Dynamics of Quality as a Strategic Variable in Complex Food Supply Chain Network Competition

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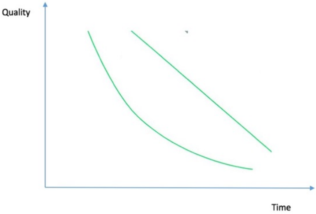
Background and Motivation

- **Food supply chains**, as noted in Yu and Nagurney (2013), are distinct from other product supply chains.
- Fresh produce is exposed to continuous and significant **change in the quality** of food products throughout the entire supply chain from the points of production/harvesting to points of demand/consumption.
- The quality of food products is **decreasing with time**, even with the use of advanced facilities and under the best **processing, handling, storage, and shipment** conditions (Sloof, Tijskens, and Wilkinson (1996) and Zhang, Habenicht, and Spieß (2003)).

Background and Motivation

- It has been discovered that the quality of fresh produce can be determined scientifically using **chemical formulae**, which include both **time** and **temperature**.
- The **initial quality** is also very important and food producers, such as farmers, have significant control over this important **strategic variable** at their production/harvesting sites.
- There are great opportunities for enhanced decision-making in this realm that can be supported by **appropriate models** and **methodological tools**.



Literature Review

- We note that early contributions focused on perishability and, in particular, on **inventory management** (see Ghare and Schrader (1963), Nahmias (1982, 2011) and Silver, Pyke, and Peterson (1998) for reviews).
- More recently, some studies have proposed integrating **more than a single supply chain network activity** (see, e.g., Zhang, Habenicht, and Spieß (2003), Widodo et al. (2006), Ahumada and Villalobos (2011), and Kopanos, Puigjaner, and Georgiadis (2012)).
- Yu and Nagurney (2013) have emphasized the need to bring greater realism to the underlying **economics** and **competition** on food supply chains.
- Monteiro (2007) studied the **traceability** in food supply chains theoretically.
- Additional modeling and methodological contributions in the **food supply chain** and **quality** domain have been made by Blackburn and Scudder (2009) and by Rong, Akkerman, and Grunow (2011).
- Besik and Nagurney (2017) formulate **short fresh produce supply chains** with the inclusion of the **dynamics of quality**, in the context of **farmers' markets**.

Contribution

- We construct a competitive supply chain network model for fresh produce under **oligopolistic competition** among the food firms, who are profit-maximizers.
- The firms have, as their **strategic variables**, not only the **product flows** on the pathways of their supply chain networks from the production/harvesting locations to the ultimate points of demand, but also the **initial quality** of the produce that they grow at their production locations.
- The consumers at the retail outlets (demand points), **differentiate the fresh produce** from the distinct firms and reflect their preferences through the **prices** that they are willing to pay which depend on quantities of the produce as well as **the average quality** of the produce associated with the firm and retail outlet pair(s).
- **Quality of the produce** reaching a destination node depends on its **initial quality** and **on the path** that it took with each particular path consisting of specific links, with particular characteristics of physical features of **time, temperature, etc**.

Preliminaries on Quality

- Fresh foods **deteriorate** since they are biological products, and, therefore, **lose quality over time**.
- The rate of **quality deterioration** can be represented as a function of the microenvironment, the gas composition, the relative humidity, and the temperature (Taoukis and Labuza (1989)).
- Labuza (1984) demonstrated that the quality of a food attribute, Q, over time t, which can correspond, depending on the fruit or vegetable, to **the color change**, **the moisture content**, **the amount of nutrition such as vitamin C**, or **the softening of the texture**, can be formulated via the differential equation:

Differential Equation of Quality Decay

$$-\frac{\partial Q}{\partial t} = -kQ^n = -Ae^{(-E/RT)}Q^n. \quad (1)$$

Here, k is the reaction rate and is defined by the Arrhenius formula, $Ae^{(-E/RT)}$, A is a pre-exponential constant, T is the temperature, E is the activation energy, and R is the universal gas constant (cf. Arrhenius (1889)).

Preliminaries on Quality

- If the reaction order n is zero, that is, $\frac{\partial Q}{\partial t} = -k$, and the **initial quality** is denoted by Q_0, we can quantify the **remaining quality** Q_t at time t (Tijskens and Polderdijk (1996)) according to:

Zero Order Quality Decay

$$Q_t = Q_0 - kt. \tag{2}$$

- Examples of fresh produce that follow a reaction order of zero include **watermelons** and **spinach**.

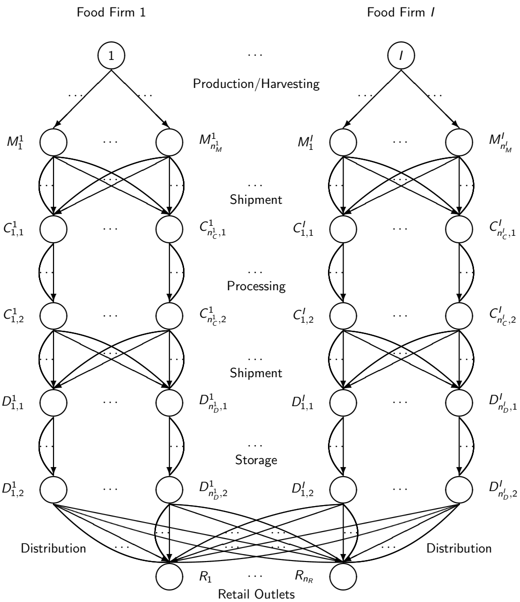
- If the reaction order is 1, known as a *first order reaction*, the quality decay function is then given by the expression:

First Order Quality Decay

$$Q_t = Q_0 e^{-kt}. \tag{3}$$

- Popular fruits that follow first order kinetics include **peaches**, and **strawberries**, as well as vegetables such as: **peas, beans**, **carrots**,

Network Topology



Quality Over a Link

- We let L^i denote the set of directed links in the supply chain network of food firm i; where $i = 1, \ldots, I$, which consists of **a set of production links**, L_1^i, and **a set of post-harvest links**, L_2^i, that is, $L^i \equiv L_1^i \cup L_2^i$.
- We allow for a distinct **initial quality** q_{0a}^i associated with each top-tiered production link $a \in L_1^i$; $i = 1, \ldots, I$, since distinct production sites of a firm may have different associated quality of the produce that is harvested because of soil conditions, investment in irrigation, types of pesticides, and fertilizers used, etc.
- We let β_b denote the **quality decay incurred on link** b, for $b \in L_2^i$, which is a factor that depends on the reaction order n, the reaction rate k_b, and the time t_b on link b, according to:

Quality Decay Over a Link

$$\beta_b \equiv \begin{cases} -k_b t_b,, & \text{if } n = 0, \forall b \in L_2^i, \forall i, \\ e^{-k_b t_b}, & \text{if } n = 1, \forall b \in L_2^i, \forall i. \end{cases} \quad (4)$$

- Here, the reaction rate is:

Reaction Rate

$$k_b = Ae^{(-E/RT_b)}, \quad \forall b \in L_2^i, \forall i. \quad (5)$$

Definition of Quality Over a Path

- We can have **multiple paths** from an origin node i to a destination node k, P_k^i denotes the set of all paths that have origin i and destination k.
- **The quality** q_p, over a path p, joining the origin node i, with a destination node k, with the incorporation of the quality deterioration of the fresh produce, is, hence:

Quality Over a Path

$$q_p \equiv \begin{cases} \mathbf{q^i_{0a}} + \sum_{b \in p \cap L_2^i} \beta_b, & \text{if } n = 0,\ p \in P_k^i,\ \forall i, k, \\ q_{0a}^i \prod_{\mathbf{b \in p \cap L_2^i}} \beta_{\mathbf{b}}, & \text{if } n = 1,\ p \in P_k^i,\ \forall i, k. \end{cases} \quad (6)$$

- Here, q_{0a}^i **is the initial quality** of the fresh produce on a top-most link a from an origin node i and in the path p under consideration.

