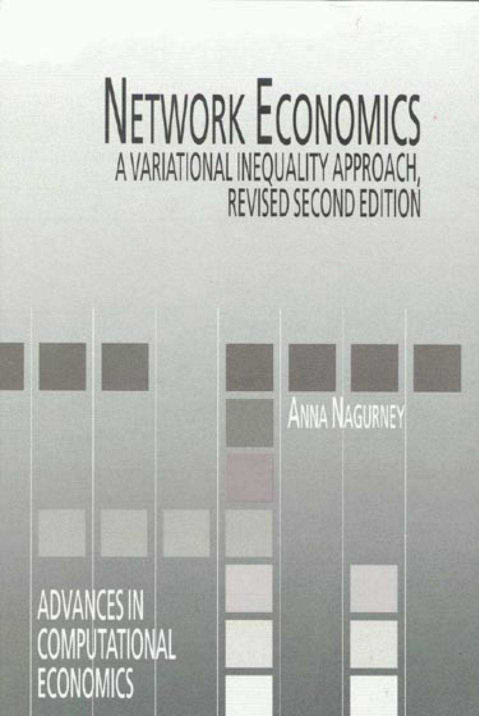
Characteristics of Networks Today Including Supply Chains

- *Large-scale nature* and complexity of network topology;

- *Congestion*; in the US we are experiencing a freight capacity crisis;

- the *interactions among networks* themselves such as in transportation versus telecommunications;

- *Dynamics* and global reach; increasing risk and uncertainty.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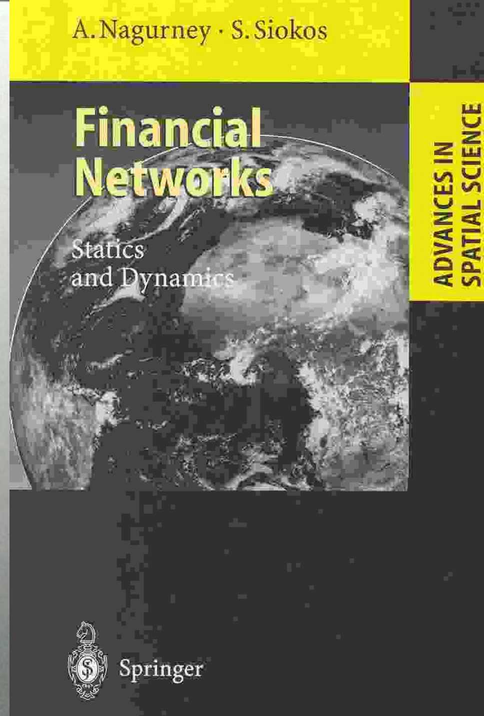
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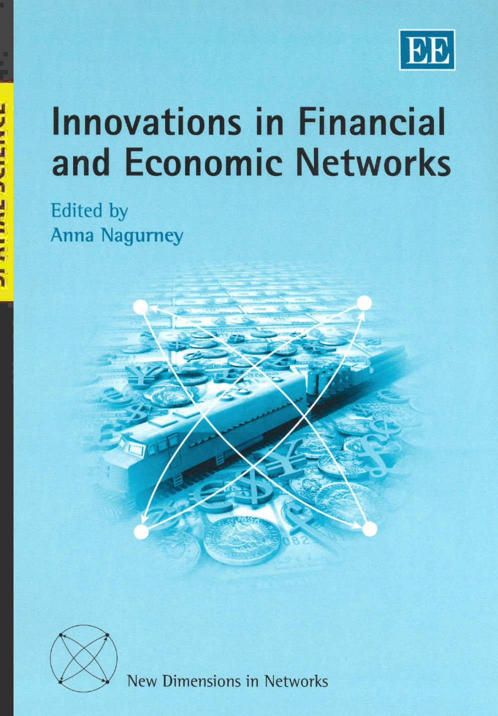
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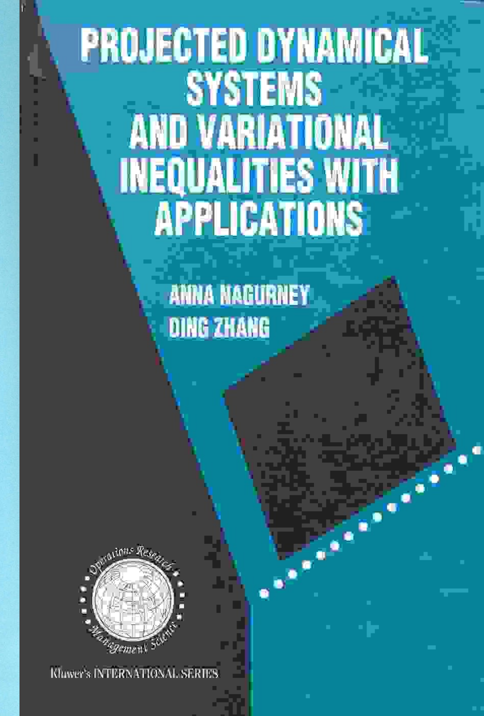
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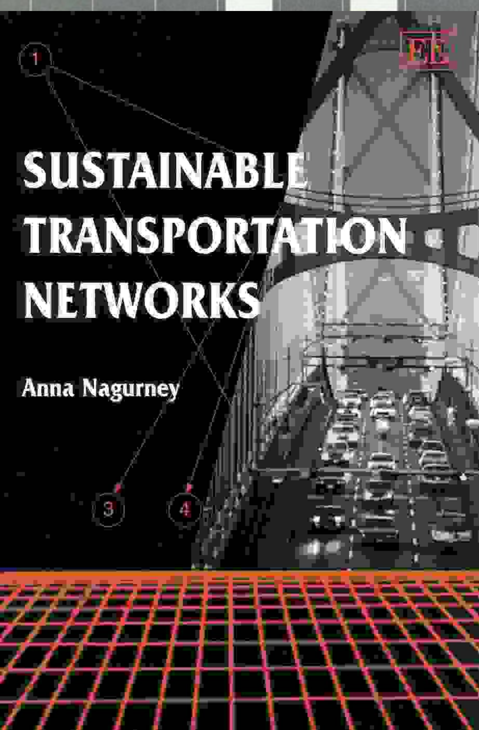
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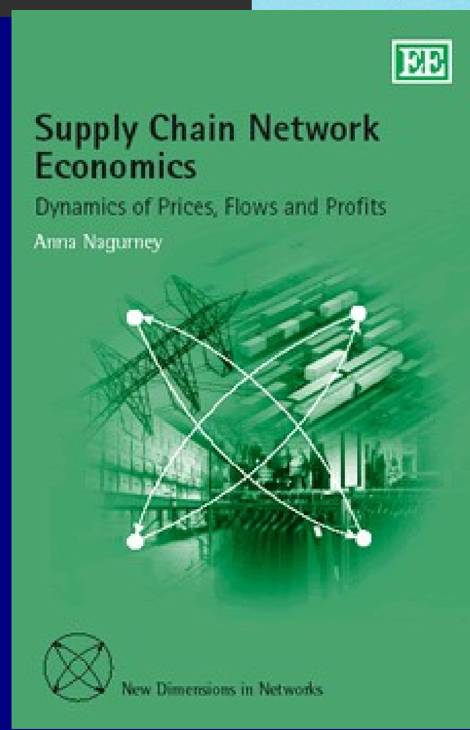
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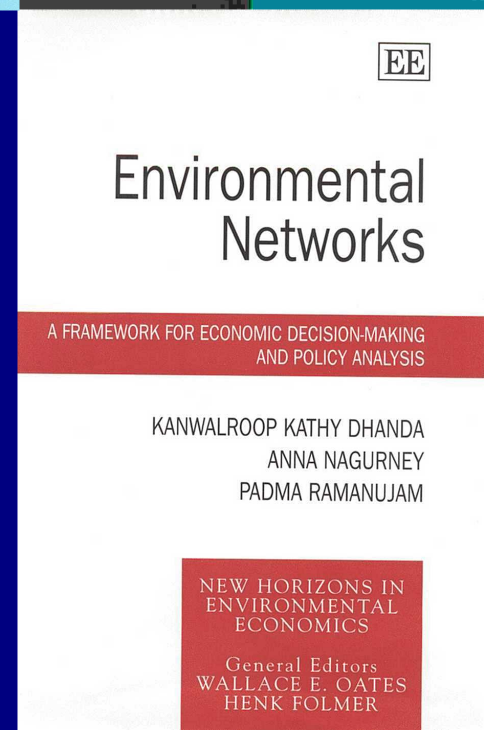
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- *alternative behaviors of the users of the network*

  – System-optimized (S-O) (centralized supply chain) versus

  – user-optimized (U-O) (decentralized supply chain),

which may lead to paradoxical phenomena (Braess Paradox and the Merger Paradox).

Network Equilibrium Problem Derived from Transportation

(U-O Problem)

Transportation Network Equilibrium User-Optimization (U-O) Problem

Consider a general network $G = [N, L]$, where N denotes the set of nodes, and L the set of directed links. Let a denote a link of the network connecting a pair of nodes, and let p denote a path consisting of a sequence of links connecting an O/D pair. P_w denotes the set of paths, assumed to be acyclic, connecting the O/D pair of nodes w and P the set of all paths.

Let x_p represent the flow on path p and f_a the flow on link a. The following conservation of flow equation must hold:

$$f_a = \sum_{p \in P} x_p \delta_{ap},$$

where $\delta_{ap} = 1$, if link a is contained in path p, and 0, otherwise. This expression states that the load on a link a is equal to the sum of all the path flows on paths p that contain (traverse) link a.

Moreover, if we let d_w denote the demand associated with O/D pair w, then we must have that

$$d_w = \sum_{p \in P_w} x_p,$$

where $x_p \geq 0$, $\forall p$, that is, the sum of all the path flows between an origin/destination pair w must be equal to the given demand d_w.

Let c_a denote the user cost associated with traversing link a, which is assumed to be continuous, and C_p the user cost associated with traversing the path p. Then

$$C_p = \sum_{a \in L} c_a \delta_{ap}.$$

In other words, the cost of a path is equal to the sum of the costs on the links comprising the path.

Network Equilibrium

The network equilibrium conditions are then given by: For each path $p \in P_w$ and every O/D pair w:

$$C_p \begin{cases} = \lambda_w, & \text{if} \quad x_p^* > 0 \\ \geq \lambda_w, & \text{if} \quad x_p^* = 0 \end{cases}$$

where λ_w is an indicator, whose value is not known a priori. These equilibrium conditions state that the user costs on all used paths connecting a given O/D pair will be minimal and equalized.

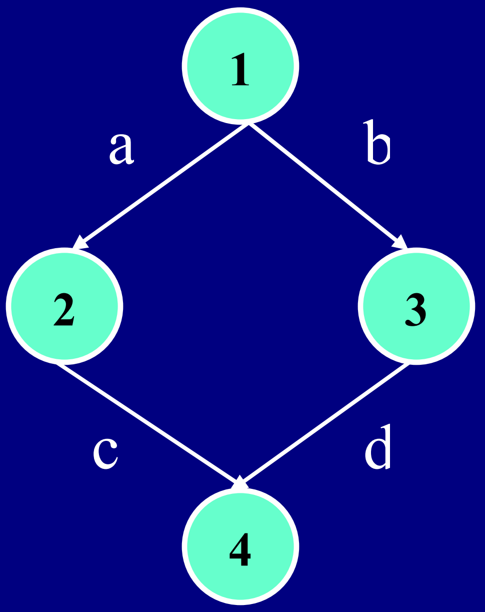
The Braess (1968) Paradox

Assume a network with a single O/D pair (1,4). There are 2 paths available to travelers: $\mathbf{p_1=(a,c)}$ and $\mathbf{p_2=(b,d)}$.

For a travel demand of **6,** the equilibrium path flows are $\mathbf{x_{p_1}^* = x_{p_2}^* = 3}$ and

The equilibrium path travel cost is

$\mathbf{C_{p_1} = C_{p_2} = 83}$.



$c_a(f_a)=10\ f_a \qquad c_b(f_b) = f_b+50$

$c_c(f_c) = f_c+50 \qquad c_d(f_d) = 10\ f_d$

Adding a Link Increases Travel Cost for All!

Adding a new link creates a new path **p_3=(a,e,d).**

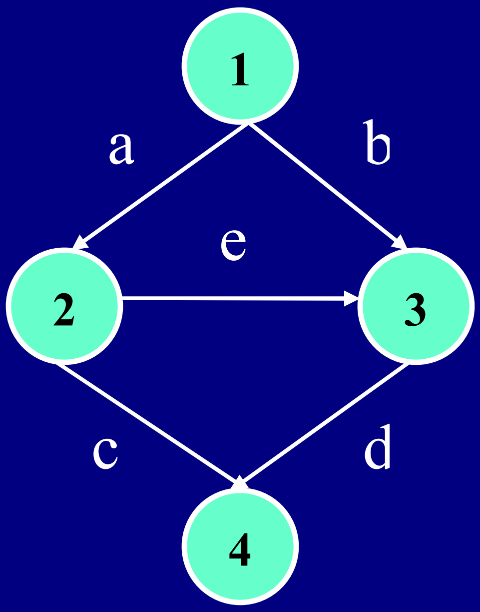
The original flow distribution pattern is no longer an equilibrium pattern, since at this level of flow the cost on path **p_3, C_{p_3}=70.**

The new equilibrium flow pattern network is

$$x_{p_1}^* = x_{p_2}^* = x_{p_3}^* = 2.$$

The equilibrium path travel costs:

$$C_{p_1} = C_{p_2} = C_{p_3} = 92.$$



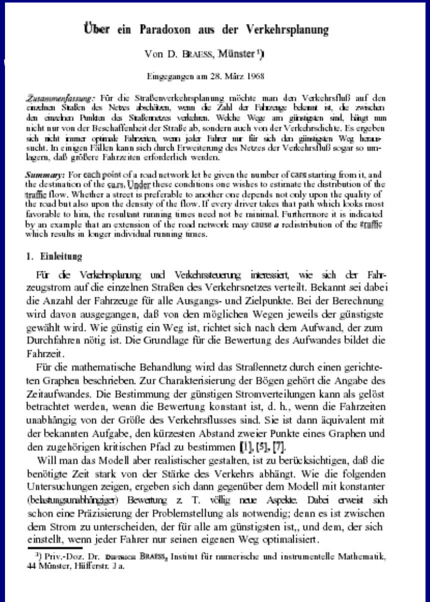
$c_e(f_e) = f_e + 10$

The 1968 Braess article has been translated from German to English and appears as

On a Paradox of Traffic Planning

by Braess, Nagurney, Wakolbinger

in the November 2005 issue of *Transportation Science*



Über ein Paradoxon aus der Verkehrsplanung

Von D. BRAESS, Münster [1])

Eingegangen am 28. März 1968

Zusammenfassung: Für die Straßenverkehrsplanung möchte man den Verkehrsfluß auf den einzelnen Straßen des Netzes abschätzen, wenn die Zahl der Fahrzeuge bekannt ist, die zwischen den einzelnen Punkten des Straßennetzes verkehren. Welche Wege am günstigsten sind, hängt nun nicht nur von der Beschaffenheit der Straße ab, sondern auch von der Verkehrsdichte. Es ergeben sich nicht immer optimale Fahrzeiten, wenn jeder Fahrer nur für sich den günstigsten Weg heraussucht. In einigen Fällen kann sich durch Erweiterung des Netzes der Verkehrsfluß sogar so umlagern, daß größere Fahrzeiten erforderlich werden.

