Fragile Networks: Identifying Vulnerabilities and Synergies in an Uncertain Age

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Tutorial – Part III

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Outline of Tutorial

- Part I: Network Fundamentals, Efficiency Measurement, and Vulnerability Analysis
- Part II: Applications and Extensions
- Part III: Mergers and Acquisitions, Network Integration, and Synergies

We have been focusing on network vulnerability and robustness analysis. We also have results in terms of *synergy* in the case of network integration as would occur in mergers and acquisitions.

In this framework we model the economic activities of each firm as a S-O problem on a network.

Relevant References

- A System-Optimization Perspective for Supply Chain Network Integration: The Horizontal Merger Case, Nagurney, *Transportation Research E* 45: (2009), pp 1-19.

- Multiproduct Supply Chain Horizontal Network Integration: Models, Theory, and Computational Results, Nagurney, Woolley, and Qiang, *International Transactions in Operational Research* 17: (2010), pp 333-349.

Mergers and Acquisitions and Supply Chain Network Synergies

Today, supply chains are more extended and complex than ever before. At the same time, the current competitive economic environment requires that firms operate efficiently, which has spurred research to determine how to utilize supply chains more effectively.

There is also a pronounced amount of merger activity. According to Thomson Financial, in the first nine months of 2007 alone, worldwide merger activity hit $3.6 trillion, surpassing the total from all of 2006 combined.

Notable examples: KMart and Sears in the retail industry in 2004 and Federated and May in 2005, Coors and Molson in the beverage industry in 2005, and the recently proposed merger between Anheuser Busch and InBev.

According to Kusstatscher and Cooper (2005) there were five major waves of of Merger & Acquisition (M &A) activity:

The First Wave: 1898-1902: an increase in horizontal mergers that resulted in many US industrial groups;

The Second Wave: 1926-1939: mainly public utilities;

The Third Wave: 1969-1973: diversification was the driving force;

The Fourth Wave: 1983-1986: the goal was efficiency;

The Fifth Wave: 1997 until the early years of the 21st century: globalization was the motto.

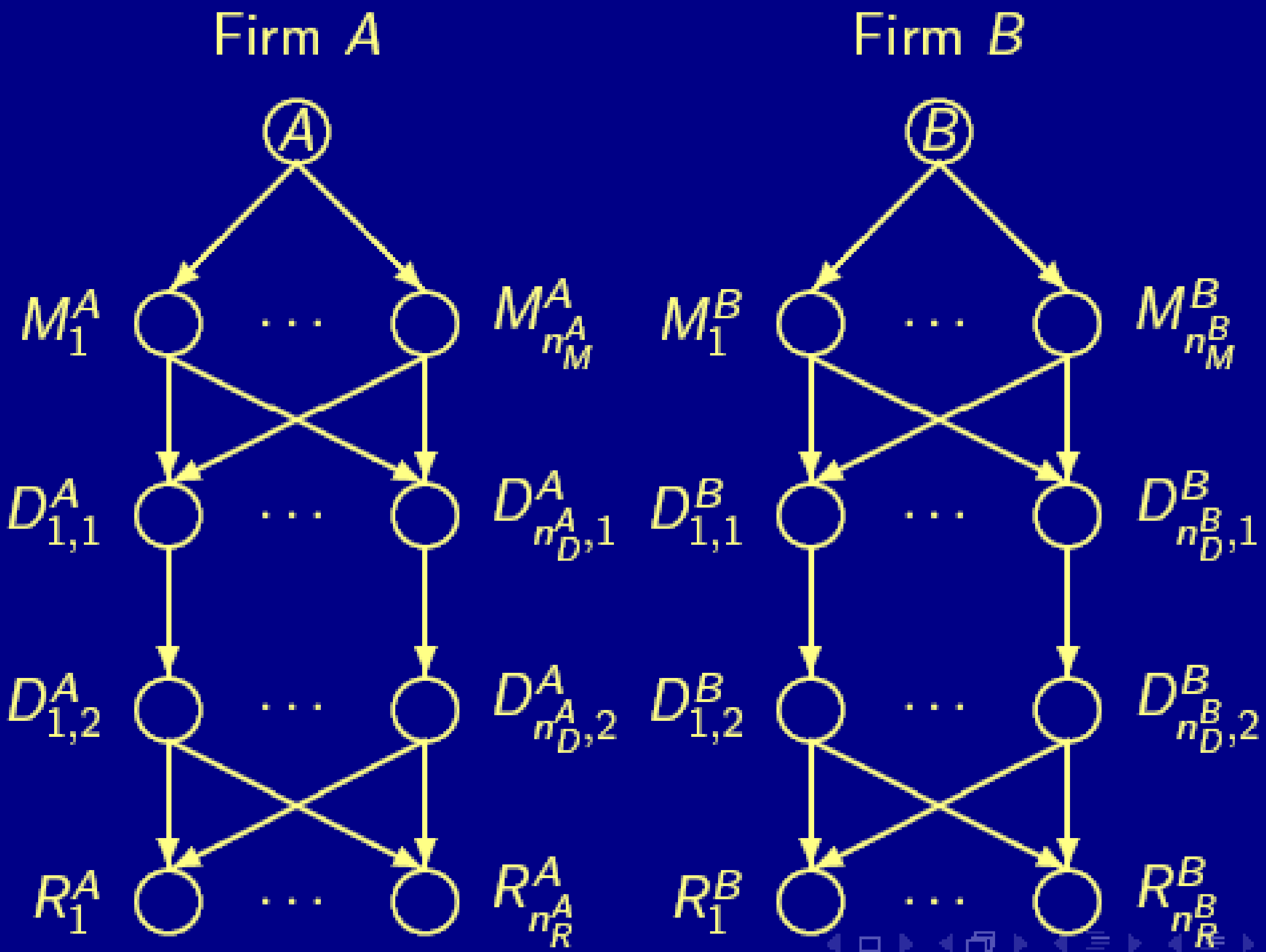
In 1998, M&As reached $2.1 trillion worldwide; in 1999, the activity exceeded $3.3 trillion, and in 2000, almost $3.5 was reached.

A survey of 600 executives involved in their companies' mergers and acquisitions (M&A) conducted by Accenture and the Economist Unit (see Byrne (2007)) found that less than half (45%) achieved expected cost-saving synergies.

