A Generalized Nash Equilibrium Model for Post-Disaster Humanitarian Relief

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Outline

- Background and Motivation
- The Game Theory Model for Post-Disaster Humanitarian Relief
- The Algorithm
- A Case Study on Hurricane Katrina
- Summary and Conclusions

Background and Motivation

The Generalized Nash Equilibrium model that we present integrates both financial donations and supply chain aspects for competing humanitarian relief organizations.

The authors of this paper are in the photo below.

Recent disasters have vividly demonstrated the importance and vulnerability of our network infrastructure systems

- The biggest blackout in North America, August 14, 2003;
- Two significant power outages in September 2003 – one in the UK and the other in Italy and Switzerland;
- The Indonesian tsunami (and earthquake), December 26, 2004;
- Hurricane Katrina, August 23, 2005;
- The Minneapolis I35 Bridge collapse, August 1, 2007;
- The Sichuan earthquake on May 12, 2008;
- The Haiti earthquake that struck on January 12, 2010 and the Chilean one on February 27, 2010;
- The triple disaster in Japan on March 11, 2011;
- Superstorm Sandy, October 29, 2012.

Hurricane Katrina in 2005

Hurricane Katrina has been called an *"American tragedy,"* in which essential services failed completely.

The Haitian and Chilean Earthquakes

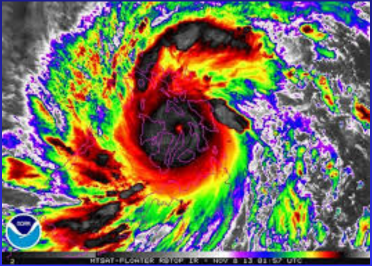
The Triple Disaster in Japan on March 11, 2011

Superstorm Sandy and Power Outages

Manhattan without power October 30, 2012 as a result of the devastation wrought by Superstorm Sandy.

Haiyan Typhoon in the Philippines in 2013

Typhoon Haiyan was a very powerful tropical cyclone that devastated portions of Southeast Asia, especially the Philippines, on November 8, 2013. It is the deadliest Philippine typhoon on record, killing at least 6,190 people in that country alone. Haiyan was also the strongest storm recorded at landfall. As of January 2014, bodies were still being found. The overall economic losses from Typhoon Haiyan totaled $10 billion.



Nepal Earthquake in 2015

The 7.8 magnitude earthquake that struck Nepal on April 25, 2015, and the aftershocks that followed, killed nearly 9,000 people and injured 22,000 others. This disaster also pushed about 700,000 people below the poverty line in the Himalayan nation, which is one of the world's poorest. About 500,000 homes were made unlivable by the quakes, leaving about three million people homeless. Much infrastructure was also badly damaged and 1/3 of the healthcare facilities devastated. According to *The Wall Street Journal*, Nepal needs $6.66 billion to rebuild.

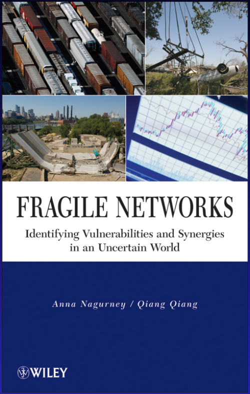
The Ebola Crisis in West Africa

According to the bbc.com and the World Health Organization, more than one year from the first confirmed case recorded on March 23, 2014, at least 11,178 people had died from Ebola in six countries; Liberia, Guinea, Sierra Leone, Nigeria, the US and Mali. The total number of reported cases was more than 27,275.

Ms. Debbie Wilson of Doctors Without Borders

On February 4, 2015, the students in my Humanitarian Logistics and Healthcare class at the Isenberg School heard Debbie Wilson, a nurse, who has worked with Doctors Without Borders, speak on her 6 weeks of experiences battling Ebola in Liberia in September and October 2014.

