A Network Economic Game Theory Model of a Service-Oriented Internet with Price and Quality Competition in Both Content and Network Provision

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The paper has been cited in:

Behzad, B., Jacobson, S.H., 2016. Asymmetric Bertrand-Edgeworth-Chamberlin competition with linear demand: A pediatric vaccine pricing model. *Service Science* 8(1), 71-84.

among other references.

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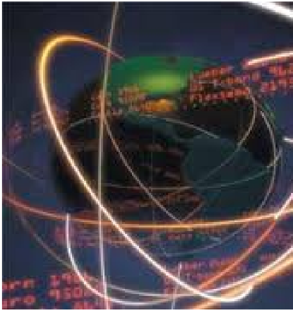

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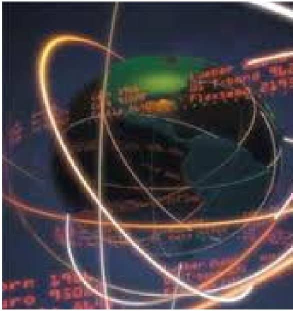

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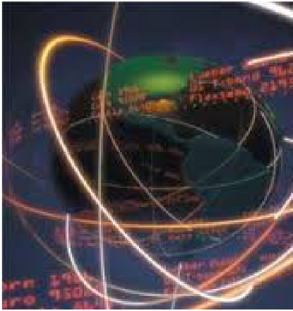

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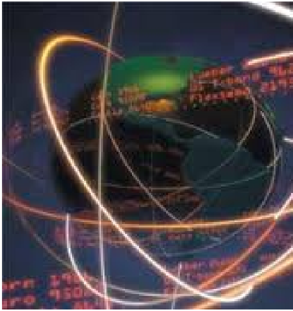

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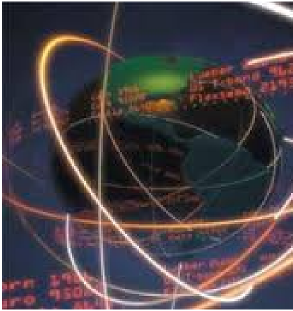

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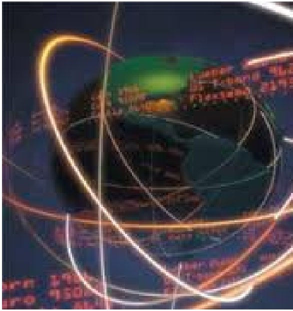

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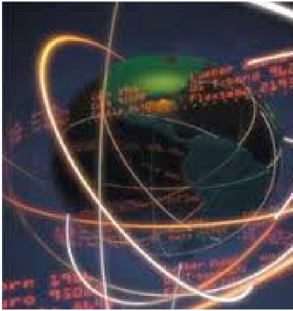

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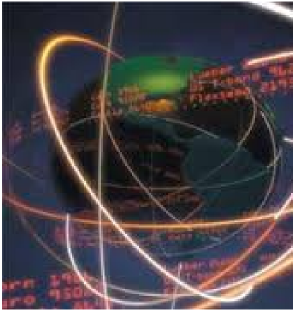

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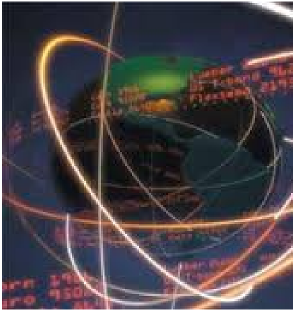

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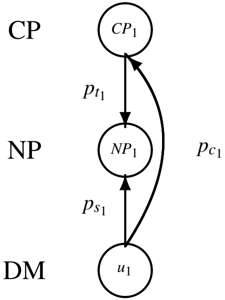
- Studied two-sided payments effects in the NGI
  - Laffont et al. (2003),
  - Hermalin and Katz (2007),
  - Musacchio et al. (2011),
  - Economides and Tag (2012).

The Basic Model Description

- The demand function: $d_{111} = d_0 - \alpha p_{s_1} - \beta p_{c_1} + \gamma q_{s_1} + \delta q_{c_1}$.



