

# A Supply Chain Network Game Theory Model of Cybersecurity Investments with Nonlinear Budget Constraints

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and Equilibrium Problems**

# Outline

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# Acknowledgements

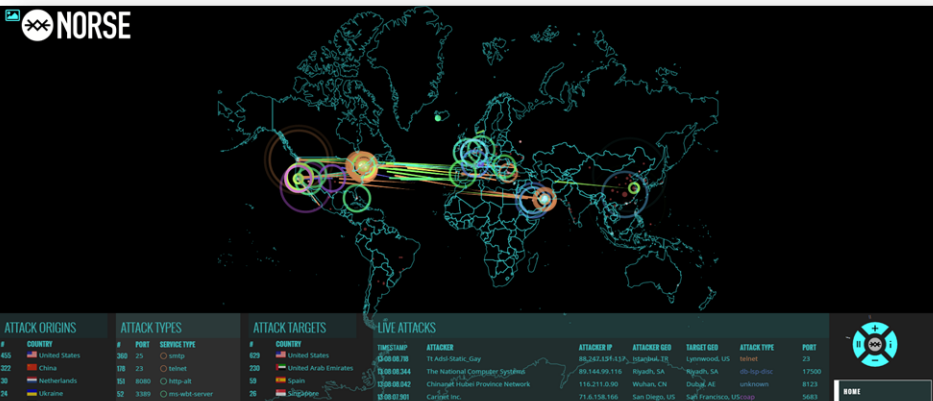
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# Introduction

- An increasingly connected world may amplify the effects of a disruption.
- **Cyber threat management** is more than a strategic imperative, it is **fundamental to business**.
- Breaches are inevitable:
  - (i) **Tangible costs** - lost funds, regulatory and legal fines, compensation, recovery - information and infrastructure rehabilitation.
  - (ii) **Intangible costs** - loss of reputation, business, competitive advantage, intellectual property, personal information.

# Cyber Attack Map



Snapshot of a real time view of cyberattacks - June 16, 2016

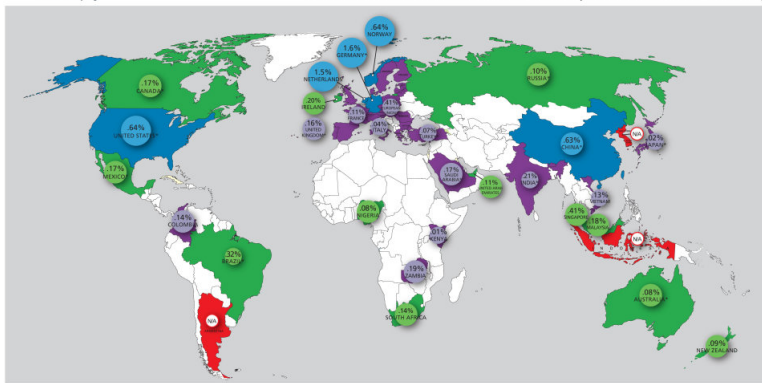
# Cost of Cybercrime

- Cybercrime climbs to 2nd most reported economic crime affecting **32% of organisations** globally (PwC Survey, 2016).
- Cost of data breaches to increase to **\$2.1 trillion** globally by 2019 - four times the estimated cost of breaches in 2015 (Forbes, 2016).
- "Cyber threats are not just increasing, but **mutating**" (Forrester Research, 2016).

# Cyber Loss as a Percent of GDP (2014)



## CYBERCRIME LOSS AS A PERCENT OF GDP (GROSS DOMESTIC PRODUCT)



Confidence Ranking: Countries Current Tracking of Cybercrime within Their Borders.