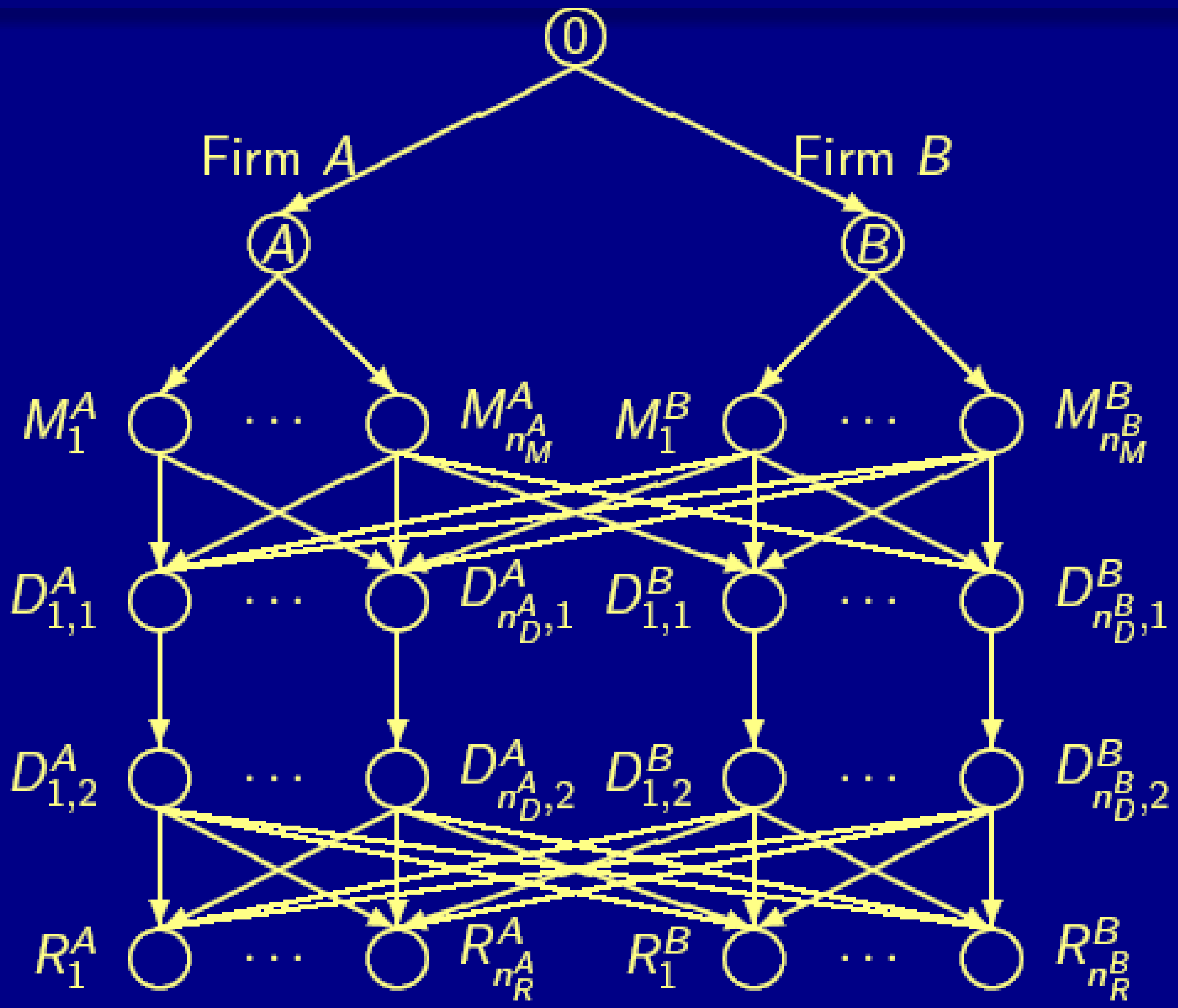
Langabeer and Seifert (2003) determined a direct correlation between how effectively supply chains of merged firms are integrated and how successful the merger is. They concluded, based on the empirical findings of Langabeer (2003), who analyzed hundreds of mergers over the preceding decade, that

Improving Supply Chain Integration between Merging Companies is the Key to Improving the Likelihood of Post-Merger Success!

Supply Chain Prior to the Merger



Supply Chain Post-Merger



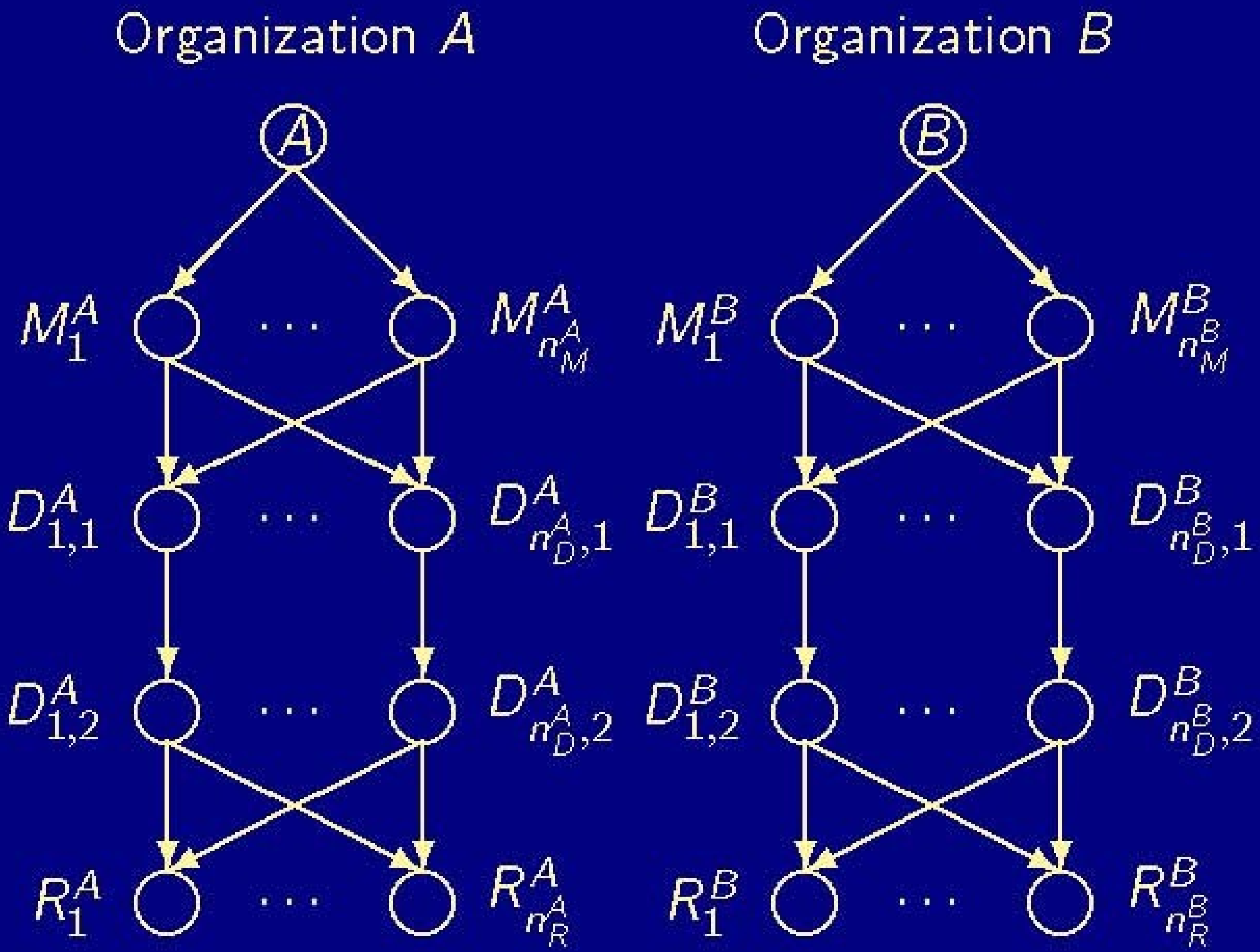
Quantifying the Synergy of the Merger

The synergy associated with the total generalized costs which captures the total generalized costs is defined as:

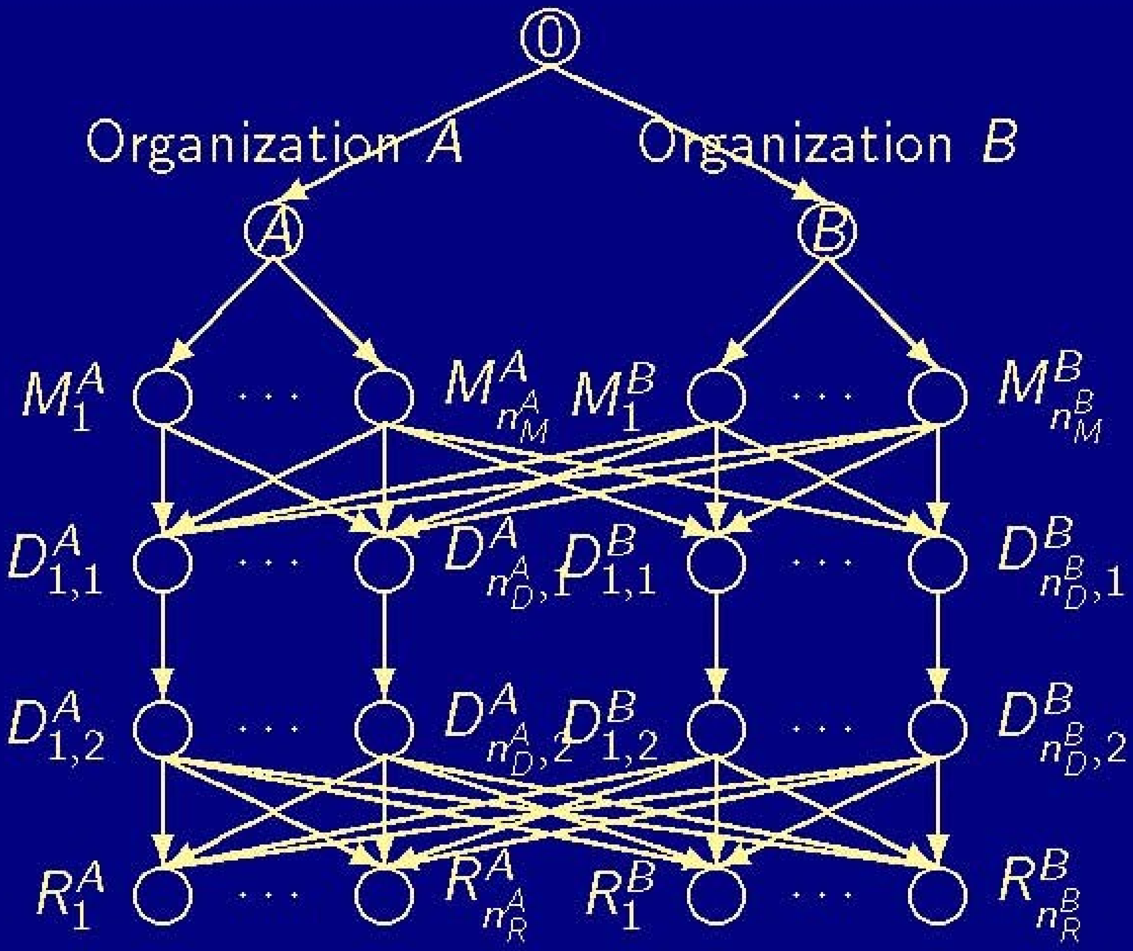
$$S^{TGC} \equiv \left[\frac{TGC^0 - TGC^1}{TGC^0}\right] \times 100\%$$

This framework can also be applied to teaming of humanitarian organizations in the case of humanitarian logistics operations; http://hlogistics.som.umass.edu

Supply Chains of Humanitarian Organizations A and B Prior to the Integration



Supply Chain Network after Humanitarian Organizations A and B Integrate their Chains



Quantifying Synergy Associated with the Integration of Multiproduct Decision-Making Organizations

The synergy based on total costs and proposed by Nagurney (2007), but now in a multiproduct context, which we denote here by $\mathcal{S}^{TC}$, can be calculated as the percentage difference between the total cost pre *vs* the total cost post the integration:

$$\mathcal{S}^{TC} \equiv [\frac{TC^0 - TC^1}{TC^0}] \times 100\%.$$

We have seen that, in fact, network design and redesign can be accomplished through the addition/deletion of nodes and links; through the alteration of link capacities, as well as through the integration of different networks.

Now we would like to highlight an explicit network design problem of specific relevance.

A Challenging Network Design Problem

The number of disasters is increasing globally, as is the number of people affected by disasters. At the same time, with the advent of increasing globalization, viruses are spreading more quickly and creating new challenges for medical and health professionals, researchers, and government officials.

Between 2000 and 2004 the average annual number of disasters was 55% higher than in the period 1994 through 1999, with 33% more humans affected in the former period than in the latter (cf. Balcik and Beamon (2008) and Nagurney and Qiang (2009)).

However, although the average number of disasters has been increasing annually over the past decade the average percentage of needs met by different sectors in the period 2000 through 2005 identifies significant shortfalls.

According to Development Initiatives (2006), based on data in the Financial Tracking System of the Office for the Coordination of Humanitarian Affairs, from 2000-2005, the average needs met by different sectors in the case of disasters were:

- 79% by the food sector;
- 37% of the health needs;
- 35% of the water and sanitation needs;
- 28% of the shelter and non-food items, and
- 24% of the economic recovery and infrastructure needs.

Hurricane Katrina in 2005

Hurricane Katrina has been called an “American tragedy,” in which essential services failed completely (Guidotti (2006)).

Haiti Earthquake in 2010

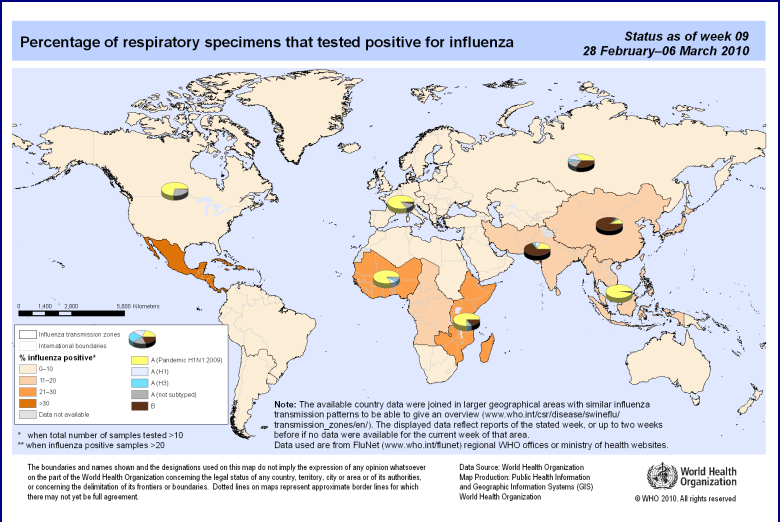
Delivering the humanitarian relief supplies (water, food, medicines, etc.) to the victims was a major logistical challenge.