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- The demand function: $d_{111} = d_0 - \alpha p_{s_1} - \beta p_{c_1} + \gamma q_{s_1} + \delta q_{c_1}$.
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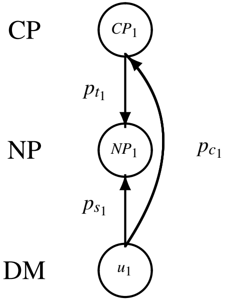

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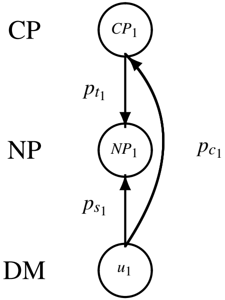

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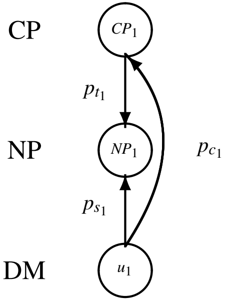

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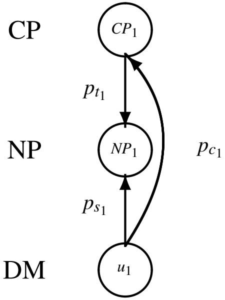
$$U_{NP_1} = U_{NP_1}(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}) = (p_{s_1}+p_{t_1})d_{111} - CS_1 = (p_{s_1}+p_{t_1}-R)d_{111} - Rq_{s_1}^2.$$



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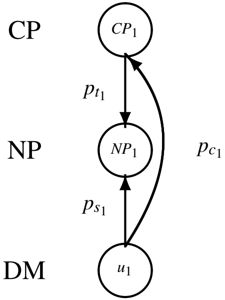

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Analysis of Two-sided Pricing in the Basic Model

$$\text{Maximize} \quad U_{CP_1}(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}) = (p_{c_1} - p_{t_1})d_{111} - Kq_{c_1}^2 .$$

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Theorem

The network provider will benefit from charging the content provider if

$$4\alpha R - \gamma^2 > 0, \; \textit{and} \; \alpha > \beta.$$

Definition: Nash Equilibrium in Price and Quality

$$U_{CP_1}(p_{c_1}^*, q_{c_1}^*, p_{s_1}^*, q_{s_1}^*) = \max_{(p_{c_1}, q_{c_1}) \in \mathcal{S}_{CP}} U_{CP_1}(p_{c_1}, q_{c_1}, p_{s_1}^*, q_{s_1}^*),$$

$$U_{NP_1}(p_{c_1}^*, q_{c_1}^*, p_{s_1}^*, q_{s_1}^*) = \max_{(p_{s_1}, q_{s_1}) \in \mathcal{S}_{NP}} U_{NP_1}(p_{c_1}^*, q_{c_1}^*, p_{s_1}, q_{s_1}).$$

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Theorem: VI Formulation of Nash Equilibrium for CP and NP

$$(-d_{111} + \beta(p^*_{c_1} - p_{t_1})) \times (p_{c_1} - p^*_{c_1}) + (2Kq^*_{c_1} + \delta(p_{t_1} - p^*_{c_1})) \times (q_{c_1} - q^*_{c_1})$$

$$+(-d_{111} + \alpha(p^*_{s_1} + p_{t_1} - R)) \times (p_{s_1} - p^*_{s_1}) + (2Rq^*_{s_1} + \gamma(R - p^*_{s_1} - p_{t_1})) \times (q_{s_1} - q^*_{s_1}) \geq 0,$$

