Supply Chain Network Competition in Price and Quality with Multiple Manufacturers and Freight Service Providers

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Outline

Background

- Manufacturers and freight service providers are fundamental decision-makers in today's globalized supply chain networks.
- The decisions that the firms make affect the prices and quality of products as well as that of the freight services provided, which, in turn, impact their own profitability.
- Quality and price have been identified empirically as critical factors in transport mode selection for product/goods delivery (cf. Floden, Barthel, and Sorkina (2010), Saxin, Lammgard, and Floden (2005)).
- Quality has also become one of the most essential factors in the success of supply chains of various products.

Motivation

- Increasingly, tough customer demands are also putting the transport system under pressure.
- The providers may offer flexibility to meet customer needs of safety, and/or traceability and, furthermore, differentiate themselves from the rest of the competition.
- Quality of freight encompasses factors such as on-time deliveries, reliability, and frequency.

Relevant Literature

- Akerlof, G.A., 1970. The market for lemons: Quality uncertainty and the market mechanism. Quarterly Journal of Economics, 84(3), 488-500.
- Spence, M., 1975. Monopoly, quality, and regulation. The Bell Journal of Economics, 6(2), 417-429.
- Sheshinski, E., 1976. Price quality and quantity regulation in monopoly situation. Economica, 43, 127-137.
- Mussa, M., Rosen, S., 1978. Monopoly and product quality. Journal of Economic Theory, 18, 301-317.

Relevant Literature

- Dixit, A., 1979. Quality and quantity competition. Review of Economic Studies, 46(4), 587-599.
- Gal-or, E., 1983. Quality and quantity competition. Bell Journal of Economics, 14, 590-600.
- Brekke, K.R., Siciliani, L., Straume, O.R., 2010. Price and quality in spatial competition. Regional Science and Urban Economics, 40, 471-480.
- Nagurney, A., Li, D., 2014. A dynamic network oligopoly model with transportation costs, product differentiation, and quality competition. Computational Economics, 44(2), 201-229.
- Nagurney, A., Li, D., Nagurney, L.S., 2014. Spatial price equilibrium with information asymmetry in quality and minimum quality standards. International Journal of Production Economics, 158, 300-313.

Contributions

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- The transportation costs differ by mode, leading to an evaluation of quality vs. costs for the freight service providers and the modes of transportation that they offer to the customers.
- We handle heterogeneity in the providers' cost functions and in the consumers' demands and do not limit ourselves to specific functional forms.
- Utilities of each manufacturing firm and freight service provider considers price and quality for not just his own products, but that of other firms or providers as well.

The Supply Chain Network Model with Price and Quality Competition

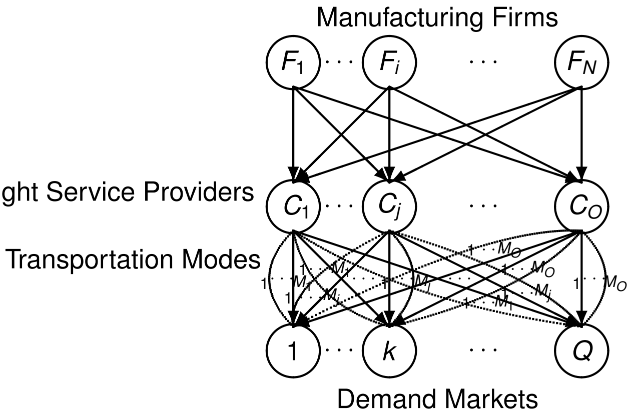

Figure : The Supply Chain Network Structure of the Game Theory Model

Supply Chain Network with Price and Quality Competition

- As in Nagurney and Li (2014), we define and quantify quality as the quality conformance level, that is, the degree to which a specific product conforms to a design or specification (Gilmore (1974), Juran and Gryna (1988)).

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- Firm F_i manufactures a product of quality q_i at the price p_i.
- The quality and price associated with freight service provider C_j retrieving the product from firm F_i and delivering it to demand market k via mode m are denoted, respectively, by q^m_{ijk}, and p^m_{ijk}; $\forall i, j, k, m$.

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- The quality and price associated with freight service provider C_j retrieving the product from firm F_i and delivering it to demand market k via mode m are denoted, respectively, by q^m_{ijk}, and $p^m_{ijk}; \forall i, j, k, m$.
- Demand is denoted by d^m_{ijk} for consumer market k, mode m coming from firm i through provider j.