**\$445 BILLION**

The annual estimated cost to the global economy from cyber crime



**200,000+**

Jobs lost in the U.S  
**150,000+**  
 Estimated in Europe



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# Major Cyberattacks

- **Hilton Worldwide**(2015) - POS terminals hacked, credit card holders' names, numbers, expiry date, and security codes stolen. Hackers shopped online (SecurityWeek, 2016).
- **TalkTalk** (2015) - Nearly 157,000 had data breached. Cost of crime was £60 m, customers chose to leave, bonuses slashed (The Guardian, 2016).
- **Sony Pictures** (2014) - 100 terabytes of sensitive data leaked, 5 Sony films put online for free, private emails, salary information of top executives, medical documents, and Sony's Twitter account also leaked. Cost of crime could be \$100 m (Reuters, 2014).



The TalkTalk website is unavailable right now

On Wednesday 21st October, we experienced an attack to our website.

A formal investigation by the Metropolitan Police Cyber Crime Unit is under way.

Webmail is working as normal, but for more information, please call 0800 083 2710 or 0141 230 0707.

Thank you for your patience.

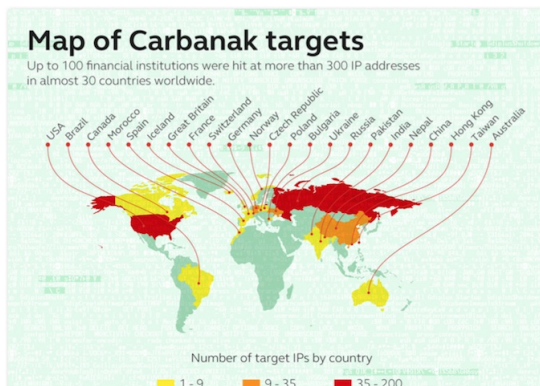
TalkTalk

If you need to contact us then you can do so on the below numbers:

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# Major Cyberattacks

- **JD Wetherspoon(2015)** - Names, email ids, birthdates and contact numbers of 656,723 customers hacked. Company became aware of the attack almost 5 months later (Telegraph, 2015).
- Kaspersky Lab reported a cyber heist (**Carbanak**) of \$1 bn when hackers infiltrated 100 banks across 30 countries over a period of 2 years.
- Other notable attacks - Target, Home Depot, Michaels Stores, Staples, eBay.



# Motivation

The **median number of days that attackers stay dormant** within a network before detection is **over 200** (Microsoft, 2015)

The majority of data breach victims surveyed, 81 percent, report they had **neither a system nor a managed security service** in place to ensure they could self-detect data breaches, **relying instead on notification from an external party.**

This was the case despite the fact that self-detected breaches take just 14.5 days to contain from their intrusion date, whereas **breaches detected by an external party take an average of 154 days to contain** (Trustwave, 2015).

# Motivation

- Growing interest in the development of **rigorous scientific tools**.
- As reported in Glazer (2015), JPMorgan was expected to double its cybersecurity spending in 2015 to \$500 million from \$250 million in 2014.
- According to Purnell (2015), the research firm Gartner reported in January 2015 that the global information security spending would increase by 7.6% in 2015 to \$790 billion.
- It is clear that making the best **cybersecurity investments is a very timely problem and issue**.

# Approach

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# Approach

- We develop a supply chain network game theory model with **competing retailers**.
- Retailers seek to individually maximize their expected revenue and minimize financial losses in case of cyber attack, along with costs associated with **cyber investments**.
- **Nonlinear budget constraints** are considered, Nash equilibrium conditions discussed, and variational inequality formulations presented.
- We also discuss how to measure the **vulnerability of a firm to cyberattacks and that of the supply chain network**, as a whole.