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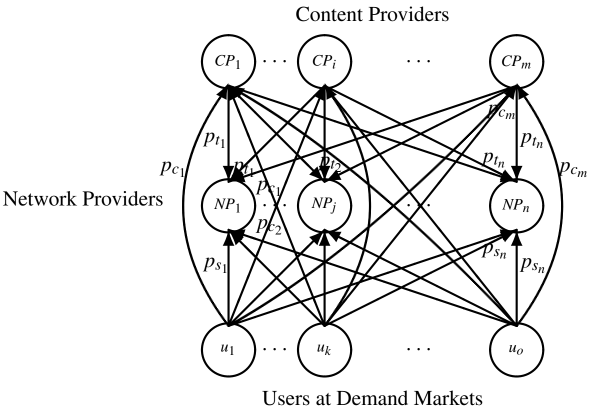
$$\forall (p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}) \in \mathcal{S}_{CP} \times \mathcal{S}_{NP}.$$

Theorem: Uniqueness of the Nash Equilibrium

The Nash equilibrium $(p^*_{c_1}, q^*_{c_1}, p^*_{s_1}, q^*_{s_1}) \in \mathcal{S}_{CP} \times \mathcal{S}_{NP}$ satisfying variational inequality is unique, if the function $F = -\nabla U(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1})$ is strictly monotone over the feasible set $\mathcal{S}_{NP} \times \mathcal{S}_{CP}$.

The Network of Oligopoly Model



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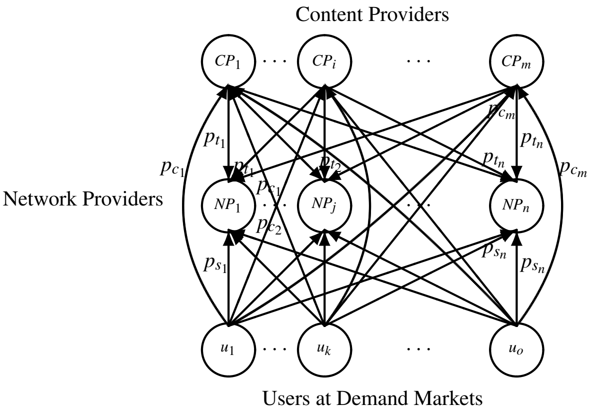

Demand function d_{ijk} for the content produced by content provider i and transmitted by network provider j to demand market k:

$$d_{ijk} = d_{ijk}(p_s, q_s, p_c, q_c), \quad \forall i, j, k.$$

The Behavior of the Providers

Content Providers

Each CP_i has a production cost CC_i:

$$CC_i = CC_i(SCP_i, q_{c_i}), \quad i = 1, \ldots, M.$$

The utility of CP_i:

$$U_{CP_i} = \sum_{j=1}^{N} (p_{c_i} - p_{t_j}) \sum_{k=1}^{O} d_{ijk} - CC_i(SCP_i, q_{c_i}).$$

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Network Providers

Each NP_j incurs a transmission cost CS_j:

$$CS_j = CS_j(TNP_j, q_{s_j}), \quad j = 1, \ldots, N.$$

The utility of NP_j:

$$U_{NP_j} = (p_{s_j} + p_{t_j})(\sum_{i=1}^{M} \sum_{k=1}^{O} d_{ijk}) - CS_j(TNP_j, q_{s_j}).$$

Nash Equilibrium

Definition: Nash Equilibrium in Price and Quality

$$U_{CP_i}(p^*_{c_i}, \hat{p}^*_{c_i}, q^*_{c_i}, \hat{q}^*_{c_i}, p^*_s, q^*_s) \geq U_{CP_i}(p_{c_i}, \hat{p}^*_{c_i}, q_{c_i}, \hat{q}^*_{c_i}, p^*_s, q^*_s), \quad \forall (p_{c_i}, q_{c_i}) \in \mathcal{K}^1_i,$$

$$\hat{p}^*_{c_i} \equiv (p^*_{c_1}, \ldots, p^*_{c_{i-1}}, p^*_{c_{i+1}}, \ldots, p^*_{c_m}) \text{ and } \hat{q}^*_{c_i} \equiv (q^*_{c_1}, \ldots, q^*_{c_{i-1}}, q^*_{c_{i+1}}, \ldots, q^*_{c_m}).$$

$$U_{NP_j}(p^*_c, q^*_c, p^*_{s_j}, \hat{p}^*_{s_j}, q^*_{s_j}, \hat{q}^*_{s_j}) \geq U_{NP_j}(p_{s_j}, p^*_c, \hat{q}^*_c, p^*_{s_j}, q_{s_j}, \hat{q}^*_{s_j}), \quad \forall (p_{s_j}, q_{s_j}) \in \mathcal{K}^2_j,$$

