To Merge or Not to Merge: Multimarket Supply Chain Network Oligopolies, Coalitions, and the Merger Paradox

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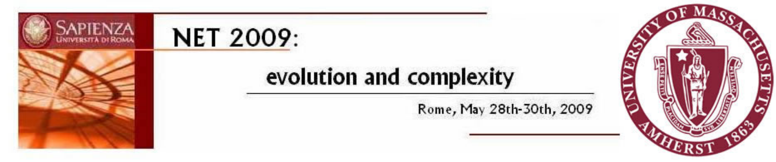

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Acknowledgements

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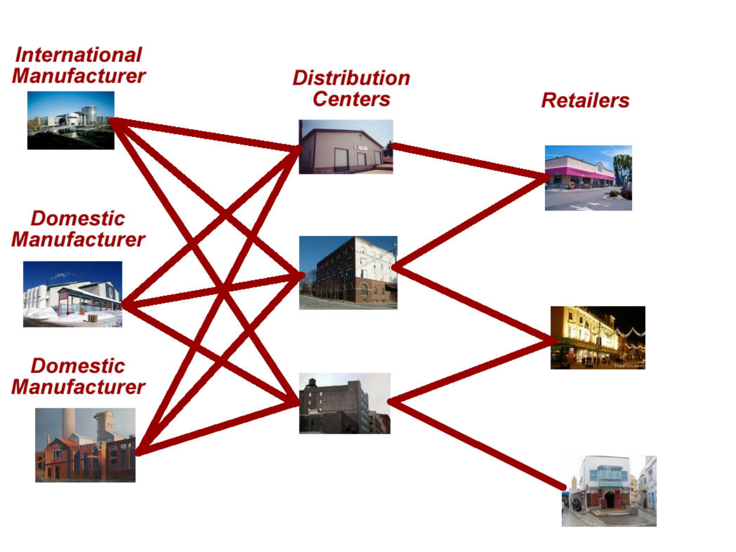
Outline

- Motivation
- Literature
- The Supply Chain Network Oligopoly Model and Special Cases
- Mergers Through Coalition Formation
- The Algorithm
- Numerical Examples and Insights Into the Merger Paradox
- Summary and Conclusions

Motivation

- Today, supply chains are more extended and complex than ever before.
- At the same time, the current competitive economic environment requires that firms operate efficiently, which has spurred research to determine how to utilize supply chains more effectively.



Depiction of a Supply Chain Network

Motivation

- There is also a pronounced amount of merger activity. According to Thomson Financial, in the first nine months of 2007 alone, worldwide merger activity hit \$3.6 trillion, surpassing the total from all of 2006 combined.
- Notable examples: KMart and Sears in the retail industry in 2004 and Federated and May in 2005, Coors and Molson in the beverage industry in 2005, and the recently proposed merger between Anheuser Busch and InBev.

kmart Sears


Coors LIGHT
MOLSON
CANADIAN


Federated
DEPARTMENT STORES, INC.
MAY

Motivation

According to Kusstatscher and Cooper (2005) there were five major waves of of Merger & Acquisition (M &A) activity:

The First Wave: 1898-1902: an increase in horizontal mergers that resulted in many US industrial groups;

The Second Wave: 1926-1939: mainly public utilities;

The Third Wave: 1969-1973: *diversification* was the driving force;

The Fourth Wave: 1983-1986: the goal was efficiency;

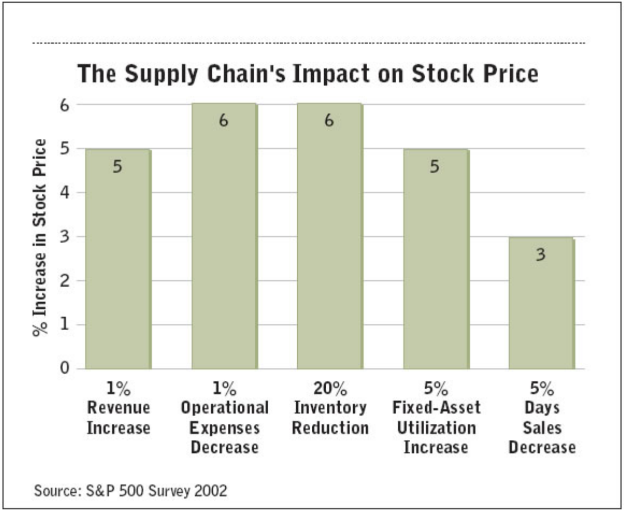
The Fifth Wave: 1997 until the early years of the 21st century: *globalization* was the motto.

In 1998, M&As reached $2.1 trillion worldwide; in 1999, the activity exceeded $3.3 trillion, and in 2000, almost $3.5 was reached.

Motivation

- A survey of 600 executives involved in their companies' mergers and acquisitions (M&A) conducted by Accenture and the Economist Unit (see Byrne (2007)) found that less than half (45%) achieved expected cost-saving synergies.
- Langabeer and Seifert (2003) determined a direct correlation between how effectively supply chains of merged firms are integrated and how successful the merger is. They concluded, based on the empirical findings of Langabeer (2003), who analyzed hundreds of mergers over the preceding decade, that

Improving Supply Chain Integration between Merging Companies is the Key
to Improving the Likelihood of Post-Merger Success!



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Motivation

- It is, therefore, worthwhile to develop tools that can better predict the associated strategic gains associated with supply chain network integration, in the context of mergers/acquisitions.
- In this presentation, we consider the modeling of supply chain network oligopolies consisting of firms who compete in a Nash-Cournot framework as well as the modeling of mergers that are formed through coalitions.
- We are also motivated by the Merger Paradox.

Examples of Oligopolies

Examples of Oligopolies:

- oil, beer, and automobile manufacturing companies in the US;
- supermarket chains in the United Kingdom;
- media outlets in Australia.

Our research on this topic is in line with our research on paradoxes on networks, in general, including for example, on the Braess (1968) Paradox where the addition of a new route to a transportation network makes everyone worse off in terms of travel time. Such a paradox has also been discovered in terms of the Internet.

Professor Braess with Nagurney and Wakolbinger at UMass Amherst after the publication of their translation in *Transportation Science* of the 1968 Braess paper from German to English.

Literature – Oligopolies and Mergers in Economics

- The topic of mergers in an oligopolistic setting has been a major issue in economics and a subject of much discussion.
- Salant, Switzer, and Reynolds (1983) pointed out that, in quantity-setting games, as we consider here, it is usually not advantageous for the merging firms unless the merger includes the vast majority of firms, in particular, 80% or more.
- According to Pepall, Richards, and Norman (1999): "What may be surprising to you is that it is, in fact, quite difficult to construct a simple economic model in which there are sizable profitability gains for the firms participating in a horizontal merger that is not a merger to monopoly."

This has come to be known as the *Merger Paradox.*

Literature – Oligopolies and Mergers in Economics

Other notable papers from the economics literature on mergers:

- ◇ Perry and Porter (1985)
- ◇ Fershtman and Judd (1987)
- ◇ Farrell and Shapiro (1990).

Meschi (1997) surveyed analytical perspectives for mergers (and acquisitions) and noted that much of the literature on this topic in economics is limited to linear cost and demand functions.

Cheong and Judd (2000) noted that numerical methods when applied to compute equilibria of merger problems associated with oligopolies may yield deeper insights and information since classical techniques from industrial organization may no longer be sufficient.