Summary: For each point of a road network let be given the number of cars starting from it, and the destination of the cars. Under these conditions one wishes to estimate the distribution of the traffic flow. Whether a street is preferable to another one depends not only upon the quality of the road but also upon the density of the flow. If every driver takes that path which looks most favorable to him, the resultant running times need not be minimal. Furthermore it is indicated by an example that an extension of the road network may cause a redistribution of the traffic which results in longer individual running times.

1. Einleitung

Für die Verkehrsplanung und Verkehrssteuerung interessiert, wie sich der Fahrzeugstrom auf die einzelnen Straßen des Verkehrsnetzes verteilt. Bekannt sei dabei die Anzahl der Fahrzeuge für alle Ausgangs- und Zielpunkte. Bei der Berechnung wird davon ausgegangen, daß von den möglichen Wegen jeweils der günstigste gewählt wird. Wie günstig ein Weg ist, richtet sich nach dem Aufwand, der zum Durchfahren nötig ist. Die Grundlage für die Bewertung des Aufwandes bildet die Fahrzeit.

Für die mathematische Behandlung wird das Straßennetz durch einen gerichteten Graphen beschrieben. Zur Charakterisierung der Bögen gehört die Angabe des Zeitaufwandes. Die Bestimmung der günstigen Stromverteilungen kann als gelöst betrachtet werden, wenn die Bewertung konstant ist, d. h., wenn die Fahrzeiten unabhängig von der Größe des Verkehrsflusses sind. Sie ist dann äquivalent mit der bekannten Aufgabe, den kürzesten Abstand zweier Punkte eines Graphen und den zugehörigen kritischen Pfad zu bestimmen [1], [5], [7].

Will man das Modell aber realistischer gestalten, ist zu berücksichtigen, daß die benötigte Zeit stark von der Stärke des Verkehrs abhängt. Wie die folgenden Untersuchungen zeigen, ergeben sich dann gegenüber dem Modell mit konstanter (belastungsunabhängiger) Bewertung z. T. völlig neue Aspekte. Dabei erweist sich schon eine Präzisierung der Problemstellung als notwendig; denn es ist zwischen dem Strom zu unterscheiden, der für alle am günstigsten ist,, und dem, der sich einstellt, wenn jeder Fahrer nur seinen eigenen Weg optimalisiert.

[1]) Priv.-Doz. Dr. DIETRICH BRAESS, Institut für numerische und instrumentelle Mathematik, 44 Münster, Hüfferstr. 3 a.

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On a Paradox of Traffic Planning

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For each point of a road network, let there be given the number of cars starting from it, and the destination of the cars. Under these conditions one wishes to estimate the distribution of traffic flow. Whether one street is preferable to another depends not only on the quality of the road, but also on the density of the flow. If every driver takes the path that looks most favorable to him, the resultant running times need not be minimal. Furthermore, it is indicated by an example that an extension of the road network may cause a redistribution of the traffic that results in longer individual running times.

Key words: traffic network planning; paradox; equilibrium; critical flows; optimal flows; existence theorem
History: Received: April 2005; revision received: June 2005; accepted: July 2005.

Translated from the original German: Braess, Dietrich. 1968. Über ein Paradoxon aus der Verkehrsplanung. *Unternehmensforschung* **12** 258–268.

1. Introduction

The distribution of traffic flow on the roads of a traffic network is of interest to traffic planners and traffic controllers. We assume that the number of vehicles per unit time is known for all origin-destination pairs. The expected distribution of vehicles is based on the assumption that the most favorable routes are chosen among all possible ones. How favorable a route is depends on its travel cost. The basis for the evaluation of cost is travel time.

The road network is modeled by a directed graph for the mathematical treatment. A (travel) time is associated with each link. The computation of the most favorable distribution can be considered solved if the travel time for each link is constant, i.e., if the time is independent of the number of vehicles on the link. In this case, it is equivalent to computing the shortest distance between two points of a graph and determining the corresponding critical (here meaning shortest) path. See Bellman (1958), von Falkenhausen (1963), and Pollack and Wiebenson (1960).

In more realistic models, however, one has to take into account that the travel time on the links will strongly depend on the traffic flow. Our investigations will show that we will encounter new effects compared to the model with flow-independent costs. Specifically, a more precise formulation of the problem will be required. We have to distinguish between flow that will be optimal for all vehicles and flow that is achieved if each user attempts to optimize his own route.

Referring to a simple model network with only four nodes, we will discuss typical features that contradict facts that seem to be plausible. Central control of traffic can be advantageous even for those drivers who think that they will discover more profitable routes for themselves. Moreover, there exists the possibility of the paradox that an extension of the road network by an additional road can cause a redistribution of the flow in such a way that increased travel time is the result.

2. Graph and Road Network

Directed graphs are used for modeling road maps, and the links, the connections between the nodes, have an orientation (Berge 1958, von Falkenhausen 1966). Two links that differ only by their direction are depicted in the figures by one line without an arrowhead.

In general, the nodes are associated with street intersections. Whenever a more detailed description is necessary, an intersection may be divided into (four) nodes with each one corresponding to an adjacent road; see Figure 2 (Pollack and Wiebenson 1960).

We will use the following notation for the nodes, links, and flows. The indices belong to finite sets. Because we use each index only in connection with one variable, we do not write the range of the indices.

The NE Paradigm is the Unifying Paradigm:

- Transportation Networks
- The Internet
- Financial Networks
- Decentralized Supply Chains.

The System-Optimization (S-O) Problem

The above discussion focused on the user-optimized (U-O) problem. We now turn to the system-optimized (S-O) problem in which a central controller, say, seeks to minimize the total cost in the network system, where the total cost is expressed as

$$\sum_{a \in L} \hat{c}_a(f_a)$$

where it is assumed that the total cost function on a link a is defined as:

$$\hat{c}_a(f_a) \equiv c_a(f_a) \times f_a,$$

subject to the conservation of flow constraints, and the nonnegativity assumption on the path flows. Here separable link costs have been assumed, for simplicity, and other total cost expressions may be used, as mandated by the particular application.

The S-O Optimality Conditions

Under the assumption of strictly increasing user link cost functions, the optimality conditions are: For each path $p \in P_w$, and every O/D pair w:

$$\hat{C}'_p \begin{cases} = \mu_w, & \text{if} \quad x_p > 0 \\ \geq \mu_w, & \text{if} \quad x_p = 0, \end{cases}$$

where $\hat{C}'_p$ denotes the marginal total cost on path p, given by:

$$\hat{C}'_p = \sum_{a \in L} \frac{\partial \hat{c}_a(f_a)}{\partial f_a} \delta_{ap}.$$

The above conditions correspond to Wardrop's second principle of travel behavior.

What is the S-O solution for the two Braess networks (before and after the addition of a new link e)?

Before the addition of the link e, we may write:

$$\hat{c}'_a = 20f_a, \quad \hat{c}'_b = 2f_b + 50,$$

$$\hat{c}'_c = 2f_c + 50, \quad \hat{c}'_d = 20f_d.$$

It is easy to see that, in this case, the S-O solution is identical to the U-O solution with $x_{p_1} = x_{p_2} = 3$ and $\hat{C}'_{p_1} = \hat{C}'_{p_2} = 116$. Furthermore, after the addition of link e, we have that $\hat{c}'_e = 2f_e + 10$. The new path p_3 is not used in the S-O solution, since with zero flow on path p_3, we have that $\hat{C}'_{p_3} = 170$ and $\hat{C}'_{p_1} = \hat{C}'_{p_2}$ remains at 116.

If the symmetry assumption does not hold for the user link costs functions, then the transportation network equilibrium conditions can no longer be reformulated as an associated optimization problem and the equilibrium conditions are formulated and solved as a *variational inequality problem!*

VI Formulation of TNE
Dafermos (1980), Smith (1979)

A traffic path flow pattern satisfies the above equilibrium conditions if and only if it satisfies the variational inequlity problem: determine $x^* \in K$, such that

$$\sum_p C_p(x^*) \times (x_p - x_p^*) \geq 0, \quad \forall x \in K.$$

Finite-dimensional variational inequality theory has been applied to-date to the wide range of equilibrium problems noted above.

In particular, the finite-dimensional variational inequality problem is to determine $x^* \in K \subset R^n$ such that

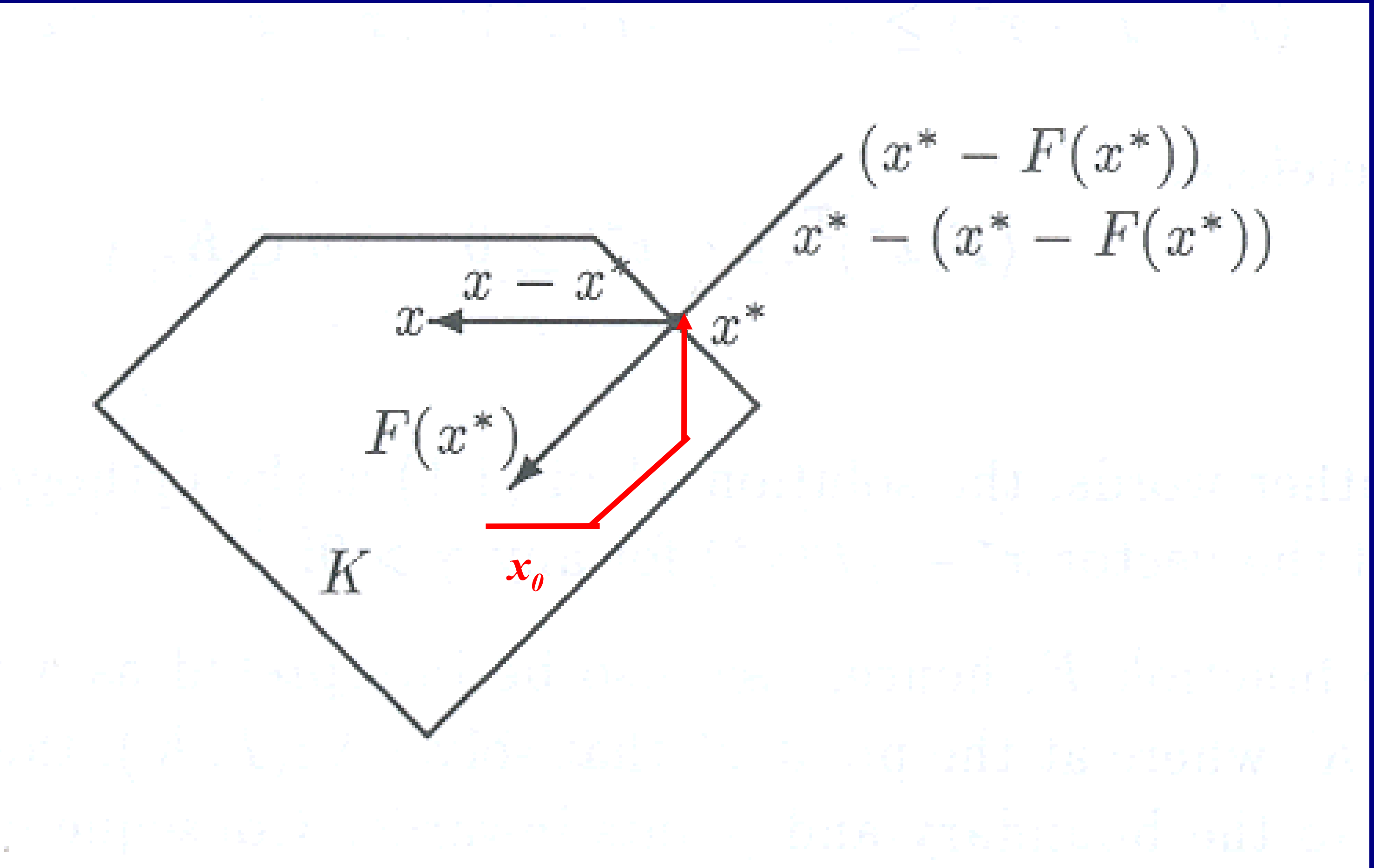
$$\langle F(x^*), x - x^* \rangle \geq 0, \quad \forall x \in K,$$

where $\langle \cdot, \cdot \rangle$ denoted the inner product in R^n and K is closed and convex.

