A Supply Chain Network Game Theory Model with Product Differentiation, Outsourcing of Production and Distribution, and Quality and Price Competition

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Background and Motivation

Outsourcing of manufacturing/production has long been noted in supply chain management and it has become prevalent in numerous manufacturing industries.

Background and Motivation

In addition to the increasing volume of outsourcing, the supply chain networks weaving the original manufacturers and the contractors are becoming increasingly complex.

The benefits of outsourcing:

- Cost reduction
- Improving manufacturing efficiency
- Diverting human and natural resources
- Obtaining benefits from supportive government policies

Background and Motivation

Quality issues in outsourced products must be of paramount concern.

- Fake heparin made by an Asian manufacturer led to recalls of drugs in over ten European countries (Payne (2008)), and resulted in the deaths of 81 citizens (Harris (2011)).

- More than 400 peanut butter products were recalled after 8 people died and more than 500 people in 43 states, half of them children, were sickened by salmonella poisoning, the source of which was a plant in Georgia (Harris (2009)).

- The suspension of the license of Pan Pharmaceuticals, the world's fifth largest contract manufacturer of health supplements, due to quality failure, caused costly consequences in terms of product recalls and credibility losses (Allen (2003)).

Background and Motivation

With the increasing volume of outsourcing and the increasing complexity of the supply chain networks associated with outsourcing, it is imperative for firms to be able to rigorously assess not only

- the possible gains due to outsourcing, but also
- the potential costs associated with disrepute (loss of reputation) resulting from the possible quality degradation due to outsourcing.

Literature Review

- Kaya, O., 2011. Outsourcing vs. in-house production: A comparison of supply chain contracts with effort dependent demand. *Omega* 39, 168-178.
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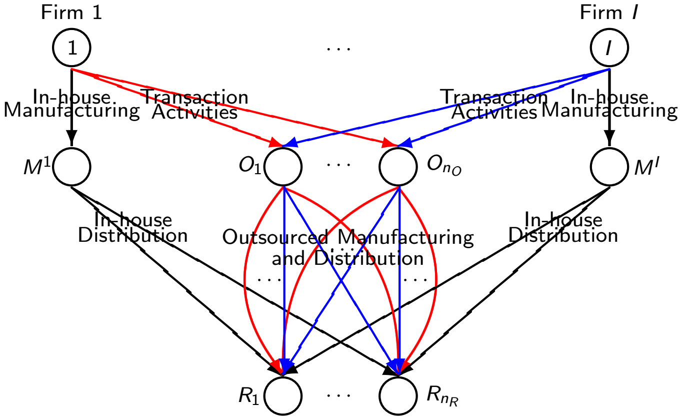
- Xiao, T., Xia, Y., Zhang, G. P., 2014. Strategic outsourcing decisions for manufacturers competing in product quality. *IIE Transactions*, 46(4), 313-329.
- Nagurney, A., Li, D., Nagurney, L. S., 2013. Pharmaceutical supply chain networks with outsourcing under price and quality competition. *International Transactions in Operational Research*, 20(6), 859-888.

Overview

- This model captures the behaviors of the firms and their potential contractors in supply chain networks with possible outsourcing of production and distribution.
- It provides each original firm with the equilibrium in-house quality level and the equilibrium make-or-buy and contractor-selection policy with the demand for its product being satisfied in multiple demand markets.
- The firms, who produce differentiated but substitutable products, seek to minimize their total cost and their weighted disrepute. They compete in Cournot fashion in in-house production quantities and quality.
- It provides each contractor its optimal quality and pricing strategy.
- The contractors compete by determining the prices that they charge the firms and the quality levels of their products to maximize their total profits.

Supply Chain Network Topology



Quality Levels

Quality level of firm i's product produced by contractor j, q_{ij}

$$0 \leq q_{ij} \leq q^U, \quad i = 1, \ldots, I; j = 1, \ldots, n_O, \tag{1}$$

where q^U is the value representing perfect quality.

Quality level of firm i's product produced by itself, q_i

$$0 \leq q_i \leq q^U, \quad i = 1, \ldots, I, \tag{2}$$

Quality Levels

Average quality level of the product of firm i, q_i'

$$q_i'(Q_i, q_i, q_i^2) = \frac{\sum_{k=1}^{n_R} \sum_{j=2}^{n} Q_{ijk} q_{i,j-1} + \sum_{k=1}^{n_R} Q_{i1k} q_i}{\sum_{k=1}^{n_R} d_{ik}}, \quad i = 1, \ldots, I. \qquad (3)$$

Q_{ijk}: the amount of firm i's product produced at manufacturing plant j, whether in-house or contracted, and delivered to demand market k, where $j = 1, \ldots, n$, $n = n_O + 1$.

d_{ik}: the demand for firm i's product at demand market k; $k = 1, \ldots, n_R$.

Q_i: the vector of firm i's product quantities, both in-house and outsourced.

q_i^2: the vector of the quality levels of firm i's outsourced products.