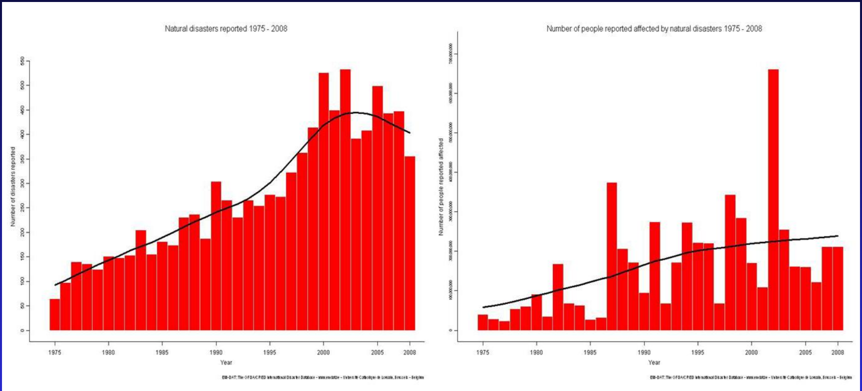
As noted in Nagurney and Qiang (2009), the number of disasters is growing as well as the number of people affected by disasters.



Hence, the development of appropriate analytical tools that can assist humanitarian organizations and nongovernmental organizations as well as governments in the various disaster management phases has become a challenge to both researchers and practitioners.

Disasters have a catastrophic effect on human lives and a region's or even a nation's resources.

Natural Disasters (1975–2008)



Game Theory Model for Post-Disaster Humanitarian Relief

The Game Theory and Disaster Relief

Although there have been quite a few optimization models developed for disaster relief there are very few game theory models.

Nevertheless, it is clear that humanitarian relief organizations and NGOs compete for financial funds from donors. Within three weeks after the 2010 earthquake in Haiti, there were 1,000 NGOs operating in that country. Interestingly, and, as noted by Ortuño et al. (2013), although the importance of donations is a fundamental difference of humanitarian logistics with respect to commercial logistics, this topic has "not yet been sufficiently studied by academics and there is a wide field for future research in this context."

Toyasaki and Wakolbinger (2014) developed perhaps the first models of financial flows that captured the strategic interaction between donors and humanitarian organizations using game theory and also included earmarked donations.

Game Theory and Disaster Relief

We developed what we believe is the first Generalized Nash Equilibrium (GNE) model for post-disaster humanitarian relief, which contains both a financial component and a supply chain component. The Generalized Nash Equilibrium problem is a generalization of the Nash Equilibrium problem (cf. Nash (1950, 1951)) in that the players' strategies, as defined by the underlying constraints, depend also on their rivals' strategies.

This presentation is based on the paper, "A Generalized Nash Equilibrium Network Model for Post-Disaster Humanitarian Relief," Anna Nagurney, Emilio Alvarez Flores, and Ceren Soylu, *Transportation Research E* **95** (2016), pp 1-18.