Supply Chain Network with Price and Quality Competition

Demand Function:

$$d_{ijk}^{m} = d_{ijk}^{m}(p_F, q_F, p_C, q_C); \forall i, j, k, m.$$

Demand depends on firm's price and quality, its competitors, and freight service providers.

The Firms' Behavior: Supply of Firm:

$$s_i(p_F, q_F, p_C, q_C) = \sum_{j=1}^{O} \sum_{k=1}^{Q} \sum_{m=1}^{M_j} d_{ijk}^{m}(p_F, q_F, p_C, q_C); \forall i.$$

The Production Cost:

$$PC_i = PC_i(s_F(p_F, q_F, p_C, q_C), q_F), \forall i$$

Supply Chain Network with Price and Quality Competition

The Utility of Firm:

$$U_{F_i}(p_F, q_F, p_C, q_C) = p_i[s_i(p_F, q_F, p_C, q_C)] - PC_i, \forall i.$$

Bounds on Quality:

$$\underline{q_i} \leq q_i \leq \bar{q}_i, \forall i.$$

$\bar{q}_i = 100$ corresponds to perfect quality conformance level. Positive lower bound corresponds $\underline{q_i}$ to a minimum quality standard.
Bounds on Price:

$$0 \leq p_i \leq \bar{p}_i, \forall i.$$

Let K_i^1 denote the feasible set for firm F_i and the bounds on price and quality hold. $K^1 \equiv \prod_{i=1}^{N} K_i^1$. Functions are continuous and continuously differentiable.

Supply Chain Network with Price and Quality Competition

The Freight Service Providers' Behavior: The Transportation Cost:

$$TC_{ijk}^m = TC_{ijk}^m(d(p_F, q_F, p_C, q_C), q_C), \forall i, j, k, m.$$

The Utility of Freight Service Provider:

$$U_{C_j} = \sum_{i=1}^{N} \sum_{k=1}^{O} \sum_{m=1}^{M_j} [p_{ijk}^m d_{ijk}^m - TC_{ijk}^m], \forall j.$$

Bounds on Quality:

$$\underline{q}_{ijk}^m \leq q_{ijk}^m \leq \bar{q}_{ijk}^m, \forall i, j, k, m.$$

Bounds on Price:

$$0 \leq p_{ijk}^m \leq \bar{p}_{ijk}^m, \forall 1, j, k, m.$$

Feasible set, K_j^2; $K^2 \equiv \prod_{j=1}^{O} K_j^2$.

The Equilibrium Conditions

Definition 1: Nash Equilibrium in Prices and Quality Levels

A price and quality level pattern $(p_F^*, q_F^*, p_C^*, q_C^*) \in K^3 \equiv \prod_{i=1}^{N} K_i^1 \times \prod_{j=1}^{O} K_j^2$, is said to constitute a Nash equilibrium if for each firm F_i; $i = 1, \ldots, N$:

$$U_{F_i}(p_i^*, \hat{p}_i^*, q_i^*, \hat{q}_i^*, p_C^*, q_C^*) \geq U_{F_i}(p_i, \hat{p}_i^*, q_i, \hat{q}_i^*, p_C^*, q_C^*), \quad \forall (p_i, q_i) \in K_i^1,$$

where

$$\hat{p}_i^* \equiv (p_1^*, \ldots, p_{i-1}^*, p_{i+1}^*, \ldots, p_N^*) \text{ and } \hat{q}_i^* \equiv (q_1^*, \ldots, q_{i-1}^*, q_{i+1}^*, \ldots, q_N^*),$$

and if for each freight service provider C_j; $j = 1, \ldots, O$:

$$U_{C_j}(p_F^*, q_F^*, p_{C_j}^*, \hat{p}_{C_j}^*, q_{C_j}^*, \hat{q}_{C_j}^*) \geq U_{C_j}(p_F^*, q_F^*, p_{C_j}, \hat{p}_{C_j}^*, q_{C_j}, \hat{q}_{C_j}^*),$$

where

$$\hat{p}_{C_j}^* \equiv (p_{C_1}^*, \ldots, p_{C_{j-1}}^*, p_{C_{j+1}}^*, \ldots, p_{C_O}^*)$$

$$\hat{q}_{C_j}^* \equiv (q_{C_1}^*, \ldots, q_{C_{j-1}}^*, q_{C_{j+1}}^*, \ldots, q_{C_O}^*).$$

