Freight Service Provision for Disaster Relief: A Competitive Network Model with Computations

Anna Nagurney

Isenberg School of Management
University of Massachusetts
Amherst, Massachusetts 01003

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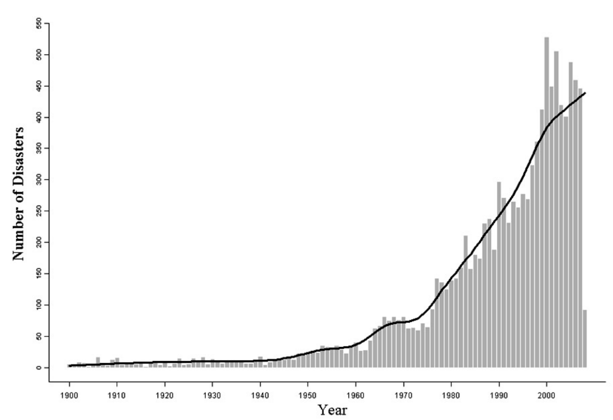
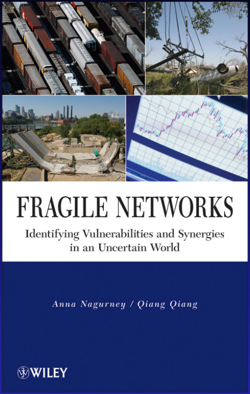
Background and Motivation

Disasters have brought an unprecedented impact on human lives in the 21st century and the number of disasters is growing. From January to October 2005, *an estimated 97,490 people were killed in disasters globally; 88,117 of them because of natural disasters*.



Frequency of disasters [Source: Emergency Events Database (2008)]

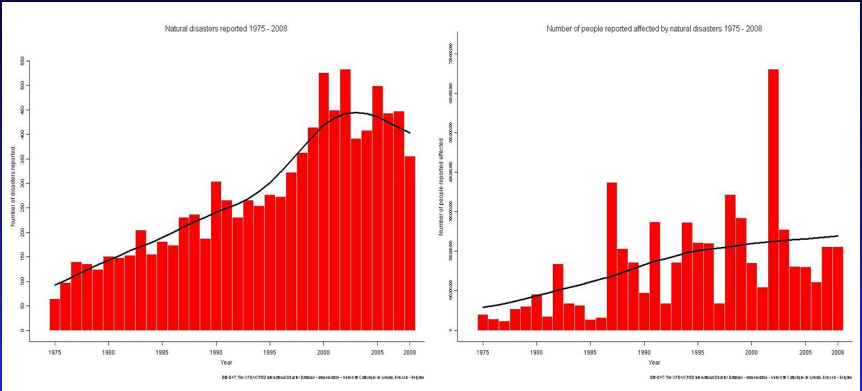
As noted in Nagurney and Qiang (2009), the number of disasters is growing as well as the number of people affected by disasters.



Hence, the development of appropriate analytical tools that can assist humanitarian organizations and nongovernmental organizations as well as governments in the various disaster management phases has become a challenge to both researchers and practitioners.

Disasters have a catastrophic effect on human lives and a region's or even a nation's resources.

Natural Disasters (1975–2008)



Recent disasters have vividly demonstrated the importance and vulnerability of our transportation and critical infrastructure systems

- The biggest blackout in North America, August 14, 2003;
- Two significant power outages in September 2003 – one in the UK and the other in Italy and Switzerland;
- The Indonesian tsunami (and earthquake), December 26, 2004;
- Hurricane Katrina, August 23, 2005;
- The Minneapolis I35 Bridge collapse, August 1, 2007;
- The Sichuan earthquake on May 12, 2008;
- The Haiti earthquake that struck on January 12, 2010 and the Chilean one on February 27, 2010;
- The triple disaster in Japan on March 11, 2011;
- Superstorm Sandy, October 29, 2012.

Hurricane Katrina in 2005


Hurricane Katrina has been called an *"American tragedy,"* in which essential services failed completely.

The Haitian and Chilean Earthquakes

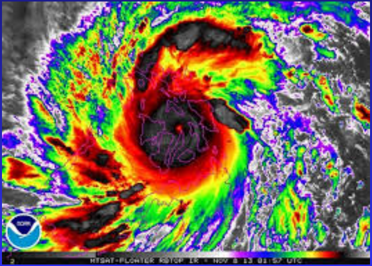
The Triple Disaster in Japan on March 11, 2011

Superstorm Sandy and Power Outages

Manhattan without power October 30, 2012 as a result of the devastation wrought by Superstorm Sandy.