A Geometric Interpretation of a Variational Inequality and a Projected Dynamical System

Dupuis and Nagurney (1993)

Nagurney and Zhang (1996)



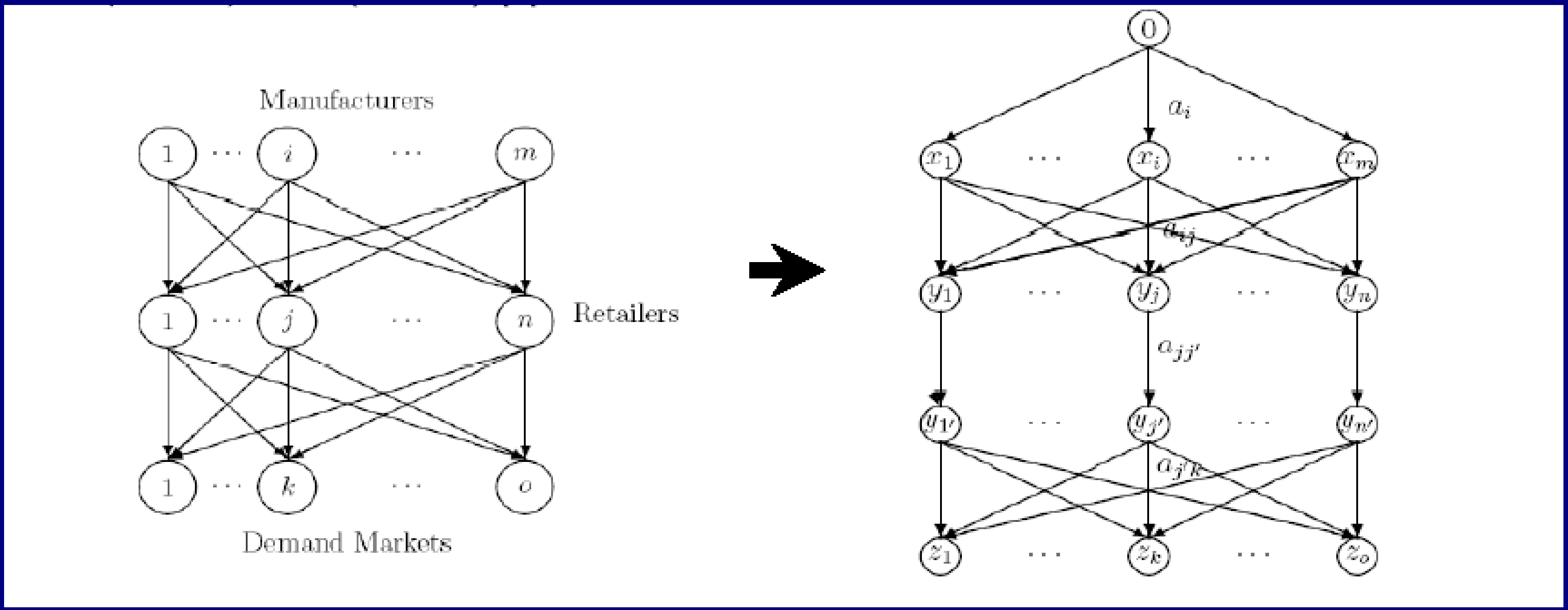
The variational inequality problem, contains, as special cases, such classical problems as:

- systems of equations
- optimization problems
- complementarity problems

and is also closely related to fixed point problems.

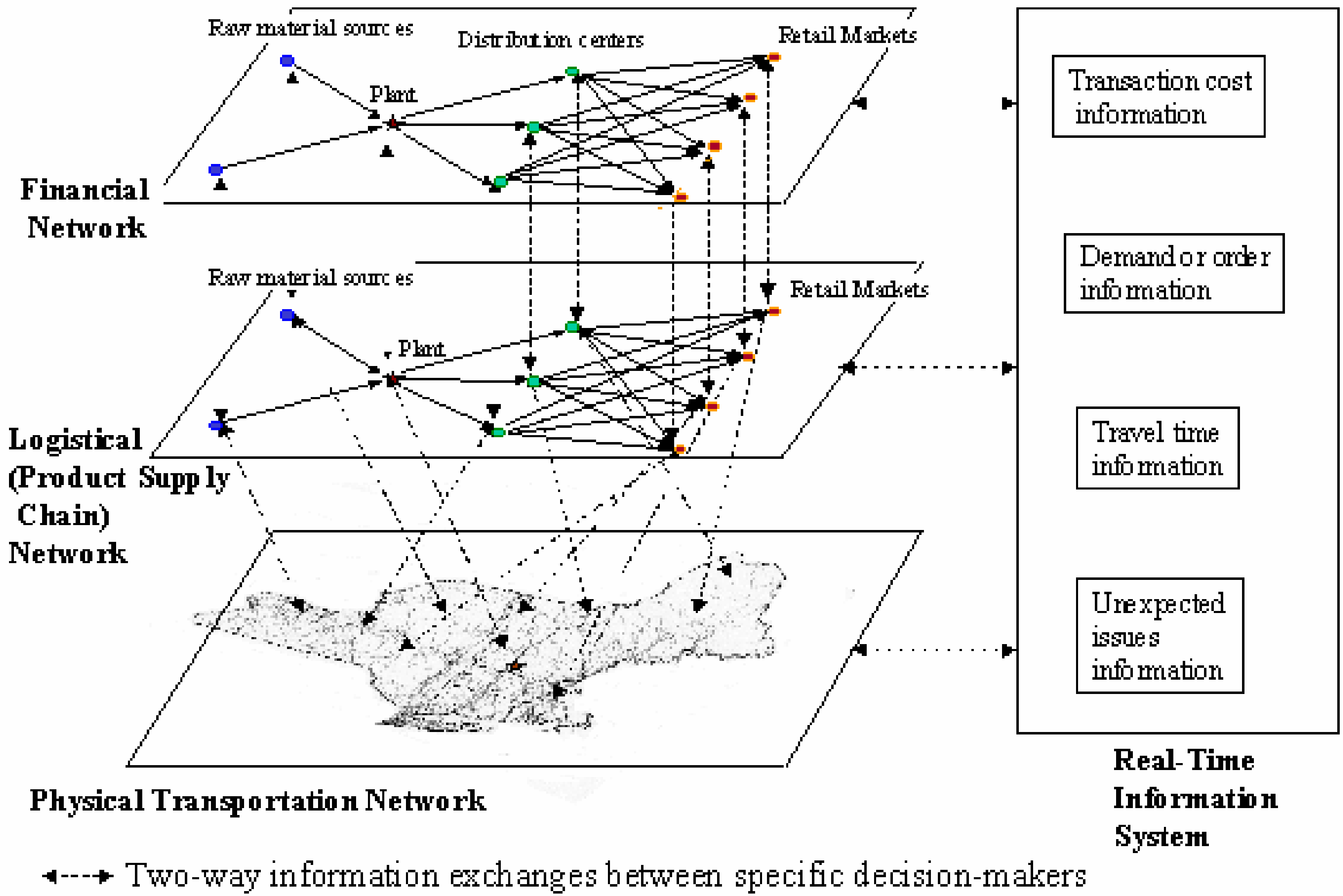
Hence, it is a unifying mathematical formulation for a variety of mathematical programming problems.

The Equivalence of Decentralized Supply Chains and Transportation Networks



Nagurney, *Transportation Research E* (2006).

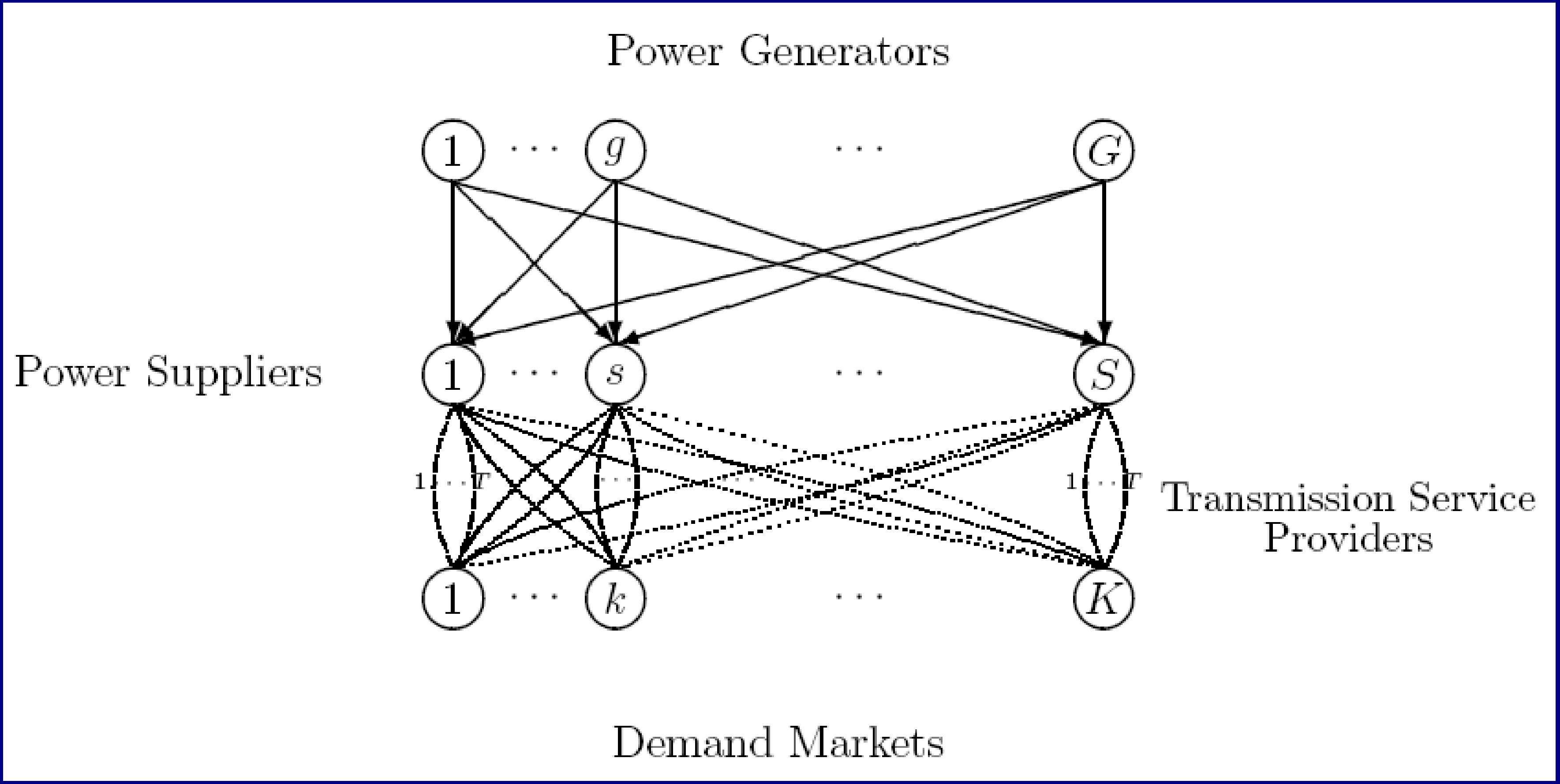
Supply Chain -Transportation Supernetwork Representation



Nagurney, Ke, Cruz, Hancock, Southworth, *Environment and Planning B* (2002).

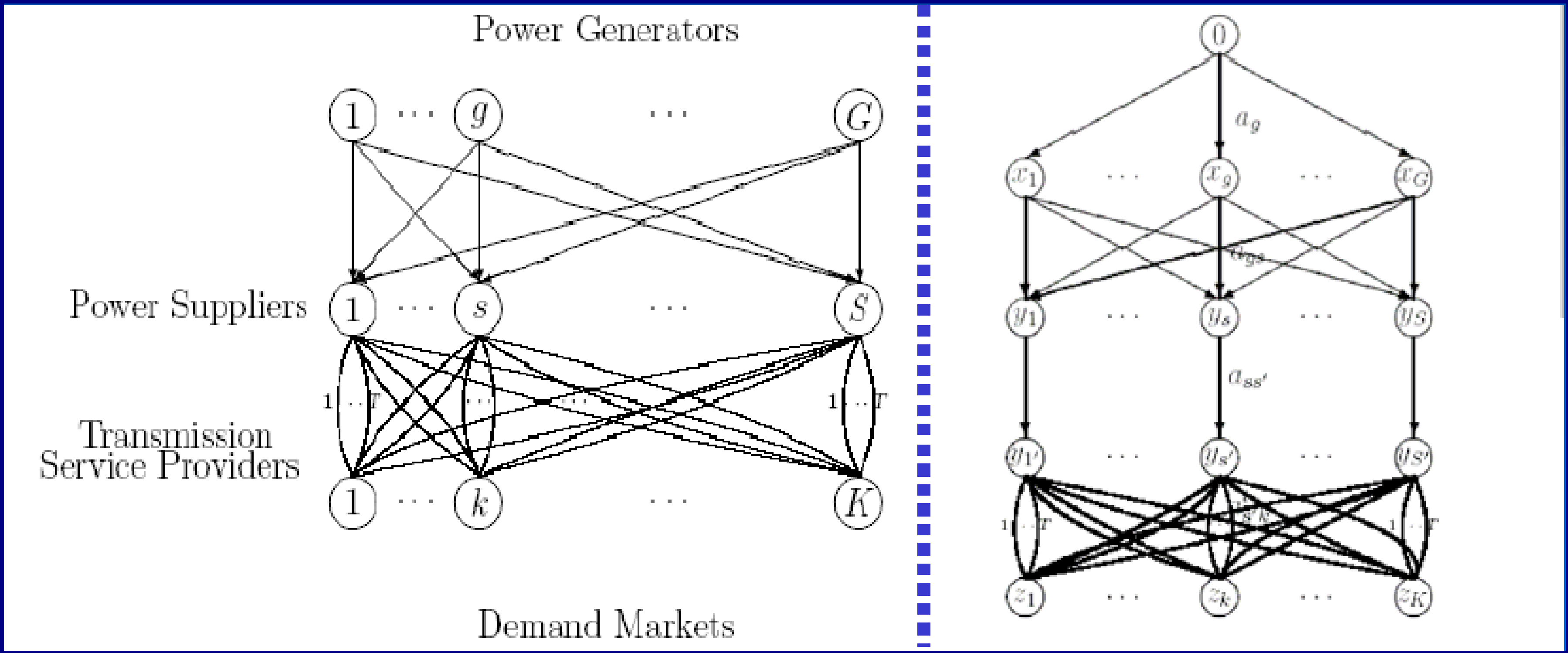
Electric Power Supply Chains

The Electric Power Supply Chain Network



Nagurney and Matsypura, *Proceedings of the CCCT* (2004).