H1N1 (Swine) Flu

As of May 2, 2010, worldwide, more than 214 countries and overseas territories or communities have reported laboratory confirmed cases of pandemic influenza H1N1 2009, including over 18,001 deaths (www.who.int).

Parts of the globe experienced serious flu vaccine shortages, both seasonal and H1N1 (swine) ones, in late 2009.

Map of Influenza Activity and Virus Subtypes



Source: World Health Organization

Underlying the delivery of goods and services in times of crises, such as in the case of disasters, pandemics, and life-threatening major disruptions, are supply chains, without which essential products do not get delivered in a timely manner, with possible increased disease, injuries, and casualties.

It is clear that better-designed supply chain networks would have facilitated and enhanced various emergency preparedness and relief efforts and would have resulted in less suffering and lives lost.

Supply Chain Networks provide the logistical backbones for the provision of products as well as services both in corporate as well as in emergency and humanitarian operations.

Here we focus on supply chains in the case of

Critical Needs Products.

Critical Needs Products

Critical needs products are those that are essential to the survival of the population, and can include, for example, vaccines, medicine, food, water, etc., depending upon the particular application.

The demand for the product should be met as nearly as possible since otherwise there may be additional loss of life.

In times of crises, a system-optimization approach is mandated since the demands for critical supplies should be met (as nearly as possible) at minimal total cost.

An Overview of the Relevant Literature

- M. J. Beckmann, C. B. McGuire, and C. B. Winsten (1956) *Studies in the Economics of Transportation*, Yale University Press, New Haven, Connecticut.
- S. C. Dafermos and F. T. Sparrow (1969) "The Traffic Assignment Problem for a General Network," *Journal of Research of the National Bureau of Standards* **73B**, 91-118.
- D. E. Boyce, H. S. Mahmassani, and A. Nagurney (2005) "A Retrospective on Beckmann, McGuire, and Winsten's *Studies in the Economics of Transportation*," *Papers in Regional Science* **84**, 85-103.
- A. Nagurney (2009), "A System-Optimization Perspective for Supply Chain Network Integration: The Horizontal Merger Case," *Transportation Research E* **45**, 1-15.

- A. Nagurney, T. Woolley, and Q. Qiang (2010) "Multiproduct Supply Chain Horizontal Network Integration: Models, Theory, and Computational Results," *International Journal of Operational Research* **17**, 333-349.
- A. Nagurney (2010) "Formulation and Analysis of Horizontal Mergers Among Oligopolistic Firms with Insights into the Merger Paradox: A Supply Chain Network Perspective," *Computational Management Science*, in press.
- A. Nagurney (2010) "Supply Chain Network Design Under Profit Maximization and Oligopolistic Competition," *Transportation Research E* **46**, 281-294.
- A. Nagurney and L. S. Nagurney (2009) "Sustainable Supply Chain Network Design: A Multicriteria Perspective," to appear in the *International Journal of Sustainable Engineering*.

This part of the tutorial is based on the paper:

"Supply Chain Network Design for Critical Needs with Outsourcing,"

A. Nagurney, M. Yu, and Q. Qiang, to appear in *Papers in Regional Science*,

where additional background as well as references can be found.

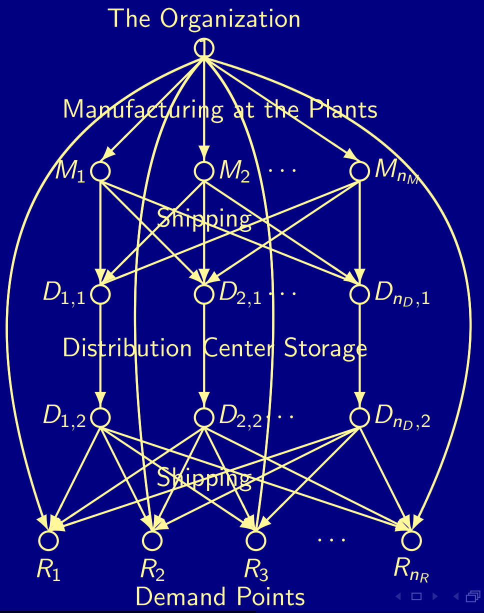
The Supply Chain Network Design Model for Critical Needs with Outsourcing

We assume that the organization (government, humanitarian one, socially responsible firm, etc.) is considering n_M manufacturing facilities/plants; n_D distribution centers, but must serve the n_R demand points.

The supply chain network is modeled as a network $G = [N, L]$, consisting of the set of nodes N and the set of links L. Let L^1 and L^2 denote the links associated with "in house" supply chain activities and the outsourcing activities, respectively. The paths joining the origin node to the destination nodes represent sequences of supply chain network activities that ensure that the product is produced and, ultimately, delivered to those in need at the demand points.

The optimization model can handle both design (from scratch) and redesign scenarios.

Supply Chain Network Topology with Outsourcing



The Links

The possible manufacturing links from the top-tiered node 1 are connected to the possible manufacturing nodes of the organization, which are denoted, respectively, by: $M_1, \ldots, M_{n_M}$.

The possible shipment links from the manufacturing nodes, are connected to the possible distribution center nodes of the organization, denoted by $D_{1,1}, \ldots, D_{n_D,1}$.

The links joining nodes $D_{1,1}, \ldots, D_{n_D,1}$ with nodes $D_{1,2}, \ldots, D_{n_D,2}$ correspond to the possible storage links.

There are possible shipment links joining the nodes $D_{1,2}, \ldots, D_{n_D,2}$ with the demand nodes: $R_1, \ldots, R_{n_R}$.

There are also outsourcing links, which may join the top node to each bottom node (or the relevant nodes for which the outsourcing activity is feasible, as in production, storage, or distribution, or a combination thereof). The organization does not control the capacities on these links since they have been established by the particular firm that corresponds to the outsource link.

The ability to outsource supply chain network activities for critical needs products provides alternative pathways for the production and delivery of products during times of crises such as disasters.

Demands, Path Flows, and Link Flows

Let d_k denote the demand at demand point k; $k = 1, \ldots, n_R$, which is a random variable with probability density function given by $\mathcal{F}_k(t)$. Let x_p represent the nonnegative flow of the product on path p; f_a denote the flow of the product on link a.