Haiyan Typhoon in the Philippines in 2013

Typhoon Haiyan was a very powerful tropical cyclone that devastated portions of Southeast Asia, especially the Philippines, on November 8, 2013. It is the deadliest Philippine typhoon on record, killing at least 6,190 people in that country alone. Haiyan was also the strongest storm recorded at landfall. As of January 2014, bodies were still being found. The overall economic losses from Typhoon Haiyan totaled $10 billion.



Nepal Earthquake in 2015

The 7.8 magnitude earthquake that struck Nepal on April 25, 2015, and the aftershocks that followed, killed nearly 9,000 people and injured 22,000 others. This disaster also pushed about 700,000 people below the poverty line in the Himalayan nation, which is one of the world's poorest. About 500,000 homes were made unlivable by the quakes, leaving about three million people homeless. Much infrastructure was also badly damaged and 1/3 of the healthcare facilities devastated. According to *The Wall Street Journal*, Nepal needs $6.66 billion to rebuild.

The Ebola Crisis in West Africa

According to bbc.com and the World Health Organization, more than one year from the first confirmed case recorded on March 23, 2014, at least 11,178 people have been reported as having died from Ebola in six countries; Liberia, Guinea, Sierra Leone, Nigeria, the US and Mali.

Ms. Debbie Wilson of Doctors Without Borders

On February 4, 2015, the students in my Humanitarian Logistics and Healthcare class at the Isenberg School heard Debbie Wilson, a nurse, who has worked with Doctors Without Borders, speak on her 6 weeks of experiences battling Ebola in Liberia in September and October 2014.

Without transportation, no needed supplies can be delivered to victims in the case of humanitarian crises or post disasters.

Hence, effective transportation is essential to humanitarian operations and disaster relief.

At the same time, it is well-recognized that costs associated with transportation are second only to personnel for humanitarian organizations. Estimates have been as high as 80% for transportation/logistics costs for humanitarian relief.

Motivation for Our Model

Although certain large humanitarian organizations have their own fleets and are responsible for the management thereof, many smaller humanitarian organizations must make use of freight service providers such as UPS, DHL, and/or others for delivery of the supplies to points of demand post a disaster or a humanitarian crisis.

Disaster relief organizations must be transparent to their constituents, including donors, in ensuring that the donated funds have been utilized in a cost-effective manner.

Motivation for Our Model

Recent research on humanitarian logistics has begun to increasingly explore important issues surrounding transportation for disaster relief.

Motivation for Our Model

Nevertheless, the majority of studies have emphasized centralized decision-making. We are unaware of any research that captures competition associated with freight service provision in this application domain.

The survival of relief organizations, especially smaller ones, some of which are established after disasters and humanitarian crises, is critically dependent on wise budgeting and financial management and, hence, the effective use of freight services is essential. Plus, an organizations very reputation, and its relationships with donors and benefactors, depend on its appropriate allocation of its financial resources.

Motivation for Our Model

Transportation in disaster relief settings and humanitarian operations has been addressed in the research literature from different perspectives, including evacuation (see, e.g., Sheffi, Mahmassani, and Powell (1982), Sherali, Carter, and Hobelka (1991), Barbarosoglu, Ozdamar, and Cevik (2002), Regnier (2008), Miller-Hooks and Sorrel (2008), Saadatseresht, Mansourian, and Taleal (2009), Vogiatzis, Walteros, and Pardalos (2013), Na and Banerjee (2015), Vogiatzis and Pardalos (2016)), the distribution of relief supplies, including last mile issues (Sheu (2007), Yi and Kumar (2007), Barbarosoglu and Arda (2004), Tzeng, Cheng, and Huang (2007), Balcik, Beamon, and Smilowitz (2008), Mete and Zabinsky (2010), Vitoriano et al. (2011), Rottkemper, Fischer, and Blecken (2012), Huang, Smilowitz, and Balcik (2012)), in the context of supply chains (Van Wassenhove (2006), Falasca and Zobel (2011), Nagurney, Yu, and Qiang (2012), Qiang and Nagurney (2012), Nagurney and Masoumi (2012), Nagurney, Masoumi, and Yu (2015), Nagurney and Nagurney (2016)).

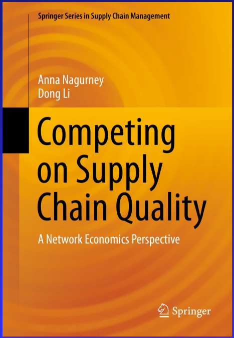
Motivation for Our Model

Also transportation has been modeled in in fleet management (see Pedraza Martinez, Stapleton, and Van Wassenhove (2010) and the references therein) and in cooperative settings to assess synergy (see Nagurney and Qiang (2012)), and in donor collections (Lodree, Carter, and Barbee (2016)).