Variational Inequality Formulation

Theorem 1: Variational Inequality Formulations of Nash Equilibrium in Prices and Quality

$(p_F^*, q_F^*, p_C^*, q_C^*) \in \mathcal{K}^3$ is a Nash equilibrium according to Definition 1 if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^{N} \frac{\partial U_{F_i}(p_F^*, q_F^*, p_C^*, q_C^*)}{\partial p_i} \times (p_i - p_i^*) - \sum_{i=1}^{N} \frac{\partial U_{F_i}(p_F^*, q_F^*, p_C^*, q_C^*)}{\partial q_i} \times (q_i - q_i^*)$$

$$-\sum_{j=1}^{O} \sum_{i=1}^{N} \sum_{k=1}^{Q} \sum_{m=1}^{M_j} \frac{\partial U_{C_j}(p_F^*, q_F^*, p_C^*, q_C^*)}{\partial p_{ijk}^m} \times (p_{ijk}^m - p_{ijk}^{m*})$$

$$-\sum_{j=1}^{O} \sum_{i=1}^{N} \sum_{k=1}^{Q} \sum_{m=1}^{M_j} \frac{\partial U_{C_j}(p_F^*, q_F^*, p_C^*, q_C^*)}{\partial q_{ijk}^m} \times (q_{ijk}^m - q_{ijk}^{m*}) \geq 0,$$

$$\forall (p_F, q_F, p_C, q_C) \in \mathcal{K}^3,$$

Variational Inequality Formulation

Standard Form

Determine $X^* \in \mathcal{K}$ where X is a vector in R^n , $F(X)$ is a continuous function such that $F(X) : X \mapsto \mathcal{K} \subset R^n$, and

$$\langle F(X^*), X - X^* \rangle \geq 0, \qquad \forall X \in \mathcal{K}.$$

We define the vector $X \equiv (p_F, q_F, p_C, q_C)$ and $F(X) \equiv (F_{p_F}, F_{q_F}, F_{p_C}, F_{q_C})$ with the i-th component of F_{p_F} and F_{q_F} given, respectively, by:

$$F_{p_i} = -\frac{\partial U_{F_i}}{\partial p_i}; \quad F_{q_i} = -\frac{\partial U_{F_i}}{\partial q_i},$$

and the (i, j, k, m)-th component of F_{p_C} and F_{q_C}, respectively, given by:

$$F_{p^m_{ijk}} = -\frac{\partial U_{C_j}}{\partial p^m_{ijk}}; \quad F_{q^m_{ijk}} = -\frac{\partial U_{C_j}}{\partial q^m_{ijk}}.$$

Existence of the Solution

Theorem 2: A Solution to the Variational Inequality

Existence of a solution to the variational inequalities is guaranteed since the feasible set $\mathcal{K}$ is compact and the function $F(X)$ in our model is continuous, under the assumptions made on the underlying functions.

Dynamics

We now propose dynamic adjustment processes for the evolution of the firms' product prices and quality levels and those of the freight service providers (carriers).

Rate of change of p_i:

$$\dot{p}_i = \begin{cases} \frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial p_i}, & \text{if} \quad 0 < p_i < \bar{p}_i \\ \max\{0, \min\{\frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial p_i}, \bar{p}_i\}\}, & \text{if} \quad p_i = 0 \text{ or } p_i = \bar{p}_i. \end{cases}$$

Rate of change of q_i:

$$\dot{q}_i = \begin{cases} \frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial q_i}, & \text{if} \quad \underline{q}_i < q_i < \bar{q}_i \\ \max\{\underline{q}_i, \min\{\frac{\partial U_{F_i}(p_F, q_F, p_C, q_C)}{\partial q_i}, \bar{q}_i\}\}, & \text{if} \quad q_i = \underline{q}_i \text{ or } q_i = \bar{q}_i. \end{cases}$$