The Competitive Fresh Produce Supply Chain Network Model with Quality

Nonnegativity Constraint of the Path Flows

For each path p, joining an origin node i with a destination node k, the following nonnegativity condition must hold:

$$x_p \geq 0, \quad \forall p \in P_k^i;\ i = 1, \ldots, I;\ k = R_1, \ldots, R_{n_R}. \tag{7}$$

Nonnegativity Constraint of the Initial Quality

The initial quality of the fresh produce on the top-most links a of an origin node i, must be nonnegative, that is:

$$q_{0a}^i \geq 0, \quad \forall a \in L_1^i;\ i = 1, \ldots, I. \tag{8}$$

Maximum Initial Quality

We assume that the quality is bounded from above by a maximum value; hence, we have that:

$$q_{0a}^i \leq \bar{q}_{0a}^i, \quad \forall a \in L_1^i;\ i = 1, \ldots, I. \tag{9}$$

The Competitive Fresh Produce Supply Chain Network Model with Quality

Link Flows

The conservation of flow equations that relate the link flows of each food firm i; $i = 1, \ldots, I$, to the path flows are given by:

$$f_l = \sum_{p \in P} x_p \delta_{lp}, \quad \forall l \in L^i; \; i = 1, \ldots, I, \tag{10}$$

where $\delta_{ap} = 1$, if link a is contained in path p, and 0, otherwise.

Capacity over the Link Flows

Link flows must satisfy capacity constraints, we have that:

$$f_l \leq u_l, \quad \forall l \in L. \tag{11}$$

Capacity over the Path Flows

Observe that, in view of the conservation of flow equations (10), we can rewrite (11) in terms of path flows as:

$$\sum_{p \in P} x_p \delta_{lp} \leq u_l, \quad \forall l \in L. \tag{12}$$

The Competitive Fresh Produce Supply Chain Network Model with Quality

Average Quality

The average quality product of firm i, at retail outlet k, is given by the expression:

$$\hat{q}_{ik} = \frac{\sum_{p \in P_k^i} q_p x_p}{\sum_{p \in P_k^i} x_p}, \quad i = 1, \ldots, I; \ k = R_1, \ldots, R_{n_R}. \tag{13}$$

Demand

The demand for food firm i's fresh food product at retail outlet k is denoted by d_{ik} and is equal to the sum of all the fresh produce flows on paths joining (i, k), so that:

$$\sum_{p \in P_k^i} x_p = d_{ik}, \quad i = 1, \ldots, I; \ k = R_1, \ldots, R_{n_R}. \tag{14}$$

Demand Price Function

We denote the demand price of food firm i's product at retail outlet k by:

$$\rho_{ik} = \rho_{ik}(d, \hat{q}), \quad i = 1, \ldots, I; \ k = R_1, \ldots, R_{n_R}. \tag{15}$$

The Competitive Fresh Produce Supply Chain Network Model with Quality

Cost of Production/Harvesting

The cost of production/harvesting at firm i's production site a depends, in general, on the initial quality q^i_{0a}, and the product flow on the production/harvesting link, that is:

$$\hat{z}_a = \hat{z}_a(f_a, q^i_{0a}), \quad \forall a \in L^i_1;\ i = 1, \ldots, I. \tag{16}$$

Operational Cost Function

We define the operational cost functions associated with the remaining links in the supply chain network as:

$$\hat{c}_b = \hat{c}_b(f), \quad \forall b \in L^i_2;\ i = 1, \ldots, I. \tag{17}$$

Vector of Path Flow Strategies

The vector of path flows associated with firm i; $i = 1, \ldots, I$ is:

$$X_i \equiv \{\{x_p\}|p \in P^i\}\} \in R^{n_{P^i}}_+, P^i \equiv \cup_{k=R_1 \ldots, R_{n_R}} P^i_k. \tag{18}$$

Vector of Initial Quality Strategies

the vector of initial quality levels, associated with firm i; $i = 1, \ldots, I$ is:

$$q^i_0 \equiv \{\{q^i_{0a}\}|a \in L^i_1\}\} \in R^{n_{L^i_1}}_+. \tag{19}$$

The Competitive Fresh Produce Supply Chain Network Model with Quality

Utility Function

The utility of firm i; $i = 1, \ldots, I$, is expressed as:

$$U_i = \sum_{k=R_1}^{R_{n_R}} \rho_{ik}(d, \hat{q}) d_{ik} - \left(\sum_{a \in L_1^i} \hat{z}_a(f_a, q_{0a}^i) + \sum_{b \in L_2^i} \hat{c}_b(f)\right). \quad (20)$$

Rewritten Demand Price Functions

In view of (6), (13), and (14), we can rewrite (15) as:

$$\hat{\rho}_{ik}(x, q_0) \equiv \rho_{ik}(d, \hat{q}), \quad i = 1, \ldots, I;\ k = R_1, \ldots, R_{n_R}. \quad (21)$$

Vector of the Profits

We can define the vector of the profits of all the firms for all firms i; $i = 1, \ldots, I$, with the I-dimensional vector $\hat{U}$, that is:

$$\hat{U} = \hat{U}(X, q_0). \quad (22)$$

Supply Chain Network Nash Equilibrium with Fresh Produce Quality

Definition 1: Supply Chain Network Nash Equilibrium with Fresh Produce Quality

A fresh produce path flow pattern and initial quality level $(X^*, q_0^*) \in K = \prod_{i=1}^{I} K_i$ *constitutes a supply chain network Nash Equilibrium with fresh produce quality if for each food firm* i*;* $i = 1, \ldots, I$*:*

$$\hat{U}_i(X_i^*, X_{-i}^*, q_0^{i*}, q_0^{-i*}) \geq \hat{U}_i(X_i, X_{-i}^*, q_0^i, q_0^{-i*}), \quad \forall (X_i, q_0^i) \in K_i, \tag{23}$$

where $X_{-i}^* \equiv (X_1^*, \ldots, X_{i-1}^*, X_{i+1}^*, \ldots, X_I^*)$, $q_0^{-i*} \equiv (q_0^{1*}, \ldots, q_0^{i-1*}, q_0^{i+1*}, \ldots, q_0^{I*})$ *and* $K_i \equiv \{(X_i, q_0^i) | X_i \in R_+^{n_{P^i}}, q_0^i \in R_+^{n_{L_1^i}},$ (9) *and* (12) *hold for* $l \in L^i\}$.