Balcik and Ak (2014) present a stochastic programming model for supplier selection in terms of framework agreements for humanitarian relief. Such agreements are long-term ones. The suppliers provide the needed relief items, purchased by the organization, and also are responsible for the transportation.

Motivation for Our Model

Our framework, in contrast, builds on the research in supply chain network equilibrium, in which there is competition among decision-makers in each tier, originating with the work of Nagurney, Dong, and Zhang (2002), and we explicitly determine the equilibrium prices. Such research in the freight context has recently examined price and quality competition among manufacturers and freight service providers.



The Competitive Freight Service Provision Model for Disaster Relief

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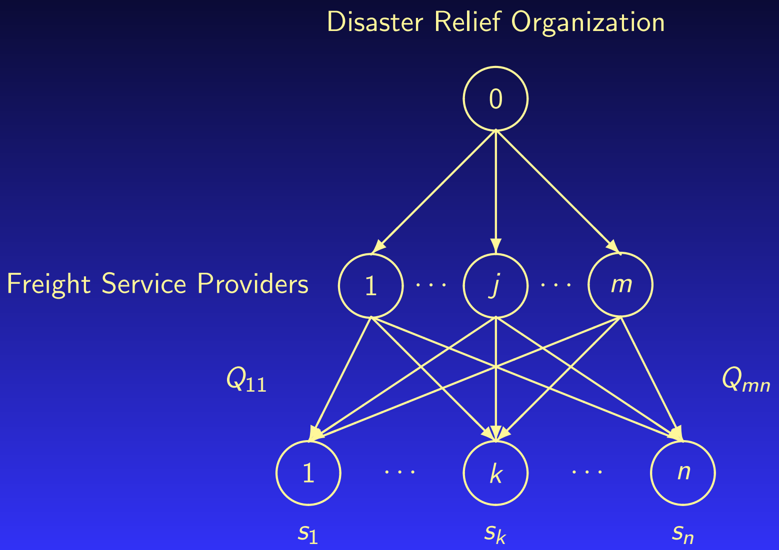

Behavior of the Disaster Relief Organization

The optimization problem faced by the disaster relief organization is as follows.

$$\text{Minimize} \quad \sum_{j=1}^{m}\sum_{k=1}^{n}\rho_{jk}^{*}Q_{jk}+\sum_{j=1}^{m}\hat{c}_j(\sum_{k=1}^{n}Q_{jk}) \tag{1}$$

subject to:

$$\sum_{j=1}^{m}Q_{jk}=s_k, \quad k=1,\ldots,n, \tag{2}$$

$$Q_{jk}\geq 0, \quad k=1,\ldots,n. \tag{3}$$

The first term in the objective function (1) corresponds to the payout to the freight service providers whereas the term following the plus sign corresponds to the total costs associated with transacting with the freight service providers. (2) ensure that relief supplies are delivered to points of demand.

Behavior of the Disaster Relief Organization

We define the feasible set $K \equiv \{Q | Q \geq 0 \text{ and satisfies } (2)\}$.

Note that the total cost functions $\hat{c}_j$; $j = 1, \ldots, m$, can also include the cost of purchasing the needed supplies, in addition to the freight service provision transaction costs. Such costs would be encumbered if the relief organization does not have the supplies in stock.

Behavior of the Disaster Relief Organization

We assume that the total cost functions $\hat{c}_j$; $j = 1, \ldots, m$, are continuously differentiable and convex. Under these assumptions, and the fact that K is convex, we know that a solution to the above optimization problem coincides with a solution to the variational inequality problem: determine $Q^* \in K$, such that

$$\sum_{j=1}^{m} \sum_{k=1}^{n} \left[\frac{\partial \hat{c}_j(\sum_{k=1}^{n} Q^*_{jk})}{\partial Q_{jk}} + \rho^*_{jk} \right] \times \left[Q_{jk} - Q^*_{jk} \right] \geq 0, \quad \forall Q \in K. \tag{4}$$

This result follows from the standard theory of variational inequalities (cf. Kinderlehrer and Stampacchia (1980) and Nagurney (1999)).

Behavior of the Freight Service Providers

The cost associated with freight service provider j delivering the relief items to demand point k is denoted by c_{jk}, where here we assume, for the sake of generality, and in order to effectively capture competition, that

$$c_{jk} = c_{jk}(Q), \quad j = 1, \ldots, m, \tag{5}$$

with the freight service provider cost functions being assumed to be continuously differentiable and convex.