The Transportation Network Equilibrium Reformulation of Electric Power Supply Chain Networks

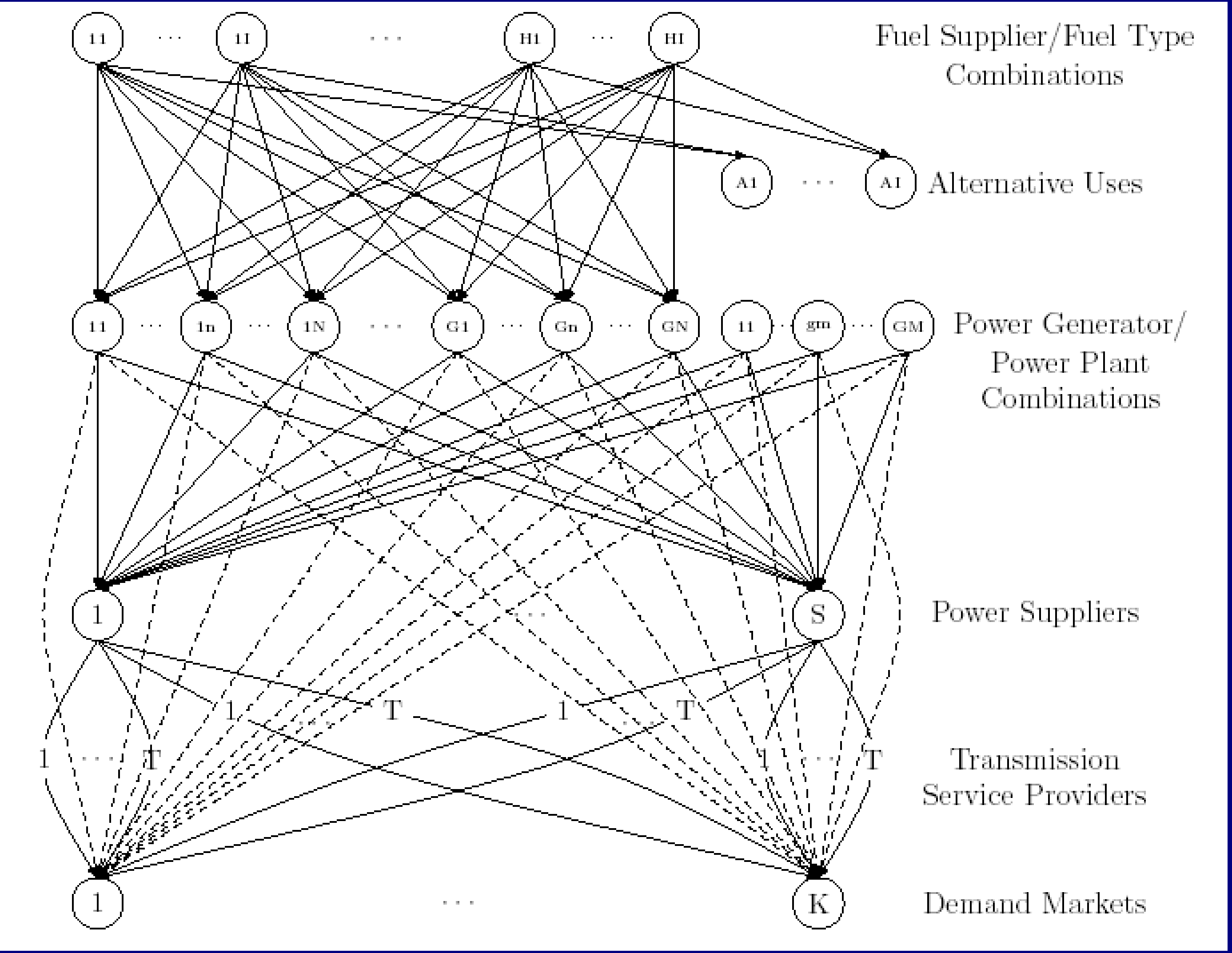


Electric Power Supply Network

Transportation Chain Network

Nagurney, Liu, Cojocaru, and Daniele, *Transportation Research E* (2007).

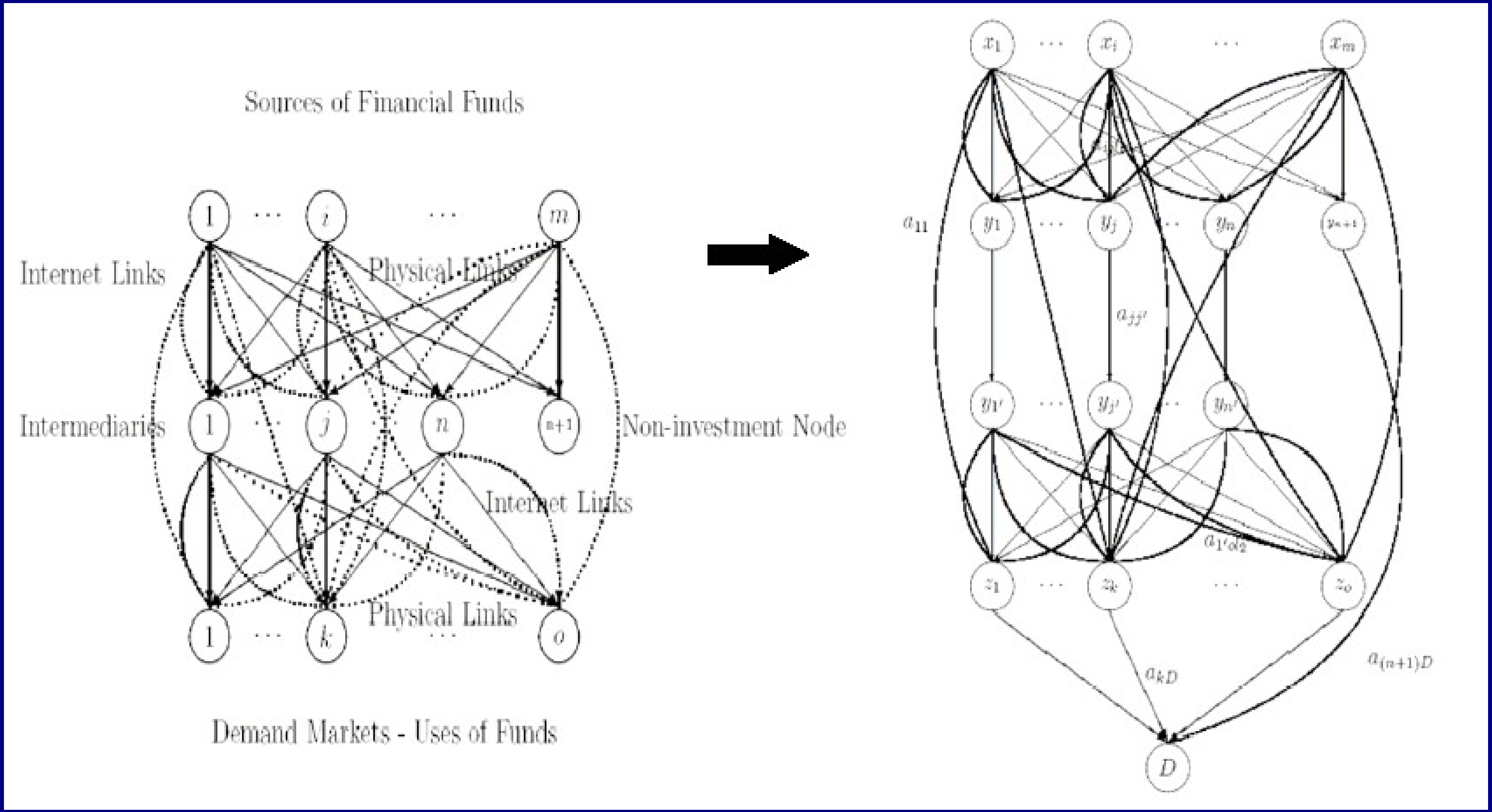
Electric Power Supply Chain Network with Fuel Suppliers



Matsypura, Nagurney, and Liu, *International Journal of Emerging Power Systems* (2007).

In 1952, Copeland wondered whether money flows like water or electricity.

The Transportation Network Equilibrium Reformulation of the Financial Network Equilibrium Model with Intermediation



Liu and Nagurney, *Computational Management Science* (2007).

We have shown that *money* as well as *electricity* flow like *transportation* and have answered questions posed fifty years ago by Copeland and by Beckmann, McGuire, and Winsten!

Additional disasters that have demonstrated the importance and the vulnerability of network systems.

Examples:

- 9/11 Terrorist Attacks, September 11, 2001;
- The biggest blackout in North America, August 14, 2003;
- Two significant power outages in September 2003 -- one in the UK and the other in Italy and Switzerland;
- Hurricane Katrina, August 23, 2005;
- The Minneapolis I35 Bridge Collapse, August 1, 2007
- The severance of the Mediterranean cable in 2008.

Our Research on Network Efficiency, Vulnerability, and Robustness

A Network Efficiency Measure for Congested Networks, Nagurney and Qiang, *Europhysics Letters,* **79**, August (2007).

A Transportation Network Efficiency Measure that Captures Flows, Behavior, and Costs with Applications to Network Component Importance Identification and Vulnerability, Nagurney and Qiang, *Proceedings of the POMS 18th Annual Conference,* Dallas, Texas (2007).

A Network Efficiency Measure with Application to Critical Infrastructure Networks, Nagurney and Qiang, *Journal of Global Optimization* (2008).

Robustness of Transportation Networks Subject to Degradable Links, Nagurney and Qiang, *Europhysics Letters,* **80**, December (2007).

A Unified Network Performance Measure with Importance Identification and the Ranking of Network Components, Qiang and Nagurney, *Optimization Letters,* **2** (2008).

A New Network Performance/Efficiency Measure with Applications to Infrastructure Networks including Supply Chains

The Nagurney and Qiang (N-Q) Network Efficiency Measure

The network performance/efficiency measure $\mathcal{E}(G,d)$, for a given network topology G and fixed demand vector d, is defined as

$$\mathcal{E}(G,d) = \frac{\sum_{w \in W} \frac{d_w}{\lambda_w}}{n_W},$$

where n_w is the number of O/D pairs in the network and λ_w is the equilibrium disutility for O/D pair w.

Nagurney and Qiang, *Europhysics Letters,* **79** (2007).

Importance of a Network Component

Definition: Importance of a Network Component

The importance, *I(g),* of a network component *gεG* is measured by the relative network efficiency drop after *g* is removed from the network:

$$I(g) = \frac{\triangle \mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}(G,d) - \mathcal{E}(G-g,d)}{\mathcal{E}(G,d)},$$

where *G-g* is the resulting network after component g is removed.

The Approach to Study the Importance of Network Components

The elimination of a link is treated in the N-Q network efficiency measure by removing that link while the removal of a node is managed by removing the links entering and exiting that node.

In the case that the removal results in no path connecting an O/D pair, we simply assign the demand for that O/D pair to an abstract path with a cost of infinity. Hence, our measure is well-defined even in the case of disconnected networks.

The measure generalizes the Latora and Marchiori network measure for complex networks.

The Latora and Marchiori (L-M) Network Efficiency Measure

Definition: The L-M Measure

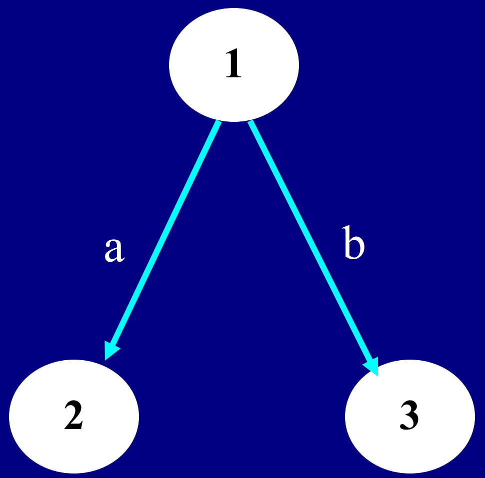
The network performance/efficiency measure, *E(G)* for a given network topology, *G*, is defined as:

$$E(G) = \frac{1}{n(n-1)} \sum_{i \neq j \in G} \frac{1}{d_{ij}}$$

where *n* is the number of nodes in the network and d_{ij} is the shortest path length between node *i* and node *j*.

Example 1

Assume a network with two O/D pairs:
$\boldsymbol{w_1}$=(1,2) and $\boldsymbol{w_2}$=(1,3) with demands:
$\boldsymbol{d_{w_1}}$=100 and $\boldsymbol{d_{w_2}}$=20.



$c_a(f_a)=0.01f_a+19$
$c_b(f_b)=0.05f_b+19$

The paths are:
for $\boldsymbol{w_1}$, $\boldsymbol{p_1=a}$; for $\boldsymbol{w_2}$, $\boldsymbol{p_2=b}$.

The equilibrium path flows are:
$\boldsymbol{x_{p_1}^*}$= 100, $\boldsymbol{x_{p_2}^*}$=20.

The equilibrium path travel costs are:
$\boldsymbol{C_{p_1}=C_{p_2}}$=20.

