

Supply Chain Network Competition in Price and Quality with Multiple Manufacturers and Freight Service Providers

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Introduction

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- Success is determined by how well the entire supply chain performs, rather than the performance of its individual entities.



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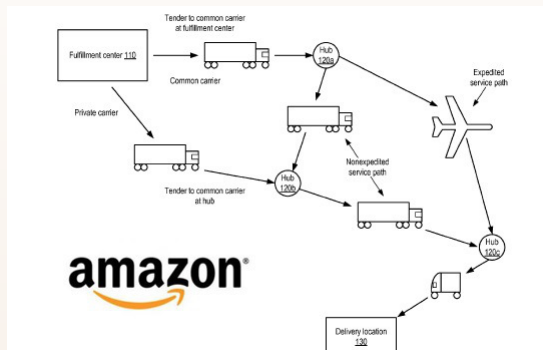
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- Quality has also become one of the most essential factors in the success of supply chains of various products.

Introduction

- Increasingly, tough customer demands are also putting the transport system under pressure. The online retailer Amazon.com recently submitted a patent (United States patent (2013)) for anticipatory shipping and speculative shipping.



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- In this paper, quality of the product is traced along the supply chain with consumers differentiating among the products offered by the manufacturers.

Contributions

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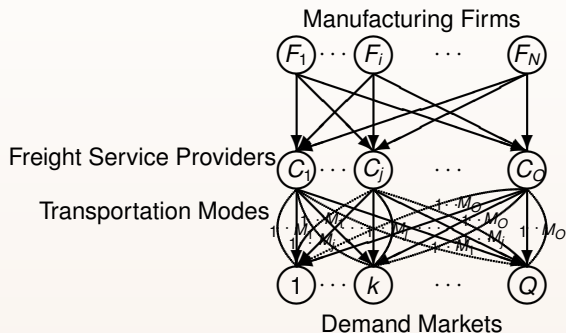
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- The transportation costs differ by mode, leading to an evaluation of quality vs. costs for the freight service providers and the modes of transportation that they offer to the customers.
- We handle heterogeneity in the providers' cost functions and in the consumers' demands and do not limit ourselves to specific functional forms.
- Utilities of each manufacturing firm and freight service provider considers price and quality for not just his own products, but that of other firms or providers as well.

The Supply Chain Network Model with Price and Quality Competition



The consumers at demand market k reveal their preferences for firm F_i 's product transported by freight service provider C_j via mode m :

The Supply Chain Network Model with Price and Quality Competition

- As in Nagurney and Li (2014), we define and quantify quality as the quality conformance level, that is, the degree to which a specific product conforms to a design or specification (Gilmore (1974), Juran and Gryna (1988)).

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- Firm F_i manufactures a product of quality q_i at the price p_i .
- The quality and price associated with freight service provider C_j retrieving the product from firm F_i and delivering it to demand market k via mode m are denoted, respectively, by q_{ijk}^m , and $p_{ijk}^m; \forall i, j, k, m$.

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- The quality and price associated with freight service provider C_j retrieving the product from firm F_i and delivering it to demand market k via mode m are denoted, respectively, by q_{ijk}^m , and $p_{ijk}^m; \forall i, j, k, m$.
- Demand is denoted by d_{ijk}^m for consumer market k , mode m coming from firm i through provider j .

The Supply Chain Network Model with Price and Quality Competition

Demand Function:

$$d_{ijk}^m = d_{ijk}^m(p_F, q_F, p_C, q_C); \forall i, j, k, m.$$

Demand depends on firm's price and quality, its competitors, and freight service providers.

The Firms' Behavior: Supply of Firm:

$$s_i(p_F, q_F, p_C, q_C) = \sum_{j=1}^O \sum_{k=1}^Q \sum_{m=1}^{M_j} d_{ijk}^m(p_F, q_F, p_C, q_C); \forall i.$$

The Production Cost:

$$PC_i = PC_i(s_F(p_F, q_F, p_C, q_C), q_F), \forall i$$

The Supply Chain Network Model with Price and Quality Competition

The Utility of Firm:

$$UF_i(p_F, q_F, p_C, q_C) = p_i[s_i(p_F, q_F, p_C, q_C)] - PC_i, \forall i.$$

Bounds on Quality:

$$\underline{q}_i \leq q_i \leq \bar{q}_i, \forall i.$$

$\bar{q}_i = 100$ corresponds to perfect quality conformance level.
Positive lower bound corresponds \underline{q}_i to a minimum quality standard.