The Behavior of the Firms

The total utility maximization objective of firm i, U_i^1

$$\text{Maximize}_{Q_i,q_i} \quad U_i^1 = -f_i(Q^1,q^1) - c_i(q^1) - \sum_{k=1}^{n_R} c_{ik}(Q^1,q^1) - \sum_{j=1}^{n_O}\sum_{k=1}^{n_R} \pi_{ijk}^* Q_{i,1+j,k}$$

$$-\sum_{j=1}^{n_O} tc_{ij}(\sum_{k=1}^{n_R} Q_{i,1+j,k}) - \omega_i dc_i(q_i'(Q_i,q_i,q_i^{2^*})) \quad (4)$$

subject to:

$$\sum_{j=1}^{n} Q_{ijk} = d_{ik}, \quad i=1,\ldots,I;\, k=1,\ldots,n_R, \quad (5)$$

$$Q_{ijk} \geq 0, \quad i=1,\ldots,I;\, j=1,\ldots,n;\, k=1,\ldots,n_R, \quad (6)$$

and (2).

"*" denotes the equilibrium solution.

All the cost functions in (4) are continuous, twice continuously differentiable, and convex.

Definition: A Cournot-Nash Equilibrium

We define the feasible set K_i as
$K_i = \{(Q_i, q_i) | Q_i \in R_+^{nn_R}$ with (5) satisfied and q_i satisfying (2)$\}$.
All K_i; $i = 1, \ldots, I$, are closed and convex. We also define the feasible set $\mathcal{K}^1 \equiv \Pi_{i=1}^I K_i$.

Definition 1

An in-house and outsourced product flow pattern and in-house quality level $(Q^*, q^{1^*}) \in \mathcal{K}^1$ is said to constitute a Cournot-Nash equilibrium if for each firm i; $i = 1, \ldots, I$,

$$U_i^1(Q_i^*, \hat{Q}_i^*, q_i^*, \hat{q}_i^*, q^{2^*}, \pi_i^*) \geq U_i^1(Q_i, \hat{Q}_i^*, q_i, \hat{q}_i^*, q^{2^*}, \pi_i^*), \quad \forall (Q_i, q_i) \in K_i, \quad (7)$$

where

$$\hat{Q}_i^* \equiv (Q_1^*, \ldots, Q_{i-1}^*, Q_{i+1}^*, \ldots, Q_I^*),$$
$$\hat{q}_i^* \equiv (q_1^*, \ldots, q_{i-1}^*, q_{i+1}^*, \ldots, q_I^*).$$

π_i: the vector of all the prices charged to firm i.

Variational Inequality Formulation

Theorem 1

Assume that, for each firm i; $i = 1, \ldots, I$, the utility function $U_i^1(Q, q^1, q^{2^*}, \pi_i^*)$ is concave with respect to its variables Q_i and q_i, and is continuous and twice continuously differentiable. Then $(Q^*, q^{1^*}) \in \mathcal{K}^1$ is a Counot-Nash equilibrium according to Definition 1 if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^{I}\sum_{h=1}^{n}\sum_{m=1}^{n_R}\frac{\partial U_i^1(Q^*, q^{1^*}, q^{2^*}, \pi_i^*)}{\partial Q_{ihm}} \times (Q_{ihm} - Q_{ihm}^*)$$

$$-\sum_{i=1}^{I}\frac{\partial U_i^1(Q^*, q^{1^*}, q^{2^*}, \pi_i^*)}{\partial q_i} \times (q_i - q_i^*) \geq 0,$$

$$\forall (Q, q^1) \in \mathcal{K}^1, \tag{8}$$

where Q is the vector of all the product quantities,

Variational Inequality Formulation

with notice that: for $h = 1$; $i = 1, \ldots, I$; $m = 1, \ldots, n_R$:

$$-\frac{\partial U_i^1}{\partial Q_{ihm}} = \left[\frac{\partial f_i}{\partial Q_{ihm}} + \sum_{k=1}^{n_R} \frac{\partial c_{ik}}{\partial Q_{ihm}} + \omega_i \frac{\partial dc_i}{\partial q_i'} \frac{\partial q_i'}{\partial Q_{ihm}} \right],$$

for $h = 2, \ldots, n$; $i = 1, \ldots, I$; $m = 1, \ldots, n_R$:

$$-\frac{\partial U_i^1}{\partial Q_{ihm}} = \left[\pi^*_{i,h-1,m} + \frac{\partial tc_{i,h-1}}{\partial Q_{ihm}} + \omega_i \frac{\partial dc_i}{\partial q_i'} \frac{\partial q_i'}{\partial Q_{ihm}} \right],$$

for $i = 1, \ldots, I$:

$$-\frac{\partial U_i^1}{\partial q_i} = \left[\frac{\partial f_i}{\partial q_i} + \frac{\partial c_i}{\partial q_i} + \sum_{k=1}^{n_R} \frac{\partial c_{ik}}{\partial q_i} + \omega_i \frac{\partial dc_i}{\partial q_i'} \frac{\partial q_i'}{\partial q_i} \right].$$

The Behavior of the Contractors

The total utility maximization objective of contractor j, U_j^2

$$\text{Maximize}_{q_j,\pi_j} \quad U_j^2 = \sum_{k=1}^{n_R}\sum_{i=1}^{I} \pi_{ijk} Q^*_{i,1+j,k} - \sum_{k=1}^{n_R}\sum_{i=1}^{I} sc_{ijk}(Q^{2^*}, q^2) - \hat{c}_j(q^2)$$

$$-\sum_{k=1}^{n_R}\sum_{i=1}^{I} oc_{ijk}(\pi) \quad (9)$$

subject to:

$$\pi_{ijk} \geq 0, \quad j = 1, \ldots, n_O; k = 1, \ldots, n_R, \quad (10)$$

and (1) for each j.