- An equilibrium is established if no food firm can unilaterally improve upon its profit by altering its product flows and initial quality at production sites in its supply chain network, given the product flows and initial quality decisions of the other firms.

Variational Inequality Formulation of the Governing Equilibrium Conditions

Theorem 1: Variational Inequality Formulation of the Governing Equilibrium Conditions

Assume that, for each food firm i; $i = 1, \ldots, I$, the profit function $\hat{U}_i(X, q_0)$ is concave with respect to the variables X_i and q_0^i, and is continuously differentiable. Then $(X^*, q_0^*) \in K$ is a supply chain network Nash Equilibrium with fresh produce quality according to Definition 1 if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^{I} \langle \nabla_{X_i} \hat{U}_i(X^*, q_0^*), X_i - X_i^* \rangle - \sum_{i=1}^{I} \langle \nabla_{q_0^i} \hat{U}_i(X^*, q_0^*), q_0^i - q_0^{i*} \rangle \geq 0, \quad \forall (X, q_0) \in K, \tag{24}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in the corresponding Euclidean space.

Variational Inequality Formulation of the Governing Equilibrium Conditions

$\nabla_{X_i} \hat{U}_i(X, q_0)$ denotes the gradient of $\hat{U}_i(X, q_0)$ with respect to X_i and $\nabla_{q_0^i} \hat{U}_i(X, q_0)$ denotes the gradient of $\hat{U}_i(X, q_0)$ with respect to q_0^i. The solution of variational inequality (22), in turn, is equivalent to the solution of the variational inequality: determine $(x^*, q_0^*, \lambda^*, \gamma^*) \in K^1$ satisfying:

$$\sum_{i=1}^{I} \sum_{k=R_1}^{R_{n_R}} \sum_{p \in P_k^i} \left[\frac{\partial \hat{Z}^i(x^*, q_0^{i*})}{\partial x_p} + \frac{\partial \hat{C}^i(x^*)}{\partial x_p} + \sum_{l \in L^i} \gamma_l^* \delta_{lp} - \hat{\rho}_{ik}(x^*, q_0^*) - \sum_{j=R_1}^{R_{n_R}} \frac{\partial \hat{\rho}_{ij}(x^*, q_0^*)}{\partial x_p} \sum_{r \in P_j^i} x_r^* \right]$$

$$\times [x_p - x_p^*] + \sum_{i=1}^{I} \sum_{a \in L_1^i} \left[\frac{\partial \hat{Z}^i(x^*, q_0^{i*})}{\partial q_{0a}^i} + \lambda_a^* - \sum_{j=R_1}^{R_{n_R}} \frac{\partial \hat{\rho}_{ij}(x^*, q_0^*)}{\partial q_{0a}^i} \sum_{r \in P_j^i} x_r^* \right] \times [q_{0a}^i - q_{0a}^{i*}]$$

$$+ \sum_{i=1}^{I} \sum_{a \in L_1^i} [\bar{q}_{0a}^i - q_{0a}^{i*}] \times [\lambda_a - \lambda_a^*] + \sum_{i=1}^{I} \sum_{l \in L^i} \left[u_l - \sum_{r \in P} x_r^* \delta_{lr} \right] \times [\gamma_l - \gamma_l^*] \geq 0, \quad \forall (x, q_0, \lambda, \gamma) \in K^1. \tag{25}$$

Variational Inequality Formulation of the Governing Equilibrium Conditions

We have $K^1 \equiv \{(x, q_0, \lambda, \gamma) | x \in R_+^{n_P}, q_0 \in R_+^{n_{L_1}}, \lambda \in R_+^{n_{L_1}}, \gamma \in R_+^{n_L}\}$ *and for each path* p; $p \in P_k^i$; $i = 1, \ldots, I$; $k = R_1, \ldots, R_{n_R}$:

$$\frac{\partial \hat{Z}^i(x, q_0^i)}{\partial x_p} \equiv \sum_{a \in L_1^i} \frac{\partial \hat{z}_a(f_a, q_{0a}^i)}{\partial f_a} \delta_{ap}, \tag{26a}$$

$$\frac{\partial \hat{C}^i(x)}{\partial x_p} \equiv \sum_{b \in L_2^i} \sum_{l \in L^i} \frac{\partial \hat{c}_b(f)}{\partial f_l} \delta_{lp}, \tag{26b}$$

$$\frac{\partial \hat{\rho}_{ij}(x, q_0)}{\partial x_p} \equiv \frac{\partial \rho_{ij}(d, \hat{q})}{\partial d_{ik}} + \frac{\partial \rho_{ij}(d, \hat{q})}{\partial \hat{q}_{ik}} \left(\frac{q_p}{\sum_{r \in P_k^i} x_r} - \frac{\sum_{r \in P_k^i} q_r x_r}{(\sum_{r \in P_k^i} x_r)^2} \right). \tag{26c}$$

For each a; $a \in L_1^i$; $i = 1, \ldots, I$,

$$\frac{\partial \hat{Z}^i(x, q_0^i)}{\partial q_{0a}^i} \equiv \frac{\partial \hat{z}_a(f_a, q_{0a}^i)}{\partial q_{0a}^i}, \tag{26d}$$

$$\frac{\partial \hat{\rho}_{ij}(x, q_0)}{\partial q_{0a}^i} \equiv \sum_{h=R_1}^{R_{n_R}} \sum_{s \in P_h^i} \frac{x_s}{\sum_{r \in P_h^i} x_r} \frac{\partial \rho_{ij}(d, \hat{q})}{\partial \hat{q}_{ih}} \frac{\partial q_s}{\partial q_{0a}^i}. \tag{26e}$$

In particular, if link a *is not included in path* s, $\frac{\partial q_s}{\partial q_{0a}^i} = 0$; *if link* a *is included in path* s, *following (6), we have:*

$$\frac{\partial q_s}{\partial q_{0a}^i} = \begin{cases} 1, & \text{if } n = 0, \\ \prod_{b \in s \cap L_2^i} \beta_b, & \text{if } n = 1. \end{cases} \tag{26f}$$

Variational Inequality Formulation of the Governing Equilibrium Conditions

Proof: (22) follows directly from Gabay and Moulin (1980); see also Dafermos and Nagurney (1987). Under the imposed assumptions, (22) holds if and only if (see, e.g., Bertsekas and Tsitsiklis (1989)) the following holds:

For each path p; $p \in P_k^i$, we have that

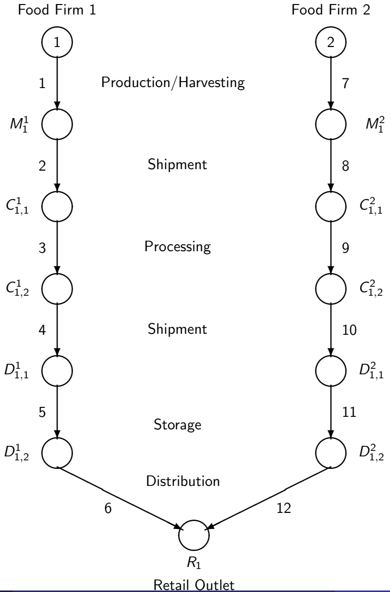
$$
\begin{aligned}
\frac{\partial \hat{U}_i}{\partial x_p} =& \frac{\partial[\sum_{j=R_1}^{R_{n_R}} \rho_{ij}(d,\hat{q})d_{ij} - (\sum_{e\in L_1^i} \hat{z}_e(f_e, q_{0e}^i) + \sum_{b\in L_2^i} \hat{c}_b(f))]}{\partial x_p} \\
=& \frac{\partial[\sum_{j=R_1}^{R_{n_R}} \rho_{ij}(d,\hat{q})d_{ij}]}{\partial x_p} - \frac{\partial[\sum_{e\in L_1^i} \hat{z}_e(f_e, q_{0e}^i)]}{\partial x_p} - \frac{\partial[\sum_{b\in L_2^i} \hat{c}_b(f)]}{\partial x_p} \\
=& \sum_{j=R_1}^{R_{n_R}} \left[\frac{\partial[\rho_{ij}(d,\hat{q})d_{ij}]}{\partial d_{ik}} \frac{\partial d_{ik}}{\partial x_p} + \frac{\partial[\rho_{ij}(d,\hat{q})d_{ij}]}{\partial \hat{q}_{ik}} \frac{\partial \hat{q}_{ik}}{\partial x_p} \right] - \sum_{a\in L_1^i} \sum_{e\in L_1^i} \frac{\partial \hat{z}_e(f_e, q_{0e}^i)}{\partial f_a} \frac{\partial f_a}{\partial x_p} \\
&- \sum_{b\in L_2^i} \sum_{l\in L^i} \frac{\partial \hat{c}_b(f)}{\partial f_l} \frac{\partial f_l}{\partial x_p} \\
=& \rho_{ik}(d,\hat{q}) + \sum_{j=R_1}^{R_{n_R}} \left[\frac{\partial \rho_{ij}(d,\hat{q})}{\partial d_{ik}} d_{ij} + \frac{\partial \rho_{ij}(d,\hat{q})}{\partial \hat{q}_{ik}} d_{ij} \left(\frac{\partial \hat{q}_{ik}}{\partial[\sum_{r\in P_k^i} q_r x_r]} \frac{\partial[\sum_{r\in P_k^i} q_r x_r]}{\partial x_p} \right.\right. \\
&\left.\left. + \frac{\partial \hat{q}_{ik}}{\partial[\sum_{r\in P_k^i} x_r]} \frac{\partial[\sum_{r\in P_k^i} x_r]}{\partial x_p} \right) \right] - \sum_{a\in L_1^i} \frac{\partial \hat{z}_a(f_a, q_{0a}^i)}{\partial f_a} \delta_{ap} - \sum_{b\in L_2^i} \sum_{l\in L^i} \frac{\partial \hat{c}_b(f)}{\partial f_l} \delta_{lp} \\
=& \rho_{ik}(d,\hat{q}) + \sum_{j=R_1}^{R_{n_R}} \left[\frac{\partial \rho_{ij}(d,\hat{q})}{\partial d_{ik}} + \frac{\partial \rho_{ij}(d,\hat{q})}{\partial \hat{q}_{ik}} \left(\frac{q_p}{\sum_{r\in P_k^i} x_r} - \frac{\sum_{r\in P_k^i} q_r x_r}{(\sum_{r\in P_k^i} x_r)^2} \right) \right] d_{ij} \\
&- \sum_{a\in L_1^i} \frac{\partial \hat{z}_a(f_a, q_{0a}^i)}{\partial f_a} \delta_{ap} - \sum_{b\in L_2^i} \sum_{l\in L^i} \frac{\partial \hat{c}_b(f)}{\partial f_l} \delta_{lp}. \qquad (27)
\end{aligned}
$$

Variational Inequality Formulation of the Governing Equilibrium Conditions

For each link a; $a \in L_1^i$, we know that:

$$\begin{aligned}
\frac{\partial \hat{U}_i}{\partial q^i_{0a}} &= \frac{\partial [\sum_{j=R_1}^{R_{n_R}} \rho_{ij}(d,\hat{q})d_{ij} - (\sum_{e \in L_1^i} \hat{z}_e(f_e, q^i_{0e}) + \sum_{b \in L_2^i} \hat{c}_b(f))]}{\partial q^i_{0a}} \\
&= \frac{\partial [\sum_{j=R_1}^{R_{n_R}} \rho_{ij}(d,\hat{q})d_{ij}]}{\partial q^i_{0a}} - \frac{\partial [\sum_{e \in L_1^i} \hat{z}_e(f_e, q^i_{0e})]}{\partial q^i_{0a}} - \frac{\partial [\sum_{b \in L_2^i} \hat{c}_b(f)]}{\partial q^i_{0a}} \\
&= \sum_{h=R_1}^{R_{n_R}} \sum_{j=R_1}^{R_{n_R}} \frac{\partial [\rho_{ij}(d,\hat{q})d_{ij}]}{\partial \hat{q}_{ih}} \frac{\partial \hat{q}_{ih}}{\partial q^i_{0a}} - \frac{\partial \hat{z}_a(f_a, q^i_{0a})}{\partial q^i_{0a}} \\
&= \sum_{h=R_1}^{R_{n_R}} \sum_{j=R_1}^{R_{n_R}} \frac{\partial \rho_{ij}(d,\hat{q})}{\partial \hat{q}_{ih}} d_{ij} \frac{\partial \hat{q}_{ih}}{\partial [\sum_{r \in P_h^i} q_r x_r]} \frac{\partial [\sum_{r \in P_h^i} q_r x_r]}{\partial q^i_{0a}} - \frac{\partial \hat{z}_a(f_a, q^i_{0a})}{\partial q^i_{0a}} \\
&= \sum_{h=R_1}^{R_{n_R}} \sum_{j=R_1}^{R_{n_R}} \frac{\partial \rho_{ij}(d,\hat{q})}{\partial \hat{q}_{ih}} d_{ij} \frac{1}{\sum_{r \in P_h^i} x_r} \sum_{s \in P_h^i} \frac{\partial [\sum_{r \in P_h^i} q_r x_r]}{\partial q_s} \frac{\partial q_s}{\partial q^i_{0a}} - \frac{\partial \hat{z}_a(f_a, q^i_{0a})}{\partial q^i_{0a}} \\
&= \sum_{h=R_1}^{R_{n_R}} \sum_{j=R_1}^{R_{n_R}} \sum_{s \in P_h^i} \frac{x_s}{\sum_{r \in P_h^i} x_r} \frac{\partial \rho_{ij}(d,\hat{q})}{\partial \hat{q}_{ih}} \frac{\partial q_s}{\partial q^i_{0a}} d_{ij} - \frac{\partial \hat{z}_a(f_a, q^i_{0a})}{\partial q^i_{0a}}.
\end{aligned} \tag{28}$$