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The Equilibrium Conditions of the Service-Oriented Internet

Theorem: Variational Inequality Formulation of Nash Equilibrium

$$\sum_{i=1}^{M}\left[-\sum_{j=1}^{N}\sum_{k=1}^{O} d_{ijk}-\sum_{j=1}^{N}\sum_{k=1}^{O}\frac{\partial d_{ijk}}{\partial p_{c_i}}\times(p^*_{c_i}-p_{t_j})+\frac{\partial f_{c_i}(SCP_i,q^*_{c_i})}{\partial SCP_i}\cdot\frac{\partial SCP_i}{\partial p_{c_i}}\right]\times(p_{c_i}-p^*_{c_i})$$

$$+\sum_{i=1}^{M}\left[-\sum_{j=1}^{N}\sum_{k=1}^{O}\frac{\partial d_{ijk}}{\partial q_{c_i}}\times(p^*_{c_i}-p_{t_j})+\frac{\partial f_{c_i}(SCP_i,q^*_{c_i})}{\partial q_{c_i}}\right]\times(q_{c_i}-q^*_{c_i})$$

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$$+\sum_{j=1}^{N}\left[-\sum_{j=1}^{N}\sum_{k=1}^{O}\frac{\partial d_{ijk}}{\partial q_{s_j}}\times(p^*_{s_j}+p_{t_j})+\frac{\partial f_{s_j}(TNP_j,q^*_{s_j})}{\partial q_{s_j}}\right]\times(q_{s_j}-q^*_{s_j})\geq 0,$$

$$\forall(p_c,q_c,p_s,q_s)\in\mathcal{K}^3.$$

Euler Method

We recall the Euler method for the solution of the Variational Inequality Problem. Specifically, iteration τ of the Euler method is given by:

$$X^{\tau+1} = p_{\mathcal{K}}(X^{\tau} - a_{\tau} F(X^{\tau})).$$

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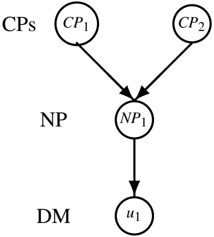
Theorem: Convergence

In the service-oriented Internet model, let $F(X) = -\nabla U(p_c, q_c, p_s, q_s)$ be strictly monotone at any equilibrium pattern. Also, assume that F is uniformly Lipschitz continuous. Then, there exists a unique equilibrium price and quality pattern $(p_c^*, q_c^*, p_s^*, q_s^*) \in \mathcal{K}$ and any sequence generated by the Euler method, where $\{a_\tau\}$ satisfies $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$ converges to $(p_c^*, q_c^*, p_s^*, q_s^*)$.

Example 1



The demand functions:

$$d_{111} = 100 - 2.8p_{s_1} - 2.1p_{c_1} + 1.3p_{c_2} + 1.62q_{s_1} + 1.63q_{c_1} - .42q_{c_2},$$

$$d_{211} = 112 - 2.8p_{s_1} + 1.3p_{c_1} - 2.7p_{c_2} + 1.62q_{s_1} - .42q_{c_1} + 1.58q_{c_2}.$$

The cost functions:

$$CC_1 = 1.7q_{c_1}^2, \qquad CC_2 = 2.4q_{c_2}^2, \qquad CS_1 = 2.2(d_{111} + d_{211} + q_{s_1}^2).$$

The utility functions, with $p_{t_1} = 33$:

$$U_{CP_1} = (p_{c_1} - p_{t_1})d_{111} - CC_1, \qquad U_{CP_2} = (p_{c_2} - p_{t_1})d_{211} - CC_2.$$

$$U_{NP_1} = (p_{s_1} + p_{t_1})(d_{111} + d_{211}) - CS_1.$$

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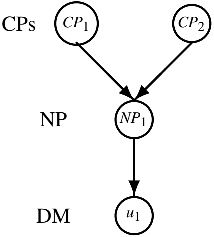