The cost functions in each contractor's utility function are continuous, twice continuously differentiable, and convex.

q_j: the vector of all the quality levels of contractor j.

π_j: the vector of all the prices charged by contractor j.

Definition: A Nash-Bertrand Equilibrium

The feasible sets are defined as
$K_j \equiv \{(q_j, \pi_j) | q_j$ satisfies (1) and π_j satisfies (10) for $j\}$, $\mathcal{K}^2 \equiv \Pi_{j=1}^{n_O} K_j$, and $\mathcal{K} \equiv \mathcal{K}^1 \times \mathcal{K}^2$. All the above-defined feasible sets are convex.

Definition 2

A quality level and price pattern $(q^{2^*}, \pi^*) \in \mathcal{K}^2$ is said to constitute a Bertrand-Nash equilibrium if for each contractor j; $j = 1, \ldots, n_O$,

$$U_j^2(Q^{2^*}, q_j^*, \hat{q}_j^*, \pi_j^*, \hat{\pi}_j^*) \geq U_j^2(Q^{2^*}, q_j, \hat{q}_j^*, \pi_j, \hat{\pi}_j^*), \quad \forall (q_j, \pi_j) \in K_j, \tag{11}$$

where

$$\hat{q}_j^* \equiv (q_1^*, \ldots, q_{j-1}^*, q_{j+1}^*, \ldots, q_{n_O}^*),$$

$$\hat{\pi}_j^* \equiv (\pi_1^*, \ldots, \pi_{j-1}^*, \pi_{j+1}^*, \ldots, \pi_{n_O}^*).$$

Variational Inequality Formulation

Theorem 2

Assume that, for each contractor j; $j = 1, \ldots, n_O$, the profit function $U_j^2(Q^{2^*}, q^2, \pi)$ is concave with respect to the variables π_j and q_j, and is continuous and twice continuously differentiable. Then $(q^{2^*}, \pi^*) \in \mathcal{K}^2$ is a Bertrand-Nash equilibrium according to Definition 2 if and only if it satisfies the variational inequality:

$$
-\sum_{l=1}^{I}\sum_{j=1}^{n_O} \frac{\partial U_j^2(Q^{2^*}, q^{2^*}, \pi^*)}{\partial q_{lj}} \times (q_{lj} - q_{lj}^*)
$$

$$
-\sum_{l=1}^{I}\sum_{j=1}^{n_O}\sum_{k=1}^{n_R} \frac{\partial U_j^2(Q^{2^*}, q^{2^*}, \pi^*)}{\partial \pi_{ljk}} \times (\pi_{ljk} - \pi_{ljk}^*) \geq 0,
$$

$$
\forall (q^2, \pi) \in \mathcal{K}^2. \tag{12}
$$

Variational Inequality Formulation

with notice that: for $j = 1, \ldots, n_O$*;* $l = 1, \ldots, I$*:*

$$-\frac{\partial U_j^2}{\partial q_{lj}} = \sum_{i=1}^{I} \sum_{k=1}^{n_R} \frac{\partial sc_{ijk}}{\partial q_{lj}} + \frac{\partial \hat{c}_j}{\partial q_{lj}},$$

and for $j = 1, \ldots, n_O$; $l = 1, \ldots, I$; $k = 1, \ldots, n_R$*:*

$$-\frac{\partial U_j^2}{\partial \pi_{ljk}} = \sum_{l=1}^{I} \sum_{k=1}^{n_R} \frac{\partial oc_{ljk}}{\partial \pi_{ljk}} - Q^*_{l,1+j,k}.$$

Definition 2

The equilibrium state of the supply chain network with product differentiation, outsourcing, and quality and price competition is one where both variational inequalities (8) and (12) hold simultaneously.

Theorem 3

The equilibrium conditions governing the supply chain network model with product differentiation, outsourcing, and quality competition are equivalent to the solution of the variational inequality problem: determine $(Q^*, q^{1^*}, q^{2^*}, \pi^*) \in \mathcal{K}$, *such that:*

$$-\sum_{i=1}^{I}\sum_{h=1}^{n}\sum_{m=1}^{n_R}\frac{\partial U_i^1(Q^*, q^{1^*}, q^{2^*}, \pi_i^*)}{\partial Q_{ihm}} \times (Q_{ihm} - Q_{ihm}^*) - \sum_{i=1}^{I}\frac{\partial U_i^1(Q^*, q^{1^*}, q^{2^*}, \pi_i^*)}{\partial q_i}$$

$$\times (q_i - q_i^*) - \sum_{l=1}^{I}\sum_{j=1}^{n_O}\frac{\partial U_j^2(Q^{2^*}, q^{2^*}, \pi^*)}{\partial q_{lj}} \times (q_{lj} - q_{lj}^*)$$

$$-\sum_{l=1}^{I}\sum_{j=1}^{n_O}\sum_{k=1}^{n_R}\frac{\partial U_j^2(Q^{2^*}, q^{2*}, \pi^*)}{\partial \pi_{ljk}} \times (\pi_{ljk} - \pi_{ljk}^*) \geq 0,$$

$$\forall (Q, q^1, q^2, \pi) \in \mathcal{K}. \tag{13}$$

Standard form VI

Determine $X^* \in \mathcal{K}$ *such that:*

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{14}$$

where $N = Inn_R + I + In_O + In_On_R$.