Supply Chains, Operations Research/Management Science, and Mergers

Mergers and acquisitions in the context of supply chains, specifically, is a topic that has been explored more recently by operations researchers:

- Gupta and Gerchak (2002)
- Langabeer and Seifert (2003)
- Soylu et al. (2006)
- Xu (2007).

Hakkinen et al. (2004) overviewed the literature on the integration of logistics after M&As and concluded that operational issues, in general, and logistics issues, in particular, have received insufficient attention; see also Herd, Saksena, and Steger (2005).

Recently, we introduced a system-optimization perspective for supply chains and proposed a cost synergy measure associated with evaluating proposed mergers:

◇ Nagurney, A. (2009) "A System-Optimization Perspective for Supply Chain Network Integration: The Horizontal Merger Case," *Transportation Research E* **45**, 1-15.

In that paper, the merger of two firms was modeled and the demands for the product at the markets, which were distinct for each firm prior to the merger, were assumed to be fixed.

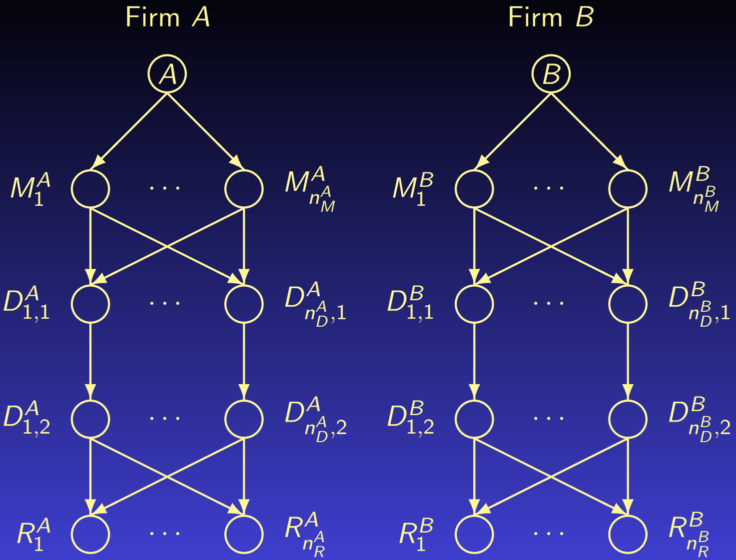


Figure 1: Case 0: Firms A and B Prior to Horizontal Merger (Nagurney (2009))

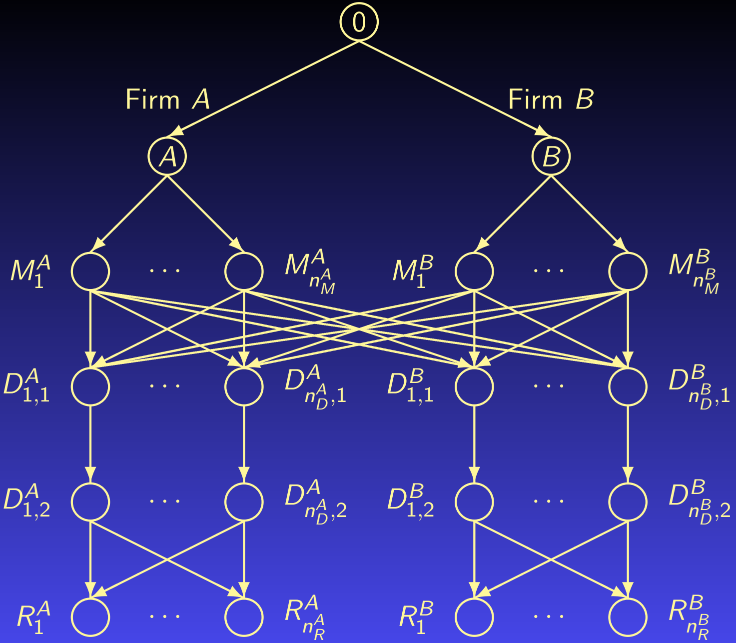


Figure 2: Case 1: Firms A and B Merge (Nagurney (2009))

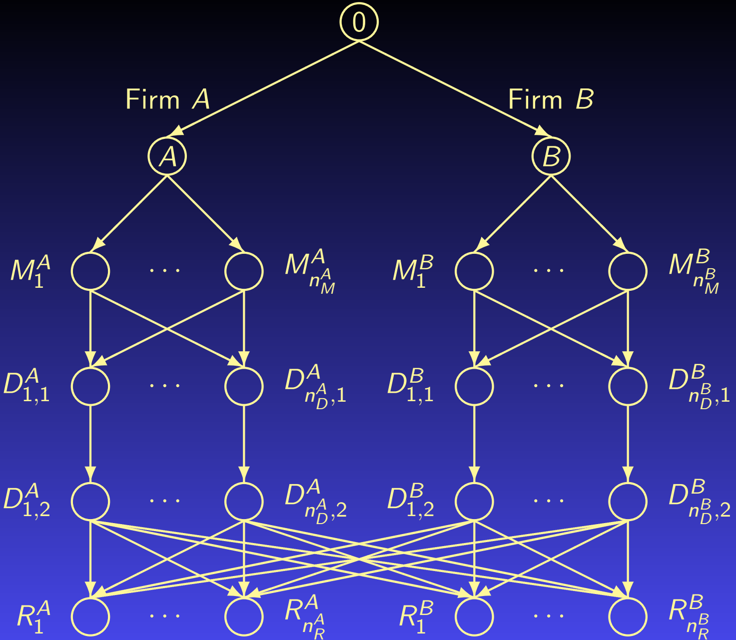


Figure 3: Case 2: Firms A and B Merge (Nagurney (2009))

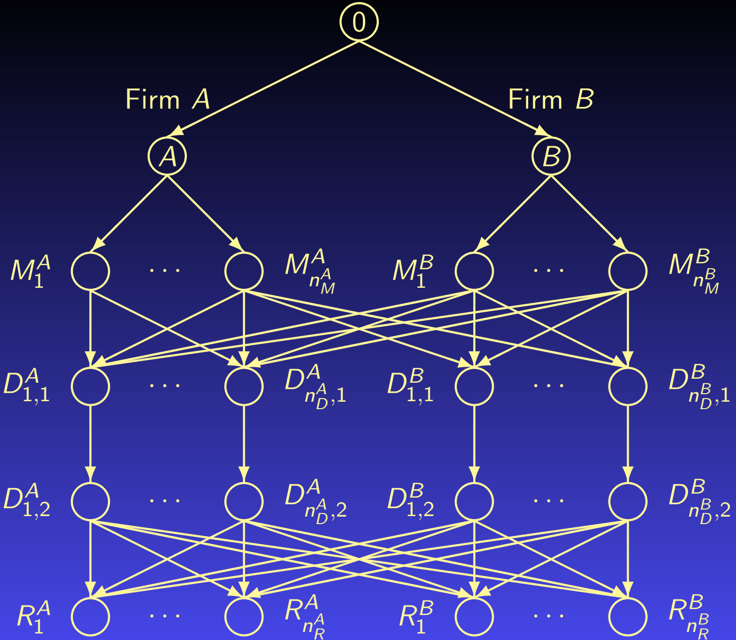


Figure 4: Case 3: Firms A and B Merge (Nagurney (2009))

Synergy Measure

The measure that we utilized in Nagurney (2009) to capture the gains, if any, associated with a horizontal merger Case i; $i = 1, 2, 3$ is as follows:

$$\mathcal{S}^i = \left[\frac{TC^0 - TC^i}{TC^0}\right] \times 100\%,$$

where TC^i is the total cost associated with the value of the objective function $\sum_{a \in L^i} \hat{c}_a(f_a)$ for $i = 0, 1, 2, 3$ evaluated at the optimal solution for Case i. Note that $\mathcal{S}^i$; $i = 1, 2, 3$ may also be interpreted as *synergy*.