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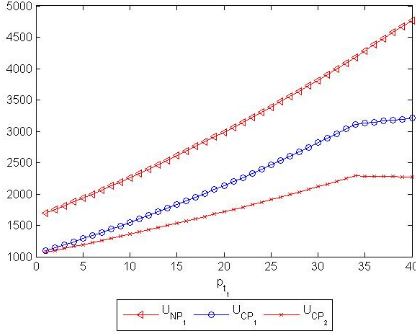
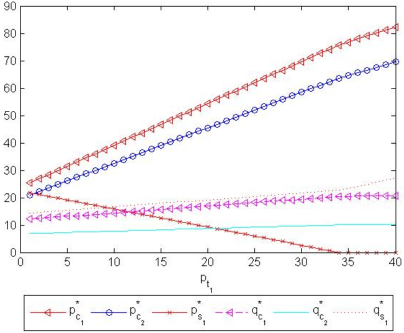
The computed equilibrium solution:

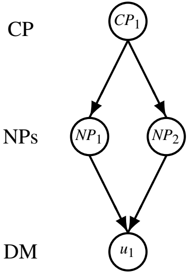
$$p_{c_1}^* = 75.68, \quad p_{c_2}^* = 63.62, \quad p_{s_1}^* = 0, \quad q_{c_1}^* = 20.46, \quad q_{c_2}^* = 10.08, \quad q_{s_1}^* = 22.68,$$

Example 1: Sensitivity Analysis





Example 2



The demand functions:

$$d_{111} = 100 - 1.8p_{s_1} + .5p_{s_2} - 1.83p_{c_1} + 1.59q_{s_1} - .6q_{s_2} + 1.24q_{c_1},$$

$$d_{121} = 100 + .5p_{s_1} - 1.5p_{s_2} - 1.83p_{c_1} - .6q_{s_1} + 1.84q_{s_2} + 1.24q_{c_1}.$$

The cost functions:

$$CS_1 = 1.7(d_{111} + q_{s_1}^2), \quad CS_2 = 1.8(d_{121} + q_{s_2}^2).$$

$$CC_1 = 1.84\left[d_{111} + d_{121} + q_{c_1}^2\right].$$

The utility functions, with $p_{t_1} = p_{t_2} = 0$:

$$U_{CP_1} = (p_{c_1} - p_{t_1})d_{111} + (p_{c_1} - p_{t_2})d_{121} - CC_1.$$

$$U_{NP_1} = (p_{s_1} + p_{t_1})d_{111} - CS_1, \qquad U_{NP_2} = (p_{s_2} + p_{t_2})d_{121} - CS_2.$$

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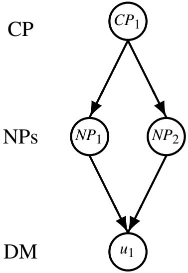

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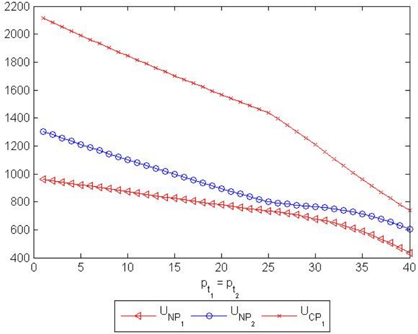
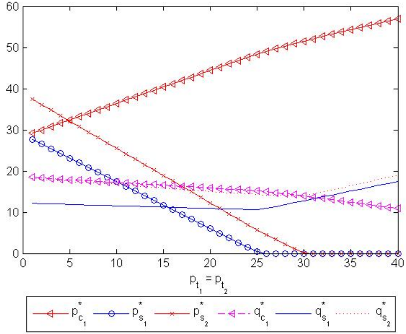
The equilibrium solution:

$$p_{c_1}^* = 29.19, \quad p_{s_1}^* = 27.66, \quad p_{s_2}^* = 37.38,$$

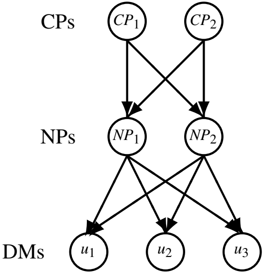
$$q_{c_1}^* = 18.43, \quad q_{s_1}^* = 12.14, \quad q_{s_2}^* = 18.18.$$