The Network Structure of the Model

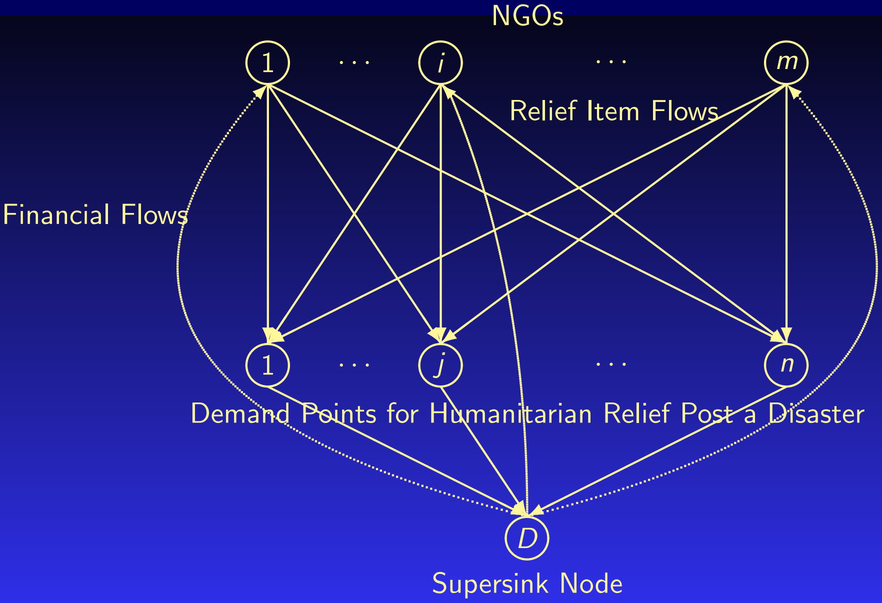


Figure 1: The Network Structure of the Game Theory Model

The Game Theory Model

We assume that each NGO i has, at its disposal, an amount s_i of the relief item that it can allocate post-disaster. Hence, we have the following conservation of flow equation, which must hold for each i; $i = 1, \ldots, m$:

$$\sum_{j=1}^{n} q_{ij} \leq s_i. \tag{1}$$

In addition, we know that the product flows for each i; $i = 1, \ldots, m$, must be nonnegative, that is:

$$q_{ij} \geq 0, \quad j = 1, \ldots, n. \tag{2}$$

Each NGO i encumbers a cost, c_{ij}, associated with shipping the relief items to location j, denoted by c_{ij}, where we assume that

$$c_{ij} = c_{ij}(q_{ij}), \quad j = 1, \ldots n, \tag{3}$$

with these cost functions being strictly convex and continuously differentiable.

The Game Theory Model

In addition, each NGO i; $i = 1, \ldots, m$, derives satisfaction or utility associated with providing the relief items to j; $j = 1, \ldots, n$, with its utility over all demand points given by $\sum_{j=1}^{n} \gamma_{ij} q_{ij}$. Here γ_{ij} is a positive factor representing a measure of satisfaction/utility that NGO i acquires through its supply chain activities to demand point j. Each NGO i; $i = 1, \ldots, m$, associates a positive weight ω_i with $\sum_{j=1}^{n} \gamma_{ij} q_{ij}$, which provides a monetization of, in effect, this component of the objective function.

The Game Theory Model

Finally, each NGO i; $i = 1, \ldots, m$, based on the media attention and the visibility of NGOs at location j; $j = 1, \ldots, n$, acquires funds from donors given by the expression

$$\beta_i \sum_{j=1}^{n} P_j(q), \tag{4}$$

where $P_j(q)$ represents the financial funds in donation dollars due to visibility of all NGOs at location j. Hence, β_i is a parameter that reflects the proportion of total donations collected for the disaster at demand point j that is received by NGO i. Expression (4), therefore, represents the financial flow on the link joining node D with node NGO i.

The Game Theory Model

Each NGO seeks to maximize its utility with the utility corresponding to the financial gains associated with the visibility through media of the relief item flow allocations, $\beta_i \sum_{j=1}^{n} P_j(q)$, plus the utility associated with the supply chain aspect of delivery of the relief items, $\sum_{j=1}^{n} \gamma_{ij} q_{ij} - \sum_{j=1}^{n} c_{ij}(q_{ij})$. The optimization problem faced by NGO i; $i = 1, \ldots, m$, is, hence,

$$\text{Maximize} \quad \beta_i \sum_{j=1}^{n} P_j(q) + \omega_i \sum_{j=1}^{n} \gamma_{ij} q_{ij} - \sum_{j=1}^{n} c_{ij}(q_{ij}) \qquad (5)$$

subject to constraints (1) and (2).

The Game Theory Model

We also have that, at each demand point j; $j = 1, \ldots, n$:

$$\sum_{i=1}^{m} q_{ij} \geq \underline{d}_j, \tag{6}$$

and

$$\sum_{i=1}^{m} q_{ij} \leq \bar{d}_j, \tag{7}$$

where $\underline{d}_j$ denotes a lower bound for the amount of the relief items needed at demand point j and $\bar{d}_j$ denotes an upper bound on the amount of the relief items needed post the disaster at demand point j.

We assume that

$$\sum_{i=1}^{m} s_i \geq \sum_{j=1}^{n} \underline{d}_j, \tag{8}$$

so that the supply resources of the NGOs are sufficient to meet the minimum financial resource needs.