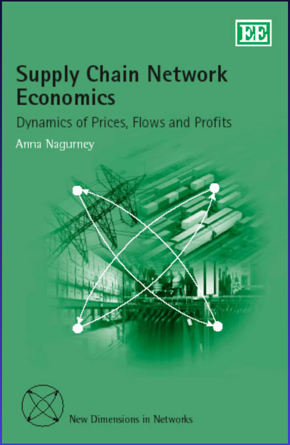
We have now extended the models in Nagurney (2009) to include:

- multicriteria decision-making in the form of environmental concerns (Nagurney and Woolley (2008))

- multiple products (Nagurney, Woolley, and Qiang (2008)), which we have also applied to model the merger of organizations for humanitarian logistical operations, to appear in the *International Transactions in Operational Research*

- noncooperative behavior / competition (Nagurney (2009b)), to appear in *Computational Management Science*.

For background on supply chain networks, equilibria, and associated dynamics, and extensive references, please see:



The Supply Chain Network Oligopoly Model

We consider a finite number of I firms, with a typical firm denoted by i, who are involved in the production, storage, and distribution of a homogeneous product and who compete noncooperatively in an oligopolistic manner.

We assume that each firm is represented as a network of its economic activities. Each firm i; $i = 1, \ldots, I$ has n^i_M manufacturing facilities/plants; n^i_D distribution centers, and serves the same n_R retail outlets/demand markets.

Let L^0_i denote the set of directed links representing the economic activities associated with firm i; $i = 1, \ldots, I$. Let $L^0 \equiv \cup_{i=1,I} L^0_i$. Let $G^0 = [N^0, L^0]$ denote the graph consisting of the set of nodes N^0 and the set of links L^0 in Figure 5.

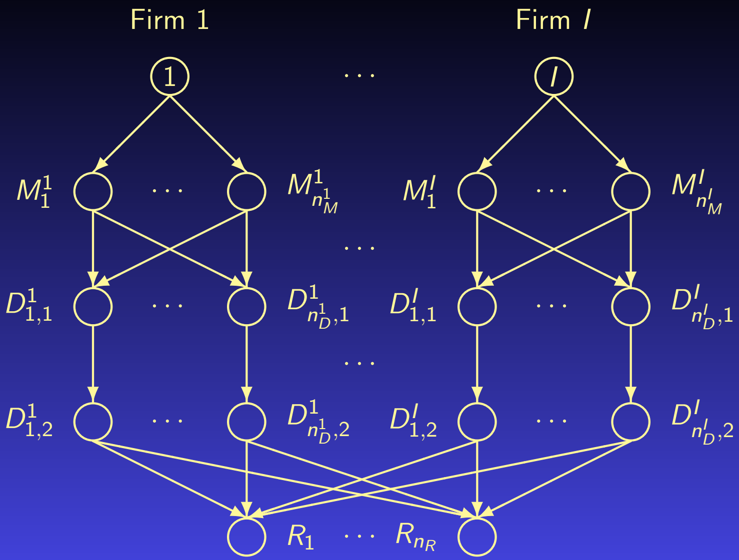


Figure 5: Supply Chain Network Structure of the Oligopoly

We denote the links by a, b, etc., and the flow of the product on link a by f_a.

Let d_{R_k} denote the demand for the product at demand market R_k; $k = 1, \ldots, n_R$. Let x_p denote the nonnegative flow of the product on path p joining (origin) node i; $i = 1, \ldots, I$ with a (destination) demand market node.

Conservation of Flow Equations

Then the following conservation of flow equations must hold:

$$\sum_{p \in P^0_{R_k}} x_p = d_{R_k}, \quad k = 1, \ldots, n_R,$$

where $P^0_{R_k}$ denotes the set of paths connecting the (origin) nodes i; $i = 1, \ldots, I$ with (destination) demand market R_k.

Also, $P^0_{R_k} = \cup_{i=1,\ldots,I} P^0_{R^i_k}$, where $P^0_{R^i_k}$ denotes the set of paths from origin node i to demand market k as in Figure 5.

P^0 denotes the set of *all* paths in Figure 5, that is, $P^0 = \cup_{k=1,\dots,n_R} P^0_{R_k}$. There are n_{P^0} paths in the network in Figure 5. P^0_i denotes the set of all paths from firm i to all the demand markets for $i = 1, \dots, I$. There are $n_{P^0_i}$ paths from the firm i node to the demand markets.

Additional Conservation of Flow Equations

We must also have the following conservation of flow equations satisfied:

$$f_a = \sum_{p \in P^0} x_p \delta_{ap}, \quad \forall a \in L^0,$$

where $\delta_{ap} = 1$ if link a is contained in path p and $\delta_{ap} = 0$, otherwise.

The path flows must be nonnegative, that is,

$$x_p \geq 0, \quad \forall p \in P^0.$$

The Total Cost, Demand Price, and Profit Functions

The total cost on link a, $\hat{c}_a$, is: $\hat{c}_a = \hat{c}_a(f), \quad \forall a \in L^0$, where f is the vector of link flows.

The demand price at demand market R_k, ρ_{R_k}, is:
$\rho_{R_k} = \rho_{R_k}(d), \quad k = 1, \ldots, n_R$, where d is the n_R-dimensional vector of demands.

The Profit Functions

The profit function u_i of firm i; $i = 1, \ldots, I$, is then:

$$u_i = \sum_{k=1}^{n_R} \rho_{R_k}(d) \sum_{p \in P^0_{R^i_k}} x_p - \sum_{a \in L^0_i} \hat{c}_a(f),$$

$$\text{or:} \quad u = u(x),$$

where x is the vector of all the path flows $\{x_p, p \in P^0\}$, and u is the I-dimensional vector of the firms' profits.

Assume that the profit functions are concave and continuously differentiable.

We consider the usual oligopolistic market mechanism in which the I firms produce and distribute the product in a noncooperative manner, each one trying to maximize its own profit. We seek to determine a nonnegative path flow pattern x for which the I firms will be in a state of equilibrium as defined below.

Definition 1: Supply Chain Network Cournot-Nash Equilibrium

A product flow pattern $x^* \in R_+^{n_{P^0}}$ is said to constitute a supply chain network Cournot-Nash equilibrium if for each firm i; $i = 1, \ldots, I$:

$$u_i(x_i^*, \hat{x}_i^*) \geq u_i(x_i, \hat{x}_i^*), \quad \forall x_i \in R_+^{n_{P_i^0}},$$

where $x_i \equiv \{\{x_p\}|p \in P_i^0\}$ and $\hat{x}_i^* \equiv (x_1^*, \ldots, x_{i-1}^*, x_{i+1}^*, \ldots, x_I^*)$.