According to (5), the cost faced by provider j in delivering the relief items to demand point k depends not only on the volume of the disaster relief items that it delivers to the demand points but also on the amounts delivered by the other freight service providers. We expect these functions to be nonlinear in order to capture congestion and also competition for resources associated with freight deliveries in compromised settings as may occur following disasters.

Behavior of the Freight Service Providers

The optimization problem faced by freight service provider j; $j = 1, \ldots, m$, is given by:

$$\text{Maximize} \quad \sum_{k=1}^{n} \rho_{jk}^{*} Q_{jk} - \sum_{k=1}^{n} c_{jk}(Q) \tag{6}$$

subject to:

$$Q_{jk} \geq 0, \quad k = 1, \ldots, n. \tag{7}$$

Since we assume that the freight service providers compete noncooperatively among one another for the product shipments, the optimality conditions of all freight service providers must satisfy the variational inequality problem (cf. Gabay and Moulin (1980), Nagurney (1999, 2006)): determine $Q^{*} \in R_{+}^{mn}$ such that:

$$\sum_{j=1}^{m} \sum_{k=1}^{n} \left[\sum_{l=1}^{n} \frac{\partial c_{jl}(Q^{*})}{\partial Q_{jk}} - \rho_{jk}^{*} \right] \times \left[Q_{jk} - Q_{jk}^{*} \right] \geq 0, \quad \forall Q \in R_{+}^{mn}. \tag{8}$$

Variational Inequality Formulation and Qualitative Properties

Definition 1: Freight Service Provision Network Equilibrium for Disaster Relief
A freight service provision network equilibrium for disaster relief is said to be established if the disaster relief product flows between the two tiers of decision-makers coincide and the product flows and prices satisfy the sum of variational inequalities (4) and (8).

According to Definition 1, the disaster relief organization as well as the freight service providers must agree on the amounts of the product shipments that they deliver to the demand points. This agreement is accomplished through the prices ρ^*_{jk}; $j = 1, \ldots, m$; $k = 1, \ldots, n$. After presenting the variational inequality formulation of the above equilibrium conditions we will demonstrate how to recover the prices.

Theorem 1: Variational Inequality Formulation of Freight Service Provision Network Equilibrium for Disaster Relief
A disaster shipment pattern $Q^* \in K$, is a freight service provision network equilibrium for disaster relief if and only if it satisfies the variational inequality problem:

$$\sum_{j=1}^{m}\sum_{k=1}^{n}\left[\frac{\partial \hat{c}_j(\sum_{k=1}^{n} Q^*_{jk})}{\partial Q_{jk}} + \sum_{l=1}^{n}\frac{\partial c_{jl}(Q^*)}{\partial Q_{jk}}\right]\times\left[Q_{jk} - Q^*_{jk}\right] \geq 0, \quad \forall Q \in K. \tag{9}$$

Note that in order to recover the equilibrium prices ρ^*_{jk}, $\forall j,k$, one just sets, according to (8): $\rho^*_{jk} = \sum_{l=1}^{n}\frac{\partial c_{jl}(Q^*)}{\partial Q_{jk}}$, $\forall j,k$.

Standard Variational Inequality Form

We now put variational inequality (9) into standard form (cf. Nagurney (1999)): determine $X^* \in \mathcal{K}$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{10}$$

where $F(X)$ is a N-dimensional vector which is a continuous function from $\mathcal{K}$ to R^N, X is an N-dimensional vector, $\mathcal{K}$ is closed and convex, and $\langle \cdot, \cdot \rangle$ denotes the inner product in N-dimensional Euclidean space. W

e define $\mathcal{K} \equiv K$, $X \equiv Q$, and component F_{jk} of $F(X)$ as:
$F_{jk}(X) \equiv \frac{\partial \hat{c}_j(\sum_{k=1}^{n} Q_{jk})}{\partial Q_{jk}} + \sum_{l=1}^{n} \frac{\partial c_{jl}(Q)}{\partial Q_{jk}}$; $j = 1, \ldots, m$; $k = 1, \ldots, n$.
Then variational inequality (9) takes on the standard form (10).

Qualitative Properties of the Freight Equilibrium Shipment Pattern for Disaster Relief

Since the feasible set K is closed and bounded, that is, it is compact, we know from the classical theory of variational inequalities that a solution to (9) is guaranteed to exist since the function $F(X)$ is continuous under our imposed assumptions that the various cost functions are continuously differentiable. Hence, the following result is immediate.