If $\mathcal{K} \equiv \mathcal{K}$, and the column vectors $X \equiv (Q, q^1, q^2, \pi)$, $F(X) \equiv (F_1(X), F_2(X), F_3(X), F_4(X))$, such that:

$$F_1(X) = \left[\frac{\partial U_i^1(Q, q^1, q^2, \pi_i)}{\partial Q_{ihm}};\ h = 1, \ldots, n; i = 1, \ldots, I; m = 1, \ldots, n_R \right],$$

$$F_2(X) = \left[\frac{\partial U_i^1(Q, q^1, q^2, \pi_i)}{\partial q_i};\ i = 1, \ldots, I \right],$$

$$F_3(X) = \left[\frac{\partial U_j^2(Q^2, q^2, \pi)}{\partial q_{lj}};\ l = 1, \ldots, I; j = 1, \ldots, n_O \right],$$

$$F_4(X) = \left[\frac{\partial U_j^2(Q^2, q^2, \pi)}{\partial \pi_{ljk}};\ l = 1, \ldots, I; j = 1, \ldots, n_O; k = 1, \ldots, n_R \right],$$

The Algorithm - The Euler Method

Iteration τ of the Euler method (see also Nagurney and Zhang (1996))

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau} F(X^{\tau})), \tag{15}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem (14).

For convergence of the general iterative scheme, which induces the Euler method, the sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \to 0$, as $\tau \to \infty$.

Explicit Formulae for Quality Levels and Contractor Prices

$$q_i^{\tau+1} = \min\{q^U, \max\{0, q_i^{\tau} - a_\tau(\frac{\partial f_i(Q^{1^\tau}, q^{1^\tau})}{\partial q_i} + \frac{\partial c_i(q^{1^\tau})}{\partial q_i} + \sum_{k=1}^{n_R} \frac{\partial c_{ik}(Q^{1^\tau}, q^{1^\tau})}{\partial q_i}$$

$$+\omega_i \frac{\partial dc_i(q_i'^{\tau})}{\partial q_i'} \frac{\partial q_i'(Q_i^{\tau}, q_i^{\tau}, q_i^{2^\tau})}{\partial q_i})\}\}; \quad (16)$$

$$q_{lj}^{\tau+1} = \min\{q^U, \max\{0, q_{lj}^{\tau} - a_\tau(\sum_{i=1}^{I}\sum_{k=1}^{n_R} \frac{\partial sc_{ijk}(Q^{2^\tau}, q^{2^\tau})}{\partial q_{lj}} + \frac{\partial \hat{c}_j(q^{2^\tau})}{\partial q_{lj}})\}\}. \quad (17)$$

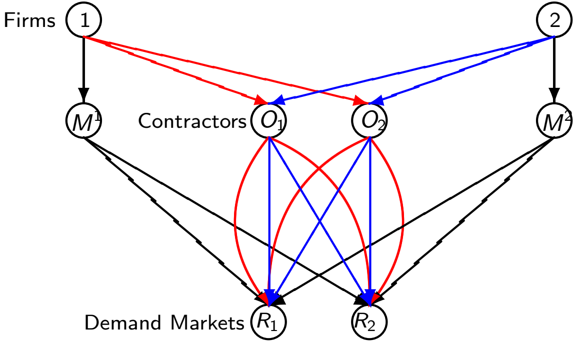
$$\pi_{ljk}^{\tau+1} = \max\{0, \pi_{ljk}^{\tau} - a_\tau(\sum_{l=1}^{I}\sum_{k=1}^{n_R} \frac{\partial oc_{ljk}(\pi^\tau)}{\partial \pi_{ljk}} - Q_{l,1+j,k}^{\tau})\}. \quad (18)$$

The Algorithm - The Euler Method

The strictly convexquadratic programming problem

$$X^{\tau+1} = \text{Minimize}_{X \in \mathcal{K}} \quad \frac{1}{2}\langle X, X\rangle - \langle X^{\tau} - a_{\tau} F(X^{\tau}), X\rangle. \qquad (19)$$

In order to obtain the values of the product flows at each iteration, we apply the exact equilibration algorithm, originated by Dafermos and Sparrow (1969) and applied to many different applications of networks with special structure (cf. Nagurney (1999) and Nagurney and Zhang (1996)).