Theorem 1: Variational Inequality Formulation

Assume that for each firm i; $i = 1, \ldots, I$, the profit function $u_i(x)$ is concave with respect to the variables x_p; $p \in P_i^0$, and is continuously differentiable. Then $x^* \in R_+^{n_{P^0}}$ is a supply chain network Cournot-Nash equilibrium if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^{I} \sum_{p \in P_i^0} \frac{\partial u_i(x^*)}{\partial x_p} \times (x_p - x_p^*) \geq 0, \quad \forall x \in R_+^{n_{P^0}},$$

or, equivalently: determine $x^* \in \mathcal{K}^0$ satisfying:

$$\sum_{i=1}^{I} \sum_{k=1}^{n_R} \sum_{p \in P^0_{R^i_k}} \left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \rho_{R_k}(x^*) - \sum_{l=1}^{n_R} \frac{\partial \rho_{R_l}(x^*)}{\partial d_{R_k}} \sum_{p \in P^0_{R^i_k}} x^*_p \right]$$

$$\times [x_p - x^*_p] \geq 0, \forall x \in \mathcal{K}^0,$$

where $\mathcal{K}^0 \equiv \{x | x \in R_+^{n_{P^0}}\}$ *and* $\frac{\partial \hat{C}_p(x)}{\partial x_p} \equiv \sum_{b \in L^0_i} \sum_{a \in L^0_i} \frac{\partial \hat{c}_b(f)}{\partial f_a} \delta_{ap}$ *for paths* $p \in P^0_i$*.*

Proof: Follows directly from Gabay and Moulin (1982) and Dafermos and Nagurney (1987). Here we have also utilized the fact that the demand price functions can be reexpressed directly as a function of path flows.

It is interesting to relate this supply chain network oligopoly model to the spatial oligopoly model proposed by Dafermos and Nagurney (1987), which is done in the following corollary.

Corollary 1: Relationship to the Spatial Oligopoly Model

Assume that that are I firms in the supply chain network oligopoly model and that each firm has a single manufacturing plant and a single distribution center. Assume also that the distribution costs from each manufacturing plant to the distribution center and the storage costs are all equal to zero. Then the resulting model is isomorphic to the spatial oligopoly model of Dafermos and Nagurney (1987) whose underlying network structure is given in Figure 6.

Proof: Follows from Dafermos and Nagurney (1987) and Nagurney (1993).

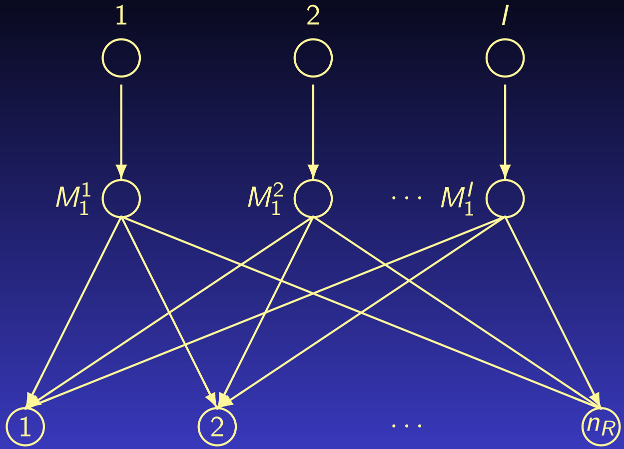


Figure 6: Network Structure of the Spatial Oligopoly

The relationship between the supply chain network oligopoly model to the classical Cournot (1838) oligopoly model is now given (see also Gabay and Moulin (1982) and Nagurney (1993)).

Corollary 2: Relationship to Classical Oligopoly Model

Assume that there is a single manufacturing plant associated with each firm in the above model, and a single distribution center. Assume also that there is a single demand market. Assume also that the manufacturing cost of each manufacturing firm depends only upon its own output. Then, if the storage and distribution cost functions are all identically equal to zero the above model collapses to the classical oligopoly model in quantity variables. Furthermore, if $I = 2$, one then obtains the classical duopoly model.

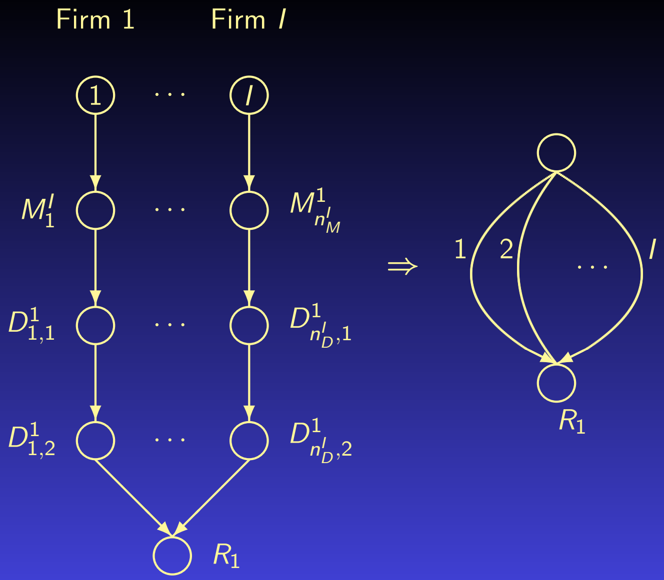


Figure 7: Network Structure of the Classical Oligopoly

Mergers Through Coalition Formation

The coalitions are formed between/among the I firms as follows.

The first $n_{1'}$ firms join to form new firm $1'$, the second group of $n_{2'}$ firms join to form firm $2'$, and so on, through the remaining $n_{I'}$ firms joining to form the I'th firm.

Associated with a coalition formation in the form of a merger, we construct a new *supersource* node to represent the new firm and we construct new links from each such supersource node, which now becomes an origin node, to the respective top-most original firm nodes. If firms do not enter into any merger/coalition we simply retain the original nodes for that firm and retain their top-most nodes as the origin nodes.

Since the newly merged firms now share resources, including their distribution centers, we now add new links from their original manufacturing nodes to the other firms' in the merger distribution center nodes and associate total cost functions with these new links.

Associated with the coalition formation resulting in a particular set of mergers is a new graph denoted by G^1, which consists of the original nodes and links as in G^0 but with the new nodes and links $[N^1, L^1]$ to represent the formation of the new firms.

This model is interesting and relevant, since not all firms in an industry necessarily need to merge when a merger occurs. Such a model also allows one to evaluate the effect of the merger on total costs and profits of firms not associated with the merger. Moreover, it allows on to explore questions regarding the merger paradox.