The Game Theory Model

Each NGO i; $i = 1, \ldots, m$, seeks to determine its optimal vector of relief items or strategies, q_i^*, that maximizes objective function (5), subject to constraints (1), (2), and (6), (7). This is the Generalized Nash Equilibrium problem for our humanitarian relief post disaster problem.

Theorem: Optimization Formulation of the Generalized Nash Equilibrium Model of Financial Flow of Funds

The above Generalized Nash Equilibrium problem, with each NGO's objective function (5) rewritten as:

$$\text{Minimize} \quad -\beta_i \sum_{j=1}^{n} P_j(q) - \omega_i \sum_{j=1}^{n} \gamma_{ij} q_{ij} + \sum_{j=1}^{n} c_{ij}(q_{ij}) \qquad (9)$$

and subject to constraints (1) and (2), with common constraints (6) and (7), is equivalent to the solution of the following optimization problem:

$$\text{Minimize} \quad -\sum_{j=1}^{n} P_j(q) - \sum_{i=1}^{m}\sum_{j=1}^{n} \frac{\omega_i \gamma_{ij}}{\beta_i} q_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n} \frac{1}{\beta_i} c_{ij}(q_{ij}) \quad (10)$$

subject to constraints: (1), (2), (6), and (7).

The Game Theory Model

Variational Inequality (VI) Formulation

The solution q^* with associated Lagrange multipliers λ_k^*, $\forall k$, for the supply constraints; Lagrange multipliers: λ_l^{1*}, $\forall l$, for the lower bound demand constraints, and Lagrange multipliers: λ_l^{2*}, $\forall k$, for the upper bound demand constraints, can be obtained by solving the VI problem: determine $(q^*, \lambda^*, \lambda^{1*}, \lambda^{2*}) \in R_+^{mn+m+2n}$:

$$\sum_{k=1}^{m}\sum_{l=1}^{n}\left[-\sum_{j=1}^{n}(\frac{\partial P_j(q^*)}{\partial q_{kl}}) - \frac{\omega_k \gamma_{kl}}{\beta_k} + \frac{1}{\beta_k}\frac{\partial c_{kl}(q_{kl}^*)}{\partial q_{kl}} + \lambda_k^* - \lambda_l^{1*} + \lambda_l^{2*}\right] \times [q_{kl} - q_{kl}^*]$$

$$+\sum_{k=1}^{m}(s_k - \sum_{l=1}^{n} q_{kl}^*) \times (\lambda_k - \lambda_k^*) + \sum_{l=1}^{n}(\sum_{k=1}^{n} q_{kl}^* - \underline{d}_l) \times (\lambda_l - \lambda_l^{1*})$$

$$+\sum_{l=1}^{n}(\bar{d}_l - \sum_{k=1}^{m} q_{kl}^*) \times (\lambda_l^2 - \lambda_l^{2*}) \geq 0, \quad \forall (q, \lambda, \lambda^1, \lambda^2) \in R_+^{mn+m+2n}, \tag{11}$$

The Algorithm

The Algorithm

Explicit Formulae for the Euler Method Applied to the Game Theory Model

We have the following closed form expression for the product flows $k = 1, \ldots, m$; $l = 1, \ldots, n$, at each iteration:

$$q_{kl}^{\tau+1}$$

$$= \max\{0, \{q_{kl}^{\tau} + a_{\tau}(\sum_{j=1}^{n}(\frac{\partial P_j(q^{\tau})}{\partial q_{kl}}) + \frac{\omega_k \gamma_{kl}}{\beta_{kl}} - \frac{1}{\beta_k}\frac{\partial c_{kl}(q_{kl}^{\tau})}{\partial q_{kl}} - \lambda_k^{\tau} + \lambda_l^{1^{\tau}} - \lambda_l^{2^{\tau}})\}\}$$

the following closed form expressions for the Lagrange multipliers associated with the supply constraints, respectively, for $k = 1, \ldots, m$:

$$\lambda_k^{\tau+1} = \max\{0, \lambda_k^{\tau} + a_{\tau}(-s_k + \sum_{l=1}^{n} q_{kl}^{\tau})\}.$$