Conservation of Flow Between Path Flows and Link Flows

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L, \tag{1}$$

that is, the total amount of a product on a link is equal to the sum of the flows of the product on all paths that utilize that link. $\delta_{ap} = 1$ if link a is contained in path p, and $\delta_{ap} = 0$, otherwise.

Supply Shortage and Surplus

Let

$$v_k \equiv \sum_{p \in P_{w_k}} x_p, \quad k = 1, \ldots, n_R, \tag{2}$$

where v_k can be interpreted as the *projected demand* at demand market k; $k = 1, \ldots, n_R$. Then,

$$\Delta_k^- \equiv \max\{0, d_k - v_k\}, \quad k = 1, \ldots, n_R, \tag{3}$$

$$\Delta_k^+ \equiv \max\{0, v_k - d_k\}, \quad k = 1, \ldots, n_R, \tag{4}$$

where Δ_k^- and Δ_k^+ represent the supply shortage and surplus at demand point k, respectively. The expected values of Δ_k^- and Δ_k^+ are given by:

$$E(\Delta_k^-) = \int_{v_k}^{\infty} (t - v_k)\mathcal{F}_k(t)d(t), \quad k = 1, \ldots, n_R, \tag{5}$$

$$E(\Delta_k^+) = \int_0^{v_k} (v_k - t)\mathcal{F}_k(t)d(t), \quad k = 1, \ldots, n_R. \tag{6}$$

The Operation Costs, Investment Costs and Penalty Costs

The total cost on a link is assumed to be a function of the flow of the product on the link. We have, thus, that

$$\hat{c}_a = \hat{c}_a(f_a), \quad \forall a \in L. \tag{7}$$

We denote the nonnegative existing capacity on a link a by $\bar{u}_a$, $\forall a \in L$. Note that the organization can add capacity to the "in house" link a; $\forall a \in L^1$. We assume that

$$\hat{\pi}_a = \hat{\pi}_a(u_a), \quad \forall a \in L^1. \tag{8}$$

The expected total penalty at demand point k; $k = 1, \ldots, n_R$, is,

$$E(\lambda_k^- \Delta_k^- + \lambda_k^+ \Delta_k^+) = \lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+), \tag{9}$$

where λ_k^- is the unit penalty of supply shortage at demand point k and λ_k^+ is that of supply surplus. Note that $\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)$ is a function of the path flow vector x.

The Supply Chain Network Design Optimization Problem

The organization seeks to determine the optimal levels of product processed on each supply chain network link (including the outsourcing links) coupled with the optimal levels of capacity investments in its supply chain network activities subject to the minimization of the total cost.

The total cost includes the total cost of operating the various links, the total cost of capacity investments, and the expected total supply shortage/surplus penalty.

The Supply Chain Network Design Optimization Problem

$$\text{Minimize} \quad \sum_{a\in L} \hat{c}_a(f_a) + \sum_{a\in L^1} \hat{\pi}_a(u_a) + \sum_{k=1}^{n_R} (\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)) \tag{10}$$

subject to: constraints (1), (2) and

$$f_a \leq \bar{u}_a + u_a, \quad \forall a \in L^1, \tag{11}$$

$$f_a \leq \bar{u}_a, \quad \forall a \in L^2, \tag{12}$$

$$u_a \geq 0, \quad \forall a \in L^1, \tag{13}$$

$$x_p \geq 0, \quad \forall p \in P. \tag{14}$$

The Feasible Set

We associate the Lagrange multiplier ω_a with constraint (11) for link $a \in L^1$ and we denote the associated optimal Lagrange multiplier by ω_a^*. Similarly, Lagrange multiplier γ_a is associated with constraint (12) for link $a \in L^2$ with the optimal multiplier denoted by γ_a^*. These two terms may also be interpreted as the price or value of an additional unit of capacity on link a. We group these Lagrange multipliers into the vectors ω and γ, respectively. Let K denote the feasible set such that

$$K \equiv \{(x, u, \omega, \gamma) | x \in R_+^{n_P}, u \in R_+^{n_{L^1}}, \omega \in R_+^{n_{L^1}}, \text{ and } \gamma \in R_+^{n_{L^2}}\}.$$

Theorem

The optimization problem is equivalent to the variational inequality problem: determine the vector of optimal path flows, the vector of optimal link capacity enhancements, and the vectors of optimal Lagrange multipliers $(x^*, u^*, \omega^*, \gamma^*) \in K$*, such that:*

$$\sum_{k=1}^{n_R} \sum_{p \in P_{w_k}} \left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} + \sum_{a \in L^1} \omega_a^* \delta_{ap} + \sum_{a \in L^2} \gamma_a^* \delta_{ap} + \lambda_k^+ P_k \left(\sum_{p \in P_{w_k}} x_p^* \right) \right.$$

$$\left. - \lambda_k^- \left(1 - P_k \left(\sum_{p \in P_{w_k}} x_p^* \right) \right) \right] \times [x_p - x_p^*]$$

$$+ \sum_{a \in L^1} \left[\frac{\partial \hat{\pi}_a(u_a^*)}{\partial u_a} - \omega_a^* \right] \times [u_a - u_a^*] + \sum_{a \in L^1} [\bar{u}_a + u_a^* - \sum_{p \in P} x_p^* \delta_{ap}] \times [\omega_a - \omega_a^*]$$

$$+ \sum_{a \in L^2} [\bar{u}_a - \sum_{p \in P} x_p^* \delta_{ap}] \times [\gamma_a - \gamma_a^*] \geq 0, \quad \forall (x, u, \omega, \gamma) \in K. \qquad (15)$$

Theorem (cont'd.)

In addition, (15) can be reexpressed in terms of links flows as: determine the vector of optimal link flows, the vectors of optimal projected demands and link capacity enhancements, and the vectors of optimal Lagrange multipliers $(f^*, v^*, u^*, \omega^*, \gamma^*) \in K^1$*, such that:*

$$\sum_{a\in L^1}\left[\frac{\partial \hat{c}_a(f_a^*)}{\partial f_a}+\omega_a^*\right]\times[f_a-f_a^*]+\sum_{a\in L^2}\left[\frac{\partial \hat{c}_a(f_a^*)}{\partial f_a}+\gamma_a^*\right]\times[f_a-f_a^*]$$

$$+\sum_{a\in L^1}\left[\frac{\partial \hat{\pi}_a(u_a^*)}{\partial u_a}-\omega_a^*\right]\times[u_a-u_a^*]$$

$$+\sum_{k=1}^{n_R}\left[\lambda_k^+P_k(v_k^*)-\lambda_k^-(1-P_k(v_k^*))\right]\times[v_k-v_k^*]+\sum_{a\in L^1}[\bar{u}_a+u_a^*-f_a^*]\times[\omega_a-\omega_a^*]$$

$$+\sum_{a\in L^2}[\bar{u}_a-f_a^*]\times[\gamma_a-\gamma_a^*]\geq 0, \quad \forall (f,v,u,\omega,\gamma)\in K^1, \qquad (16)$$

where $K^1 \equiv \{(f,v,u,\omega,\gamma) | \exists x \geq 0,$ *and* $(1),(2),(13),$ *and* (14) *hold, and* $\omega \geq 0, \quad \gamma \geq 0\}.$

Applications to Vaccine Production

Consider a vaccine manufacturer who is gearing up for next year's production of H1N1 (swine) flu vaccine. Governments around the world are beginning to contract with this company for next year's flu vaccine.