Simple Illustrative Examples



Simple Illustrative Examples

- The cost of production/harvesting is **higher for Food Firm 1** since it uses **better machinery** and has invested more into the necessary chemicals to maintain **the soil quality**, with the top-tiered link cost functions being:

$$\hat{z}_1(f_1, q_{01}^1) = f_1^2 + 8f_1 + 3q_{01}^1, \quad \hat{z}_7(f_7, q_{07}^2) = f_7^2 + 3q_{07}^2.$$

- There are ten additional links, belonging to the sets L_2^1 and L_2^2, in the supply chain network and their **total link cost functions** are:

$$\hat{c}_2(f_2) = 5f_2^2 + 10f_2, \ \hat{c}_3(f_3) = 2f_3^2, \ \hat{c}_4(f_4) = 2f_4^2 + f_4, \ \hat{c}_5(f_5) = 3f_5^2, \ \hat{c}_6(f_6) = f_6^2 + f_6,$$

$$\hat{c}_8(f_8) = f_8^2 + f_8, \ \hat{c}_9(f_9) = 3f_9^2 + f_9, \ \hat{c}_{10}(f_{10}) = 2f_{10}^2, \ \hat{c}_{11}(f_{11}) = 6f_{11}^2 + f_{11},$$

$$\hat{c}_{12}(f_{12}) = 6f_{12}^2 + f_{12}.$$

- The total link cost functions are constructed according to the assumptions made for Food Firm 1, Food Firm 2, and Retail Outlet R_1.

Quality Decay for the Illustrative Examples

Link b	Hours	Temperature (Celsius)	β_b ($n = 0$)	β_b ($n = 1$)
2	**48**	**22**	**-0.1784**	**0.8366**
3	10	22	-0.0372	0.9635
4	12	10	-0.0167	0.9835
5	10	22	-0.0372	0.9635
6	10	22	-0.0372	0.9635
8	**4**	**22**	**-0.0149**	**0.9852**
9	2	22	-0.0074	0.9926
10	2	5	-0.0004	0.9995
11	8	22	-0.0297	0.9707
12	2	22	-0.0149	0.9852

- The shipment time is longer for Food Firm 1 than for Food Firm 2 because of their respective distances to their processing facilities, which can be seen from the time difference between link 2 and link 8.

Example 1a: Linear Quality Decay (Zero Order Kinetics)

- Since there exists one path for each food firm:

$$d^*_{11} = x^*_{p_1}, \; d^*_{21} = x^*_{p_2}.$$

- In a zero order quality decay function, the reaction order $n = 0$, and the quality q_p over a path p can be determined by the appropriate formula for $n = 0$.

- The initial quality variables are q^1_{01} and q^2_{07}.

  The demand price functions are:

$$\hat{\rho}_{11}(x, q_0) \equiv \rho_{11}(d, \hat{q}) = -2x_{p_1} - x_{p_2} + \frac{q_{p_1} x_{p_1}}{x_{p_1}} + 100$$

and

$$\hat{\rho}_{21}(x, q_0) \equiv \rho_{21}(d, \hat{q}) = -3x_{p_2} - x_{p_1} + \frac{q_{p_2} x_{p_2}}{x_{p_2}} + 90,$$

with the path quality q_p for the two paths constructed according to zero order decay function, for $n = 0$ for $i = 1, 2$, given by:

$$q_{p_1} = q^1_{01} + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 = q^1_{01} - 0.3066,$$

$$q_{p_2} = q^2_{07} + \beta_8 + \beta_9 + \beta_{10} + \beta_{11} + \beta_{12} = q^2_{07} - 0.0674.$$

Example 1a: Linear Quality Decay (Zero Order Kinetics)

- In order to obtain the equilibrium path flows and the equilibrium initial quality levels that satisfy variational inequality (25), the following expressions must be equal to 0:

$$\frac{\partial \hat{Z}^1(x^*, q_{01}^{1*})}{\partial x_{p_1}} + \frac{\partial \hat{C}^1(x^*)}{\partial x_{p_1}} - \hat{\rho}_{11}(x^*, q_0^*) - \frac{\partial \hat{\rho}_{11}(x^*, q_0^*)}{\partial x_{p_1}} x_{p_1}^* = 0, \tag{29}$$

$$\frac{\partial \hat{Z}^1(x^*, q_{01}^{1*})}{\partial q_{01}^1} - \frac{\partial \hat{\rho}_{11}(x^*, q_0^*)}{\partial q_{01}^1} x_{p_1}^* = 0, \tag{30}$$

$$\frac{\partial \hat{Z}^2(x^*, q_{07}^{2*})}{\partial x_{p_2}} + \frac{\partial \hat{C}^2(x^*)}{\partial x_{p_2}} - \hat{\rho}_{21}(x^*, q_0^*) - \frac{\partial \hat{\rho}_{21}(x^*, q_0^*)}{\partial x_{p_2}} x_{p_2}^* = 0, \tag{31}$$

$$\frac{\partial \hat{Z}^2(x^*, q_{07}^{2*})}{\partial q_{07}^2} - \frac{\partial \hat{\rho}_{21}(x^*, q_0^*)}{\partial q_{07}^2} x_{p_2}^* = 0. \tag{32}$$

Example 1a: Linear Quality Decay (Zero Order Kinetics)

- Grouping the terms above corresponding to each equation (29) – (32) we obtain the following system of equations:

$$32x^*_{p_1} + x^*_{p_2} - q^{1*}_{01} = 79.6934,$$

$$3 - x^*_{p_1} = 0,$$

$$x^*_{p_1} + 44x^*_{p_2} - q^{2*}_{07} = 85.9326,$$

$$3 - x^*_{p_2} = 0,$$

with solution:

$$x^*_{p_1} = 3, \ x^*_{p_2} = 3, \quad q^{1*}_{01} = 19.3066, \ q^{2*}_{07} = 49.0674.$$

- The path quality levels are: $q_{p_1} = 19$ and $q_{p_2} = 49$, the demand prices are: $\rho_{11} = 110$ and $\rho_{21} = 127$, with Food Firm 1 enjoying a profit (in dollars) of $\hat{U}_1(X^*, q^*_0) = 87.0000$ and Food Firm 2 a profit of $\hat{U}_2(X^*, q^*_0) = 51.0000$.