The Algorithm

The following closed form expressions are for the Lagrange multipliers associated with the lower bound demand constraints, respectively, for $l = 1, \ldots, n$:

$$\lambda_l^{1\,\tau+1} = \max\{0, \lambda_l^{1\,\tau} + a_\tau(-\sum_{k=1}^{n} q_{kl}^{\tau} + \underline{d}_l)\}.$$

The following closed form expressions are for the Lagrange multipliers associated with the upper bound demand constraints, respectively, for $l = 1, \ldots, n$:

$$\lambda_l^{2\,\tau+1} = \max\{0, \lambda_l^{2\,\tau} + a_\tau(-\bar{d}_l + \sum_{k=1}^{m} q_{kl}^{\tau})\}.$$

Hurricane Katrina Case Study

Hurricane Katrina Case Study

Making landfall in August of 2005, Katrina caused extensive damages to property and infrastructure, left 450,000 people homeless, and took 1,833 lives in Florida, Texas, Mississippi, Alabama, and Louisiana (Louisiana Geographic Information Center (2005)).

Given the hurricane's trajectory, most of the damage was concentrated in Louisiana and Mississippi. In fact, 63% of all insurance claims were in Louisiana, a trend that is also reflected in FEMA's post-hurricane damage assessment of the region (FEMA (2006)).

Hurricane Katrina Case Study

The total damage estimates range from \$105 billion (Louisiana Geographic Information Center (2005)) to \$150 billion (White (2015)), making Hurricane Katrina not only a far-reaching and costly disaster, but also a very challenging environment for providing humanitarian assistance.

We consider 3 NGOs: the Red Cross, the Salvation Army, and Others.

The structure of the P_j functions is as follows:

$$P_j(q) = k_j \sqrt{\sum_{i=1}^{m} q_{ij}}.$$

The weights are:

$$\omega_1 = \omega_2 = \omega_3 = 1,$$

with $\gamma_{ij} = 950$ for $i = 1, 2, 3$ and $j = 1, \ldots, 10$.

Hurricane Katrina Case Study

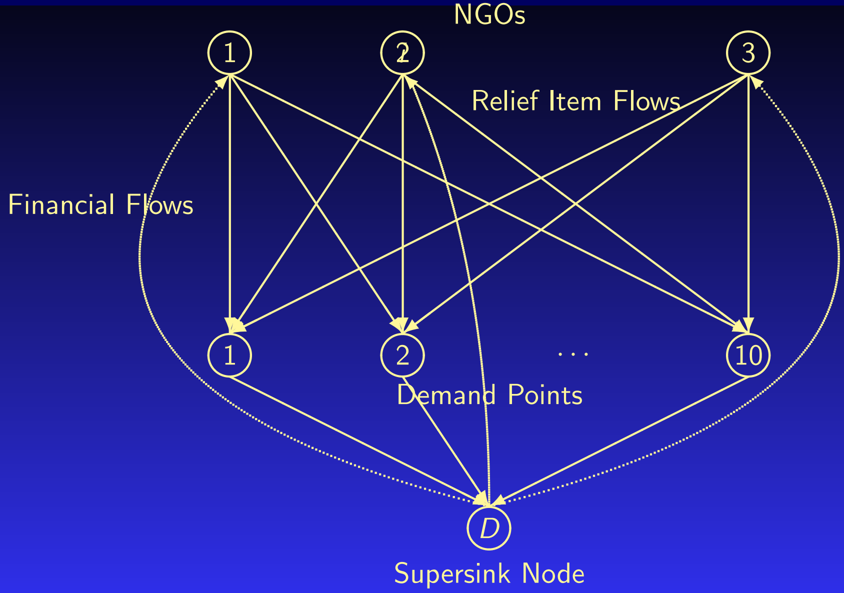


Figure 2: Hurricane Katrina Relief Network Structure

Hurricane Katrina Case Study

Hurricane Katrina Demand Point Parameters					
Parish	Node j	k_j	$\underline{d}_j$	$\bar{d}_j$	p_j (in %)
St. Charles	1	8	16.45	50.57	2.4
Terrebonne	2	16	752.26	883.82	6.7
Assumption	3	7	106.36	139.24	1.9
Jefferson	4	29	742.86	1,254.89	19.5
Lafourche	5	6	525.53	653.82	1.7
Orleans	6	42	1,303.99	1,906.80	55.9
Plaquemines	7	30	33.28	62.57	57.5
St. Barnard	8	42	133.61	212.43	78.4
St. James	9	9	127.53	166.39	1.2
St. John the Baptist	10	7	19.05	52.59	6.7