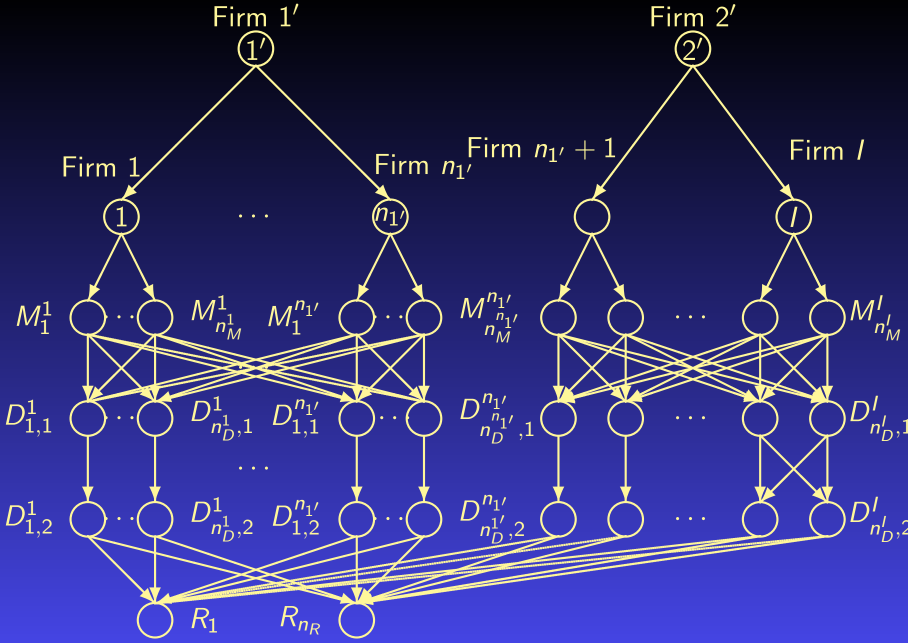


Figure 8: Mergers of the First $n_{1'}$ Firms and the Next $n_{2'}$ Firms

The Conservation of Flow Equations Post-Mergers

Define $P^1_{R^i_k}$ as the set of paths joining origin node i with demand market R_k, where $i = 1', 2', \ldots, I'$ with the proviso that we relabel the origin nodes of the unmerged firms accordingly.

Let x_p now denote the nonnegative flow on a path joining the (origin) node i with a demand market node. Then:

$$\sum_{p \in P^1_{R_k}} x_p = d_{R_k}, \quad k = 1, \ldots, n_R,$$

where $P^1_{R_k} = \cup_{i=1',\ldots,I'} P^1_{R^i_k}$. Let P^1_i denote the set of all paths emanating from node i to the demand markets for $i = 1', \ldots, I'$.

The link conservation of flow equations now take the form:

$$f_a = \sum_{p \in P^1} x_p \delta_{ap}, \quad \forall a \in L^1, \text{ where } P^1 = \cup_{k=1,\ldots,n_R} P^1_{R_k}.$$

The path flows are nonnegative: $x_p \geq 0, \quad \forall p \in P^1$.

Total Cost, Demand, and Profit Functions Post-Merger

Again, we assume that the new links that correspond to the merger have total cost functions associated with them; hence,

$$\hat{c}_a = \hat{c}_a(f), \quad \forall a \in L^1,$$

and the new total cost functions on the new links have properties corresponding to those as in the original links. Of course, we retain the original total cost functions on the original links.

The demand price functions remain as before.

The profit now for the firms, with the firms renumbered as: $i = 1', \ldots, I'$ can be expressed as:

$$u_i = \sum_{k=1}^{n_R} \rho_{R_k}(d) \sum_{p \in P^1_{R^i_k}} x_p - \sum_{a \in L^1_i} \hat{c}_a(f),$$

where L^1_i denotes that subset of links in L^1 corresponding to firm i; $i = 1', \ldots, I'$.

We can now adapt Definition 1 to the merger/coalition setting, in which firms $1', \ldots, I'$ compete with one another in a Cournot-Nash setting until the equilibrium is attained. We impose the same assumptions on the utility functions here as were imposed on the utility functions in Theorem 1.

Definition 2: Merger Supply Chain Cournot-Nash Equilibrium

A product flow pattern $x^* \in R_+^{n_{P^1}}$ is said to constitute a supply chain network Cournot-Nash equilibrium for the particular merger, due to coalition formation, if for firm i; $i = 1', \ldots, I'$:

$$u_i(x_i^*, \hat{x}_i^*) \geq u_i(x_i, \hat{x}_i^*), \quad \forall x_i \in R_+^{n_{P_i^1}},$$

where now, w.l.o.g. $x_i \equiv \{\{x_p\} | p \in P_i^1\}$ and $\hat{x}_i^* \equiv (x_{1'}^*, \ldots, x_{i-1}^*, x_{i+1}^*, \ldots, x_{I'}^*)$.

Theorem 2: Variational Inequality Formulation

Assume that for each firm i; $i = 1', \ldots, I'$, $u_i(x)$ is concave with respect to the variables x_p; $p \in P_i^1$, and is continuously differentiable. Then $x^* \in R_+^{n_{P^1}}$ is a merger supply chain Cournot-Nash equilibrium according to Definition 2 if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^{I'} \sum_{p \in P_i^1} \frac{\partial u_i(x^*)}{\partial x_p} \times (x_p - x_p^*) \geq 0, \quad \forall x \in R_+^{n_{P^1}},$$

or, equivalently, determine $x^* \in \mathcal{K}^1$ satisfying: $\forall x \in \mathcal{K}^1$:

$$\sum_{i=1}^{I'} \sum_{k=1}^{n_R} \sum_{p \in P^1_{R_k^i}} \left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \rho_{R_k}(x^*) - \frac{\partial \rho_{R_l}(x^*)}{\partial d_{R_k}} \sum_{p \in P^1_{R_k^i}} x_p^* \right] \times [x_p - x_p^*] \geq 0,$$

$\mathcal{K}^1 \equiv \{x | x \in R_+^{n_{P^1}}\}$, $\frac{\partial \hat{C}_p(x)}{\partial x_p} \equiv \sum_{b \in L_i^1} \sum_{a \in L_i^1} \frac{\partial \hat{c}_b(f)}{\partial f_a} \delta_{ap}$, $\forall p \in P_i^1$.

The Algorithm

Recall the Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993). Specifically, at an iteration τ of the Euler method (see also Nagurney and Zhang (1996b)) one computes:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau} F(X^{\tau})),$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem: determine $X^* \in \mathcal{K}$ such that

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

where $\langle \cdot, \cdot \rangle$ is the inner product in n-dimensional Euclidean space, $X \in R^n$, and $F(X)$ is an n-dimensional function from $\mathcal{K}$ to R^n, with $F(X)$ being continuous.

The Algorithm (cont'd.)

Both variational inequality problems (pre and post the merger(s)) can be put into the above standard form (see also Nagurney (1993)).

As shown in Dupuis and Nagurney (1993); see also Nagurney and Zhang (1996), for convergence of the general iterative scheme, which induces the Euler method, among other methods, the sequence $\{a_\tau\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \rightarrow 0$, as $a_\tau \rightarrow \infty$.

Explicit Formulae for the Solution of the Pre-Merger Supply Chain Network

The iterative step above yields for the pre-merger problem: $\forall i, \forall k, \forall p \in P^0_{R^i_k}$:

$$x_p^{\tau+1} = \max\{0, x_p^\tau + a_\tau(\rho_{R_k}(x^\tau) - \sum_{l=1}^{n_R} \frac{\partial \rho_l(x^\tau)}{\partial d_{R_k}} \sum_{p \in P^0_{R^i_k}} x_p^\tau - \frac{\partial \hat{C}_p(x^\tau)}{\partial x_p})\}.$$

Explicit Formulae for the Solution of the Merger Supply Chain Network

In the case of the merger supply chain network problem, the iterative step becomes: $\forall i, \forall k, \forall p \in P^1_{R^i_k}$:

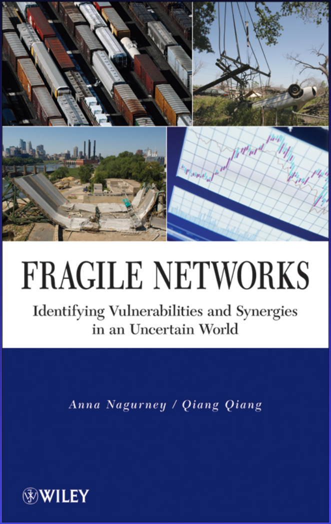
$$x_p^{\tau+1} = \max\{0, x_p^\tau + a_\tau(\rho_{R_k}(x^\tau) - \sum_{l=1}^{n_R} \frac{\partial \rho_l(x^\tau)}{\partial d_{R_k}} \sum_{p \in P^1_{R^i_k}} x_p^\tau - \frac{\partial \hat{C}_p(x^\tau)}{\partial x_p})\}.$$

Both explicit formulae above are similar to the iterative step of the Euler method for elastic demand traffic network equilibrium problems (cf. Nagurney and Zhang (1996)).