## Important References:

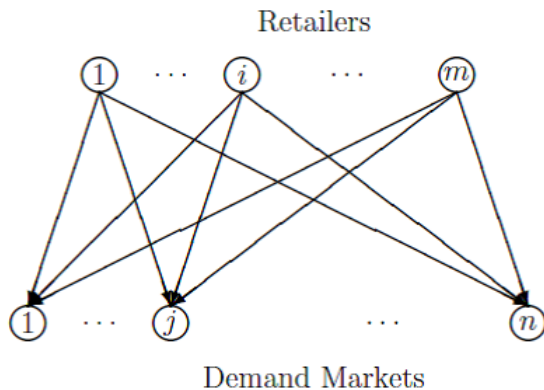
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# The Supply Chain Game Theory Model of Cybersecurity Investments with Nonlinear Budget Constraints

**Network Topology:** Bipartite Structure



# The Supply Chain Game Theory Model of Cybersecurity Investments with Nonlinear Budget Constraints

**Network Security,  $s_i$ :**

$$0 \leq s_i \leq u_{s_i} \quad i = 1, \dots, m.$$

$u_{s_i} < 1$ : Upper bound on security level of firm  $i$ .

**Average Network Security of the Chain,  $\bar{s}$ :**

$$\bar{s} = \frac{1}{m} \sum_{i=1}^m s_i.$$

**Probability of a Successful Cyberattack on  $i$ ,  $p_i$ :**

$$p_i = (1 - s_i)(1 - \bar{s}), \quad i = 1, \dots, m.$$

**Vulnerability,  $v_i$ :**

$v_i = (1 - s_i)$ ,  $i = 1, \dots, m$ . Vulnerability of network,  $\bar{v} = (1 - \bar{s})$ .

# The Supply Chain Game Theory Model of Cybersecurity Investments with Nonlinear Budget Constraints

**Investment Cost Function to Acquire Security  $s_i$ ,  $h_i(s_i)$ :**

$$h_i(s_i) = \alpha_i \left( \frac{1}{\sqrt{(1-s_i)}} - 1 \right), \quad \alpha_i > 0, \quad i = 1, \dots, m.$$

$\alpha_i$  quantifies size and needs of retailer  $i$ ;  $h_i(0) = 0 =$  insecure retailer, and  $h_i(1) = \infty =$  complete security at infinite cost.

**Nonlinear Budget Constraint:**

$$\alpha_i \left( \frac{1}{\sqrt{(1-s_i)}} - 1 \right) \leq B_i, \quad i = 1, \dots, m.$$

Each retailer cannot exceed his allocated cybersecurity budget,  $B_i$ .

# The Supply Chain Game Theory Model of Cybersecurity Investments with Nonlinear Budget Constraints

Incurred financial damage if attack successful:  $D_i$ .

**Expected Financial Damage after Cyberattack for Firm  $i$ ;  $i = 1, \dots, m$ :**

$$D_i p_i, \quad D_i \geq 0.$$

The **demand for the product at demand market  $j$**  must satisfy the following conservation of flow equation:

$$d_j = \sum_{i=1}^m Q_{ij}, \quad j = 1, \dots, n,$$

where

$$0 \leq Q_{ij} \leq \bar{Q}_{ij}, \quad i = 1, \dots, m; j = 1, \dots, n.$$

# The Supply Chain Game Theory Model of Cybersecurity Investments with Nonlinear Budget Constraints

In view of the demand, we can define **demand price functions**

$$\hat{\rho}_j(Q, s) \equiv \rho_j(d, \bar{s}), \forall j$$

. The consumers reflect their preferences through vector of demands and supply chain network security.

**Profit of Retailer  $i, i = 1, \dots, m$  in absence of cyberattack and investments,  $f_i$ :**

$$f_i(Q, s) = \sum_{j=1}^n \hat{\rho}_j(Q, s) Q_{ij} - c_i \sum_{j=1}^n Q_{ij} - \sum_{j=1}^n c_{ij}(Q_{ij}),$$

$Q_{ij}$  : Quantity from  $i$  to  $j$ ;  $c_i$  : Cost of processing at  $i$ ;  $c_{ij}$  : Cost of transactions from  $i$  to  $j$ .

# The Supply Chain Game Theory Model of Cybersecurity Investments with Nonlinear Budget Constraints

**Expected Utility**  $i, i = 1, \dots, m$ :

$$E(U_i) = (1 - p_i)f_i(Q, s) + p_i(f_i(Q, s) - D_i) - h_i(s_i).$$

Each  $E(U_i(s))$  is strictly concave with respect to  $s_i$  and each  $h_i(s_i)$  is strictly convex.