Firms
1
2
M^1
Contractors
O_1
O_2
M^2
Demand Markets
R_1
R_2

Example 1

The production cost functions at the in-house manufacturing plants are:

$$f_1(Q^1, q^1) = (Q_{111} + Q_{112})^2 + 1.5(Q_{111} + Q_{112}) + 2(Q_{211} + Q_{212}) + .2q_1(Q_{111} + Q_{112}),$$

$$f_2(Q^1, q^1) = 2(Q_{211} + Q_{212})^2 + .5(Q_{211} + Q_{212}) + (Q_{111} + Q_{112}) + .1q_2(Q_{211} + Q_{212}).$$

The total transportation cost functions for the in-house manufactured products are:

$$c_{11}(Q_{111}) = Q_{111}^2 + 5Q_{111}, \quad c_{12}(Q_{112}) = 2.5Q_{112}^2 + 10Q_{112},$$

$$c_{21}(Q_{211}) = .5Q_{211}^2 + 3Q_{211}, \quad c_{22}(Q_{212}) = 2Q_{212}^2 + 5Q_{212}.$$

The in-house total quality cost functions for the two original firms are given by:

$$c_1(q_1) = (q_1 - 80)^2 + 10, \quad c_2(q_2) = (q_2 - 85)^2 + 20.$$

The transaction cost functions are:

$$tc_{11}(Q_{121} + Q_{122}) = .5(Q_{121} + Q_{122})^2 + 2(Q_{121} + Q_{122}) + 100,$$

$$tc_{12}(Q_{131} + Q_{132}) = .7(Q_{131} + Q_{132})^2 + .5(Q_{131} + Q_{132}) + 150,$$

$$tc_{21}(Q_{221} + Q_{222}) = .5(Q_{221} + Q_{222})^2 + 3(Q_{221} + Q_{222}) + 75,$$

$$tc_{22}(Q_{221} + Q_{222}) = .75(Q_{231} + Q_{232})^2 + .5(Q_{231} + Q_{232}) + 100.$$

The contractors' total cost functions of production and distribution are:

$$sc_{111}(Q_{121}, q_{11}) = .5Q_{121}q_{11}, \quad sc_{112}(Q_{122}, q_{11}) = .5Q_{122}q_{11},$$

$$sc_{121}(Q_{131}, q_{12}) = .5Q_{131}q_{12}, \quad sc_{122}(Q_{132}, q_{12}) = .5Q_{132}q_{12},$$

$$sc_{211}(Q_{221}, q_{21}) = .3Q_{221}q_{21}, \quad sc_{212}(Q_{222}, q_{21}) = .3Q_{222}q_{21},$$

$$sc_{221}(Q_{231}, q_{22}) = .25Q_{231}q_{22}, \quad sc_{222}(Q_{232}, q_{22}) = .25Q_{232}q_{22}.$$

Example 1

The total quality cost functions of the contractors are:

$$\hat{c}_1(q_{11}, q_{21}) = (q_{11} - 75)^2 + (q_{21} - 75)^2 + 15,$$

$$\hat{c}_2(q_{12}, q_{22}) = 1.5(q_{12} - 75)^2 + 1.5(q_{22} - 75)^2 + 20.$$

The contractors' opportunity cost functions are:

$$oc_{111}(\pi_{111}) = (\pi_{111} - 10)^2, \quad oc_{121}(\pi_{121}) = .5(\pi_{121} - 5)^2,$$

$$oc_{112}(\pi_{112}) = .5(\pi_{112} - 5)^2, \quad oc_{122}(\pi_{122}) = (\pi_{122} - 15)^2,$$

$$oc_{211}(\pi_{211}) = 2(\pi_{211} - 20)^2, \quad oc_{221}(\pi_{221}) = .5(\pi_{221} - 5)^2,$$

$$oc_{212}(\pi_{212}) = .5(\pi_{212} - 5)^2, \quad oc_{222}(\pi_{222}) = (\pi_{222} - 15)^2.$$

The original firms' disrepute cost functions are:

$$dc_1(q_1') = 100 - q_1', \quad dc_2(q_2') = 100 - q_2',$$

where

$$q_1' = \frac{Q_{121}q_{11} + Q_{131}q_{12} + Q_{111}q_1 + Q_{122}q_{11} + Q_{132}q_{12} + Q_{112}q_1}{d_{11} + d_{12}},$$

and

$$q_2' = \frac{Q_{221}q_{21} + Q_{231}q_{22} + Q_{211}q_2 + Q_{222}q_{21} + Q_{232}q_{22} + Q_{212}q_2}{d_{21} + d_{22}}.$$

ω_1 and ω_2 are 1. q^U is 100.

Example 1

The Euler method converges in 255 iterations and yields the following equilibrium solution.
The computed product flows are:

$$Q^*_{111} = 13.64, \quad Q^*_{121} = 26.87, \quad Q^*_{131} = 9.49, \quad Q^*_{112} = 9.34, \quad Q^*_{122} = 42.85,$$

$$Q^*_{132} = 47.81, \quad Q^*_{211} = 16.54, \quad Q^*_{221} = 47.31, \quad Q^*_{231} = 11.16, \quad Q^*_{212} = 12.65,$$

$$Q^*_{222} = 62.90, \quad Q^*_{232} = 74.45.$$

The computed quality levels of the original firms and the contractors are:

$$q^*_1 = 77.78, \quad q^*_2 = 83.61, \quad q^*_{11} = 57.57, \quad q^*_{12} = 65.45,$$

$$q^*_{21} = 58.47, \quad q^*_{22} = 67.87.$$

The equilibrium prices are:

$$\pi^*_{111} = 23.44, \quad \pi^*_{112} = 47.85, \quad \pi^*_{121} = 14.49, \quad \pi^*_{122} = 38.91,$$

$$\pi^*_{211} = 31.83, \quad \pi^*_{212} = 67.90, \quad \pi^*_{221} = 16.16, \quad \pi^*_{222} = 52.23.$$

The total costs of the original firms' are, respectively, 11,419.90 and 24,573.94, with their incurred disrepute costs being 36.32 and 34.69. The profits of the contractors are 567.84 and 440.92. The values of q'_1 and q'_2 are, respectively, 63.68 and 65.31.