Note that the variational inequality problems of concern can also be reformulated in link flow variables. However, we have provided formulations in path flow variables, since, computationally, these lead to the above simple and explicit formulae.

Bertsekas and Gafni (1982) also proposed projection methods in path flow variables for the traffic assignment problem, along with convergence results. It is also worth noting that both of the above iterative steps can be implemented on parallel architectures (see also Bertsekas and Tsitsiklis (1989) and Nagurney (1996)).

Our New Book


FRAGILE NETWORKS
Identifying Vulnerabilities and Synergies in an Uncertain World
Anna Nagurney / Qiang Qiang
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Numerical Examples

We present three sets of numerical oligopoly examples of increasing complexity.

In Set 1, we present mergers associated with oligopoly examples consisting of four firms where each firm has a single manufacturing plant and a single distribution center and there is a single retailer/demand market that each of the firms competes in.

In Set 2, we again considered such oligopoly problems but, unlike the problems in Set 1, the total cost functions on the new merger links are no longer equal to zero.

In Set 3, we compute solutions to the mergers of more complex supply chain networks with multiple demand markets.

We implemented the Euler method for the pre and post-merger supply chain network problems. The codes were implemented in FORTRAN and the computer used for the computations was a Unix system at the University of Massachusetts at Amherst. The convergence tolerance was: $|X^{\tau+1} - X^{\tau}| \leq .001$ for all the examples. The sequence $\{a_\tau\}$ used was: $.1\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \ldots, \}$.

Problem Set 1

In this set, we solved oligopolistic supply chain network problems both prior to and and post various mergers.

Example 1.1

The original/baseline problem, Example 1.1, consisted of four oligopolistic firms (cf. Figure 9). For simplicity, we let all the total cost functions on the links representing this baseline problem be equal and given by:

$$\hat{c}_a = 2f_a^2 + f_a, \quad \forall a \in L_i^0; i = 1, 2, 3, 4.$$

The demand price function at the single demand market was given by:

$$\rho_{R_1} = -d_{R_1} + 200.$$

Denote the paths by p_1, p_2, p_3, and p_4 corresponding to firm 1 through firm 4 with each path originating in its top-most firm node and ending in the demand market node (Fig. 9).

The Firms

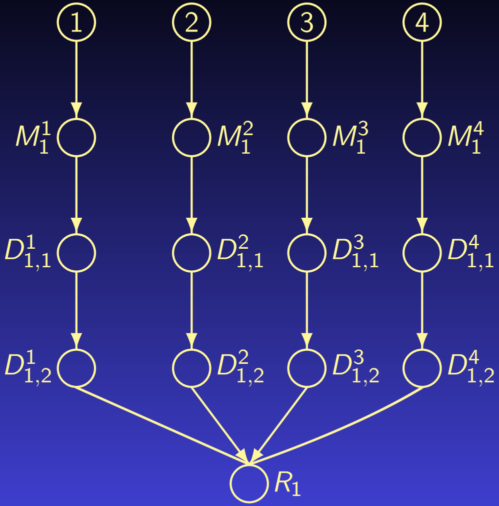

Figure 9: Network Structure of the Four Firm Oligopoly for Problem Sets 1 and 2

The Euler method converged to the equilibrium solution:

$$x^*_{p_1} = x^*_{p_2} = x^*_{p_3} = x^*_{p_4} = 9.33, \quad d^*_{R_1} = 37.33.$$

The demand market price was: $\rho_{R_1} = 162.67$. The total cost was: $2,936.50$; the total revenue was: $6,072.79$, and the total profit was: $3,136.29$. Each firm in this four firm oligopoly, hence, earned an individual profit of: 784.07.

Example 1.1a

We considered the case of the first two firms in Example 1.1 merging. Recall that, according to Salant, Switzer, and Reynolds (1983), in a Cournot oligopoly, it is not usually advantageous for quantity-setting firms to merge unless almost all of them merge. We assumed in this merger example, as well as in the remainder of the examples in Problem Set 1, that the total costs on the new links associated with the particular merger were all identically equal to zero.

Refer to Figure 10 for the supply chain network topology associated with this Example. Path p_1 now originates in node $1'$ but follows then the same sequence of nodes as path p_1 in Example 1.1; the same for path p_2. Paths p_3 and p_4 remained as in Example 1.1. There were two additional paths associated with new firm $1'$ and we denote these cross-hauling paths, respectively, by paths p_5 and p_6.

Merger 1

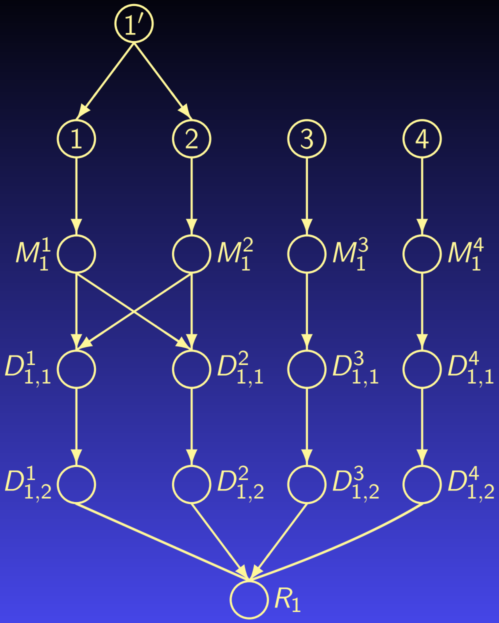

Figure 10: Supply Chain Network Structure for Example 1.1a

The computed equilibrium solution was now:

$$x^*_{p_1} = x^*_{p_2} = 0.00, \quad x^*_{p_3} = x^*_{p_4} = 9.14, \quad x^*_{p_5} = x^*_{p_6} = 11.16$$

with an equilibrium demand $d^*_{R_1} = 40.60$. The demand market price was: $\rho_{R_1} = 159.40$. The total cost was: $2,971.56$. The total revenue was: $6,472.11$, and the total profit was: $3,500.54$. Each firm in the merged firm earned a profit of: $1,038.00$, whereas each of the two unmerged firms earned a profit of: 712.04. Hence, each of the "insiders" gained considerably, whereas the firms that did not merge (the "outsiders") now had lower profits than in Example 1.1.

Hence, through computations, we were able to construct a simple counterexample to the ideas set forth in Salant, Switzer, and Reynolds (1983) but in the more general framework of supply chain network oligopolies.

Example 1.1b

We next considered the merger of the first three firms in the oligopoly in Example 1.1. The resulting supply chain network structure post the merger is given in Figure 11. Again, as in Example 1.1a, we assumed that the total cost functions on all the new links establishing the merger were identically equal to zero.