Example 1b: Exponential Quality Decay (First Order Kinetics)

- The product now has an **exponential quality decay** with a **reaction order** $n = 1$.
- The quality levels of the paths are constructed and the β_b values in Table 1, for $n = 1$ for $i = 1, 2$, yielding:

$$q_{p_1} = q_{01}^1 \times \beta_2 \times \beta_3 \times \beta_4 \times \beta_5 \times \beta_6 = (q_{01}^1)(0.7359),$$

$$q_{p_2} = q_{07}^2 \times \beta_8 \times \beta_9 \times \beta_{10} \times \beta_{11} \times \beta_{12} = (q_{07}^2)(0.9418).$$

- We now proceed to solve the equations (29) – (32) for this example, with the following terms for paths p_1 and p_2 presented for completeness and convenience:

$$-\hat{\rho}_{11}(x^*, q_0^*) = 2x_{p_1}^* + x_{p_2}^* - \frac{(q_{01}^{1*})(0.7359)(x_{p_1}^*)}{x_{p_1}^*} - 100 = 2x_{p_1}^* + x_{p_2}^* - (q_{01}^{1*})(0.7359) - 100,$$

$$-\frac{\partial \hat{\rho}_{11}(x^*, q_0^*)}{\partial q_{01}^1} x_{p_1} = -0.7359 x_{p_1}^*,$$

$$-\hat{\rho}_{21}(x^*, q_0^*) = 3x_{p_2}^* + x_{p_1}^* - \frac{(q_{07}^{2*})(0.9418)(x_{p_2}^*)}{x_{p_2}^*} - 90 = 3x_{p_2}^* + x_{p_1}^* - (q_{07}^{2*})(0.9418) - 90,$$

$$-\frac{\partial \hat{\rho}_{21}(x^*, q_0^*)}{\partial q_{07}^2} x_{p_2} = -0.9418 x_{p_2}^*.$$

Example 1b: Exponential Quality Decay (First Order Kinetics)

- We obtain the following system of equations:

$$32x^*_{p_1} + x^*_{p_2} - 0.7359q^{1*}_{01} = 80,$$

$$3 - 0.7359x^*_{p_1} = 0,$$

$$x^*_{p_1} + 44x^*_{p_2} - 0.9418q^{2*}_{07} = 86,$$

$$3 - 0.9418x^*_{p_2} = 0.$$

- Straightforward calculations yield the following equilibrium path flows and equilibrium initial quality levels:

$$x^*_{p_1} = 4.0766,\ x^*_{p_2} = 3.1854, \quad q^{1*}_{01} = 72.8857,\ q^{2*}_{07} = 61.8329.$$

- The path quality levels are, $q_{p_1} = 53.6366$ and $q_{p_2} = 58.2342$.
- We obtain the following equilibrium demand prices at the retail outlet for Food Firm 1 and Food Firm 2, respectively:

$$\rho_{11} = 142.30,\ \rho_{21} = 134.60.$$

- The profits of the food firms are calculated, in dollars, as:

$$\hat{\mathbf{U}}_1(\mathbf{X}^*, \mathbf{q}^*_0) = \mathbf{47.2497}, \hat{U}_2(X^*, q^*_0) = 37.7258.$$

The Euler Method

Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993) is a solution methodology of the Variational Inequality Problem. Specifically, iteration τ of the Euler method is given by:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau}F(X^{\tau})), \tag{33}$$

The Euler method, the sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \to 0$, as $\tau \to \infty$.

The Euler Method Explicit Formulae

For each path $p \in P_j^i$, $\forall i, j$, compute:

$$x_p^{\tau+1} = \max\{0, x_p^\tau + \alpha_\tau(\hat{\rho}_{ik}(x^\tau, q_0^\tau) + \sum_{j=R_1}^{R_{n_R}} \frac{\partial \hat{\rho}_{ij}(x^\tau, q_0^\tau)}{\partial x_p} \sum_{r \in P_j^i} x_r^\tau - \frac{\partial \hat{Z}^i(x^\tau, q_0^{i\tau})}{\partial x_p} - \frac{\partial \hat{C}^i(x^\tau)}{\partial x_p} - \sum_{l \in L^i} \gamma_l^\tau \delta_{lp})\}. \quad (34)$$

For each initial quality level $a \in L_1^i$, $\forall i$, in turn, compute:

$$q_{0a}^{i\tau+1} = \max\{0, q_{0a}^{i\tau} + \alpha_\tau(\sum_{j=R_1}^{R_{n_R}} \frac{\partial \hat{\rho}_{ij}(x^\tau, q_0^\tau)}{\partial q_{0a}^i} \sum_{r \in P_j^i} x_r^\tau - \frac{\partial \hat{Z}^i(x^\tau, q_0^{i\tau})}{\partial q_{0a}^i} - \lambda_a^\tau)\}. \quad (35)$$

The Lagrange multiplier for each top-most link $a \in L_1^i$; $i = 1, \ldots, I$, associated with the initial quality bounds is computed as:

$$\lambda_a^{\tau+1} = \max\{0, \lambda_a^\tau + \alpha_\tau(q_{0a}^{i\tau} - \bar{q}_{0a}^i)\}. \quad (36)$$

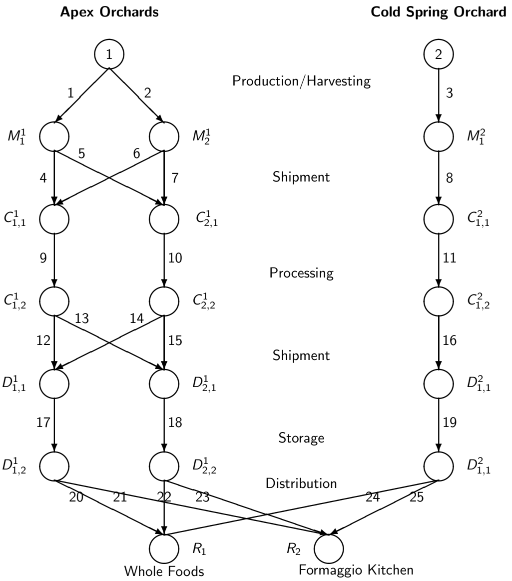
The Lagrange multiplier for each link $l \in L^i$; $i = 1, \ldots, I$, associated with the link capacities is computed according to:

$$\gamma_l^{\tau+1} = \max\{0, \gamma_l^\tau + \alpha_\tau(\sum_{r \in P} x_r^\tau \delta_{lr} - u_l)\}. \quad (37)$$

A Case Study of Peaches

- We focus on the **peach market** in the United States, specifically in **Western Massachusetts**.
- It is noted that, in 2015, the United States peach production was **825,415 tons** in volume, and 606 million dollars in worth (USDA NASS (2016), Zhao et al. (2017)).
- We selected two orchards from Western Massachusetts: **Apex Orchards** and **Cold Spring Orchard**, located, respectively, in Shelburne, MA and Belchertown, MA.
- The orchards sell their peaches to two retailers, **Whole Foods**, located in Hadley, MA, and **Formaggio Kitchen**, located in Cambridge, MA.
- The mode of **transportation** for both of the orchards is **trucks**.
- The **color change attribute** of peaches, in the form of **browning**, follows **a first-order**, that is, an **exponential decay function**.