Theorem 2: Existence of a Freight Service Provision Equilibrium Shipment Pattern
A solution $X^* \in K$ to variational inequality (9) is guaranteed to exist.

Qualitative Properties of the Freight Equilibrium Shipment Pattern for Disaster Relief

In addition, under the assumption of strict monotonicity of $F(X)$, we have the following result, which also comes from classical variational inequality theory.

Theorem 3: Uniqueness of a Freight Service Provision Equilibrium Pattarn
If $F(X)$ is strictly monotone, that is,

$$\langle F(X^1)-F(X^2), X^1-X^2\rangle > 0, \quad \forall X^1, X^2 \in \mathcal{K}, \quad X^1 \neq X^2, \tag{11}$$

then X^* satisfying (9) is unique.

We know that if the Jacobian of $F(X)$, $\nabla F(X)$, is positive definite over $\mathcal{K}$, then $F(X)$ is strictly monotone.

An Illustrative Example and Variant

An Illustrative Example and Variant

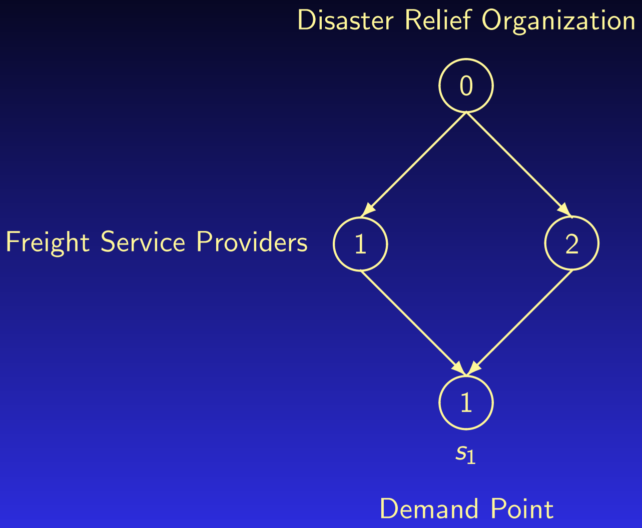

Figure 2: Network Topology for the Illustrative Example

An Illustrative Example and Variant

The data are as follows. The total costs faced by the disaster relief organization in transacting with the two freight service providers are: $\hat{c}_1(Q_{11}) = Q_{11}^2$ and $\hat{c}_2(Q_{21}) = Q_{21}^2$. The cost faced by freight service provider 1 is: $c_{11}(Q_{11}) = 5Q_{11}^2$ and that faced by freight service provider 2 is: $c_{21}(Q_{21}) = 3Q_{21}^2$.

The organization wishes to have 100 units of the disaster relief item delivered to demand point 1; hence, $s_1 = 100$.

Variational inequality (9) takes on the following form for this problem: determine $Q^* = (Q_{11}^*, Q_{21}^*) \in \mathcal{K}$, where $\mathcal{K} \equiv \{Q_{11} \geq 0, Q_{21} \geq 0 \text{ and } (2) \text{ holds}\}$ such that:

$$[12Q_{11}^*] \times [Q_{11} - Q_{11}^*] + [8Q_{21}^*] \times [Q_{21} - Q_{21}^*] \geq 0, \quad \forall Q \in \mathcal{K}. \quad (12)$$

An Illustrative Example and Variant

It is easy to see (cf. Nagurney (1999)) that (12) can be solved (indeed, variational inequality (12) is, in fact, equivalent to the solution of an optimization problem since for this problem $\nabla F(X)$ is symmetric) as a system of equations:

$$12Q_{11}^* = 8Q_{21}^*$$

$$Q_{11}^* + Q_{21}^* = 100.$$

Clearly, $Q_{11}^* = 40$ and $Q_{21}^* = 60$ is the solution.

The prices, as discussed above, are recovered as: $\rho_{11}^* = 400$, since $\rho_{11}^* = \frac{\partial c_{11}(Q_{11}^*)}{\partial Q_{11}} = 10Q_{11}^* = 10(40) = 400$, and $\rho_{21}^* = 360$, since $\rho_{21}^* = \frac{\partial c_{21}(Q_{21}^*)}{\partial Q_{21}} = 6Q_{21}^* = 6(60) = 360$.
This solution, which is unique, corresponds to the organization encumbering costs of 42,800 for delivery of the disaster relief items with freight service provider 1 having a profit of 8,000 and freight service provider a profit of 10,800.