Bounds on Price:

$$0 \leq p_i \leq \bar{p}_i, \forall i.$$

Let K_i^1 denote the feasible set for firm F_i

The Supply Chain Network Model with Price and Quality Competition

The Freight Service Providers' Behavior: The Transportation Cost:

$$TC_{ijk}^m = TC_{ijk}^m(d(p_F, q_F, p_C, q_C), q_C), \forall i, j, k, m.$$

The Utility of Freight Service Provider:

$$UC_j = \sum_{i=1}^N \sum_{k=1}^O \sum_{m=1}^{M_j} [p_{ijk}^m d_{ijk}^m - TC_{ijk}^m], \forall j.$$

Bounds on Quality:

$$\underline{q}_{ijk}^m \leq q_{ijk}^m \leq \bar{q}_{ijk}^m, \forall i, j, k, m.$$

Bounds on Price:

$$0 \leq p_{ijk}^m \leq \bar{p}_{ijk}^m, \forall i, j, k, m.$$

The Equilibrium Conditions

Definition 1: Nash Equilibrium in Prices and Quality Levels

A **price and quality** level pattern $(p_F^*, q_F^*, p_C^*, q_C^*) \in K^3 \equiv \prod_{i=1}^N K_i^1 \times \prod_{j=1}^O K_j^2$, is said to constitute a Nash equilibrium if for **each firm** $F_i; i = 1, \dots, N$:

$$U_{F_i}(p_i^*, \hat{p}_i^*, q_i^*, \hat{q}_i^*, p_C^*, q_C^*) \geq U_{F_i}(p_i, \hat{p}_i^*, q_i, \hat{q}_i^*, p_C^*, q_C^*), \quad \forall (p_i, q_i) \in K_i^1,$$

where

$$\hat{p}_i^* \equiv (p_1^*, \dots, p_{i-1}^*, p_{i+1}^*, \dots, p_N^*) \text{ and } \hat{q}_i^* \equiv (q_1^*, \dots, q_{i-1}^*, q_{i+1}^*, \dots, q_N^*),$$

and if **for each freight service provider** $C_j; j = 1, \dots, O$:

$$U_{C_j}(p_F^*, q_F^*, p_{C_j}^*, \hat{p}_{C_j}^*, q_{C_j}^*, \hat{q}_{C_j}^*) \geq U_{C_j}(p_F^*, q_F^*, p_{C_j}, \hat{p}_{C_j}^*, q_{C_j}, \hat{q}_{C_j}^*),$$

where

$$\hat{p}_{C_j}^* \equiv (p_{C_1}^*, \dots, p_{C_{j-1}}^*, p_{C_{j+1}}^*, \dots, p_{C_O}^*) \text{ and } \hat{q}_{C_j}^* \equiv (q_{C_1}^*, \dots, q_{C_{j-1}}^*, q_{C_{j+1}}^*, \dots, q_{C_O}^*).$$

Variational Inequality Formulation

Theorem 1: Variational Inequality Formulations of Nash Equilibrium in Prices and Quality

$(p_F^*, q_F^*, p_C^*, q_C^*) \in \mathcal{K}^3$ is a Nash equilibrium according to Definition 1 if and only if it satisfies the variational inequality:

$$\begin{aligned}
 & - \sum_{i=1}^N \frac{\partial U_{F_i}(p_F^*, q_F^*, p_C^*, q_C^*)}{\partial p_i} \times (p_i - p_i^*) - \sum_{i=1}^N \frac{\partial U_{F_i}(p_F^*, q_F^*, p_C^*, q_C^*)}{\partial q_i} \times (q_i - q_i^*) \\
 & - \sum_{j=1}^O \sum_{i=1}^N \sum_{k=1}^Q \sum_{m=1}^{M_j} \frac{\partial U_{C_j}(p_F^*, q_F^*, p_C^*, q_C^*)}{\partial p_{ijk}^m} \times (p_{ijk}^m - p_{ijk}^{m*}) \\
 & - \sum_{j=1}^O \sum_{i=1}^N \sum_{k=1}^Q \sum_{m=1}^{M_j} \frac{\partial U_{C_j}(p_F^*, q_F^*, p_C^*, q_C^*)}{\partial q_{ijk}^m} \times (q_{ijk}^m - q_{ijk}^{m*}) \geq 0, \\
 & \forall (p_F, q_F, p_C, q_C) \in \mathcal{K}^3,
 \end{aligned}$$