Network Topology of the Case Study



Parameters for the Calculation of Quality Decay

Link b	Hours	Temperature (Celsius)	β_b ($n = 1$)
4	1	23	0.9961
5	2	23	0.9922
6	2	23	0.9922
7	1	23	0.9961
8	2	27	0.9913
9	3	18	0.9906
10	3	18	0.9906
11	4	25	0.9836
12	1	23	0.9961
13	2	23	0.9922
14	2	23	0.9922
15	1	23	0.9961
16	3	27	0.9870
17	**48**	**1**	**1.0000**
18	**72**	**1**	**1.0000**
19	**96**	**18**	**0.7397**
20	2	27	0.9913
21	4	27	0.9827
22	1	27	0.9956
23	4	27	0.9827
24	0.5	27	0.9978
25	4	27	0.9827

Cost Functions, Capacities and Upper Bounds for the Numerical Examples

Table: Total Production / Harvesting Cost Functions, Link Capacities, and Upper Bounds on Initial Quality

Link a	$\hat{z}_a(f_a, q^i_{0a})$	u_a	$\bar{q}^i_{0a}$
1	$.002f_1^2 + f_1 + 0.7q^1_{01} + .01(q^1_{01})^2$	**200**	**98**
2	$.002f_2^2 + f_2 + 0.7q^1_{02} + .01(q^1_{02})^2$	**200**	**95**
3	$.002f_3^2 + f_3 + 0.5q^2_{03} + .001(q^1_{03})^2$	**150**	**90**

Table: Total Operational Link Cost Functions and Link Capacities

Link b	$\hat{c}_b(f)$	u_b
4	$.001f_4^2 + .7f_4$	150
5	$.002f_5^2 + .7f_5$	150
6	$.001_6^2 + .5f_6$	120
7	$.002f_7^2 + .5f_7$	120
8	$.002f_8^2 + .9f_8$	100
9	$.0025f_9^2 + 1.2f_9$	200
10	$.0025f_{10}^2 + 1.2_{10}$	200
11	$.0026f_{11}^2 + 1.5f_{11}$	150
12	$.001f_{12}^2 + .6f_{12}$	150
13	$.002f_{13}^2 + .6f_{13}$	150
14	$.001f_{14}^2 + .6f_{14}$	150
15	$.002f_{15}^2 + .6f_{15}$	150
16	$.002f_{16}^2 + .6f_{16}$	120
17	$.003f_{17}^2 + .5f_{17}$	**150**
18	$.0037f_{18}^2 + .9f_{18}$	**150**
19	$.002f_{19}^2 + .7f_{19}$	**120**
20	$.002f_{20}^2 + .6f_{20}$	150
21	$.003f_{21}^2 + .7f_{21}$	120
22	$.002f_{22}^2 + .6f_{22}$	150
23	$.003f_{23}^2 + .7f_{23}$	100
24	$.002f_{24}^2 + .6f_{24}$	100
25	$.003f_{25}^2 + .7f_{25}$	100

- We report the total **production / harvesting cost functions**, **the upper bounds on the initial quality**, the total operational cost functions, and the link flow capacities.
- The Euler method is implemented in FORTRAN and a Linux system at the University of Massachusetts used for the computations.
- The data is gathered from Sumner and Murdock (2017) and Dris and Jain (2007), in which the authors made a sample cost analysis.
- The **time horizon**, under consideration, is that of **a week**.
- The Euler method is implemented in FORTRAN and a Linux system at the University of Massachusetts used for the computations.
- The sequence is, $a_\tau = \{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \ldots\}$, with the convergence tolerance being 10^{-7}.

Example 1 - Baseline

- It is known that both retailers sell **high quality food** products, with **Formaggio Kitchen** selling peaches at a higher price due to its emphasis on **quality**.
- Through conversations at the retailers, we concluded that **Apex Orchards** sell their **peaches** at a higher price.

Demand Price Functions of Apex Orchards:

$$\rho_{11} = -.02d_{11} - .01d_{21} + 0.008\hat{q}_{11} + \mathbf{20},$$

$$\rho_{12} = -.02d_{12} - .01d_{22} + \mathbf{0.01}\hat{q}_{12} + 22.$$

Demand Price Functions of Cold Spring Orchard:

$$\rho_{21} = -.02d_{21} - .015d_{11} + \mathbf{0.008}\hat{q}_{21} + \mathbf{18},$$

$$\rho_{22} = -.02d_{22} - .015d_{12} + 0.01\hat{q}_{22} + 19.$$

Equilibrium Flows, Equilibrium Initial Quality, and the Equilibrium Lagrange Multipliers

Link a	f_a^*	q_{0a}^{i*}	γ_a^*	λ_a^*
1	**133.43**	**97.54**	**0.00**	**0.00**
2	166.57	95.00	0.00	0.05
3	**100.00**	**65.61**	**0.00**	**0.00**

Table: Equilibrium Link Flows, Equilibrium Initial Quality, and the Equilibrium Production Site Lagrange Multipliers

Table: Equilibrium Link Flows and the Equilibrium Link Lagrange Multipliers

Link b	f_b^*	γ_b^*
4	69.31	0.00
5	64.12	0.00
6	90.33	0.00
7	76.24	0.00
8	**100.00**	**6.53**
9	159.64	0.00
10	140.36	0.00
11	100.00	0.00
12	81.96	0.00
13	77.68	0.00
14	68.04	0.00
15	72.32	0.00
16	100.00	0.00
17	**150.00**	**6.95**
18	**150.00**	**6.19**
19	100.00	0.00
20	64.84	0.00
21	85.16	0.00
22	65.10	0.00
23	84.90	0.00
24	47.80	0.00
25	52.20	0.00

Equilibrium Prices, Demands, Average Quality and Profits

Equilibrium Prices at the Demand Markets:

$$\rho_{11} = 17.67, \ \boldsymbol{\rho_{12} = 19.00}, \ \rho_{21} = 15.47, \ \rho_{22} = 15.86.$$

Equilibrium Demands:

$$d^*_{11} = 129.95, \ \mathbf{d^*_{12} = 170.05}, \ d^*_{21} = 47.80, \ d^*_{22} = 52.20.$$

Average Quality:

$$\mathbf{\hat{q}_{11} = 93.40}, \ \hat{q}_{12} = 92.56, \ \hat{q}_{21} = 46.60, \ \hat{q}_{22} = 45.90.$$

Profits:

$$\mathbf{U_1 = 3,302.01}, \quad U_2 = 787.65.$$

Example 2 - Disruption Scenario 1

- We now consider a **disruption scenario** in which a **natural disaster** has significantly affected the **capacity** of the orchard production sites of both orchards.
- Such an incident occurred in **2016 in the Northeast of the United States** when extreme weather in terms of cold temperatures "decimated" the **peach crop**.
- We now have the following capacities on the production/harvesting links:

$$u_1 = 100, \ u_2 = 150, \ u_3 = 80.$$

Link a	f_a^*	q_{0a}^{i*}	γ_a^*	λ_a^*
1	**100.00**	**75.54**	**8.17**	0.00
2	**150.00**	**75.54**	**8.18**	0.00
3	**80.00**	**11.02**	**7.78**	0.00