By applying the general theoretical model to the company's data, the firm can determine whether it needs to expand its facilities (or not), how much of the vaccine to produce where, how much to store where, and how much to have shipped to the various demand points. Also, it can determine whether it should outsource any of its vaccine production and at what level.

The firm by solving the model with its company-relevant data can then ensure that the price that it receives for its vaccine production and delivery is appropriate and that it recovers its incurred costs and obtains, if negotiated correctly, an equitable profit.

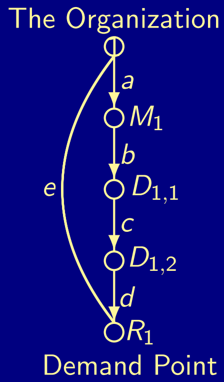
Applications to Emergency Preparedness and Humanitarian Logistics

A company can, using the model, prepare and plan for an emergency such as a natural disaster in the form of a hurricane and identify where to store a necessary product (such as food packets, for example) so that the items can be delivered to the demand points in a timely manner and at minimal total cost.

In August 2005 Hurricane Katrina hit the US and this natural disaster cost immense damage with repercussions that continue to this day. While US state and federal officials came under severe criticism for their handling of the storm's aftermath, Wal-Mart had prepared in advance and through its logistical efficiencies had dozens of trucks loaded with supplies for delivery before the hurricane even hit landfall.

Numerical Examples

Consider the supply chain network topology in which the organization is considering a single manufacturing plant, a single distribution center for storing the critical need product and is to serve a single demand point. The links are labeled, that is, a, b, c, d, and e, with e denoting the outsourcing link.



Example 1

The total cost functions on the links were:

$$\hat{c}_a(f_a) = .5f_a^2 + f_a, \quad \hat{c}_b(f_b) = .5f_b^2 + 2f_b, \quad c_c(f_c) = .5f_c^2 + f_c,$$

$$\hat{c}_d(f_d) = .5f_d^2 + 2f_d, \quad \hat{c}_e(f_e) = 5f_e.$$

The investment capacity cost functions were:

$$\hat{\pi}_a(u_a) = .5u_a^2 + u_a, \quad \forall a \in L^1.$$

The existing capacities were: $\bar{u}_a = 0, \quad \forall a \in L^1$, and $\bar{u}_e = 2$.

The demand for the product followed a uniform distribution on the interval $[0, 10]$ so that:

$$P_1(\sum_{p \in P_{w_1}} x_p) = \frac{\sum_{p \in P_{w_1}} x_p}{10}.$$

The penalties were: $\lambda_1^- = 10, \quad \lambda_1^+ = 0.$

Example 2

Example 2 had the same data as Example 1 except that we now increased the penalty associated with product shortage from 10 to 50, that is, we now set $\lambda_1^- = 50$.

Example 3

Example 3 had the same data as Example 2 except that $\bar{u}_a = 3$ for all the links $a \in L^1$. This means that the organization does not have to construct its supply chain activities from scratch as in Examples 1 and 2 but does have some existing capacity.

Example 4

Example 4 had the total cost functions on the links given by:

$$\hat{c}_a(f_a) = f_a^2, \quad \hat{c}_b(f_b) = f_b^2, \quad c_c(f_c) = f_c^2, \quad \hat{c}_d(f_d) = f_d^2, \quad \hat{c}_e(f_e) = 100f_e.$$

The investment capacity cost functions were: $\hat{\pi}_a(u_a) = u_a^2, \quad \forall a \in L^1$.

The existing capacities were: $\bar{u}_a = 10, \quad \forall a \in L$.

We assumed that the demand followed a uniform distribution on the interval $[10, 20]$ so that

$$P_1(\sum_{p \in P_{w_1}} x_p) = \frac{\sum_{p \in P_{w_1}} x_p - 10}{10}.$$

The penalties were: $\lambda_1^- = 1000, \quad \lambda_1^+ = 10.$

The Solutions

Example 1

The path flow solution was: $x^*_{p_1} = 0.00, \quad x^*_{p_2} = 2.00$, which corresponds to the link flow pattern:

$$f^*_a = f^*_b = f^*_c = f^*_d = 0.00, f^*_e = 2.00.$$

The capacity investments were: $u^*_a = 0.00, \quad \forall a \in L^1$.
The optimal Lagrange multipliers were: $\omega^*_a = 1.00, \quad \forall a \in L^1$, $\gamma^*_e = 3.00$.

Since the current capacities in the "in-house" supply chain links are zero, it is more costly to expand them than to outsource. Consequently, the organization chooses to outsource the product for production and delivery.

The Solutions (cont'd.)

Example 2

The path flow solution was: $x^*_{p_1} = 2.31, \quad x^*_{p_2} = 2.00$, which corresponds to the link flow pattern:

$$f^*_a = f^*_b = f^*_c = f^*_d = 2.31, f^*_e = 2.00.$$

The capacity investments were: $u^*_a = 2.31, \quad \forall a \in L^1$.
The optimal Lagrange multipliers were: $\omega^*_a = 3.31, \quad \forall a \in L^1$, $\gamma^*_e = 23.46$.

Since the penalty cost for under-supplying is increased, the organization increased its "in-house" capacity and product output.

The Solutions (cont'd.)

Example 3

The path flow solution was: $x^*_{p_1} = 3.23, \quad x^*_{p_2} = 2.00$, which corresponds to the link flow pattern:

$$f^*_a = f^*_b = f^*_c = f^*_d = 3.23, f^*_e = 2.00.$$

The capacity investments were: $u^*_a = 0.23, \quad \forall a \in L^1$.
The optimal Lagrange multipliers were: $\omega^*_a = 1.23, \quad \forall a \in L^1$, $\gamma^*_e = 18.84$.

Given the existing capacities in the "in-house" supply chain links, the organization chooses to supply more of the critical product from its manufacturer and distributor.

The Solutions (cont'd.)

Example 4

The path flow solution was: $x^*_{p_1} = 11.25, \quad x^*_{p_2} = 7.66$, which corresponds to the link flow pattern:

$$f^*_a = f^*_b = f^*_c = f^*_d = 11.25, f^*_e = 7.66.$$

The capacity investments were: $u^*_a = 1.25, \quad \forall a \in L^1$.
The optimal Lagrange multipliers were: $\omega^*_a = 2.50, \quad \forall a \in L^1$, $\gamma^*_e = 0.00$.

Since the penalty cost for under-supplying is much higher than that of over-supplying, the organization needs to both expand the "in-house" capacities and to outsource the production and delivery of the product to the demand point.