Example 1 - Sensitivity Analysis

We conduct sensitivity analysis by varying the weights that the firms impose on the disrepute, ω, which is the vector of ω_i; $i = 1, 2$, with
$\omega = (0, 0), (1000, 1000), (2000, 2000), (3000, 3000), (4000, 4000), (5000, 5000)$.

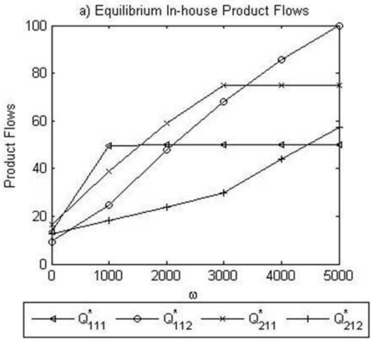


Figure: Equilibrium In-house Product Flows as ω Increases for Example 1

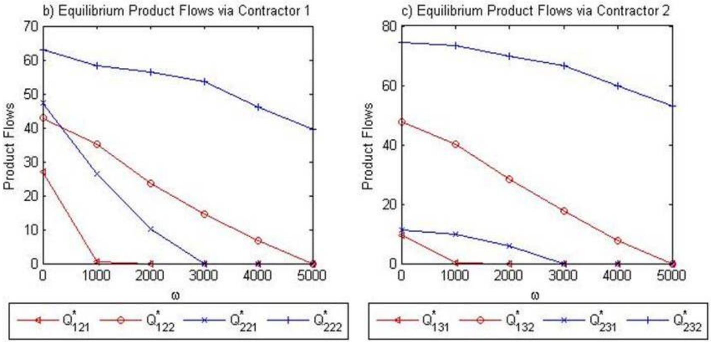


Figure: Equilibrium Outsourced Product Flows as ω Increases for Example 1

Example 1 - Sensitivity Analysis

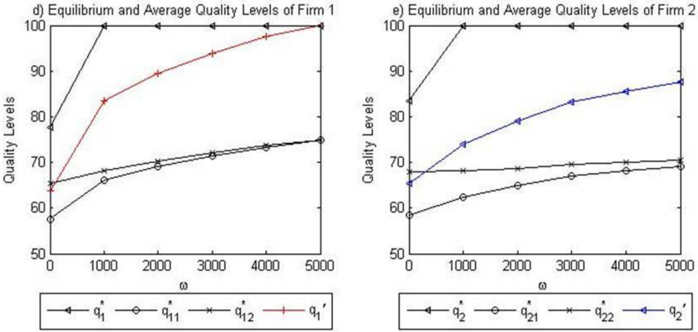


Figure: Equilibrium and Average Quality Levels as ω Increases for Example 1

Example 1 - Sensitivity Analysis

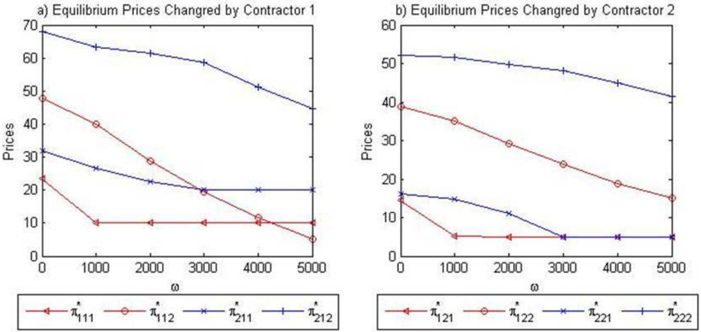


Figure: Equilibrium Prices as ω Increases for Example 1

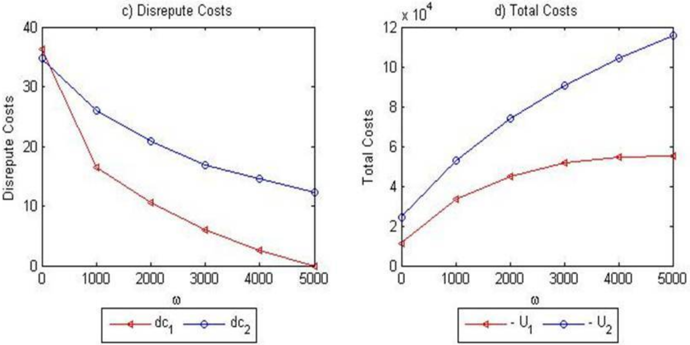


Figure: The Disrepute Costs and Total Costs of the Firms as ω Increases for Example 1

Example 2

In Example 2, both firms consider quality levels as variables affecting their in-house transportation costs. The transportation cost functions of the original firms, hence, now depend on in-house quality levels as follows:

$$c_{11}(Q_{111}, q_1) = Q_{111}^2 + 1.5Q_{111}q_1, \quad c_{12}(Q_{112}, q_1) = 2.5Q_{112}^2 + 2Q_{112}q_1,$$

$$c_{21}(Q_{211}, q_2) = .5Q_{211}^2 + 3Q_{211}q_2, \quad c_{22}(Q_{212}, q_2) = 2Q_{212}^2 + 2Q_{212}q_2.$$

The remaining data are identical to those in Example 1.