Dynamics

Rate of change of p_{ijk}^m:

$$\dot{p}_{ijk}^m = \begin{cases} \frac{\partial U_{C_j}(p_F, q_F, p_C, q_C)}{\partial p_{ijk}^m}, & \text{if} \quad 0 < p_{ijk}^m < \bar{p}_{ijk}^m \\ \max\{0, \min\{\frac{\partial U_{C_j}(p_F, q_F, p_C, q_C)}{\partial p_{ijk}^m}, \bar{p}_{ijk}^m\}\}, & \text{if} \quad p_{ijk}^m = 0 \text{ or } \bar{p}_{ijk}^m. \end{cases}$$

Rate of change of q_{ijk}^m:

$$\dot{q}_{ijk}^m = \begin{cases} \frac{\partial U_{C_j}(p_F, q_F, p_C, q_C)}{\partial q_{ijk}^m}, & \text{if} \quad \underline{q}_{ijk}^m < q_{ijk}^m < \bar{q}_{ijk}^m \\ \max\{\underline{q}_{ijk}^m, \min\{\frac{\partial U_{C_j}(p_F, q_F, p_C, q_C)}{\partial q_{ijk}^m}, \bar{q}_{ijk}^m\}\}, & \text{if} \quad q_{ijk}^m = \underline{q}_{ijk}^m \text{ or } \bar{q}_{ijk}^m. \end{cases}$$

Dynamics

Ordinary Differential Equation (ODE) for the adjustment processes of the prices and quality levels of firms and freight service providers, in vector form:

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \quad X(0) = X^0.$$

The projection operator:

$$\Pi_{\mathcal{K}}(X, -F(X)) = \lim_{\delta \to 0} \frac{P_{\mathcal{K}}(X - \delta F(X)) - X}{\delta},$$

with $P_{\mathcal{K}}$ denoting the projection map:

$$P_{\mathcal{K}}(X) = \text{argmin}_{z \in \mathcal{K}} \|X - z\|.$$

Dynamics

Theorem 3

According to Dupuis and Nagurney (1993) X^* solves the variational inequality problem if and only if it is a stationary point of the ODE, that is,

$$\dot{X} = 0 = \Pi_{\mathcal{K}}(X^*, -F(X^*)).$$

This theorem demonstrates that the necessary and sufficient condition for a product and freight service price and quality level pattern $X^* = (p_F^*, q_F^*, p_C^*, q_C^*)$ to be a Nash equilibrium, according to Definition 1, is that $X^* = (p_F^*, q_F^*, p_C^*, q_C^*)$ is a stationary point of the adjustment processes defined by ODE, that is, X^* is the point at which $\dot{X} = 0$.

Explicit Formulae for the Euler Method Applied to the Multitiered Supply Chain Network Problem

Closed form expressions of price and quality of firms:

$$p_i^{\tau+1} = \max\Bigg\{0\,,\min\Bigg\{\bar{p}_i\,, p_i^\tau + a_\tau\,[\sum_{j=1}^{O}\sum_{k=1}^{Q}\sum_{m=1}^{M_j} d_{ijk}^m(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau)$$

$$+p_i^\tau \sum_{j=1}^{O}\sum_{k=1}^{Q}\sum_{m=1}^{M_j} \frac{\partial d_{ijk}^m(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau)}{\partial p_i}$$

$$-\sum_{l=1}^{N} \frac{\partial PC_i(s_F(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau), q_F^\tau)}{\partial s_l} \times \frac{\partial s_l(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau)}{\partial p_i}]\Bigg\}\Bigg\},$$

$$q_i^{\tau+1} = \max\Bigg\{\underline{q}_i\,,\min\Bigg\{\bar{q}_i\,, q_i^\tau + a_\tau\,[p_i^\tau \sum_{j=1}^{O}\sum_{k=1}^{Q}\sum_{m=1}^{M_j} \frac{\partial d_{ijk}^m(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau)}{\partial q_i}$$

$$-\sum_{l=1}^{N} \frac{\partial PC_i(s_F(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau), q_F^\tau)}{\partial s_l} \times \frac{\partial s_l(p_F^\tau, q_F^\tau, p_C^\tau, q_C^\tau)}{\partial q_i} - \frac{\partial PC_i(s_F^\tau, q_F^\tau)}{\partial q_i}]\Bigg\}\Bigg\}.$$