Feasible Set:  $K \equiv \prod_{i=1}^m K^i$ , where

$K^i \equiv \{(Q_i, s_i) | 0 \leq Q_i \leq \bar{Q}_{ij}; 0 \leq s_i \leq u_{s_i}, \text{ and budget constraint}\}$

## Definition 1: A Supply Chain Nash Equilibrium in Product Transactions and Security Levels

We seek to determine a nonnegative product transaction and security level pattern  $(Q^*, s^*) \in K$  for which the  $m$  retailers will be in a state of equilibrium as defined below.

### Definition 1: Nash Equilibrium in Cybersecurity Levels

A product transaction and security level pattern  $(Q^*, s^*) \in K$  is said to constitute a supply chain Nash equilibrium if for each retailer  $i; i = 1, \dots, m$ :

$$E(U_i(Q_i^*, s_i^*, \hat{Q}_i^*, \hat{s}_i^*)) \geq E(U_i(Q_i, s_i, \hat{Q}_i^*, \hat{s}_i^*)), \quad \forall (Q_i, s_i) \in K_i^1,$$

where

$$\hat{Q}_i^* \equiv (Q_1^*, \dots, Q_{i-1}^*, Q_{i+1}^*, \dots, Q_m^*); \hat{s}_i^* \equiv (s_1^*, \dots, s_{i-1}^*, s_{i+1}^*, \dots, s_m^*).$$



# Nonlinear Budget Constraints in the Feasible Set

In our model, unlike in many network equilibrium problems from congested urban transportation networks to supply chains and financial networks, the feasible set contains nonlinear constraints.

## Lemma 1

Let  $h_i$  be a convex function for all retailers  $i; i = 1, \dots, m$ . The feasible set  $K$  is then convex.

# Variational Inequality Formulation

## Theorem 1: Variational Inequality Formulation

$(Q^*, s^*) \in K$  is a Nash equilibrium if and only if it satisfies the variational inequality,

$$\begin{aligned}
 & - \sum_{i=1}^m \sum_{j=1}^n \frac{\partial E(U_i(Q^*, s^*))}{\partial Q_{ij}} \times (Q_{ij} - Q_{ij}^*) \\
 & - \sum_{i=1}^m \frac{\partial E(U_i(Q^*, s^*))}{\partial s_i} \times (s_i - s_i^*) \geq 0, \forall (Q, s) \in K,
 \end{aligned}$$

or, equivalently,

# Variational Inequality Formulation

$(Q^*, s^*) \in K$  is a Nash equilibrium if and only if it satisfies the variational inequality,

$$\begin{aligned} & \sum_{i=1}^m \sum_{j=1}^n \left[ c_i + \frac{\partial c_{ij}(Q_{ij}^*)}{\partial Q_{ij}} - \hat{\rho}_j(Q^*, s^*) - \sum_{k=1}^n \frac{\hat{\rho}_k(Q^*, s^*)}{\partial Q_{ij}} Q_{ik}^* \right] \times (Q_{ij} - Q_{ij}^*) \\ & + \sum_{i=1}^m \left[ \frac{\partial h_i(s_i^*)}{\partial s_i} - \sum_{k=1}^n \frac{\partial \hat{\rho}_k(Q^*, s^*)}{\partial s_i} Q_{ik}^* \right. \\ & \left. - \left( 1 - \sum_{k=1}^m \frac{s_k^*}{m} + \frac{1 - s_i^*}{m} \right) D_i \right] \times (s_i - s_i^*) \geq 0, \forall (Q, s) \in K. \end{aligned}$$

# Existence

## Theorem 2: Existence

*A solution  $(Q^*, s^*)$  to the variational inequality is guaranteed to exist.*

The result follows from the classical theory of variational inequalities (see Kinderlehrer and Stampacchia (1980)) since the feasible set  $K$  is compact, and the function that enters the variational inequality is continuous.