Variational Inequality Formulation

Standard Form

Determine $X^* \in \mathcal{K}$ where X is a vector in R^n , $F(X)$ is a continuous function such that $F(X) : X \mapsto \mathcal{K} \subset R^n$, and

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}.$$

We define the vector $X \equiv (p_F, q_F, p_C, q_C)$ and $F(X) \equiv (F_{p_F}, F_{q_F}, F_{p_C}, F_{q_C})$ with the i -th component of F_{p_F} and F_{q_F} given, respectively, by:

$$F_{p_i} = -\frac{\partial U_{F_i}}{\partial p_i}; \quad F_{q_i} = -\frac{\partial U_{F_i}}{\partial q_i},$$

and the (i, j, k, m) -th component of F_{p_C} and F_{q_C} , respectively, given by:

$$F_{p_{ijk}^m} = -\frac{\partial U_{C_j}}{\partial p_{ijk}^m}; \quad F_{q_{ijk}^m} = -\frac{\partial U_{C_j}}{\partial q_{ijk}^m}.$$

Existence of the Solution

Theorem 2: A Solution to the Variational Inequality Discussed here Exists

Existence of a solution to the variational inequalities discussed earlier is guaranteed since the feasible set \mathcal{K} is compact and the function $F(X)$ in our model is continuous, under the assumptions made on the underlying functions. Hence, the following theorem is immediate from the classical theory of variational inequalities (see Kinderlehrer and Stampacchia (1980)).

The Dynamics

We now propose dynamic adjustment processes for the evolution of the firms' product prices and quality levels and those of the freight service providers (carriers).

Rate of change of p_i :

$$\dot{p}_i = \begin{cases} \frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial p_i}, & \text{if } 0 < p_i < \bar{p}_i \\ \max \left\{ 0, \min \left\{ \frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial p_i}, \bar{p}_i \right\} \right\}, & \text{if } p_i = 0 \text{ or } p_i = \bar{p}_i. \end{cases}$$

Rate of change of q_i :

$$\dot{q}_i = \begin{cases} \frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial q_i}, & \text{if } \underline{q}_i < q_i < \bar{q}_i \\ \max \left\{ \underline{q}_i, \min \left\{ \frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial q_i}, \bar{q}_i \right\} \right\}, & \text{if } q_i = \underline{q}_i \text{ or } q_i = \bar{q}_i. \end{cases}$$

The Dynamics

Rate of change of p_{ijk}^m :

$$\dot{p}_{ijk}^m = \begin{cases} \frac{\partial U_{C_j}(p_F, q_F, p_C, q_C)}{\partial p_{ijk}^m}, & \text{if } 0 < p_{ijk}^m < \bar{p}_{ijk}^m \\ \max \left\{ 0, \min \left\{ \frac{\partial U_{C_j}(p_F, q_F, p_C, q_C)}{\partial p_{ijk}^m}, \bar{p}_{ijk}^m \right\} \right\}, & \text{if } p_{ijk}^m = 0 \text{ or } \bar{p}_{ijk}^m. \end{cases}$$

Rate of change of q_{ijk}^m :

$$\dot{q}_{ijk}^m = \begin{cases} \frac{\partial U_{C_j}(p_F, q_F, p_C, q_C)}{\partial q_{ijk}^m}, & \text{if } \underline{q}_{ijk}^m < q_{ijk}^m < \bar{q}_{ijk}^m \\ \max \left\{ \underline{q}_{ijk}^m, \min \left\{ \frac{\partial U_{C_j}(p_F, q_F, p_C, q_C)}{\partial q_{ijk}^m}, \bar{q}_{ijk}^m \right\} \right\}, & \text{if } q_{ijk}^m = \underline{q}_{ijk}^m \text{ or } \bar{q}_{ijk}^m. \end{cases}$$