There are now nine paths joining node $1'$ to demand market node R_1 in Figure 11.

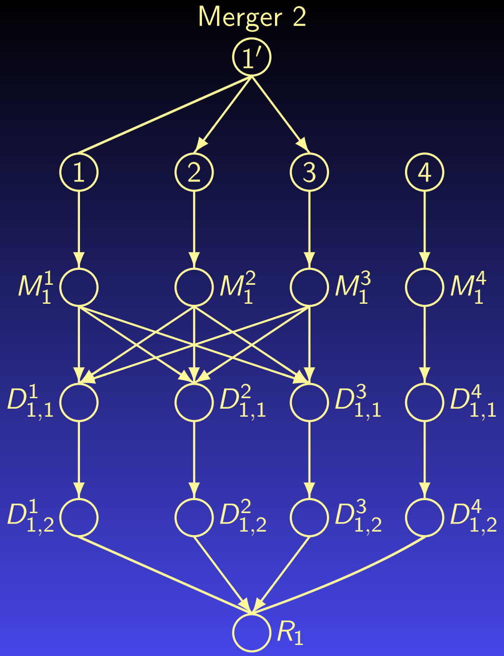

Figure 11: Supply Chain Network Structure for Example 1.1b

The computed equilibrium solution was as follows. Each of the original paths associated with the original first three firms prior to the merger (but extended to include node $1'$) had flow equal to zero, whereas the flow on each of the new paths resulting from the merger was: 5.22. The flows on the path for the fourth firm, which did not enter into the merger was: 9.15. The demand $d^*_{R_1} = 40.44$ and the demand market price $\rho_{R_1} = 159.56$. The total cost was now: $2,758.85$. The total revenue was now: $6,453.71$, and the total profit was: $3,694.86$.

Each of the firms in the three-firm merger now earned a profit of: 980.45, whereas the unmerged firm earned a profit of: 753.49. Hence, for the fourth firm, from a profit perspective, it was better when three firms, rather than only two, merged. However, for the first two firms, their individual profit was higher when they did not merge with the third firm but only merged with one another as in Example 1.1a.

Example 1.1c

This example consisted of all the four firms in Example 1.1 merging to form a monopoly. Again, we assumed that all the total link cost functions on the new links were equal to zero; the original total cost functions were retained as was the demand market price function (as we had also done in Examples 1.1a through 1.1c). The supply chain network topology for this merger is given in Figure 8. There are now sixteen paths joining node $1'$ to node R_1.

Merger 3

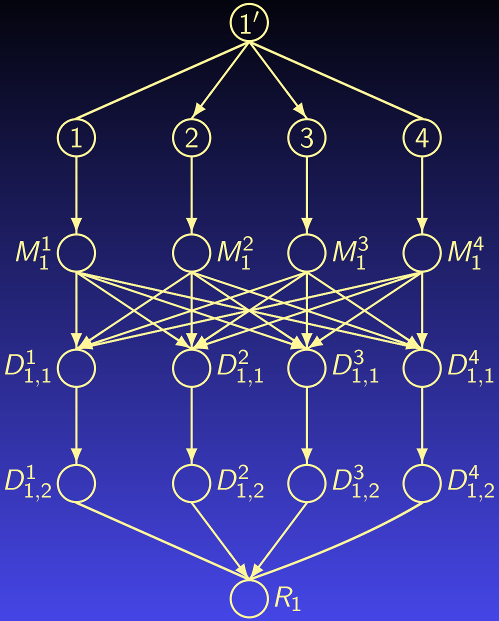

Figure 12: Supply Chain Network Structure for Example 1.1c

The computed equilibrium solution was as follows. The equilibrium path flows on all the original firm paths (cf. Figure 5 but extended to node $1'$ as in Figure 12) were equal to zero. The flow on each of the cross-hauling paths, of which there were twelve such paths, was equal to 3.28. The equilibrium demand was: 39.38 and the demand market price was: 160.62. The total cost was now: $2,444.58$. The total revenue was: $6,326.91$, and the profit was: $3,882.33$.

Each firm in the monopoly earned an individual profit of: 970.58. Hence, as predicted by economic theory, the total profit in the monopoly was the highest of all the examples reported above.

Example 1.1d

For completeness, we also investigated the merger in the case of a merger of the first two firms and the merger of the next two firms in Example 1.1 yielding the supply chain network topology in Figure 13. We retained the original functions as in Example 1.1 and assigned zero total costs to all the new links.

Merger 4

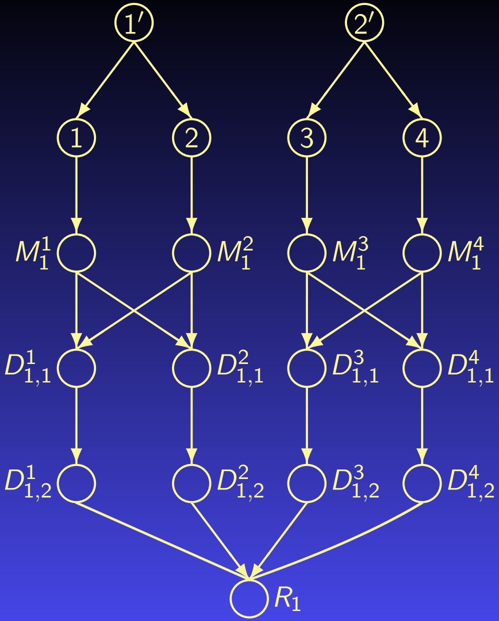

Figure 13: Supply Chain Network Structure for Example 1.1d

The computed equilibrium solution was now as follows. The flows on the cross-hauling paths for each new firms were all equal to: 10.94 with the other path flows all equal to zero. The equilibrium demand was: $d^*_{R_1} = 43.74$ and the demand market price was: 156.26. The total cost was: $3,000.82$, the total revenue was: $6,835.27$, and the profit was: $3,834.45$.

Each firm individually earned a profit of: 958.61, which is lower than that earned in the case of a merger to a monopoly as in Example 1.1c.

In Table 1, we present a summary of the results for Examples 1.1, 1.1a through 1.1d.

Table 1: Summary of Results for Problem Set 1: Examples 1.1 (Four Firm Oligopoly) and Examples 1.1a Through 1.1d (Post-merger)

Measure	Ex. 1.1	Ex. 1.1a	Ex. 1.1b	Ex. 1.1c	Ex. 1.1d
Profit	3,136.29	3,500.54	3,694.86	3,882.33	3,834.45
Total Cost	2,936.50	2,971.56	2,758.85	2,444.58	3,000.82
Revenue	6,072.79	6,472.11	6,453.71	6,326.91	6,835.27
Equil. Dem.	37.33	40.60	40.44	39.38	43.74
Equil. Price	162.67	159.40	159.56	160.62	156.26

In Example 1.1a, 2 merged firms gain; unmerged firms lose out.
in Example 1.1b, 3 merged firms gain; unmerged firm loses out.
In Example 1.1c, 4 merged firms gain.
In Example 1.1d, each firm in the mergers of two sets of 2 firms gains.

Problem Set 2

In the second problem set, we, again, used Example 1.1 as a baseline and we constructed Examples 2.1a through 2.1d to mimic Examples 1.1a through 1.1d, respectively, with the proviso, however, that the new links for the particular mergers no longer had associated zero total costs, but, rather, now had associated total cost functions as on the original links.