Importance and Ranking of Links and Nodes

Link	Importance Value from Our Measure	Importance Ranking from Our Measure
a	0.8333	1
b	0.1667	2

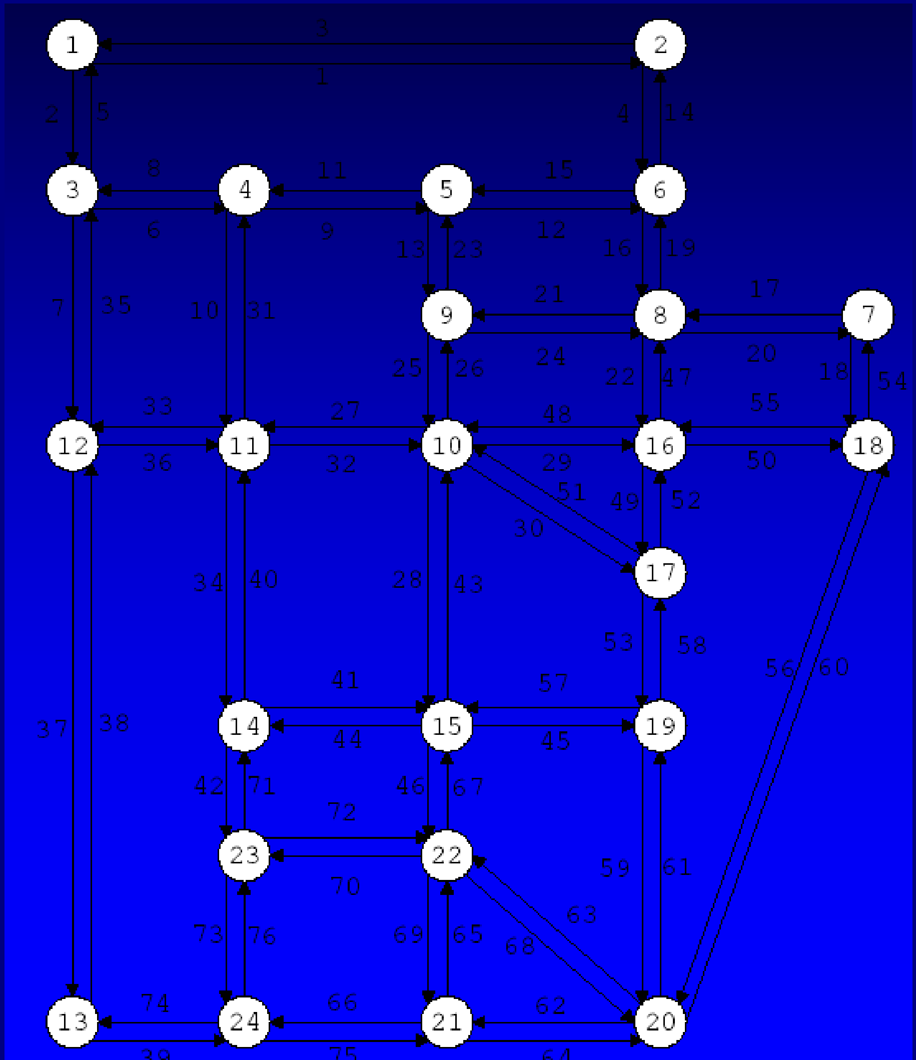
Node	Importance Value from Our Measure	Importance Ranking from Our Measure
1	1	1
2	0.8333	2
3	0.1667	3

Example - Sioux Falls Network

The network data are from LeBlanc, Morlok, and Pierskalla (1975).

The network has 528 O/D pairs, 24 nodes, and 76 links.

The user link cost functions are of Bureau of Public Roads (BPR) form.



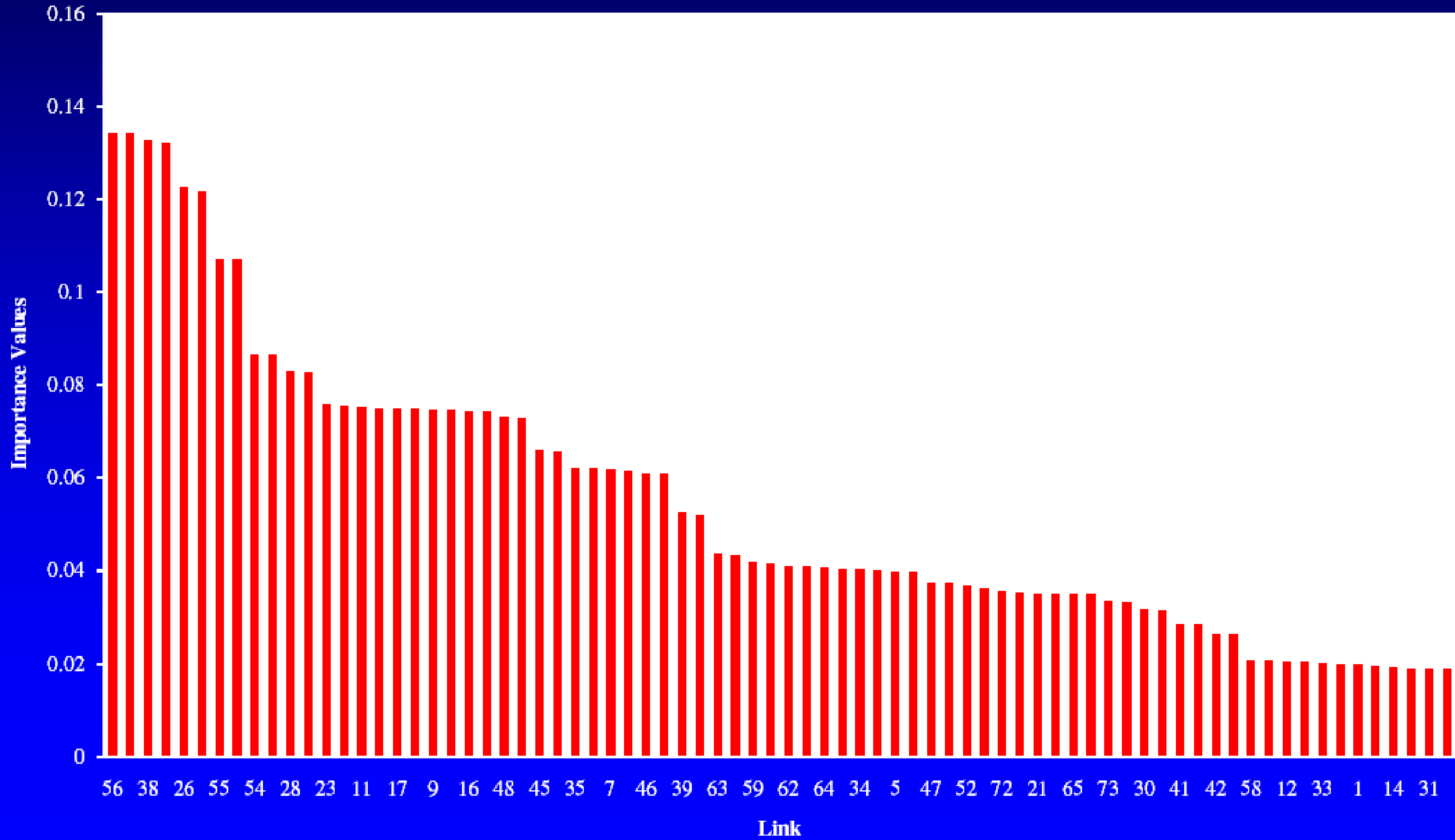
BPR Link Cost Functions

The Bureau of Public Roads (BPR) link cost functional form is:

$$c_a(f_a) = t_a^0 \left[1 + k\left(\frac{f_a}{u_a}\right)^{\beta} \right] \qquad \forall a \in L$$

where *k* and *β* are greater than zero and the *u's* are the practical capacities on the links.

Example - Sioux Falls Network Link Importance Rankings



The Advantages of the N-Q Network Efficiency Measure

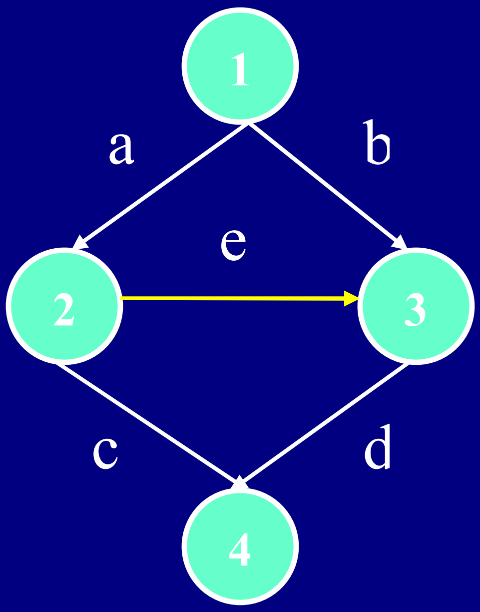
- The measure captures demands, flows, costs, and behavior of users, in addition to network topology;
- The resulting importance definition of network components is applicable and well-defined even in the case of disconnected networks;
- It can be used to identify the importance (and ranking) of either nodes, or links, or both; and
- It can be applied to assess the efficiency/performance of a wide range of network systems.
- It is applicable also to elastic demand networks (Qiang and Nagurney, *Optimization Letters* (2008)).
- It has been extended to dynamic networks (Nagurney and Qiang, *Netnomics*, in press).

The focus of the robustness of networks (and complex networks) has been on the impact of different network measures when facing the removal of nodes on networks.

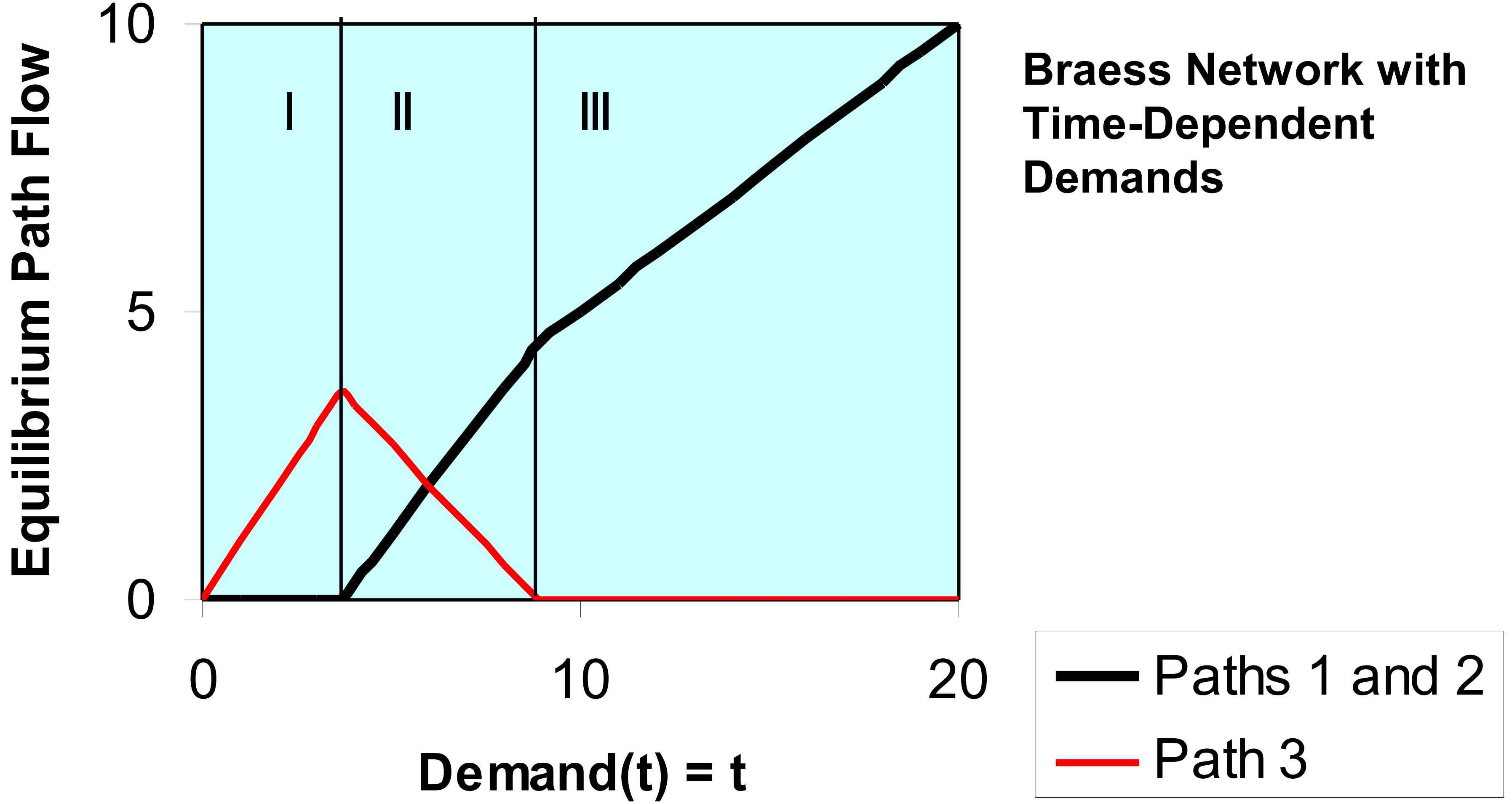
We focus on the *degradation of links through reductions in their capacities* and the effects on the induced costs in the presence of known demands and different functional forms for the links.

The Time-Dependent (Demand-Varying) Braess Paradox and Evolutionary Variational Inequalities

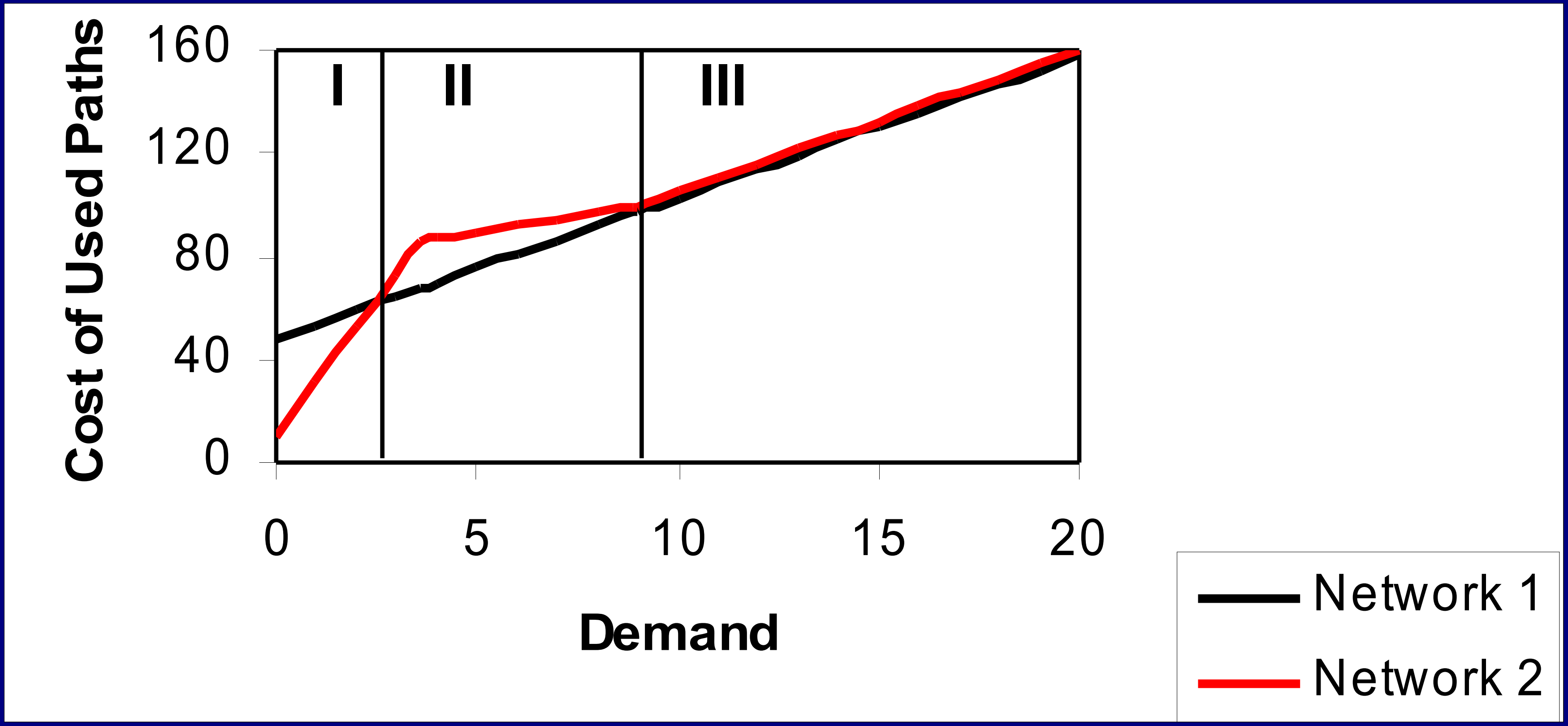
Recall the Braess Network
where we add the link e.