Table: Equilibrium Link Flows, Equilibrium Initial Quality, and the Equilibrium Production Site Lagrange Multipliers

Equilibrium Link Flows and the Equilibrium Link Lagrange Multipliers

Link b	f_b^*	γ_b^*
4	50.28	0.00
5	49.72	0.00
6	82.76	0.00
7	67.24	0.00
8	80.00	0.00
9	133.04	0.00
10	116.96	0.00
11	80.00	0.00
12	80.78	0.00
13	52.25	0.00
14	69.22	0.00
15	47.75	0.00
16	80.00	0.00
17	**150.00**	**0.17**
18	100.00	0.00
19	80.00	0.00
20	67.2	0.00
21	82.79	0.00
22	37.46	0.00
23	62.54	0.00
24	37.67	0.00
25	42.33	0.00

Equilibrium Prices, Demands, Average Quality and Profits

Equilibrium Prices at the Demand Markets:

$$\rho_{11} = 18.12, \ \boldsymbol{\rho_{12} = 19.40}, \ \rho_{21} = 15.74, \ \rho_{22} = 16.05.$$

Equilibrium Demands:

$$d^*_{11} = 104.67, \ \mathbf{d^*_{12} = 145.33}, \ d^*_{21} = 37.67, \ d^*_{22} = 42.33.$$

Average Quality:

$$\mathbf{\hat{q}_{11} = 73.42}, \ \hat{q}_{12} = 72.68, \ \hat{q}_{21} = 7.83, \ \hat{q}_{22} = 7.71.$$

Profits:

$$\mathbf{U_1 = 2,984.07}, \quad U_2 = 675.72.$$

Example 3 - Disruption Scenario 2

- We consider a **disruption** that affects **transportation** in that the **links** 5 **and** 6 associated with the supply chain network of **Apex Orchards** are no longer available.
- This can occur and has occurred in western Massachusetts as a result of flooding.
- We now have the following capacities on the production/harvesting links:

$$\mathbf{u_5 = 0},\ \mathbf{u_6 = 0}.$$

Link a	f_a^*	q_{0a}^{i*}	γ_a^*	λ_a^*
1	150.00	84.50	0.00	0.00
2	120.00	84.50	0.00	0.00
3	100.00	65.59	0.00	0.00

Table: Equilibrium Link Flows, Equilibrium Initial Quality, and the Equilibrium Production Site Lagrange Multipliers

Equilibrium Link Flows and the Equilibrium Link Lagrange Multipliers

Link b	f_b^*	γ_b^*
4	150.00	6.94
5	**0.00**	**78.33**
6	**0.00**	**79.92**
7	120.00	**7.27**
8	100.00	**6.75**
9	150.00	0.00
10	120.00	0.00
11	100.00	0.00
12	85.36	0.00
13	64.64	0.00
14	64.64	0.00
15	55.36	0.00
16	100.00	0.00
17	150.00	**0.40**
18	120.00	**6.19**
19	100.00	0.00
20	66.22	0.00
21	83.78	0.00
22	48.52	0.00
23	71.48	0.00
24	47.86	0.00
25	52.14	0.00

Equilibrium Prices, Demands, Average Quality and Profits

Equilibrium Prices at the Demand Markets:

$$\rho_{11} = 17.89, \ \boldsymbol{\rho_{12} = 19.19}, \ \rho_{21} = 15.69, \ \rho_{22} = 16.09.$$

Equilibrium Demands:

$$d^*_{11} = 114.74, \ \mathbf{d^*_{12} = 155.26}, \ d^*_{21} = 47.86, \ d^*_{22} = 52.14.$$

Average Quality:

$$\hat{\mathbf{q}}_{11} = \mathbf{82.32}, \ \hat{q}_{12} = 81.46, \ \hat{q}_{21} = 46.59, \ \hat{q}_{22} = 45.88.$$

Profits:

$$\mathbf{U_1 = 3,074.72}, \quad U_2 = 811.35.$$

Conclusions

- We constructed a general framework for the modeling, analysis, and computation of solutions to **competitive fresh produce supply chain networks** in which food firm owners seek to **maximize their profits** while determining both the **initial quality** of the fresh produce with associated costs as well as the **fresh produce flows** along pathways of their supply chain network through the various activities of harvesting, processing, storage, and distribution.
- We utilize **explicit formulae** associated with **quality deterioration** on the supply chain network links which are a function of physical characteristics, including **temperature and time**.
- The governing **Nash Equilibrium** conditions are stated and alternative **variational inequality formulations** provided, along with existence results.
- **Stylistic examples** are provided to illustrate the framework and a case study on **peaches**, consisting of numerical examples under **status quo and disruption scenarios**, is then presented, along with the computed equilibrium patterns.

Some References

- Ahumada, O., Villalobos, J. R., 2011. A tactical model for planning the production and distribution of fresh produce. *Annals of Operations Research* 190, 339-358.
- Besik, D., Nagurney, A., 2017. Quality in fresh produce supply chains with application to farmers' markets. *Socio-Economic Planning Sciences* 60, 62-76.
- Blackburn, J., Scudder, G., 2009. Supply chain strategies for perishable products: The case of fresh produce. *Production and Operations Management* 18(2), 129-137.
- Ghare, P., Schrader, G., 1963. A model for an exponentially decaying inventory. *Journal of Industrial Engineering* 14, 238-243.
- Kopanos, G.M., Puigjaner, L., Georgiadis, M.C., 2012. Simultaneous production and logistics operations planning in semicontinuous food industries. *Omega* 40, 634-650.
- Monteiro, D.M.S., 2007. Theoretical and empirical analysis of the economics of traceability adoption in food supply chains. Ph.D. Thesis, University of Massachusetts Amherst, Amherst, MA.
- Nahmias, S., 1982. Perishable inventory theory: A review. *Operations Research* 30(4), 680-708.
- Nahmias, S., 2011. **Perishable Inventory Systems**. Springer, New York.
- Silver, E.A., Pyke, D.F., Peterson, R., 1998. **Inventory Management and Production Planning and Scheduling**, third edition. John Wiley & Sons, New York.
- Sloof, M., Tijskens, L.M.M., Wilkinson, E.C., 1996. Concepts for modelling the quality of perishable products. *Trends in Food Science & Technology* 7(5), 165-171.
- Widodo, K.H., Nagasawa, H., Morizawa, K., Ota, M., 2006. A periodical flowering-harvesting model for delivering agricultural fresh products. *European Journal of Operational Research* 170, 24-43.
- Yu M, Nagurney A. Competitive food supply chain networks with application to fresh produce. *European Journal of Operational Research* 2013; 224(2): 273-282.
- Zhang, G., Habenicht, W., Spieß, W.E.L., 2003. Improving the structure of deep frozen and chilled food chain with tabu search procedure. *Journal of Food Engineering* 60, 67-79.

THANK YOU!

For more information: https://supernet.isenberg.umass.edu/