The Dynamics

Ordinary Differential Equation (ODE) for the adjustment processes of the prices and quality levels of firms and freight service providers, in vector form:

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \quad X(0) = X^0.$$

The projection operator:

$$\Pi_{\mathcal{K}}(X, -F(X)) = \lim_{\delta \rightarrow 0} \frac{P_{\mathcal{K}}(X - \delta F(X)) - X}{\delta},$$

with $P_{\mathcal{K}}$ denoting the projection map:

$$P_{\mathcal{K}}(X) = \operatorname{argmin}_{z \in \mathcal{K}} \|X - z\|.$$

The Dynamics

Theorem 3

X^* solves the variational inequality problem if and only if it is a stationary point of the ODE, that is,

$$\dot{X} = 0 = \Pi_{\mathcal{K}}(X^*, -F(X^*)).$$

This theorem demonstrates that the necessary and sufficient condition for a product and freight service price and quality level pattern $X^* = (p_F^*, q_F^*, p_C^*, q_C^*)$ to be a Nash equilibrium, according to Definition 1, is that $X^* = (p_F^*, q_F^*, p_C^*, q_C^*)$ is a stationary point of the adjustment processes defined by ODE, that is, X^* is the point at which $\dot{X} = 0$.

Explicit Formulae of the Euler Method

Closed form expressions of price and quality of firms:

$$\begin{aligned}
 p_i^{\tau+1} = \max & \left\{ 0, \min \left\{ \bar{p}_i, p_i^{\tau} + a_{\tau} \left[\sum_{j=1}^O \sum_{k=1}^Q \sum_{m=1}^{M_j} d_{ijk}^m(p_F^{\tau}, q_F^{\tau}, p_C^{\tau}, q_C^{\tau}) \right. \right. \right. \\
 & \left. \left. \left. + p_i^{\tau} \sum_{j=1}^O \sum_{k=1}^Q \sum_{m=1}^{M_j} \frac{\partial d_{ijk}^m(p_F^{\tau}, q_F^{\tau}, p_C^{\tau}, q_C^{\tau})}{\partial p_i} \right. \right. \right. \\
 & \left. \left. \left. - \sum_{l=1}^N \frac{\partial PC_i(s_F(p_F^{\tau}, q_F^{\tau}, p_C^{\tau}, q_C^{\tau}), q_F^{\tau})}{\partial s_l} \times \frac{\partial s_l(p_F^{\tau}, q_F^{\tau}, p_C^{\tau}, q_C^{\tau})}{\partial p_i} \right] \right\} \right\}, \\
 q_i^{\tau+1} = \max & \left\{ \underline{q}_i, \min \left\{ \bar{q}_i, q_i^{\tau} + a_{\tau} \left[p_i^{\tau} \sum_{j=1}^O \sum_{k=1}^Q \sum_{m=1}^{M_j} \frac{\partial d_{ijk}^m(p_F^{\tau}, q_F^{\tau}, p_C^{\tau}, q_C^{\tau})}{\partial q_i} \right. \right. \right. \\
 & \left. \left. \left. - \sum_{l=1}^N \frac{\partial PC_i(s_F(p_F^{\tau}, q_F^{\tau}, p_C^{\tau}, q_C^{\tau}), q_F^{\tau})}{\partial s_l} \times \frac{\partial s_l(p_F^{\tau}, q_F^{\tau}, p_C^{\tau}, q_C^{\tau})}{\partial q_i} - \frac{\partial PC_i(s_F^{\tau}, q_F^{\tau})}{\partial q_i} \right] \right\} \right\}.
 \end{aligned}$$

Explicit Formulae of the Euler Method

Closed form expressions of price and quality of freight service providers:

$$\begin{aligned}
 p_{ijk}^{m(\tau+1)} = & \max \left\{ 0, \min \left\{ \bar{p}_{ijk}^m, p_{ijk}^{m\tau} + a_\tau [d_{ijk}^m(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau) \right. \right. \\
 & + \sum_{l=1}^N \sum_{s=1}^Q \sum_{t=1}^{M_j} \frac{\partial d_{ljs}^t(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau)}{\partial p_{ijk}^m} \times p_{ljs}^{t\tau} \\
 & \left. \left. - \sum_{l=1}^N \sum_{s=1}^Q \sum_{t=1}^{M_j} \left(\sum_{r=1}^N \sum_{v=1}^Q \sum_{w=1}^{M_v} \frac{\partial TC_{ljs}^t(d(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau), q_C^\tau)}{\partial d_{rvw}^z} \times \frac{\partial d_{rvw}^z(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau)}{\partial p_{ijk}^m} \right) \right\} \right\}, \\
 q_{ijk}^{m(\tau+1)} = & \max \left\{ q_{ijk}^m, \min \left\{ \bar{q}_{ijk}^m, q_{ijk}^{m\tau} + a_\tau \left[\sum_{l=1}^N \sum_{s=1}^Q \sum_{t=1}^{M_j} \frac{\partial d_{ljs}^t(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau)}{\partial q_{ijk}^m} \times p_{ljs}^{t\tau} \right. \right. \\
 & \left. \left. - \sum_{l=1}^N \sum_{s=1}^Q \sum_{t=1}^{M_j} \left(\sum_{r=1}^N \sum_{v=1}^Q \sum_{w=1}^{M_v} \frac{\partial TC_{ljs}^t(d(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau), q_C^\tau)}{\partial d_{rvw}^z} \times \frac{\partial d_{rvw}^z(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau)}{\partial q_{ijk}^m} \right) \right. \right. \\
 & \left. \left. - \sum_{l=1}^N \sum_{s=1}^Q \sum_{t=1}^{M_j} \frac{\partial TC_{ljs}^t(d^\tau, q_C^\tau)}{\partial q_{ijk}^m} \right] \right\} \right\}.
 \end{aligned}$$

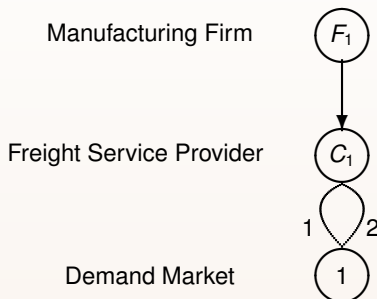
Convergence

Theorem 4

In our multitiered supply chain network game theory model, assume that $F(X) = -\nabla U(p_F, q_F, p_C, q_C)$ is strictly monotone. Also, assume that F is uniformly Lipschitz continuous. Then, there exists a unique equilibrium price and quality pattern $(p_F^*, q_F^*, p_C^*, q_C^*) \in \mathcal{K}$ and any sequence generated by the Euler method as given by the closed form expressions, where $\{a_\tau\}$ satisfies $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \rightarrow 0$, as $\tau \rightarrow \infty$ converges to $(p_F^*, q_F^*, p_C^*, q_C^*)$.

Example 1

The supply chain network topology is depicted as here:



The **demand** functions are:

$$d_{111}^1 = 43 - 1.62p_{111}^1 + 1.6q_{111}^1 - 1.45p_1 + 1.78q_1 + .03p_{111}^2 - .2q_{111}^2,$$

$$d_{111}^2 = 52 - 1.75p_{111}^2 + 1.21q_{111}^2 - 1.45p_1 + 1.78q_1 + .03p_{111}^1 - .2q_{111}^1.$$

Example 1

The **supply** of manufacturing firm F_1 is :

$$s_1 = d_{111}^1 + d_{111}^2$$

The **transportation costs** of the freight service provider C_1 for modes 1 and 2 are:

$$TC_{111}^1 = .5d_{111}^1 + (q_{111}^1)^2,$$

$$TC_{111}^2 = .45d_{111}^2 + .54(q_{111}^2)^2 + .0035d_{111}^2q_{111}^2.$$

The **utility** of freight service provider C_1 is:

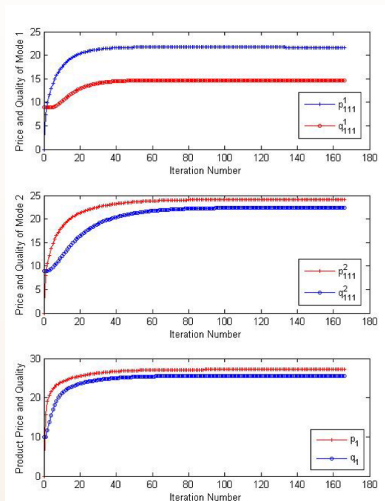
$$U_{C_1} = p_{111}^1 d_{111}^1 + p_{111}^2 d_{111}^2 - TC_{111}^1 - TC_{111}^2,$$