An Illustrative Example and Variant

We now consider the following scenario. Suppose that only freight service provider 1 is available to offer its services. The relief organization still must have the 100 units delivered. Clearly, $Q^*_{11} = 100$ with the price charged by the freight service provider 1 being: $1,000$.

The organization now, in the absence of competition, incurs a cost of $110,000$ since the price $\rho^*_{11} = 1,000$ with the freight service provider 1 obtaining a profit of 50,000.

A Cooperative Model and the Price of Anarchy

Relationship to a Cooperative System-Optimized Model

In the cooperative, system-optimized model, the problem becomes that of minimizing the total costs with the costs consisting of those of the disaster relief organization and those of the freight service providers.

Thus, the cooperative, system-optimized model in which the costs to society are minimized is given by:

$$\text{Minimize} \quad \sum_{j=1}^{m} \hat{c}_j(\sum_{k=1}^{n} Q_{jk}) + \sum_{j=1}^{m}\sum_{k=1}^{n} c_{jk}(Q) \tag{13}$$

subject to previous demand and nnnegativity constraints.

Under the previously imposed assumptions on the cost functions, we know that an optimal solution to (13), coincides with the solution to the variational inequality given below. This follows from the standard theory of variational inequalities (see Nagurney (1999)) since the objective function in (13) is convex and continuously differentiable and the feasible set $\mathcal{K}$ is convex.

Theorem 4: Variational Inequality Formulation of the Cooperative System-Optimized Freight Service Provision Network Model

A solution $Q^* \in \mathcal{K}$ is an optimal solution to the above cooperative, system-optimized freight service provision network model for disaster relief if and only if it also satisfies the variational inequality:

$$\sum_{j=1}^{m}\sum_{k=1}^{n}\left[\frac{\partial \hat{c}_j(\sum_{k=1}^{n} Q^*_{jk})}{\partial Q_{jk}} + \sum_{h=1}^{m}\sum_{l=1}^{n}\frac{\partial c_{hl}(Q^*)}{\partial Q_{jk}}\right] \times \left[Q_{jk} - Q^*_{jk}\right] \geq 0,$$

$$\forall Q \in \mathcal{K}. \qquad (14)$$

Price of Anarchy

Moreover, we can introduce a *price of anarchy* (see Roughgarden (2005)) in this new setting, where the price $\mathcal{P}$ is defined below:

$$\mathcal{P} = \frac{\text{TC(Equilibrium Solution)}}{\text{TC(System-Optimized Solution)}}, \tag{15}$$

where the total cost TC$=\sum_{j=1}^{m} \hat{c}_j(\sum_{k=1}^{n} Q_{jk}) + \sum_{j=1}^{m} \sum_{k=1}^{n} c_{jk}(Q)$ is evaluated at the equilibrium solution satisfying variational inequality (9) in the numerator of (15) and at the system-optimized solution, satisfying variational inequality (14), in the denominator of (15).

The Algorithm

The Algorithm

The algorithm that we use to compute solutions to variational inequality (9) is the projection method of Bertsekas and Gafni (1982). The algorithm therein was applied to the traffic network equilibrium problem with fixed demands and is path-based, rather than link-based, as is the projection method of Dafermos (1980).

The Algorithm

Specifically, in referring to Figure 1, the paths joining the origin node 0 with each demand point k; $k = 1, \ldots, n$, have a special structure. Moreover, if we assign costs of zero to the topmost links in the network in Figure 1, and each link (j, k) joining a freight service node j with demand point k is assigned a "user" link cost of:

$$\left[\frac{\partial \hat{c}_j(\sum_{k=1}^{n} Q_{jk})}{\partial Q_{jk}} + \sum_{l=1}^{n} \frac{\partial c_{jl}(Q)}{\partial Q_{jk}} \right]$$

then the variational inequality (9) can be viewed as a solution to a traffic or transportation network equilibrium problem (Dafermos (1980), Patriksson (1994), Nagurney (1999)), with the equilibrium solution Q^*_{jk}; $j = 1, \ldots, m$; $k = 1, \ldots, n$, flowing on the respective path p_{jk}, originating at node 0, and connecting freight service node j and demand point node k. The demand for an O/D pair $(0, k)$ is equal to s_k; $k = 1, \ldots, n$.

The Algorithm

Specifically, the path-based projection method here takes the form below, where $F(X)$ is as in (10).

Path-Based Projection Method

Step 0: Initialization

Start with an $X^0 \in \mathcal{K}$. Set $\tau := 1$ and select β, where β is a step size that is sufficiently small. Set $\tau := 1$, and go to Step 1.