Example 2: Sensitivity Analysis





Example 3



The utility functions of the content providers:

$$U_{CP_1} = (p_{c_1} - p_{t_1})(d_{111} + d_{112} + d_{113}) + (p_{c_1} - p_{t_2})(d_{121} + d_{122} + d_{123}) - CC_1,$$

$$U_{CP_2} = (p_{c_2} - p_{t_1})(d_{211} + d_{212} + d_{213}) + (p_{c_2} - p_{t_2})(d_{221} + d_{222} + d_{223}) - CC_2.$$

The utility functions, with $p_{t_1} = 23$ and $p_{t_2} = 21$:

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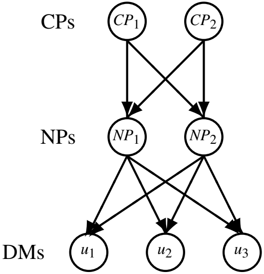

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The equilibrium solution:

$$p^*_{c_1} = 40.57, \quad p^*_{c_2} = 41.49, \quad p^*_{s_1} = 8.76, \quad p^*_{s_2} = 5.35,$$

$$q^*_{c_1} = 13.96, \quad q^*_{c_2} = 12.76 \quad q^*_{s_1} = 36.67, \quad q^*_{s_2} = 12.15,$$

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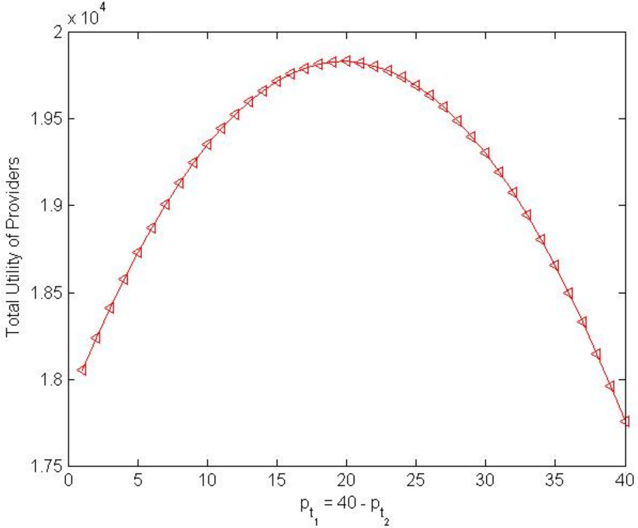

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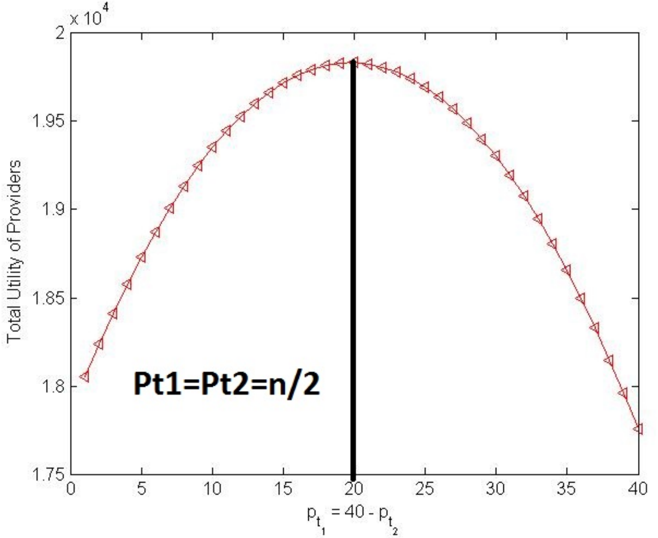

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THANK YOU!

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