The Algorithm – The Euler Method

The Algorithm

At an iteration τ of the Euler method (see Dupuis and Nagurney (1993) and Nagurney and Zhang (1996)) one computes:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau} F(X^{\tau})), \tag{17}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem: determine $X^* \in \mathcal{K}$ such that

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{18}$$

where $\langle \cdot, \cdot \rangle$ is the inner product in n-dimensional Euclidean space, $X \in R^n$, and $F(X)$ is an n-dimensional function from $\mathcal{K}$ to R^n, with $F(X)$ being continuous.

The sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \to 0$, as $\tau \to \infty$.

Explicit Formulae for (17) to the Supply Chain Network Design Variational Inequality (15)

$$x_p^{\tau+1} = \max\{0, x_p^\tau + a_\tau(\lambda_k^-(1 - P_k(\sum_{p \in P_{w_k}} x_p^\tau)) - \lambda_k^+ P_k(\sum_{p \in P_{w_k}} x_p^\tau)$$

$$-\frac{\partial \hat{C}_p(x^\tau)}{\partial x_p} - \sum_{a \in L^1} \omega_a^\tau \delta_{ap} - \sum_{a \in L^2} \gamma_a^\tau \delta_{ap})\}, \forall p \in P; \tag{19}$$

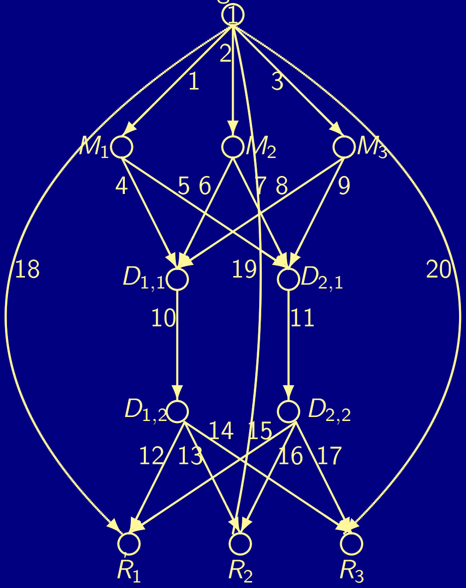
$$u_a^{\tau+1} = \max\{0, u_a^\tau + a_\tau(\omega_a^\tau - \frac{\partial \hat{\pi}_a(u_a^\tau)}{\partial u_a})\}, \quad \forall a \in L^1; \tag{20}$$

$$\omega_a^{\tau+1} = \max\{0, \omega_a^\tau + a_\tau(\sum_{p \in P} x_p^\tau \delta_{ap} - \bar{u}_a - u_a^\tau)\}, \quad \forall a \in L^1; \tag{21}$$

$$\gamma_a^{\tau+1} = \max\{0, \gamma_a^\tau + a_\tau(\sum_{p \in P} x_p^\tau \delta_{ap} - \bar{u}_a)\}, \quad \forall a \in L^2. \tag{22}$$

Additional Numerical Examples

The Organization



Example 5

The demands at the three demand points followed a uniform probability distribution on the intervals $[0, 10]$, $[0, 20]$, and $[0, 30]$, respectively:

$$P_1(\sum_{p \in P_{w_1}} x_p) = \frac{\sum_{p \in P_{w_1}} x_p}{10}, \quad P_2(\sum_{p \in P_{w_2}} x_p) = \frac{\sum_{p \in P_{w_2}} x_p}{20},$$

$$P_3(\sum_{p \in P_{w_3}} x_p) = \frac{\sum_{p \in P_{w_3}} x_p}{30},$$

where $w_1 = (1, R_1)$, $w_2 = (1, R_2)$, and $w_3 = (1, R_3)$.
The penalties were:

$$\lambda_1^- = 50, \quad \lambda_1^+ = 0; \quad \lambda_2^- = 50, \quad \lambda_2^+ = 0; \quad \lambda_3^- = 50, \quad \lambda_3^+ = 0.$$

The capacities associated with the three outsourcing links were:

$$\bar{u}_{18} = 5, \quad \bar{u}_{19} = 10, \quad \bar{u}_{20} = 5.$$

We set $\bar{u}_a = 0$ for all links $a \in L^1$.

Table 1: Total Cost Functions and Solution for Example 5

Link a	$\hat{c}_a(f_a)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	ω_a^*	γ_a^*
1	$f_1^2 + 2f_1$	$.5u_1^2 + u_1$	1.34	1.34	2.34	–
2	$.5f_2^2 + f_2$	$.5u_2^2 + u_2$	2.47	2.47	3.47	–
3	$.5f_3^2 + f_3$	$.5u_3^2 + u_3$	2.05	2.05	3.05	–
4	$1.5f_4^2 + 2f_4$	$.5u_4^2 + u_4$	0.61	0.61	1.61	–
5	$f_5^2 + 3f_5$	$.5u_5^2 + u_5$	0.73	0.73	1.73	–
6	$f_6^2 + 2f_6$	$.5u_6^2 + u_6$	0.83	0.83	1.83	–
7	$.5f_7^2 + 2f_7$	$.5u_7^2 + u_7$	1.64	1.64	2.64	–
8	$.5f_8^2 + 2f_8$	$.5u_8^2 + u_8$	1.67	1.67	2.67	–
9	$f_9^2 + 5f_9$	$.5u_9^2 + u_9$	0.37	0.37	1.37	–
10	$.5f_{10}^2 + 2f_{10}$	$.5u_{10}^2 + u_{10}$	3.11	3.11	4.11	–
11	$f_{11}^2 + f_{11}$	$.5u_{11}^2 + u_{11}$	2.75	2.75	3.75	–
12	$.5f_{12}^2 + 2f_{12}$	$.5u_{12}^2 + u_{12}$	0.04	0.04	1.04	–
13	$.5f_{13}^2 + 5f_{13}$	$.5u_{13}^2 + u_{13}$	0.00	0.00	0.45	–

Table 2: Total Cost Functions and Solution for Example 5 (continued)

Link a	$\hat{c}_a(f_a)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	ω_a^*	γ_a^*
14	f_{14}^2	$.5u_{14}^2 + u_{14}$	3.07	3.07	4.07	–
15	$f_{15}^2 + 2f_{15}$	$.5u_{15}^2 + u_{15}$	0.00	0.00	0.45	–
16	$.5f_{16}^2 + 3f_{16}$	$.5u_{16}^2 + u_{16}$	0.00	0.00	0.45	–
17	$.5f_{17}^2 + 2f_{17}$	$.5u_{17}^2 + u_{17}$	2.75	2.75	3.75	–
18	$10f_{18}$	–	5.00	–	–	14.77
19	$12f_{19}$	–	10.00	–	–	13.00
20	$15f_{20}$	–	5.00	–	–	16.96

Note that the optimal supply chain network design for Example 5 is, hence, as the initial topology but with links 13, 15, and 16 removed since those links have zero capacities and associated flows. Note that the organization took advantage of outsourcing to the full capacity available.

Example 6

Example 6 had the identical data to that in Example 5 except that we now assumed that the organization had capacities on its supply chain network activities where $\bar{u}_a = 10$, for all $a \in L^1$.