Example 2.1a

This merger corresponded to the merger of the four oligopolistic firms depicted in Figure 10.

The computed equilibrium path flows were now as follows. The flows on all the paths corresponding to the new firm $1'$ were equal to 4.00 whereas the flow on the path of each unmerged firm was: 9.47. The equilibrium demand was: 34.94 and the demand price was: 165.05. The total cost was: $2,743.91$. The revenue was: $5,768.47$, and the profit was: $3,024.55$.

The profit for each merged firm was now: 704.57 and the profit for each unmerged firm was: 807.70.

It is quite interesting to compare these results with those obtained in Example 1.1a, in which the firms in the merger profited substantially whereas those who were not in the merger lost out as compared to the individual profits in the four firm oligopoly Example 1.1 prior to the mergers. Note that in this merger example, in contrast to Example 1.1a, there were now non-zero total cost functions associated with the merger links.

In contrast to the results obtained for Example 1.1a, the firms now in the merger had the individual profits reduced from 998.23 to 704.57, whereas the two firms who did not enter into the merger each had its profits raised from 752.04 to 807.70.

Example 2.1b

This merger corresponds to the network in Figure 11. The equilibrium path flows associated with the newly merged firm were all equal to: 2.65. There were nine such path flows. The path flow for the unmerged firm was: 9.56. The equilibrium demand was: 33.41 and the equilibrium demand price was now: 166.59. The total cost was: $2,530.71$. The revenue was: $5,565.32$, and the profit was: $3,034.61$.

The profit for the unmerged firm was: 823.24. The profit for each of the three merged firms was: 737.12. In this example, the outsider clearly gained by not entering into the merger.

Example 2.1c

This merger corresponds to the supply chain network in Figure 12 and represents a merger of the four firms in Example 1.1 to a monopoly. The equilibrium path flows were now all equal to: 1.95 and the equilibrium demand was: 31.20 with an equilibrium price of: 168.80. The total cost was: $2,225.56$. The revenue was: $5,266.03$, and the profit was: $3,040.47$.

Hence, the profit of each of the original firms was: 760.10, which is lower than pre-merger but, understandable, since we now, unlike in Example 1.1c, have non-zero total cost functions associated with the merger links.

Example 2.1d

This merger corresponds to the network in Figure 13. Each of the path flows was now: 4.06 with a demand of 32.50 and a demand price of: 167.50. The total cost was: $2,506.12$. The revenue was: $5,443.74$, and the profit was: $2,937.62$.

The individual profit was, thus, 734.40, which is lower than in Example 1.1d, in which the merger links have zero associated total cost functions.

In Table 2, we present a summary of the results for Examples 1.1, 2.1a through 2.1d.

Table 2: Summary of Results for Problem Set 2: Examples 1.1 (Four Firm Oligopoly) and Examples 2.1a Through 2.1d (Post-merger)

Measure	Ex. 1.1	Ex. 2.1a	Ex. 2.1b	Ex. 2.1c	Ex. 2.1d
Profit	3,136.29	3,024.55	3,034.61	3,040.47	2,937.62
Total Cost	2,936.50	2,743.91	2,530.71	2,225.56	2,506.12
Revenue	6,072.79	5,768.47	5,565.32	5,266.03	5,443.74
Equil. Dem.	37.33	34.94	33.41	31.20	32.50
Equil. Price	162.67	165.05	166.59	168.80	167.50

In Example 2.1a, 2 merged firms lose; unmerged firms gain.
in Example 2.1b, 3 merged firms lose; unmerged firm gains.
In Example 2.1c, 4 merged firms lose.
In Example 2.1d, each firm in the mergers of two sets of 2 firms loses.

Problem Set 3

In this set, we solved pre- and post-merger problems consisting of four firms and two demand markets. These examples were more complex than those reported above.

Example 3.1

In this example, we, again considered four firms competing in an oligopolistic manner. The total cost functions on each of the links was given by:

$$\hat{c}_a = 2f_a^2 + f_a, \quad \forall a \in L^0.$$

However, rather than a single demand market, we now had two demand markets. The demand price function associated with the first demand market was as previously. The demand price function for the second demand market was:

$$\rho_{R_2} = -d_{R_2} + 100.$$

The Cournot-Nash equilibrium solution consisted of each firm in the oligopoly supplying demand market R_1 an amount 9.33 of the product and each supplying demand market R_2 an amount: 0.00 of the product.

The total cost was: $2,935.94$, the revenue was: $6,072.54$, and the profit, hence, was: $3,136.59$ with each firm earning a profit of: 784.15. The demand market price at R_1 was: 162.67 and at R_2: 100.00.

Example 3.1.1

We then assumed that the demand for the product at the second demand market increased, so that now:

$$\rho_{R_2} = -d_{R_2} + 300,$$

with the remainder of the data unchanged.

The new computed equilibrium solution was: each firm in the oligopoly produced and shipped an amount 1.91 of the product to demand market R_1 and an amount: 13.00 to demand market R_2. The total cost was now: $6,954.69$. The revenue was: $14,365.24$, and the profit was: $7,410.55$, with each individual firm earning a profit of: $1,852.64$. The demand price at demand market R_1 was: 192.37 and the demand price at R_2 was: 248.00.

Example 3.2

Example 3.2 was a partial merger problem. We assumed that the first two firms in Example 3.1.1 formed a coalition and merged. Also, we assumed that the new links associated with the merger (the top-most links and the added distribution links between the first two original firms) had total cost functions that were all equal to zero. The total cost was now: $7,425.73$, the revenue was: $15,902.39$, and the profit was: $8,476.66$.

The newly merged firm supplied demand market R_1 at an amount of 4.90 on each of its two cross-hauling (new) paths, and supplied demand market R_2 at a level of: 14.47 on the same two paths. Each of the unmerged two firms supplied R_1 at a level of 1.53 and R_2 at an amount: 13.09 of the product. The demand market price at R_1 was: 187.47 and was: 244.89 at R_2. Note that the profit now was substantially higher than that in Example 3.1.1.

The profit of each of the two unmerged firms was now: $1,804.13$ whereas the profit of each merged firm was: $2,432.20$, a value significantly higher than pre-merger.

Again, we have constructed a relatively simple example for which the "insiders" in the merger gain. We were able to accomplish this through the powerful tool of computational methods and numerical experimentation.

Managerial Insights

By capturing the full network economic activities of the underlying supply chain of firms we were able to construct a model which better reflects the reality and supporting intuition:

Even the merger of two firms (out of four) can yield sizable profits, provided that the costs associated with the merger are relatively low.

It is imperative to capture the costs associated with any merger in the full network context in order to ascertain any potential synergies as well as the possible profts.

Summary and Conclusions

- We have presented supply chain network oligopoly models, prior to and post mergers formed through coalitions.
- The variational inequality formulations of the governing equilibrium conditions for both models were given.
- We utilized the Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993), for the computation of the equilibrium path flows and reported extensive numerical examples.
- Through computations, we were able to demonstrate that, contrary to the *Merger Paradox*, we can construct examples that demonstrate that firms can gain through mergers even when the merger is not a merger to a monopoly (or almost).
- The network approach to mergers, through the perspective of supply chains, provides a powerful graphical approach and illustrates the types of connections that are possible.

Thank You!

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