$$0 \leq p_{111}^2 \leq 70, \quad 9 \leq q_{111}^2 \leq 100.$$

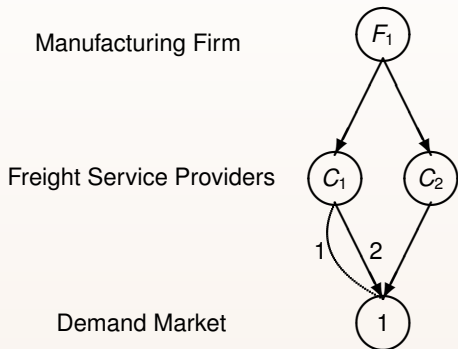
The **equilibrium solution**, after 166 iterations, is:

$$p_{111}^{1*} = 21.68, p_{111}^{2*} = 24.16, p_1^* = 27.18, q_{111}^{1*} = 14.58, q_{111}^{2*} = 22.43, q_1^* = 25.59.$$

Trajectories: Example 1



Example 2



Example 2

The **demand** functions are:

$$d_{111}^1 = 43 - 1.62p_{111}^1 + 1.6q_{111}^1 - 1.45p_1 + 1.78q_1 + .03p_{111}^2 - .2q_{111}^2 + .04p_{121}^1 - .1q_{121}^1,$$

$$d_{111}^2 = 52 - 1.75p_{111}^2 + 1.21q_{111}^2 - 1.45p_1 + 1.78q_1 + .03p_{111}^1 - .2q_{111}^1 + .04p_{121}^1 - .1q_{121}^1,$$

$$d_{121}^1 = 47 - 1.79p_{121}^1 + 1.41q_{121}^1 - 1.45p_1 + 1.78q_1 + .03p_{111}^1 - .2q_{111}^1 + .04p_{111}^2 - .1q_{111}^2.$$

The **transportation costs** of freight service provider C_1 are:

$$TC_{111}^1 = .5d_{111}^1 + (q_{111}^1)^2 + .045d_{121}^1,$$

$$TC_{111}^2 = .45d_{111}^2 + .54(q_{111}^2)^2 + .005d_{111}^2q_{111}^2,$$

and that of freight service provider C_2 is:

$$TC_{121}^1 = .64d_{121}^1 + .76(q_{121}^1)^2.$$

The **utility** of C_2 is:

$$U_{C_2} = p_{121}^1 d_{121}^1 - TC_{121}^1.$$

$$0 \leq p_{121}^1 \leq 65, \quad 12 \leq q_{121}^1 \leq 100.$$

Example 2: Result

The **equilibrium solution**, computed after 218 iterations, is:

$$\begin{aligned} p_{111}^{1*} &= 45.69, & p_{111}^{2*} &= 45.32, & p_{121}^{1*} &= 44.82, & p_1^* &= 53.91, \\ q_{111}^{1*} &= 31.69, & q_{111}^{2*} &= 41.32, & q_{121}^{1*} &= 41.24, & q_1^* &= 78.43. \end{aligned}$$

The utility of manufacturing firm F_1 is 961.39 and that of freight service providers C_1 and C_2 are 4753.06 and 2208.92, respectively.

The inclusion of an **additional freight service provider** helps to **increase the total demand**. So that, manufacturing firm F_1 **increases his quality level and, consequently, his price**.

Variant of Example 2

The **demand** functions are:

$$d_{111}^1 = 43 - 1.44p_{111}^1 + 1.53q_{111}^1 - 1.82p_1 + 1.21q_1 + .03p_{111}^2 - .2q_{111}^2 + .04p_{121}^1 - .1q_{121}^1,$$

$$d_{111}^2 = 52 - 1.49p_{111}^2 + 1.65q_{111}^2 - 1.82p_1 + 1.21q_1 + .03p_{111}^1 - .2q_{111}^1 + .04p_{121}^1 - .1q_{121}^1,$$

$$d_{121}^1 = 47 - 1.57p_{121}^1 + 1.64q_{121}^1 - 1.82p_1 + 1.21q_1 + .03p_{111}^1 - .2q_{111}^1 + .04p_{111}^2 - .1q_{111}^2.$$