Table 3: Total Cost Functions and Solution for Example 6

Link a	$\hat{c}_a(f_a)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	ω_a^*	γ_a^*
1	$f_1^2 + 2f_1$	$.5u_1^2 + u_1$	1.84	0.00	0.00	–
2	$.5f_2^2 + f_2$	$.5u_2^2 + u_2$	4.51	0.00	0.00	–
3	$.5f_3^2 + f_3$	$.5u_3^2 + u_3$	3.85	0.00	0.00	–
4	$1.5f_4^2 + 2f_4$	$.5u_4^2 + u_4$	0.88	0.00	0.00	–
5	$f_5^2 + 3f_5$	$.5u_5^2 + u_5$	0.97	0.00	0.00	–
6	$f_6^2 + 2f_6$	$.5u_6^2 + u_6$	1.40	0.00	0.00	–
7	$.5f_7^2 + 2f_7$	$.5u_7^2 + u_7$	3.11	0.00	0.00	–
8	$.5f_8^2 + 2f_8$	$.5u_8^2 + u_8$	3.47	0.00	0.00	–
9	$f_9^2 + 5f_9$	$.5u_9^2 + u_9$	0.38	0.00	0.00	–

Table 4: Total Cost Functions and Solution for Example 6 (continued)

Link a	$\hat{c}_a(f_a)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	ω_a^*	γ_a^*
10	$.5f_{10}^2 + 2f_{10}$	$.5u_{10}^2 + u_{10}$	5.75	0.00	0.00	–
11	$f_{11}^2 + f_{11}$	$.5u_{11}^2 + u_{11}$	4.46	0.00	0.00	–
12	$.5f_{12}^2 + 2f_{12}$	$.5u_{12}^2 + u_{12}$	0.82	0.00	0.00	–
13	$.5f_{13}^2 + 5f_{13}$	$.5u_{13}^2 + u_{13}$	0.52	0.00	0.00	–
14	f_{14}^2	$.5u_{14}^2 + u_{14}$	4.41	0.00	0.00	–
15	$f_{15}^2 + 2f_{15}$	$.5u_{15}^2 + u_{15}$	0.00	0.00	0.00	–
16	$.5f_{16}^2 + 3f_{16}$	$.5u_{16}^2 + u_{16}$	0.05	0.00	0.00	–
17	$.5f_{17}^2 + 2f_{17}$	$.5u_{17}^2 + u_{17}$	4.41	0.00	0.00	–
18	$10f_{18}$	–	5.00	–	–	10.89
19	$12f_{19}$	–	10.00	–	–	11.59
20	$15f_{20}$	–	5.00	–	–	11.96

Note that links 13 and 16 now have positive associated flows although at very low levels.

Example 7

Example 7 had the same data as Example 6 except that we changed the probability distributions so that we now had:

$$P_1\left(\sum_{p \in P_{w_1}} x_p\right) = \frac{\sum_{p \in P_{w_1}} x_p}{110},$$

$$P_2\left(\sum_{p \in P_{w_2}} x_p\right) = \frac{\sum_{p \in P_{w_2}} x_p}{120},$$

$$P_3\left(\sum_{p \in P_{w_3}} x_p\right) = \frac{\sum_{p \in P_{w_3}} x_p}{130}.$$

Table 5: Total Cost Functions and Solution for Example 7

Link a	$\hat{c}_a(f_a)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	ω_a^*	γ_a^*
1	$f_1^2 + 2f_1$	$.5u_1^2 + u_1$	4.23	0.00	0.00	–
2	$.5f_2^2 + f_2$	$.5u_2^2 + u_2$	9.06	0.00	0.00	–
3	$.5f_3^2 + f_3$	$.5u_3^2 + u_3$	8.61	0.00	0.00	–
4	$1.5f_4^2 + 2f_4$	$.5u_4^2 + u_4$	2.05	0.00	0.00	–
5	$f_5^2 + 3f_5$	$.5u_5^2 + u_5$	2.18	0.00	0.00	–
6	$f_6^2 + 2f_6$	$.5u_6^2 + u_6$	3.28	0.00	0.00	–
7	$.5f_7^2 + 2f_7$	$.5u_7^2 + u_7$	5.77	0.00	0.00	–
8	$.5f_8^2 + 2f_8$	$.5u_8^2 + u_8$	7.01	0.00	0.00	–
9	$f_9^2 + 5f_9$	$.5u_9^2 + u_9$	1.61	0.00	0.00	–
10	$.5f_{10}^2 + 2f_{10}$	$.5u_{10}^2 + u_{10}$	12.34	2.34	3.34	–
11	$f_{11}^2 + f_{11}$	$.5u_{11}^2 + u_{11}$	9.56	0.00	0.00	–
12	$.5f_{12}^2 + 2f_{12}$	$.5u_{12}^2 + u_{12}$	5.82	0.00	0.00	–
13	$.5f_{13}^2 + 5f_{13}$	$.5u_{13}^2 + u_{13}$	2.38	0.00	0.00	–

Table 6: Total Cost Functions and Solution for Example 7 (continued)

Link a	$\hat{c}_a(f_a)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	ω_a^*	γ_a^*
14	f_{14}^2	$.5u_{14}^2 + u_{14}$	4.14	0.00	0.00	–
15	$f_{15}^2 + 2f_{15}$	$.5u_{15}^2 + u_{15}$	2.09	0.00	0.00	–
16	$.5f_{16}^2 + 3f_{16}$	$.5u_{16}^2 + u_{16}$	2.75	0.00	0.00	–
17	$.5f_{17}^2 + 2f_{17}$	$.5u_{17}^2 + u_{17}$	4.72	0.00	0.00	–
18	$10f_{18}$	–	5.00	–	–	34.13
19	$12f_{19}$	–	10.00	–	–	31.70
20	$15f_{20}$	–	5.00	–	–	29.66

The optimal supply chain network design for Example 7 has the initial topology since there are now positive flows on all the links. It is also interesting to note that there is a significant increase in production volumes by the organization at its manufacturing plants.

References - for Further Reading

Link to Network Economics course materials as well as several other related courses conducted by Nagurney on her Fulbright in Austria:
http://supernet.som.umass.edu/austria_lectures/fulmain.html

Overview article on Network Economics by Nagurney:
http://supernet.som.umass.edu/articles/NetworkEconomics.pdf

Background article on the importance of the Beckmann, McGuire, and Winsten book, *Studies in the Economics of Transportation*:
http://tsap.civil.northwestern.edu/boyce_pubs/retrospective_on_beckmann.pdf

Preface to the translation of the Braess (1968) article and the translation:
http://tsap.civil.northwestern.edu/bouce_pubs/preface_to.pdf
http://homepage.rub.de/Dietrich.Braess/Paradox-BNW.pdf

Link to numerous articles on network modeling and applications, vulnerability and robustness analysis, as well as network synergy:
http://supernet.som.umass.edu/dart.html

Link to books of interest: http://supernet.som.umass.edu/bookser.html

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