Step 1: Computation

Compute X^τ by solving the variational inequality subproblem:

$$\langle X^\tau + (\beta F(X^{\tau-1} - X^{\tau-1}), X - X^{\tau-1}) \rangle \geq 0, \quad \forall X \in \mathcal{K}. \qquad (16)$$

Step 2: Convergence Verification

If $|X_l^\tau - X_l^{\tau-1}| \leq \epsilon$, for all l, with $\epsilon > 0$, a prespecified tolerance, then stop; else, set $\tau := \tau + 1$, and go to Step 1.

The Algorithm

It is well-known that this projection method is guaranteed to converge to the solution of variational inequality (9), if $F(X)$ is strongly monotone, that is:

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq \gamma \|X^1 - X^2\|^2, \quad \forall X^1, X^2 \in \mathcal{K}, \quad (17)$$

with $\gamma > 0$, and Lipschitz continuous, that is:

$$\|F(X^1) - F(X^2)\| \leq L\|X^1 - X^2\|, \quad X^1, X^2 \in \mathcal{K} \qquad (18)$$

where $L > 0$.

Case Study Inspired by the Ebola Healthcare Crisis

Case Study Inspired by the Ebola Healthcare Crisis

In 2014 and 2015, the world was transfixed by the Ebola healthcare crisis that hit western Africa, notably, the countries of: Liberia, Sierra Leone, and Guinea. This contagious disease, with numerous deaths, put immense pressures on the healthcare systems of these countries, which already had been challenged.

No vaccines were available and medical professionals were in dire need of supplies including personal protective equipment. According to the Centers for Disease Control and Prevention (2016), as of December 27, 2015, based on World Health Organization (WHO) kept statistics, there were 2,536 deaths in Guinea attributed to Ebola, 3,955 deaths in Sierra Leone, and 4,806 deaths in Liberia, with confirmed cases, respectively, of: 3,351, 8,704, and 3,151, and with suspected, probable, and confirmed cases, respectively, of: 3,804, 14,122, and 10,666.

Case Study Inspired by the Ebola Healthcare Crisis

According to a report by the World Health Organization (2015), over 800 health care workers contracted Ebola during this crisis. In the WHO report, the term "health worker includes not only clinical staff, but all those who worked in health services, including drivers, cleaners, burial teams, and community-based workers amongst others.

Case Study Inspired by the Ebola Healthcare Crisis

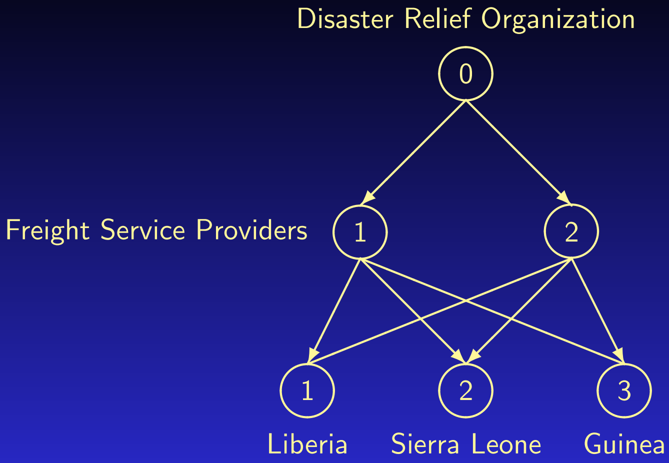

Figure 3: Network Topology for the Ebola Case Study

Case Study Inspired by the Ebola Healthcare Crisis

We used The World Bank (2016) data to identify the cost of transport of a container of 20 feet, which can hold 1360 cubic feet of supplies, via ship from the US to these countries. We then multiplied the cost by 14, as per the United States Department of Commerce (2016), to obtain an estimated cost for air freight since time was of the essence since, as noted earlier, healthcare workers were also contracting Ebola.

The disaster relief organization wishes to ship 10,000 PPE items to each of the three destinations.

We initialized the projection method so that the flows for each demand point were equally distributed among the paths connecting the origin node to that demand point. We set $\beta = 1$. The convergence tolerance ϵ eas set to 10^{-4}.

The data were estimated to be as follows.

The disaster relief organization was faced with the following total costs:

$$\hat{c}_1 = 4.50 \times (Q_{11} + Q_{12} + Q_{13}), \quad \hat{c}_2 = 4.25 \times (Q_{21} + Q_{22} + Q_{23}).$$

In our case study, we assume that the relief organization has to purchase the PPE items and, hence, the $\hat{c}_j$; $j = 1, 2$, cost functions include also the purchase cost. The total cost associated with freight service provider 1, $\hat{c}_1$, is higher than that for freight service provider 2, $\hat{c}_2$, since it does not have as much experience with the former provider and the transfer cost is higher per unit.