Table 1: Demand Point Data for the Generalized Nash Equilibrium Problem for Hurricane Katrina

Hurricane Katrina Case Study

We then estimated the cost of providing aid to the Parishes as a function of the total damage in the area and the supply chain efficiency of each NGO. We assume that these costs follow the structures observed by Van Wassenhove (2006) and randomly generate a number based on his research with a mean of $\hat{p} = .8$ and standard deviation of $s = \sqrt{\frac{.8(.2)}{3}}$.

We denote the corresponding coefficients by π_i. Thus, each NGO i; $i = 1, 2, 3$, incurs costs according the the following functional form:

$$c_{ij}(q_{ij}) = \left(\pi_i q_{ij} + \frac{1}{1 - p_j}\right)^2.$$

Hurricane Katrina Case Study

Data Parameters for NGOs Providing Aid					
NGO	i	π_i	γ_{ij}	β_i	s_i
Others	1	.82	950	.355	1,418
Red Cross	2	.83	950	.55	2,200
Salvation Army	3	.81	950	.095	382

Table 2: NGO Data for the Generalized Nash Equilibrium Problem for Hurricane Katrina

Hurricane Katrina Case Study

Generalized Nash Equilibrium Product Flows			
Demand Point	Others	Red Cross	Salvation Army
St. Charles	17.48	28.89	4.192
Terrebonne	267.023	411.67	73.57
Assumption	49.02	77.26	12.97
Jefferson	263.69	406.68	72.45
Lafourche	186.39	287.96	51.18
Orleans	463.33	713.56	127.1
Plaquemines	21.89	36.54	4.23
St. Barnard	72.31	115.39	16.22
St. James	58.67	92.06	15.66
St. John the Baptist	18.2	29.99	4.40

Table 3: Flows to Demand Points under Generalized Nash Equilibrium

Hurricane Katrina Case Study

The total utility obtained through the above flows for the Generalized Nash Equilibrium for Hurricane Katrina is $9,257,899$, with the Red Cross capturing 3,022,705, the Salvation Army 3,600,442.54, and Others 2,590,973. It is interesting to see that, despite having the lowest available supplies, the Salvation Army is able to capture the largest part of the total utility. This is due to the fact that the costs of providing aid grow at a nonlinear rate, so even if the Salvation Army was less efficient and used all of its available supplies, it will not be capable of providing the most expensive supplies.

In addition, we have that the Red Cross, the Salvation Army, and Others receive 2,200.24, 1418.01, and 382.31 million in donations, respectively. Also, notice how the flows meet at least the lower bound, even if doing so is very expensive due to the damages to the infrastructure in the region.

Hurricane Katrina Case Study

Furthermore, the above flow pattern behaves in a way that, after the minimum requirements are met, any additional supplies are allocated in the most efficient way. For example, only the minimum requirements are met in New Orleans Parish, while the upper bound is met for St. James Parish.

The Nash Equilibrium Solution

If we remove the shared constraints, we obtain a Nash Equilibrium solution, and we can compare the outcomes of the humanitarian relief efforts for Hurricane Katrina under the Generalized Nash Equilibrium concept and that under the Nash Equilibrium concept.

The Nash Equilibrium Solution

Nash Equilibrium Product Flows			
Demand Point	Others	Red Cross	Salvation Army
St. Charles	142.51	220.66	38.97
Terrebonne	142.50	220.68	38.93
Assumption	142.51	220.66	38.98
Jefferson	142.38	220.61	38.74
Lafourche	142.50	220.65	38.98
Orleans	141.21	219.59	37.498
Plaquemines	141.032	219.28	37.37
St. Barnard	138.34	216.66	34.59
St. James	142.51	220.65	38.58
St. John the Baptist	145.51	220.66	38.98

Table 4: Flows to Demand Points under Nash Equilibrium

The Nash Equilibrium Solution

Under the Nash Equilibrium, the NGOs obtain a higher utility than under the Generalized Nash Equilibrium. Specifically, of the total utility $10,346,005.44$, 2,804,650 units are received by the Red Cross, 5,198,685 by the Salvation Army, and 3,218,505 are captured by all other NGOs.