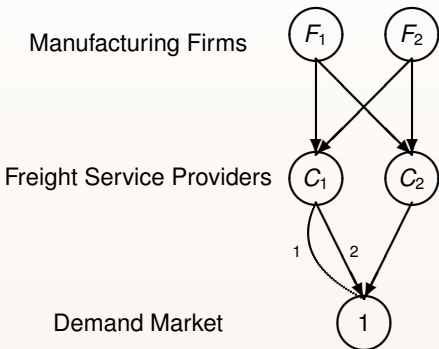
The **equilibrium solution**, computed after 553 iterations, is:

$$p_{111}^{1*} = 8.71, \quad p_{111}^{2*} = 63.17, \quad p_{121}^{1*} = 16.22, \quad p_1^* = 24.80,$$

$$q_{111}^{1*} = 9.00, \quad q_{111}^{2*} = 93.15, \quad q_{121}^{1*} = 16.92, \quad q_1^* = 23.67.$$

Quality levels offered by the freight service providers take on higher values than their prices **as opposed to** a vice versa situation in the case of **Example 2**.

Example 3



Example 3

The **demand** functions for manufacturing firm F_1 are:

$$d_{111}^1 = 43 - 1.62p_{111}^1 + 1.6q_{111}^1 - 1.45p_1 + 1.78q_1 + .08p_2 - .04q_2 + .03p_{111}^2 - .2q_{111}^2 + .04p_{121}^1 - .1q_{121}^1,$$

$$d_{111}^2 = 52 - 1.75p_{111}^2 + 1.21q_{111}^2 - 1.45p_1 + 1.78q_1 + .08p_2 - .04q_2 + .03p_{111}^1 - .2q_{111}^1 + .04p_{121}^1 - .1q_{121}^1,$$

$$d_{121}^1 = 47 - 1.79p_{121}^1 + 1.41q_{121}^1 - 1.45p_1 + 1.78q_1 + .08p_2 - .04q_2 + .03p_{111}^1 - .2q_{111}^1 + .04p_{111}^2 - .1q_{111}^2,$$

and that of manufacturing firm F_2 are:

$$d_{211}^1 = 51 - 1.57p_{211}^1 + 1.26q_{211}^1 - 1.65p_2 + 1.98q_2 + .08p_1 - .04q_1 + .04p_{211}^2 - .1q_{211}^2 + .02p_{221}^1 - .12q_{221}^1,$$

$$d_{211}^2 = 44 - 1.63p_{211}^2 + 1.21q_{211}^2 - 1.65p_2 + 1.98q_2 + .08p_1 - .04q_1 + .04p_{211}^1 - .1q_{211}^1 + .02p_{221}^1 - .12q_{221}^1,$$

$$d_{221}^1 = 56 - 1.46p_{221}^1 + 1.41q_{221}^1 - 1.65p_2 + 1.98q_2 + .08p_1 - .04q_1 + .04p_{211}^1 - .1q_{211}^1 + .02p_{211}^2 - .12q_{211}^2.$$

Example 3

The supply of F_1 is similar to that in Example 2 and that of manufacturing firm F_2 is:

$$s_2 = d_{211}^1 + d_{211}^2 + d_{221}^1.$$

The utility of manufacturing firm F_2 is:

$$U_{F_2} = p_2 s_2 - PC_2,$$

and the price and quality of his product are constrained in the following manner:

$$0 \leq p_2 \leq 95, \quad 8 \leq q_2 \leq 100.$$

The utility of C_1 is:

$$U_{C_1} = p_{111}^1 d_{111}^1 + p_{111}^2 d_{111}^2 + p_{211}^1 d_{211}^1 + p_{211}^2 d_{211}^2 - TC_{111}^1 - TC_{111}^2 - TC_{211}^1 - TC_{211}^2,$$

and that of C_2 is:

$$U_{C_2} = p_{121}^1 d_{121}^1 + p_{221}^1 d_{221}^1 - TC_{121}^1 - TC_{221}^1.$$