Example 2

The Euler method converges in 298 iterations and yields the following equilibrium solution.
The computed product flows are:

$$Q^*_{111} = 0.00, \quad Q^*_{121} = 36.42, \quad Q^*_{131} = 13.58, \quad Q^*_{112} = 0.00, \quad Q^*_{122} = 46.42,$$

$$Q^*_{132} = 53.58, \quad Q^*_{211} = 0.00, \quad Q^*_{221} = 60.13, \quad Q^*_{231} = 14.87, \quad Q^*_{212} = 3.83,$$

$$Q^*_{222} = 65.50, \quad Q^*_{232} = 80.68.$$

The computed quality levels of the original firms and the contractors are:

$$q^*_1 = 80, \quad q^*_2 = 80.90, \quad q^*_{11} = 54.29, \quad q^*_{12} = 63.81, \quad q^*_{21} = 56.16, \quad q^*_{22} = 67.04.$$

The equilibrium prices are:

$$\pi^*_{111} = 28.21, \quad \pi^*_{112} = 51.42, \quad \pi^*_{121} = 18.58, \quad \pi^*_{122} = 41.79,$$

$$\pi^*_{211} = 35.03, \quad \pi^*_{212} = 70.50, \quad \pi^*_{221} = 19.87, \quad \pi^*_{222} = 55.34.$$

The total costs of the original firms are, respectively, 13,002.64 and 27,607.44, with incurred disrepute costs of 41.45 and 38.80. The profits of the contractors are, respectively, 967.96 and 656.78. The average quality levels of the original firms, q'_1 and q'_2, are 58.55 and 61.20.

Example 2 - Sensitivity Analysis

We also conduct sensitivity analysis by varying the weights associated with the disrepute, ω, for
$\omega = (0, 0), (1000, 1000), (2000, 2000), (3000, 3000), (4000, 4000), (5000, 5000)$.