# Uniqueness

We define the  $(mn + m)$ -dimensional column vector  $X \equiv (Q, s)$  and the  $(mn + m)$ -dimensional column vector  $F(X) = (F^1(X), F^2(X))$  with the  $(i,j)$ -th component,  $F_{ij}^1$  of  $F^1(X)$  is  $\frac{\partial E(U_i(Q^*, s^*))}{\partial Q_{ij}}$ , and  $i$ -th component  $F_i^2$  of  $F^2(X)$  is  $\frac{\partial E(U_i(Q^*, s^*))}{\partial s_i}$ .

## Theorem 3: Uniqueness

*A solution  $(Q^*, s^*)$  to the variational inequality is unique if  $F(X)$  and  $X \equiv (Q, s)$  is strictly monotone (see Kinderlehrer and Stampacchia (1980)).*

# Variational Inequality Formulation with Lagrange Multipliers

Feasible set:  $\mathcal{K} \equiv \prod_{i=1}^m \mathcal{K}_i^1 \times R_+^m$ ,  
 where  $\mathcal{K}_i^1 \equiv \{(Q_i, s_i) | 0 \leq Q_i \leq \bar{Q}_{ij}, \forall j; 0 \leq s_i \leq u_{s_i}\}$ .

## Theorem 4: Alternative Variational Inequality Formulation

A vector  $(Q^*, s^*, \lambda^*)$  in feasible set,  $\mathcal{K}$ , containing non-negativity constraints is an equilibrium solution if and only if it satisfies the following variational inequality,

$$\begin{aligned}
 & - \sum_{i=1}^m \sum_{j=1}^n \frac{\partial E(U_i(Q^*, s^*))}{\partial Q_{ij}} \times (Q_{ij} - Q_{ij}^*) \\
 & - \sum_{i=1}^m \frac{\partial E(U_i(Q^*, s^*))}{\partial s_i} \times (s_i - s_i^*) \\
 & + \sum_{i=1}^m [B_i - \alpha_i \left( \frac{1}{\sqrt{1 - s_i}} - 1 \right)] \times (\lambda_i - \lambda_i^*) \geq 0, \forall (Q, s, \lambda) \in \mathcal{K},
 \end{aligned}$$

or, equivalently,

$$\begin{aligned}
 & \sum_{i=1}^m \sum_{j=1}^n \left[ c_i + \frac{\partial c_{ij}(Q_{ij}^*)}{\partial Q_{ij}} - \hat{\rho}_j(Q^*, s^*) - \sum_{k=1}^n \frac{\partial \hat{\rho}_k(Q^*, s^*)}{\partial Q_{ij}} Q_{ik}^* \right] \times (Q_{ij} - Q_{ij}^*) \\
 & \quad + \sum_{i=1}^m \left[ \frac{\partial h_i(s_i^*)}{\partial s_i} - \sum_{k=1}^n \frac{\partial \hat{\rho}_k(Q^*, s^*)}{\partial s_i} Q_{ik}^* \right. \\
 & \quad \left. - \left( 1 - \sum_{k=1}^m \frac{s_k^*}{m} + \frac{1 - s_i^*}{m} \right) D_i \right] + \frac{\lambda_i^*}{2} \alpha_i (1 - s_i^*)^{-\frac{3}{2}} \times (s_i - s_i^*) \\
 & \quad + \sum_{i=1}^m \left[ B_i - \alpha_i \left( \frac{1}{\sqrt{1 - s_i}} - 1 \right) \right] \times (\lambda_i - \lambda_i^*) \geq 0, \forall (Q, s, \lambda) \in \mathcal{K}.
 \end{aligned}$$

# Assumption

## The Slater Condition:

There exists a Slater vector  $\tilde{X}_i \in K_1^i$  for each  $i = 1, \dots, m$ , such that  $g_i(\tilde{X}_i) < 0$ .