Under this product flow pattern, there are total donations of 3,760.73, of which 2,068.4 are donated to the Red Cross, 357.27 to the Salvation Army, and 1,355 to the other players.

The Nash Equilibrium Solution

It is clear that there is a large contrast between the flow patterns under the Generalized Nash and Nash Equilibria. For example, the Nash Equilibrium flow pattern results in about $500 million less in donations.

While this has strong implications about how collaboration between NGOs can be beneficial for their fundraising efforts, the differences in the general flow pattern highlights a much stronger point.

Additional Insights

Under the Nash Equilibrium, NGOs successfully maximize their utility. Overall, the Nash Equilibrium solution leads to an increase of utility of roughly 21% when compared to the flow patterns under the Generalized Nash Equilibrium. But they do so at the expense of those in need. In the Nash Equilibrium, each NGO chooses to supply relief items such that costs can be minimized. On the surface, this might be a good thing, but recall that, given the nature of disasters, it is usually more expensive to provide aid to demand points with the greatest needs.

Additional Insights

With this in mind, one can expect oversupply to the demand points with lower demand levels, and undersupply to the most affected under a purely competitive scheme. This behavior can be seen explicitly in the results summarized in the Tables.

For example, St. Charles Parish receives roughly 795% of its upper demand, while Orleans Parish only receives about 30.5% of its minimum requirements. That means that much of the 21% in 'increased' utility is in the form of waste.

In contrast, the flows under the Generalized Nash Equilibrium guarantee that minimum requirements will be met and that there will be no waste; that is to say, *as long as there is a coordinating authority that can enforce the upper and lower bound constraints, the humanitarian relief flow patterns under this bounded competition will be significantly better than under untethered competition.*

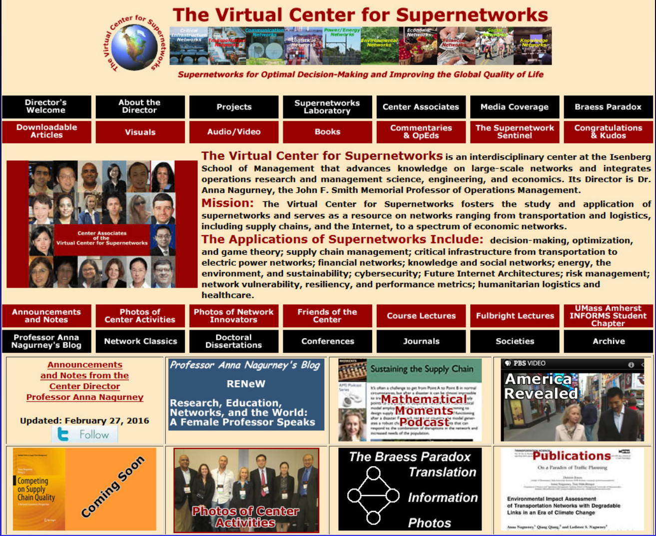
Additional Insights

In addition, we found that changes to the values in the functional form result in changes in the product flows, but the general behavioral differences are robust to changes in the coefficients: β_i, γ_{ij}, k_j, $\forall i, j$, and the bounds on upper and lower demand estimates.

Summary and Conclusions

- We presented a Generalized Nash Equilibrium model, with a special case being a Nash Equilibrium model, for disaster relief with supply chain and financial fund aspects for each NGO's objective function.
- Each NGO obtains utility from providing relief to demand points post a disaster and also seeks to minimize costs but can gain in financial donations based on the visibility of the NGOs in terms of product deliveries to the demand points.
- A case study based on Hurricane Katrina was discussed.
- All the models were network-based and provide new insights in terms of disaster relief and management.

THANK YOU!



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