The Solution of an Evolutionary (Time-Dependent) Variational Inequality for the Braess Network with Added Link (Path)



In Demand Regime I, only the new path is used.
In Demand Regime II, the Addition of a New Link (Path) Makes Everyone Worse Off!
In Demand Regime III, only the original paths are used.



Network 1 is the Original Braess Network - Network 2 has the added link.

The new link is NEVER used after a certain demand is reached even if the demand approaches infinity.

Hence, in general, except for a limited range of demand, building the new link is a complete waste!

In the paper:

"Modeling of Supply Chain Risk Under Disruptions with Performance Measurement and Robustness Analysis," invited for the volume **Managing Supply Chain Risk and Vulnerability: Tools and Methods for Supply Chain Decision Makers**, Wu and Blackhurst, editors, Springer

we have extended the performance measure of Nagurney and Qiang to handle disruptions in supply chains under risk and uncertainty.

Contributions of This Research

- Developed a multi-tiered, multi transportation modal supply chain network with interactions among various decision-makers.
- The model captures the supply-side risks together with uncertain demand.
- The mean-variance approach is used to model individual's attitude towards risks.
- Developed a weighted measure to study the supply chain network performance.

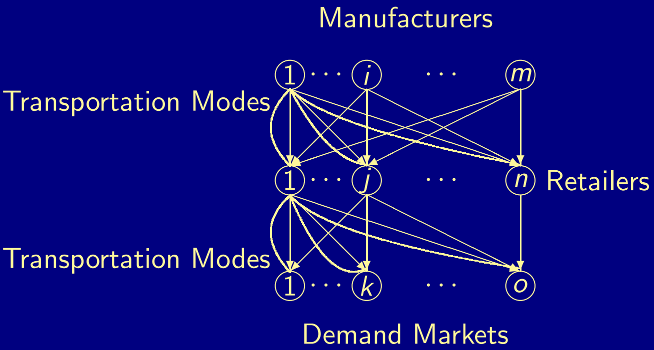


Figure: The Multitiered Network Structure of the Supply Chain

Assumptions

- Manufacturers and retailers are multicriteria decision-makers
- Manufacturers and retailers try to
  - Maximize profit
  - Minimize risk
  - Individual weight assigned to the risk level according to decision maker's attitude towards risk
- Nash Equilibrium

For each manufacturer i, there is a random parameter α_i that reflects the impact of disruption to his production cost function. The expected production cost function is given by:

$$\hat{F}_i(Q^1) \equiv \int f_i(Q^1, \alpha_i) d\mathcal{F}_i(\alpha_i), \quad i = 1, \ldots, m.$$

The variance of the above production cost function is denoted by $VF_i(Q^1)$ where $i = 1, \ldots, m$.
We assume that each manufacturer has g types of transportation modes available to ship the product to the retailers, the cost of which is also subject to disruption impacts. The expected transportation cost function is given by:

$$\hat{C}_{ij}^u(q_{ij}^u) \equiv \int_{\beta_{ij}^u} c_{ij}^u(q_{ij}^u, \beta_{ij}^u) d\mathcal{F}_{ij}^u(\beta_{ij}^u), \; i = 1, \ldots, m; \; j = 1, \ldots, n; \; u = 1, \ldots, g.$$

We further denote the variance of the above transportation cost function as $VC_{ij}^u(Q^1)$ where $i = 1, \ldots, m$; $j = 1, \ldots, n$; $u = 1, \ldots, g$.

Manufacturer's Maximization Problem

$$\text{Maximize} \quad \sum_{j=1}^{n}\sum_{u=1}^{g}\rho_{1ij}^{u*}q_{ij}^{u} - \hat{F}_i(Q^1) - \sum_{j=1}^{n}\sum_{u=1}^{g}\hat{C}_{ij}^{u}(q_{ij}^{u})$$

$$-\theta_i\left[\sum_{i=1}^{m}VF_i(Q^1) + \sum_{j=1}^{n}\sum_{u=1}^{g}VC_{ij}^{u}(q_{ij}^{u})\right]$$

Nonnegative weight θ_i is assigned to the variance of the cost functions for each manufacturer to reflect his attitude towards disruption risks.

We assume that for each manufacturer, the production cost function and the transaction cost function without disruptions are continuously differentiable and convex. Hence, the optimality conditions for all manufacturers simultaneously (cf. Bazaraa, Sherali, and Shetty (1993) and Nagurney (1999)) can be expressed as the following VI:

The Optimal Conditions for All Manufacturers

Determine $Q^{1*} \in R_+^{mng}$ satisfying:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{u=1}^{g}[\frac{\partial \hat{F}_i(Q^{1*})}{\partial q_{ij}^u} + \frac{\partial \hat{C}_{ij}^u(q_{ij}^{u*})}{\partial q_{ij}^u} + \theta_i(\frac{\partial VF_i(Q^{1*})}{\partial q_{ij}^{u*}}$$

$$+\frac{\partial VC_{ij}^u(q_{ij}^{u*})}{\partial q_{ij}^{u*}}) - \rho_{1ij}^{u*}] \times [q_{ij}^u - q_{ij}^{u*}] \geq 0, \quad \forall Q^1 \in R_+^{mng}.$$

A random risk/disruption related random parameter η_j is associated with the handling cost of retailer j. The expected handling cost is:

$$\hat{C}_j^1(Q^1, Q^2) \equiv \int c_j(Q^1, Q^2, \eta_j) d\mathcal{F}_j(\eta_j), \; j = 1, \ldots, n$$

The variance of the handling cost function is denoted by $VC_j^1(Q^1, Q^2)$ where $j = 1, \ldots, n$.

Retailer's Maximization Problem

The objective function for distributor j; $j = 1, \ldots, n$ can be expressed as follows:

$$\text{Maximize} \;\; \sum_{k=1}^{o} \sum_{v=1}^{h} \rho_{2jk}^{v*} q_{jk}^{v} - \hat{C}_j^1(Q^1, Q^2) - \sum_{i=1}^{m} \sum_{u=1}^{g} \rho_{1ij}^{u*} q_{ij}^{u} - \varpi_j VC_j^1(Q^1, Q^2)$$

subject to:

$$\sum_{k=1}^{o} \sum_{v=1}^{h} q_{jk}^{v} \leq \sum_{i=1}^{m} \sum_{u=1}^{g} q_{ij}^{u}$$

and the nonnegativity constraints: $q_{ij}^u \geq 0$ for all i, j, and u; $q_{jk}^v \geq 0$ for all j, k, and v.

We assume that, for each retailer, the handling cost without disruptions is continuously differentiable and convex.

The Optimal Conditions for All Retailers

Determine $(Q^{1*}, Q^{2*}, \gamma^*) \in R_+^{mng+noh+n}$ satisfying:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{u=1}^{g}\left[\frac{\partial \hat{C}_j^1(Q^{1*}, Q^{2*})}{\partial q_{ij}^u} + \rho_{1ij}^{u*} + \varpi_j \frac{\partial VC_j^1(Q^{1*}, Q^{2*})}{\partial q_{ij}^u} - \gamma_j^*\right]$$

$$\times \left[q_{ij}^u - q_{ij}^{u*}\right] + \sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{v=1}^{h}[-\rho_{2jk}^{v*} + \gamma_j^* + \frac{\partial \hat{C}_j^1(Q^{1*}, Q^{2*})}{\partial q_{jk}^v}$$

$$+\varpi_j \frac{\partial VC_j^1(Q^{1*}, Q^{2*})}{\partial q_{jk}^v}] \times \left[q_{jk}^v - q_{jk}^{v*}\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{u=1}^{g} q_{ij}^{u*} - \sum_{k=1}^{o}\sum_{v=1}^{h} q_{jk}^{v*}\right]\times\left[\gamma_j - \gamma_j^*\right] \geq 0, \ \forall (Q^1, Q^2, \gamma) \in R_+^{mng+noh+n}$$

The Market Stochastic Economic Equilibrium Conditions

For any retailer with associated demand market k; $k = 1, \ldots, o$:

$$\hat{d}_k(\rho_3^*) \begin{cases} \leq \sum_{j=1}^{o} \sum_{v=1}^{h} q_{jk}^{v*}, & \text{if} \quad \rho_{3k}^* = 0, \\ = \sum_{j=1}^{o} \sum_{v=1}^{h} q_{jk}^{v*}, & \text{if} \quad \rho_{3k}^* > 0, \end{cases}$$

$$\rho_{2jk}^{v*} + c_{jk}^{v}(Q^{2*}) \begin{cases} \geq \rho_{3k}^*, & \text{if} \quad q_{jk}^{v*} = 0, \\ = \rho_{3k}^*, & \text{if} \quad q_{jk}^{v*} > 0. \end{cases}$$

The above market equilibrium conditions are equivalent to the following VI problem, after taking the expected value and summing over all retailers/demand markets k:

Equivalent VI Problem

Determine $(Q^{2*}, \rho_3^*) \in R_+^{noh+o}$ satisfying:

$$\sum_{k=1}^{o}(\sum_{j=1}^{n}\sum_{v=1}^{h} q_{jk}^{v*} - \hat{d}_k(\rho_3^*)) \times [\rho_{3k} - \rho_{3k}^*]$$

$$+\sum_{k=1}^{o}\sum_{j=1}^{n}\sum_{v=1}^{h}(\rho_{2jk}^{v*}+c_{jk}^{v}(Q^{2*})-\rho_{3k}^*)\times[q_{jk}^{v}-q_{jk}^{v*}] \geq 0, \ \forall \rho_3 \in R_+^o, \forall Q^2 \in R_+^{noh},$$

where ρ_3 is the o-dimensional vector with components: $\rho_{31}, \ldots, \rho_{3o}$ and Q^2 is the noh-dimensional vector.

Remark:

We are interested in the cases where the expected demands are positive, that is, $\hat{d}_k(\rho_3) > 0$, $\forall \rho_3 \in R^o_+$ for $k = 1, \ldots, o$. Furthermore, we assume that the unit transaction costs: $c^v_{jk}(Q^2) > 0$, $\forall j, k$, $\forall Q^2 \neq 0$.
Under the above assumptions, we can show that $\rho^*_{3k} > 0$ and $\hat{d}_k(\rho^*_3) = \sum_{j=1}^n \sum_{k=1}^o \sum_{v=1}^h q^{v*}_{jk}$, $\forall k$.

Definition: Supply Chain Network Equilibrium with Uncertainty and Expected Demands

The equilibrium state of the supply chain network with disruption risks and expected demands is one where the flows of the product between the tiers of the decision-makers coincide and the flows and prices satisfy the sum of conditions of manufacturers, distributors, and demand markets.

Theorem: VI Formulation of the Supply Chain Network Equilibrium with Uncertainty and Expected Demands

Determine $(Q^{1*}, Q^{2*}, \gamma^*, \rho_3^*) \in R_+^{mng+noh+n+o}$ *satisfying:*

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{u=1}^{g}[\frac{\partial \hat{F}_i(Q^{1*})}{\partial q_{ij}^u} + \frac{\partial \hat{C}_{ij}^u(q_{ij}^{u*})}{\partial q_{ij}^u} + \theta_i(\frac{\partial VF_i(Q^{1*})}{\partial q_{ij}^u} + \frac{\partial VC_{ij}^u(q_{ij}^{u*})}{\partial q_{ij}^u})$$

$$+\frac{\partial \hat{C}_j^1(Q^{1*}, Q^{2*})}{\partial q_{ij}^u} + \varpi_j \frac{\partial VC_j^1(Q^{1*}, Q^{2*})}{\partial q_{ij}^u} - \gamma_j^*] \times [q_{ij}^u - q_{ij}^{u*}]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{v=1}^{h}[\frac{\partial \hat{C}_j^1(Q^{1*}, Q^{2*})}{\partial q_{jk}^v} + \varpi_j \frac{\partial VC_j^1(Q^{1*}, Q^{2*})}{\partial q_{jk}^v}$$

$$+\gamma_j^* + c_{jk}^v(Q^{2*}) - \rho_{3k}^*] \times \left[q_{jk}^v - q_{jk}^{v*}\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{u=1}^{g} q_{ij}^{u*} - \sum_{k=1}^{o}\sum_{v=1}^{h} q_{jk}^{v*}\right] \times \left[\gamma_j - \gamma_j^*\right] + \sum_{k=1}^{o}(\sum_{j=1}^{n}\sum_{v=1}^{h} q_{jk}^{v*} - \hat{d}_k(\rho_3^*)) \times [\rho_{3k} - \rho_{3k}^*] \geq 0,$$

$$\forall (Q^1, Q^2, \gamma, \rho_3) \in R_+^{mng+noh+n+o}.$$

where $\mathcal{K} \equiv \{(Q^1, Q^2, \gamma, \rho_3) \| (Q^1, Q^2, \gamma, \rho_3) \in R_+^{mng+noh+n+o}\}$.

A Supply Chain Network Performance Measure

The supply chain network performance measure, $\mathcal{E}$, for a given supply chain, and expected demands: $\hat{d}_k$; $k = 1, 2, \ldots, o$, is defined as follows:

$$\mathcal{E} \equiv \frac{\sum_{k=1}^{o} \frac{\hat{d}_k}{\rho_{3k}}}{o},$$

where o is the number of demand markets in the supply chain network, and $\hat{d}_k$ and ρ_{3k} denote, respectively, the expected equilibrium demand and the equilibrium price at demand market k.