It is a sufficient condition for strong duality to hold for a convex optimization problem. Informally, Slater's condition states that the feasible region must have an interior point.



# The Algorithm

**The Euler Method:** At each iteration  $\tau$ , one solves the following problem:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau}F(X^{\tau})),$$

where  $P_{\mathcal{K}}$  is the projection operator and  $F$  is the function that enters the Variational Inequality,  $\langle F(X^*), X - X^* \rangle \geq 0$ , where  $X \equiv (Q, s, \lambda)$ .

As established in Dupuis and Nagurney (1993), for convergence of the general iterative scheme, which induces the Euler method, the sequence  $\{a_{\tau}\}$  must satisfy:  $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$ ,  $a_{\tau} > 0$ ,  $a_{\tau} \rightarrow 0$ , as  $\tau \rightarrow \infty$ .

# Explicit Formulae for the Euler Method Applied to the Game Theory Model

Closed form expression for the product transactions,  
 $i = 1, \dots, m; j = 1, \dots, n$ :

$$Q_{ij}^{\tau+1} = \max\{0, \min\{\bar{Q}_{ij}, Q_{ij}^{\tau} + a_{\tau}(\hat{\rho}_j(Q^{\tau}, s^{\tau}) + \sum_{k=1}^n \frac{\partial \hat{\rho}_k(Q^{\tau}, s^{\tau})}{\partial Q_{ij}} Q_{ik}^{\tau} - c_i - \frac{\partial c_{ij}(Q_{ij}^{\tau})}{\partial Q_{ij}})\}\}$$

Closed form expression for security levels and Lagrange multipliers for  $i = 1, \dots, m$ :

$$s_i^{\tau+1} = \max\{0, \min\{u_{s_i}, s_i^{\tau} + a_{\tau}(\sum_{k=1}^n \frac{\partial \hat{\rho}_k(Q^{\tau}, s^{\tau})}{\partial s_i} Q_{ik}^{\tau} - \frac{\partial h_i(s_i^{\tau})}{\partial s_i^{\tau}} + (1 - \sum_{j=1}^m \frac{s_j^{\tau}}{m} + \frac{1 - s_i}{m})D_i) - \frac{\lambda_i^{\tau}}{2} \alpha_i (1 - s_i^{\tau})^{\frac{-3}{2}}\}\},$$

$$\lambda_i^{\tau+1} = \max\{0, \lambda_i^{\tau} + a_{\tau}(B_i + \alpha_i(\frac{1}{\sqrt{1 - s_i^{\tau}}} - 1))\}.$$

# Numerical Examples

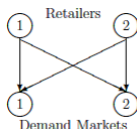
Convergence Criterion:  $\epsilon = 10^{-4}$ .

The Euler method was considered to have converged if, at a given iteration, the absolute value of the difference of each product transaction and each security level differed from its respective value at the preceding iteration by no more than  $\epsilon$ .

Sequence  $a_\tau$ :  $.1(1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \dots)$ .

Initial Values: We initialized the Euler method by setting each product transaction  $Q_{ij} = 1.00, \forall i, j$ , the security level of each retailer  $s_i = 0.00, \forall i$ , and the Lagrange multiplier for each retailers budget constraint  $\lambda_i = 0.00, \forall i$ . The capacities  $\bar{Q}_{ij}$  were set to 100 for all  $i, j$ .

# Example 1



Cost functions:

$$c_1 = 5, \quad c_2 = 10,$$

$$c_{11}(Q_{11}) = .5Q_{11}^2 + Q_{11}, \quad c_{12}(Q_{12}) = .25Q_{12}^2 + Q_{12},$$

$$c_{21}(Q_{21}) = .5Q_{21}^2 + 2, \quad c_{22}(Q_{22}) = .25Q_{22}^2 + Q_{22}$$

Demand price functions:

$$\rho_1(d, \bar{s}) = -d_1 + .1\left(\frac{s_1 + s_2}{2}\right) + 100, \quad \rho_2(d, \bar{s}) = -.5d_2 + .2\left(\frac{s_1 + s_2}{2}\right) + 200.$$

Damage parameters:  $D_1 = 50, D_2 = 70$ . Budgets:  $B_1 = B_2 = 2.5$ .