Case Study Inspired by the Ebola Healthcare Crisis

The freight service provider total costs, in turn, are estimated to be the following:

For freight service provider 1:

$$c_{11} = .0001Q_{11}^2 + 18.48Q_{11}, \quad c_{12} = .001Q_{12}^2 + 16.59Q_{12},$$

$$c_{13} = .001Q_{13}^2 + 12.81Q_{13};$$

For freight service provider 2:

$$c_{21} = .001Q_{21}^2 + 18.48Q_{21}, \quad c_{22} = .0001Q_{22}^2 + 16.59Q_{22}.$$

$$c_{23} = .01Q_{23}^2 + 12.81Q_{23}.$$

The nonlinear terms in the cost functions faced by the freight service provider capture the risk associated with transporting the supplies to the points of demand.

Case Study Inspired by the Ebola Healthcare Crisis

The computed solution is:

$$Q_{11}^* = 8,976.31, \quad Q_{12}^* = 796.43, \quad Q_{13}^* = 9,079.99,$$

$$Q_{21}^* = 1,023.69, \quad Q_{22}^* = 9,203.57, \quad Q_{23}^* = 920.01.$$

The prices charged by the freight service providers are:

$$\rho_{11}^* = 20.28, \quad \rho_{12}^* = 18.18, \quad \rho_{13}^* = 30.97,$$

$$\rho_{21}^* = 20.53, \quad \rho_{22}^* = 18.43, \quad \rho_{23}^* = 31.23.$$

Case Study Inspired by the Ebola Healthcare Crisis

The value of the objective function of the disaster relief organization is: 829,254.38. The payout to the freight service providers for transport is: 697,041.25, which means that 84% is for transport. This is reasonable since about 80% of disaster relief organizations' budgets are towards transportation in disasters. The value of freight service provider 1's objective function, which coincides with his profits, is: 91,137.94 and that of freight service provider 2 is: 17,982.72. The equilibrium conditions are satisfied.

From the results, we see that freight service provider 1 delivers the bulk (the majority) of the PPE supplies to Liberia and Guinea, whereas freight service provider delivers the bulk of the supplies to Sierra Leone.

A Variant

We now proceed to investigate the following scenario. The demand for PPEs in Liberia has increased due to the spread of Ebola and emphasis on containment. The data remain as in the above example except that now s_1 has doubled to: $20,000$.

The new computed equilibrium product shipment pattern is:

$$Q_{11}^* = 18,067.12, \quad Q_{12}^* = 795.92, \quad Q_{13}^* = 9,079.99,$$

$$Q_{21}^* = 1,932.88, \quad Q_{22}^* = 9,204.08, \quad Q_{23}^* = 920.01.$$

The new freight service provider prices are:

$$\rho_{11}^* = 22.09, \quad \rho_{12}^* = 18.18, \quad \rho_{13}^* = 30.97$$

$$\rho_{21}^* = 22.35, \quad \rho_{22}^* = 18.43, \quad \rho_{23}^* = 31.21.$$

Case Study Inspired by the Ebola Healthcare Crisis

The total cost faced by the disaster relief organization is now 1,113,372.63 with the payout to the freight service providers being: 936,386.88. The percentage of the organization's total cost that this payout entails is 84%.

Freight service provider 1 now has a profit of 115,721.75 and freight service provider 2 a profit of 20,671.77. Since now both freight service provider 1 and freight service provider 2 transport a greater volume of the PPE supplies to Liberia, the prices that they charge have increased and their profits have as well. Freight service provider 1 again dominates the shipments to Liberia and Guinea, whereas freight service provider carries the bulk of the PPEs to Sierra Leone.

Summary and Conclusions

- We presented a game theory model of freight service provision for disaster relief.
- the governing equilibrium conditions were formualted as a variational inequality problem and also analyzed quaalitatively in terms of existence and uniqueness.
- We also provided a cooperative syste-optimized version of the model along with the price of anarchy.
- Illustrative numerical examples were provided were showed the impact of competition on prices.
- An effective computational procedure was provided which exploits the special network structure of the problem.
- A case study inpspired by the Ebola healthcare crisis in West Africa demonstrated the algorithm and the model.
- Future research may include multiple organizations competing for freight service provision in disaster relief and well as coopeation

THANK YOU!

For more information, see: http://supernet.isenberg.umass.edu