Assume that all the random parameters take on a given threshold probability value; say, for example, 95%. Moreover, assume that all the cumulative distribution functions for random parameters have inverse functions. Hence, we have that: $\alpha_i = \mathcal{F}_i^{-1}(.95)$, for $i = 1, \ldots, m$; $\beta_{ij}^u = {\mathcal{F}_{ij}^u}^{-1}(.95)$, for $i = 1, \ldots, m$; $j = 1, \ldots, n$, and so on.

Supply Chain Robustness Measurement

Let $\mathcal{E}_w$ denote the supply chain performance measure with random parameters fixed at a certain level as described above. Then, the supply chain network robustness measure, $\mathcal{R}$, is given by the following:

$$\mathcal{R} = \mathcal{E}^0 - \mathcal{E}_w,$$

where $\mathcal{E}^0$ gauges the supply chain performance based on the supply chain model, but with weights related to risks being zero.

$\mathcal{E}^0$ examines the "base" supply chain performance while $\mathcal{E}_w$ assesses the supply chain performance measure at some prespecified uncertainty level. If their difference is small, a supply chain maintains its functionality well and we consider the supply chain to be robust.

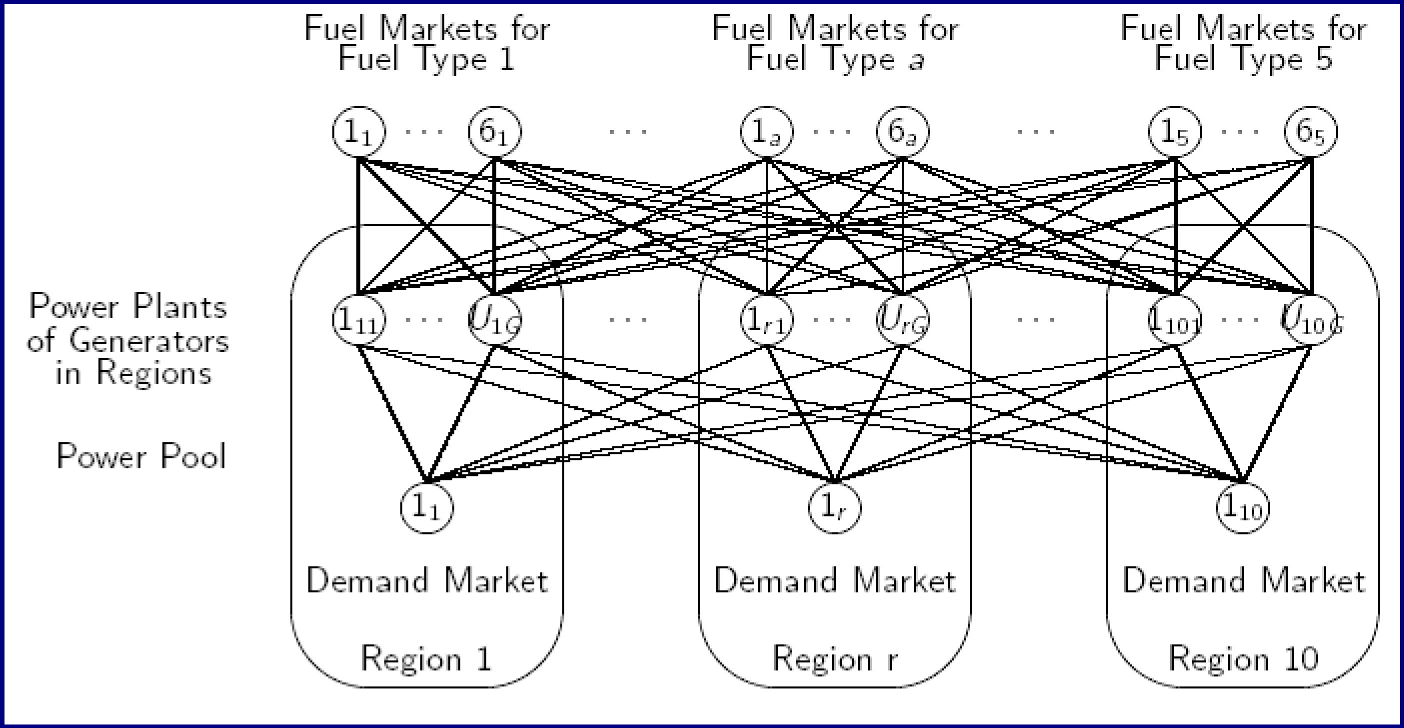
Where Are We Now?

An Integrated Electric Power Supply Chain and Fuel Market Network Framework: Theoretical Modeling with Empirical Analysis for New England, Liu and Nagurney (2007).

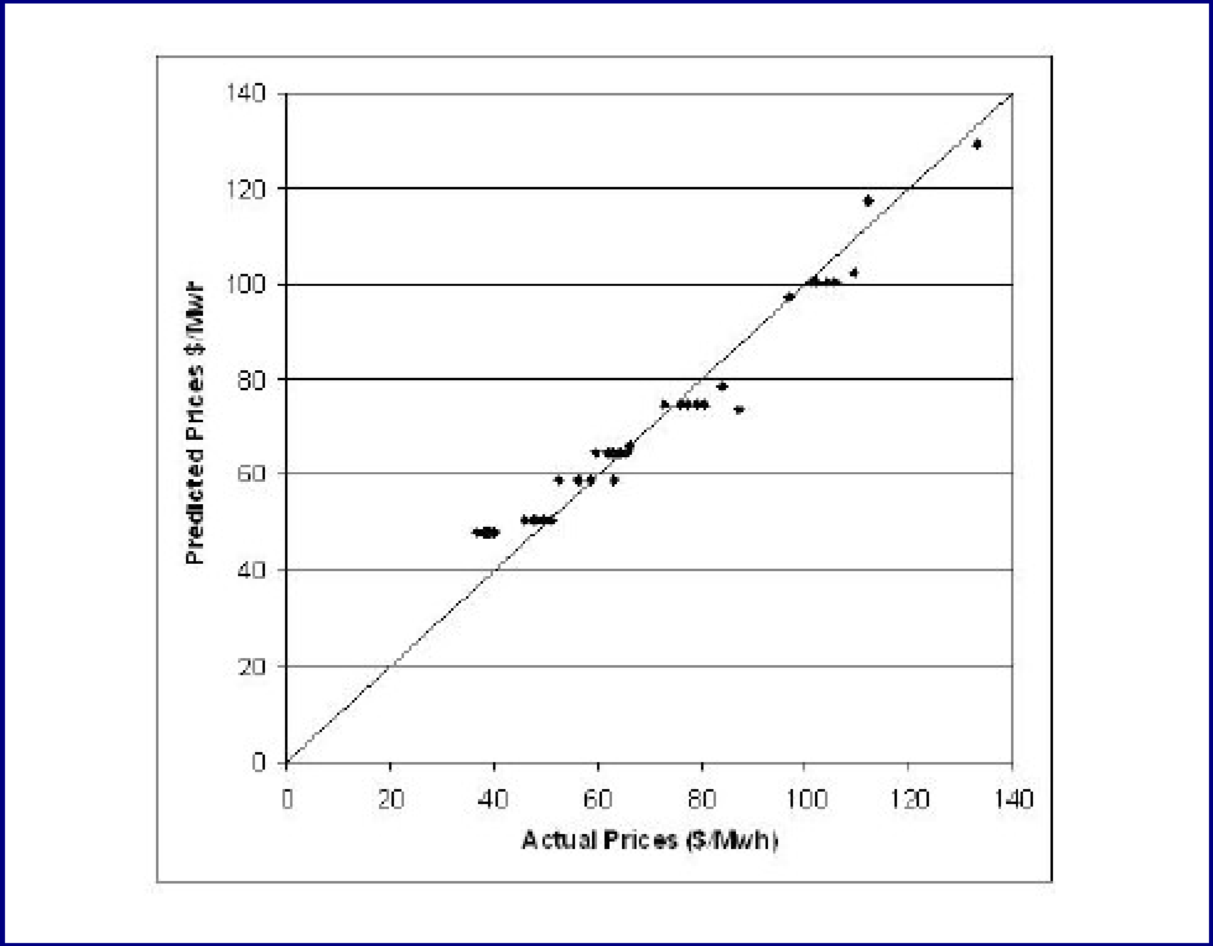
Empirical Case Study

- New England electric power market and fuel markets
- 82 generators who own and operate 573 power plants
- 5 types of fuels: natural gas, residual fuel oil, distillate fuel oil, jet fuel, and coal
- Ten regions (R=10): 1. Maine, 2. New Hampshire, 3. Vermont, 4. Connecticut(excluding Southwest Connecticut), 5. Southwest Connecticut(excluding Norwalk-Stamford area), 6. Norwalk-Stamford area, 7. Rhode Island, 8. Southeast Massachusetts, 9. West and Central Massachusetts, 10. Boston/Northeast Massachusetts
- Hourly demand/price data of July 2006 ($24 \times 31 = 744$ scenarios)
- 6 blocks (L1 = 94 hours, and Lw = 130 hours; w = 2, ..., 6)

The New England Electric Power Supply Chain Network with Fuel Suppliers



Predicted Prices vs. Actual Prices ($/Mwh)



Our New Book

Fragile Networks: Identifying Vulnerabilities and Synergies in an Uncertain World

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and
Qiang (Patrick) Qiang
Wiley & Sons

July 2009

Mergers and Acquisitions and Supply Chain Network Synergies

- Today, supply chains are more extended and complex than ever before. At the same time, the current competitive economic environment requires that firms operate efficiently, which has spurred research to determine how to utilize supply chains more effectively.
- There is also a pronounced amount of merger activity. According to Thomson Financial, in the first nine months of 2007 alone, worldwide merger activity hit $3.6 trillion, surpassing the total from all of 2006 combined.
- Notable examples: KMart and Sears in the retail industry in 2004 and Federated and May in 2005, Coors and Molson in the beverage industry in 2005, and the recently proposed merger between Anheuser Busch and InBev.

kmart Sears


Coors
LIGHT
MOLSON
CANADIAN


Federated
DEPARTMENT STORES, INC.
MAY

According to Kusstatscher and Cooper (2005) there were five major waves of of Merger & Acquisition (M &A) activity:

The First Wave: 1898-1902: an increase in horizontal mergers that resulted in many US industrial groups;

The Second Wave: 1926-1939: mainly public utilities;

The Third Wave: 1969-1969: *diversification* was the driving force;

The Fourth Wave: 1983-1986: the goal was efficiency;

The Fifth Wave: 1997 until the early years of the 21st century: *globalization* was the motto.

In 1998, M&As reached $2.1 trillion worldwide; in 1999, the activity exceeded $3.3 trillion, and in 2000, almost $3.5 was reached.

- A survey of 600 executives involved in their companies' mergers and acquisitions (M&A) conducted by Accenture and the Economist Unit (see Byrne (2007)) found that less than half (45%) achieved expected cost-saving synergies.

- Langabeer and Seifert (2003) determined a direct correlation between how effectively supply chains of merged firms are integrated and how successful the merger is. They concluded, based on the empirical findings of Langabeer (2003), who analyzed hundreds of mergers over the preceding decade, that

Improving Supply Chain Integration between Merging Companies is the Key to Improving the Likelihood of Post-Merger Success!

Mergers and Acquisitions and Supply Chain Network Synergies

Recently, we introduced a system-optimization perspective for supply chains in which firms are engaed in multiple activities of production, storage, and distribution to the demand markets and proposed a cost synergy measure associated with evaluating proposed mergers:

- Nagurney, A. (2009) "A System-Optimization Perspective for Supply Chain Network Integration: The Horizontal Merger Case," *Transportation Research E* **45**, pp. 1-15.

In that paper, the merger of two firms was modeled and the demands for the product at the markets, which were distinct for each firm prior to the merger, were assumed to be fixed.

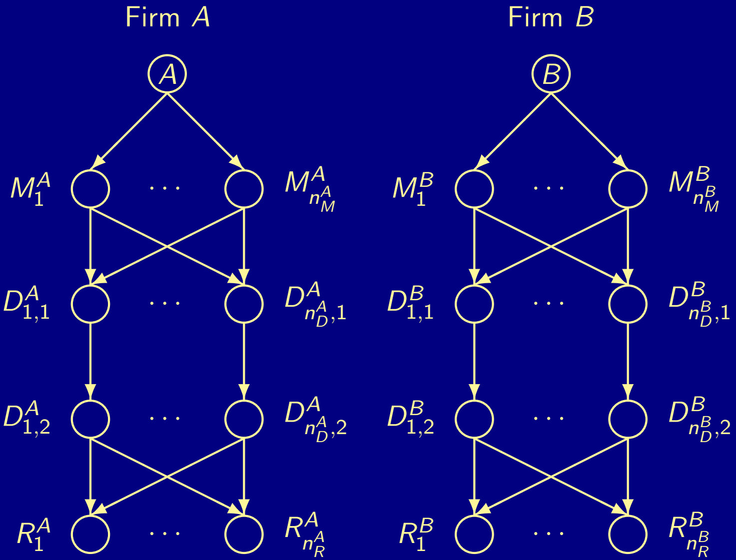

Figure 1: Case 0: Firms A and B Prior to Horizontal Merger (Nagurney (2009))

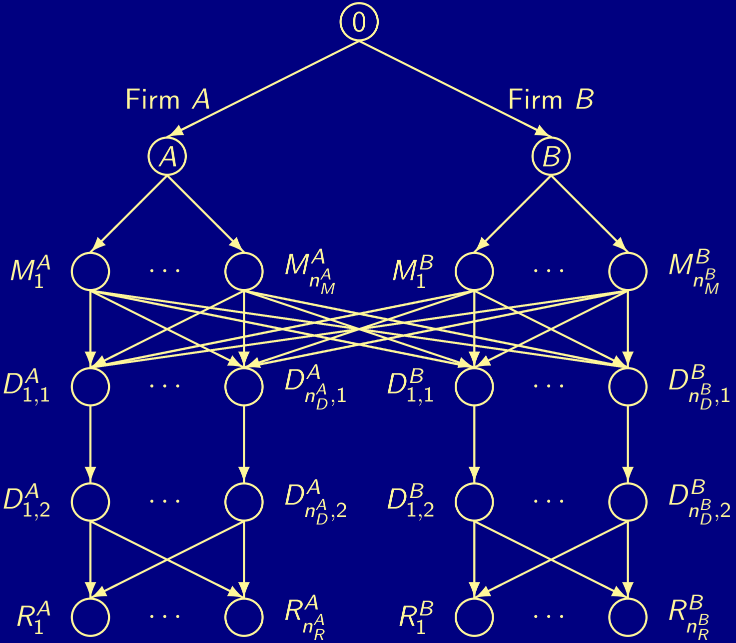

Figure 2: Case 1: Firms A and B Merge (Nagurney (2009))