Example 3: Result

The [equilibrium solution](#), computed after 231 iterations, is:

$$\begin{array}{llll}
 p_{111}^{1*} = 40.20, & p_{111}^{2*} = 40.72, & p_{121}^{1*} = 39.79, & p_1^* = 48.08, \\
 p_{211}^{1*} = 51.17, & p_{211}^{2*} = 42.88, & p_{221}^{1*} = 69.18, & p_2^* = 50.89, \\
 q_{111}^{1*} = 27.73, & q_{111}^{2*} = 37.76, & q_{121}^{1*} = 36.53, & q_1^* = 66.25, \\
 q_{211}^{1*} = 37.64, & q_{211}^{2*} = 29.42, & q_{221}^{1*} = 63.97. & q_2^* = 75.65.
 \end{array}$$

Due to the [added competition at the manufacturers' level](#), the quality and price of the product manufactured at firm F_1 have declined as compared to Example 2.

Variant of Example 3: Result

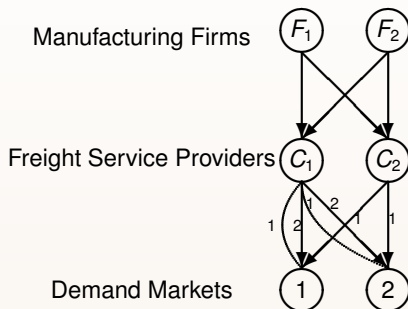
The **equilibrium solution**, computed after 568 iterations, is:

$$\begin{array}{llll}
 p_{111}^{1*} = 8.30, & p_{111}^{2*} = 64.70, & p_{121}^{1*} = 15.54, & p_1^* = 25.02, \\
 p_{211}^{1*} = 28.70, & p_{211}^{2*} = 18.47, & p_{221}^{1*} = 36.15, & p_2^* = 21.38, \\
 q_{111}^{1*} = 9.00, & q_{111}^{2*} = 96.71, & q_{121}^{1*} = 16.16, & q_1^* = 22.71, \\
 q_{211}^{1*} = 28.34, & q_{211}^{2*} = 17.19, & q_{221}^{1*} = 38.55, & q_2^* = 19.24.
 \end{array}$$

At equilibrium, the utilities of manufacturing firms F_1 and F_2 are 2037.45 and 1511.87, and that of freight service providers C_1 and C_2 are 1729.44 and 737.02.

Based on the variant's solution, the utilities of the freight service providers (focus on quality) are lower than the utilities of the manufacturers (focus on price). This is directly connected to the transportation costs which increase in order to ensure high quality

Example 4



We consider **competition** at the manufacturers' level, the freight service providers' level, and between modes of a particular service provider, wherein all these players are competing to satisfy the demands at two different demand markets.

Example 4: Result

The equilibrium solution, after 254 iterations, is:

$$\begin{array}{cccc}
 p_{111}^{1*} = 56.79, & p_{111}^{2*} = 55.45, & p_{112}^{1*} = 72.96, & p_{112}^{2*} = 36.93, \\
 p_{121}^{1*} = 55.19, & p_{122}^{1*} = 53.55, & p_{211}^{1*} = 62.77, & p_{211}^{2*} = 53.28, \\
 p_{212}^{1*} = 72.94, & p_{212}^{2*} = 65.91, & p_{221}^{1*} = 76.15, & p_{222}^{1*} = 83.73, \\
 p_1^* = 63.76, & p_2^* = 64.90, & q_1^* = 100.00, & q_2^* = 100.00, \\
 q_{111}^{1*} = 39.53, & q_{111}^{2*} = 51.20, & q_{112}^{1*} = 74.61, & q_{112}^{2*} = 23.54, \\
 q_{121}^{1*} = 50.93, & q_{122}^{1*} = 51.05, & q_{211}^{1*} = 46.25, & q_{211}^{2*} = 36.72, \\
 q_{212}^{1*} = 76.89, & q_{212}^{2*} = 69.56, & q_{221}^{1*} = 61.18, & q_{222}^{1*} = 94.70.
 \end{array}$$

The **price and quality levels have gone up as well as utilities** for both manufacturers and carriers as compared to Example 3 since there are **two demand markets** to be satisfied now as opposed to one.

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- The discrete-time algorithm, also serving as an approximation to the continuous time trajectories, yields an equilibrium price and quality patterns.
- we then provided solutions to a series of numerical examples - small to large scenarios and their variants.
- In the scenarios, quality was given more importance and in the variants, prices were given more importance.

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