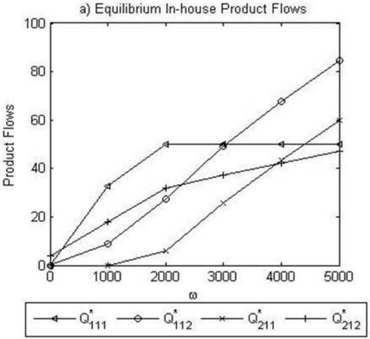


Figure: Equilibrium In-house Product Flows as ω Increases for Example 2

Example 2 - Sensitivity Analysis

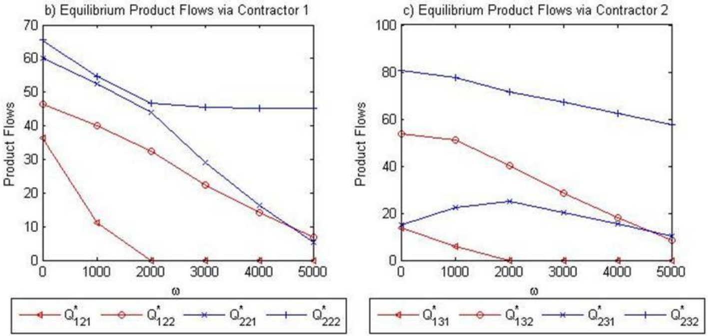


Figure: Equilibrium Outsourced Product Flows as ω Increases for Example 2

Example 2 - Sensitivity Analysis

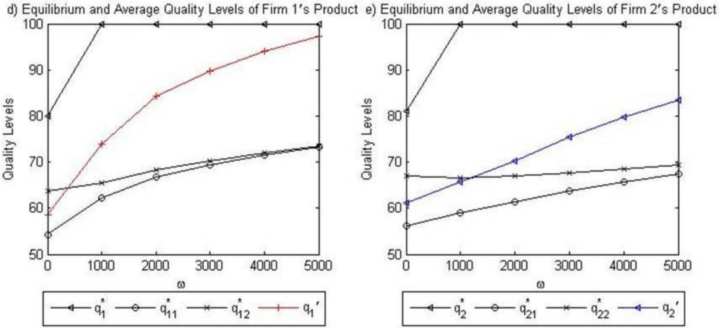


Figure: Equilibrium and Average Quality Levels as ω Increases for Example 2

Example 2 - Sensitivity Analysis

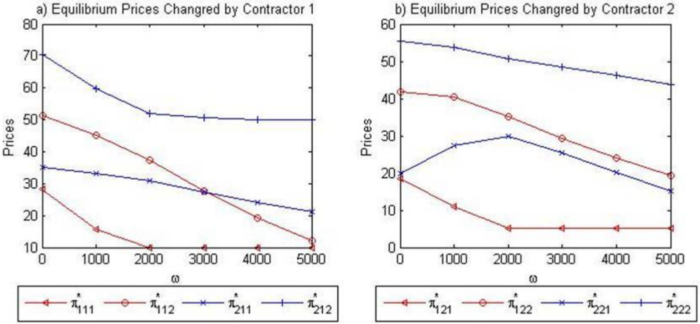


Figure: Equilibrium Prices as ω Increases for Example 2

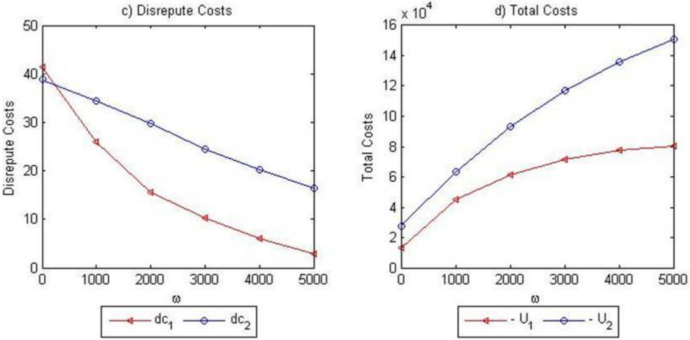


Figure: The Disrepute Costs and Total Costs of the Firms as ω Increases for Example 2

Example 3

In Example 3, we consider the scenario that the competition between the firms are getting more intense.

The total in-house transportation cost functions of the two firms now become:

$$c_{11}(Q_{111}, Q_{211}, q_1) = Q_{111}^2 + 1.5Q_{111}q_1 + 7Q_{211},$$

$$c_{12}(Q_{112}, Q_{212}.q_1) = 2.5Q_{112}^2 + 2Q_{112}q_1 + 10Q_{212},$$

$$c_{21}(Q_{211}, Q_{111}, q_2) = .5Q_{211}^2 + 3Q_{211}q_2 + 8Q_{111},$$

$$c_{22}(Q_{212}, Q_{112}, q_2) = 2Q_{212}^2 + 2Q_{212}q_2 + 10Q_{112}.$$

The remaining data are identical to those in Example 2.

Example 3

The total costs of firm 1 and firm 2 associated with different ω values are displayed. The total cost of firm 1 increases monotonically, whether ω_1 or ω_2 increases.

Table: Total Costs of Firm 1 with Different Sets of ω_1 and ω_2

ω	$\omega_1 = 0$	$\omega_1 = 1000$	$\omega_1 = 2000$	$\omega_1 = 3000$	$\omega_1 = 4000$	$\omega_1 = 5000$
$\omega_2 = 0$	12,999.09	45,135.09	61,322.22	71,463.36	77,437.89	80,462.63
$\omega_2 = 1000$	13,218.71	45,348.05	61,535.18	71,676.32	77,650.85	80,675.60
$\omega_2 = 2000$	13,425.67	45,571.40	61,758.53	71,899.67	77,874.20	80,898.94
$\omega_2 = 3000$	13,666.29	45,812.52	61,999.65	72,140.79	78,115.32	81,114.01
$\omega_2 = 4000$	14,091.85	46,034.08	62,221.20	72,362.34	78,336.88	81,361.62
$\omega_2 = 5000$	14,091.85	46,239.00	62,426.12	72,567.26	78,541.80	81,566.54

Table: Total Costs of Firm 2 with Different Sets of ω_1 and ω_2

ω	$\omega_1 = 0$	$\omega_1 = 1000$	$\omega_1 = 2000$	$\omega_1 = 3000$	$\omega_1 = 4000$	$\omega_1 = 5000$
$\omega_2 = 0$	27,585.65	28,203.96	28,561.15	28,798.10	29,005.24	29,187.92
$\omega_2 = 1000$	62,896.33	63,626.00	63,983.19	64,220.14	64,427.28	64,609.96
$\omega_2 = 2000$	92,753.88	93,312.11	93,669.30	93,906.25	94,113.39	94,296.07
$\omega_2 = 3000$	116,378.40	116,981.94	117,339.13	117,576.08	117,783.22	117,965.90
$\omega_2 = 4000$	135,237.43	135,872.91	136,230.10	136,467.05	136,674.19	136,856.87
$\omega_2 = 5000$	150,231.01	150,886.51	151,243.69	151,480.65	151,687.79	151,870.47

Summary and Conclusions

- We developed a supply chain network game theory model with product differentiation, outsourcing, quantity and quality competition among multiple firms, and price and quality competition among their potential contractors.
- We modeled the impacts of quality on in-house and outsourced production and transportation and on the reputation of each firm.
- This model provides the optimal make-or-buy as well as contractor selection decisions and the optimal in-house quality level for each original firm.
- It also provides the optimal quality and pricing strategy for each contractor.
- We provided solutions to a series of numerical examples, accompanied by sensitivity analysis.

Thank you!

- For more information, please visit http://supernet.isenberg.umass.edu.