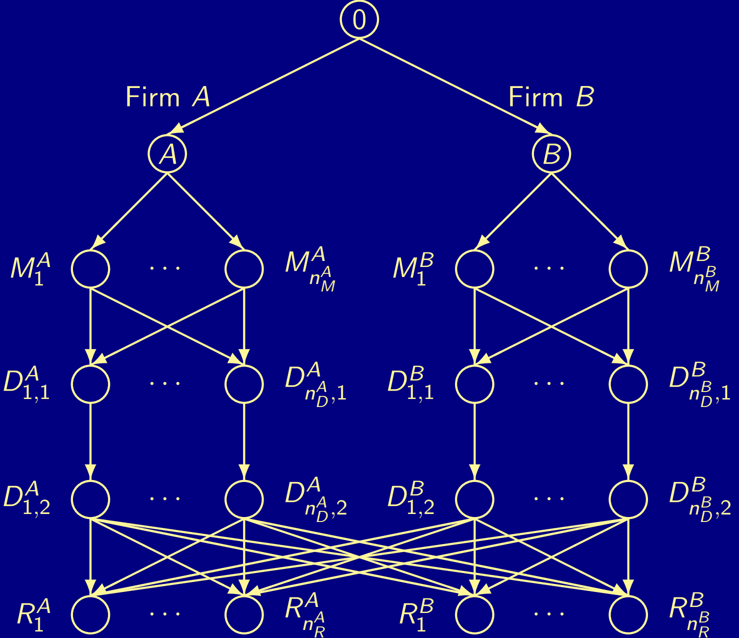

Figure 3: Case 2: Firms A and B Merge (Nagurney (2009))

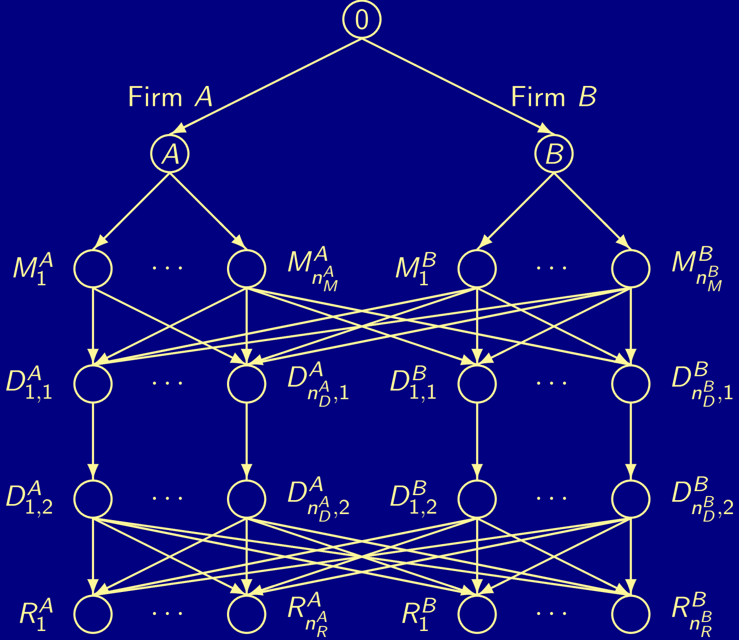

Figure 4: Case 3: Firms A and B Merge (Nagurney (2009))

Synergy Measure

The measure that we utilized in Nagurney (2009) to capture the gains, if any, associated with a horizontal merger Case i; $i = 1, 2, 3$ is as follows:

$$\mathcal{S}^i = \left[\frac{TC^0 - TC^i}{TC^0} \right] \times 100\%,$$

where TC^i is the total cost associated with the value of the objective function $\sum_{a \in L^i} \hat{c}_a(f_a)$ for $i = 0, 1, 2, 3$ evaluated at the optimal solution for Case i. Note that $\mathcal{S}^i$; $i = 1, 2, 3$ may also be interpreted as *synergy*.

This framework can also be applied to teaming of humanitarian organizations in the case of humanitarian logistics operations.

http://hlogistics.som.umass.edu

The Supply Chain Network Oligopoly Model (Nagurney (2008b))

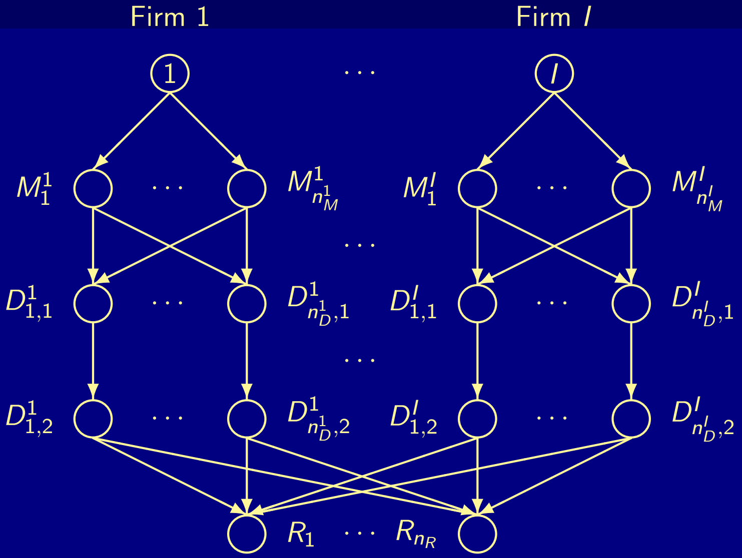

Figure 5: Supply Chain Network Structure of the Oligopoly

Assume that the profit functions are concave and continuously differentiable.

We consider the usual oligopolistic market mechanism in which the I firms produce and distribute the product in a noncooperative manner, each one trying to maximize its own profit. We seek to determine a nonnegative path flow pattern x for which the I firms will be in a state of equilibrium as defined below.

Definition: Supply Chain Network Cournot-Nash Equilibrium

A product flow pattern $x^* \in R_+^{n_{P^0}}$ is said to constitute a supply chain network Cournot-Nash equilibrium if for each firm i; $i = 1, \ldots, I$:

$$u_i(x_i^*, \hat{x}_i^*) \geq u_i(x_i, \hat{x}_i^*), \quad \forall x_i \in R_+^{n_{P_i^0}},$$

where $x_i \equiv \{\{x_p\} | p \in P_i^0\}$ and $\hat{x}_i^* \equiv (x_1^*, \ldots, x_{i-1}^*, x_{i+1}^*, \ldots, x_I^*)$.

Theorem: Variational Inequality Formulation

Assume that for each firm i; $i = 1, \ldots, I$, the profit function $u_i(x)$ is concave with respect to the variables x_p; $p \in P_i^0$, and is continuously differentiable. Then $x^* \in R_+^{n_{P^0}}$ is a supply chain network Cournot-Nash equilibrium if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^{I} \sum_{p \in P_i^0} \frac{\partial u_i(x^*)}{\partial x_p} \times (x_p - x_p^*) \geq 0, \quad \forall x \in R_+^{n_{P^0}},$$

or, equivalently: determine $x^* \in \mathcal{K}^0$ *satisfying:*

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\sum_{p\in P^0_{R^i_k}}\left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p}-\rho_{R_k}(x^*)-\sum_{l=1}^{n_R}\frac{\partial \rho_{R_l}(x^*)}{\partial d_{R_k}}\sum_{p\in P^0_{R^i_k}}x^*_p\right]$$

$$\times[x_p-x^*_p]\geq 0,\ \forall x\in\mathcal{K}^0,$$

where $\mathcal{K}^0 \equiv \{x|x\in R_+^{n_{P^0}}\}$ *and* $\frac{\partial \hat{C}_p(x)}{\partial x_p}\equiv \sum_{b\in L^0_i}\sum_{a\in L^0_i}\frac{\partial \hat{c}_b(f)}{\partial f_a}\delta_{ap}$ *for paths* $p\in P^0_i$.

Proof: Follows directly from Gabay and Moulin (1982) and Dafermos and Nagurney (1987). Here we have also utilized the fact that the demand price functions can be reexpressed directly as a function of path flows.

It is interesting to relate this supply chain network oligopoly model to the spatial oligopoly model proposed by Dafermos and Nagurney (1987), which is done in the following corollary.

Corollary: Relationship to the Spatial Oligopoly Model

Assume that that are I firms in the supply chain network oligopoly model and that each firm has a single manufacturing plant and a single distribution center. Assume also that the distribution costs from each manufacturing plant to the distribution center and the storage costs are all equal to zero. Then the resulting model is isomorphic to the spatial oligopoly model of Dafermos and Nagurney (1987) whose underlying network structure is given in Figure 6.

Proof: Follows from Dafermos and Nagurney (1987) and Nagurney (1993).

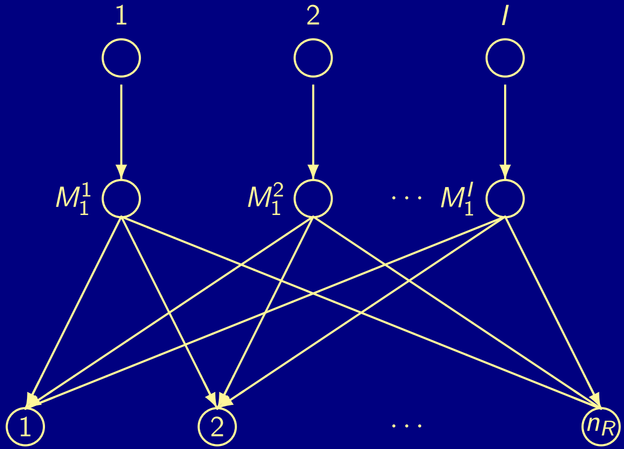

Figure 6: Network Structure of the Spatial Oligopoly

The relationship between the supply chain network oligopoly model to the classical Cournot (1838) oligopoly model is now given (see also Gabay and Moulin (1982) and Nagurney (1993)).

Corollary: Relationship to Classical Oligopoly Model

Assume that there is a single manufacturing plant associated with each firm in the above model, and a single distribution center. Assume also that there is a single demand market. Assume also that the manufacturing cost of each manufacturing firm depends only upon its own output. Then, if the storage and distribution cost functions are all identically equal to zero the above model collapses to the classical oligopoly model in quantity variables. Furthermore, if $I = 2$*, one then obtains the classical duopoly model.*

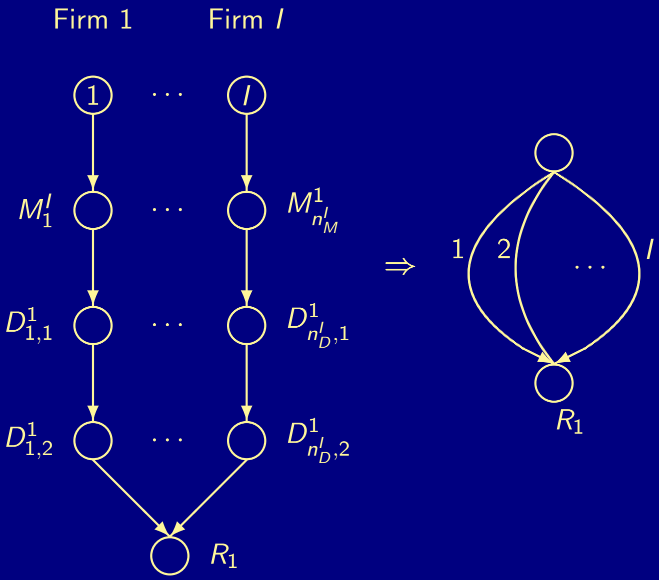

Figure 7: Network Structure of the Classical Oligopoly

Mergers Through Coalition Formation

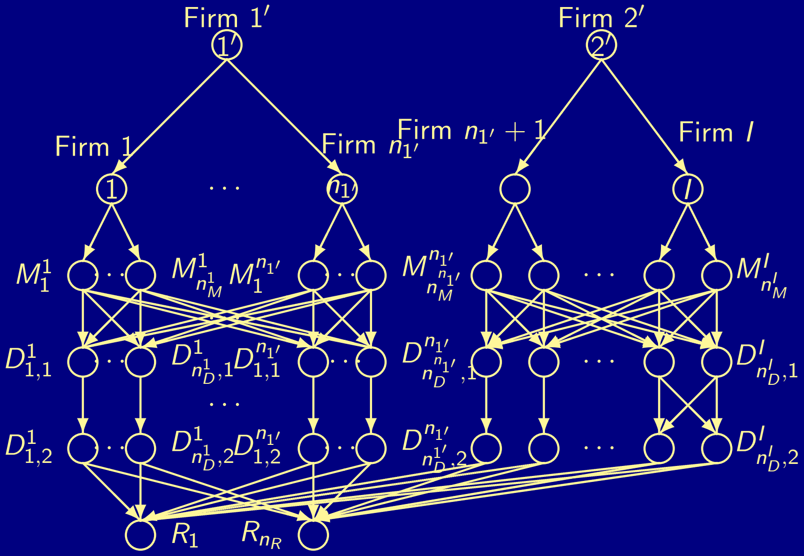

Figure 6: Mergers of the First $n_{1'}$ Firms and the Next $n_{2'}$ Firms

Summary and Conclusions

In this talk we have demonstrated the richness of network concepts for quantifying synergies as well as vulnerabilities associated with supply chain networks in the global economy.

The need for performance metrics as well as analytics has never been more profound nor more feasible.

By focusing on interdisciplinary research and practice we can better identify which nodes and links in supply chain networks truly matter and whether or not to participate in any merger based on potential and predetermined synergies.

Thank You!

For more information, see http://supernet.som.umass.edu