Investment cost functions:

$$h_1(s_1) = \frac{1}{\sqrt{(1 - s_1)}} - 1, \quad h_2(s_2) = \frac{1}{\sqrt{(1 - s_2)}} - 1$$

# Example 1

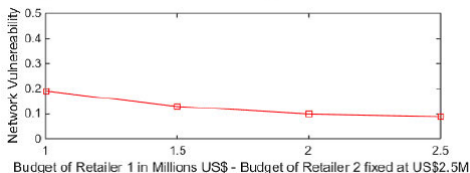
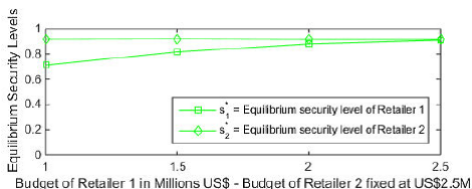
## Results:

Solution	Ex.1
$Q_{11}^*$	24.27
$Q_{12}^*$	98.34
$Q_{21}^*$	21.27
$Q_{22}^*$	93.34
$d_1^*$	45.55
$d_2^*$	191.68
$s_1^*$	.91
$s_2^*$	.91
$\bar{s}^*$	.91
$\lambda_1^*$	0.00
$\lambda_2^*$	0.00
$\rho_1(d_1^*, \bar{s}^*)$	54.55
$\rho_2(d_2^*, \bar{s}^*)$	104.34
$E(U_1)$	8137.38
$E(U_2)$	7213.49

## Example 1: Sensitivity Analysis

Base results showed that Retailer 1 has .21 (in millions) in unspent cybersecurity funds whereas Retailer 2 has .10(in millions). Hence, the associated Lagrange multipliers are 0.

For sensitivity analysis, we kept the budget of Retailer 2 fixed at 2.5 (in millions of US dollars), and we varied the budget of Retailer 1 in increments of .5.



## Example 2

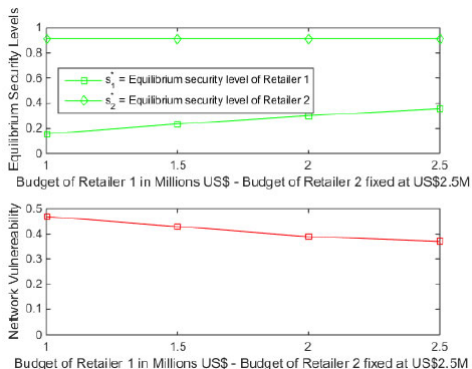
Example 2 was constructed from Example 1, except that the investment cost function of Retailer 1 was changed to:  $h_1(s_1) = 10 \frac{1}{\sqrt{(1-s_1)}} - 1$ .

Solution	Ex.2
$Q_{11}^*$	24.27
$Q_{12}^*$	98.31
$Q_{21}^*$	21.27
$Q_{22}^*$	93.31
$d_1^*$	45.53
$d_2^*$	191.62
$s_1^*$	.36
$s_2^*$	.91
$\bar{s}^*$	.63
$\lambda_1^*$	3.68
$\lambda_2^*$	1.06
$\rho_1(d_1^*, \bar{s}^*)$	54.53
$\rho_2(d_2^*, \bar{s}^*)$	104.32
$E(U_1)$	8122.77
$E(U_2)$	7207.47

## Example 2: Sensitivity Analysis

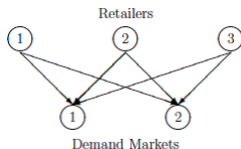
Base results showed that budgets were fully spent, so the Lagrange multipliers are no more 0. Retailer 1 invests less in security. Network vulnerability increased to .37.

For sensitivity analysis, Budget of Retailer 2 fixed at 2.5 and the budget of Retailer 1 varied in increments of .5.





## Example 3



Example 3 was constructed from Example 1 with the following for Retailer 3. Cost functions:

$$c_3 = 3$$

$$c_{31}(Q_{31}) = Q_{31}^2 + 2Q_{31}, \quad c_{32}(Q_{32}) = Q_{32}^2 + 4Q_{32}$$

Damage parameters:  $D_3 = 80$ . Budgets:  $B_3 = 3.0$ .

Investment cost functions:

$$h_3(s_3) = 3\left(\frac{1}{\sqrt{1-s_3}} - 1\right)$$

# Example 3

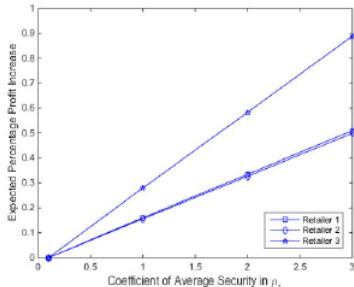
## Results:

$Q_{11}^*$	20.80
$Q_{12}^*$	89.48
$Q_{21}^*$	17.80
$Q_{22}^*$	84.48
$Q_{31}^*$	13.87
$Q_{32}^*$	35.40
$d_1^*$	52.48
$d_2^*$	209.36
$s_1^*$	.90
$s_2^*$	.91
$s_3^*$	.74
$\bar{s}^*$	.85
$\lambda_1^*$	0.00
$\lambda_2^*$	0.00
$\lambda_3^*$	0.00
$\rho_1(d_1^*, \bar{s}^*)$	47.61
$\rho_2(d_2^*, \bar{s}^*)$	95.49
$E(U_1)$	6655.13
$E(U_2)$	5828.82
$E(U_3)$	2262.26

## Example 3: Sensitivity Analysis

Base results showed that addition of Retailer 3 caused profits for all to drop, demands increase, and network vulnerability increase. Budgets were not exhausted. Retailer 3 turned out to be a “free rider”.

For sensitivity analysis, demand price function coefficient for demand market 1 increased to 1.0, 2.0, and 3.0, and the percent increase in expected profits of the retailers reported.



## Example 4

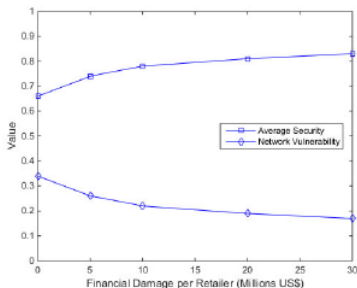
Example 4 constructed from Example 3. All damages at 0.00.

$Q_{11}^*$	20.80
$Q_{12}^*$	89.48
$Q_{21}^*$	17.80
$Q_{22}^*$	84.47
$Q_{31}^*$	13.87
$Q_{32}^*$	35.40
$d_1^*$	52.47
$d_2^*$	209.30
$s_1^*$	.82
$s_2^*$	.81
$s_3^*$	.34
$\bar{s}^*$	.66
$\lambda_1^*$	0.00
$\lambda_2^*$	0.00
$\lambda_3^*$	0.00
$\rho_1(d_1^*, \bar{s}^*)$	47.60
$\rho_2(d_2^*, \bar{s}^*)$	95.48
$E(U_1)$	6652.45
$E(U_2)$	5828.10
$E(U_3)$	2264.24

## Example 4: Sensitivity Analysis

Base results showed that budgets were not fully spent due to: (i) information asymmetry, (ii) no damages.

For sensitivity analysis, damages for all are increased to 5.00, 10.00, followed by increments of 10.00 through 30.00



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- Various data instances are evaluated through the algorithm, with relevant managerial insights and sensitivity analysis.
- The generalized framework of cybersecurity investments in a supply chain network game theory context with nonlinear budget constraints is a **novel contribution to the literature of both variational inequalities and game theory, and cybersecurity investments.**

Thank You!

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