Explicit Formulae for the Euler Method Applied to the Multitiered Supply Chain Network Problem

Closed form expressions of price and quality of freight service providers:

$$p_{ijk}^{m(\tau+1)} = \max\Bigg\{0\,,\min\Bigg\{\bar{p}_{ijk}^{m}\,,p_{ijk}^{m\tau} + a_\tau\,[d_{ijk}^{m}(p_F^\tau,q_F^\tau,p_C^\tau,q_C^\tau)$$

$$+\sum_{l=1}^{N}\sum_{s=1}^{Q}\sum_{t=1}^{M_j}\frac{\partial d_{ljs}^{t}(p_F^\tau,q_F^\tau,p_C^\tau,q_C^\tau)}{\partial p_{ijk}^{m}}\times p_{ljs}^{t\tau}$$

$$-\sum_{l=1}^{N}\sum_{s=1}^{Q}\sum_{t=1}^{M_j}(\sum_{r=1}^{N}\sum_{v=1}^{O}\sum_{w=1}^{Q}\sum_{z=1}^{M_v}\frac{\partial TC_{ljs}^{t}(d(p_F^\tau,q_F^\tau,p_C^\tau,q_C^\tau),q_C^\tau)}{\partial d_{rvw}^{z}}\times\frac{\partial d_{rvw}^{z}(p_F^\tau,q_F^\tau,p_C^\tau,q_C^\tau)}{\partial p_{ijk}^{m}})]\Bigg\}\Bigg\},$$

$$q_{ijk}^{m(\tau+1)} = \max\Bigg\{\underline{q}_{ijk}^{m}\,,\min\Bigg\{\bar{q}_{ijk}^{m}\,,q_{ijk}^{m\tau} + a_\tau\,[\sum_{l=1}^{N}\sum_{s=1}^{Q}\sum_{t=1}^{M_j}\frac{\partial d_{ljs}^{t}(p_F^\tau,q_F^\tau,p_C^\tau,q_C^\tau)}{\partial q_{ijk}^{m}}\times p_{ljs}^{t\tau}$$

$$-\sum_{l=1}^{N}\sum_{s=1}^{Q}\sum_{t=1}^{M_j}(\sum_{r=1}^{N}\sum_{v=1}^{O}\sum_{w=1}^{Q}\sum_{z=1}^{M_v}\frac{\partial TC_{ljs}^{t}(d(p_F^\tau,q_F^\tau,p_C^\tau,q_C^\tau),q_C^\tau)}{\partial d_{rvw}^{z}}\times\frac{\partial d_{rvw}^{z}(p_F^\tau,q_F^\tau,p_C^\tau,q_C^\tau)}{\partial q_{ijk}^{m}})$$

$$-\sum_{l=1}^{N}\sum_{s=1}^{Q}\sum_{t=1}^{M_j}\frac{\partial TC_{ljs}^{t}(d^\tau,q_C^\tau)}{\partial q_{ijk}^{m}}]\Bigg\}\Bigg\}.$$

Convergence

Theorem 4

In our multitiered supply chain network game theory model, assume that $F(X) = -\nabla U(p_F, q_F, p_C, q_C)$ is strictly monotone. Also, assume that F is uniformly Lipschitz continuous. Then, there exists a unique equilibrium price and quality pattern $(p_F^*, q_F^*, p_C^*, q_C^*) \in \mathcal{K}$ and any sequence generated by the Euler method as given by the closed form expressions, where $\{a_\tau\}$ satisfies $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$ converges to $(p_F^*, q_F^*, p_C^*, q_C^*)$.

Example 1

Figure : The Supply Chain Network Topology for Example 1

The demand function for demand market 1 is:

$$d^1_{111} = 43 - 1.62p^1_{111} + 1.6q^1_{111} - 1.45p_1 + 1.78q_1.$$

The supply of F_1 is:

$$s_1 = d^1_{111}.$$

Example 1

The production cost and utility of manufacturing firm F_1 is:

$$PC_1 = 1.55(s_1 + 1.15q_1^2), \qquad U_{F_1} = p_1 s_1 - PC_1.$$

The quality and price of the firm are bounded as per the following constraints:

$$0 \leq p_1 \leq 80, \qquad 10 \leq q_1 \leq 100.$$

The transportation cost of freight service provider C_1 is:

$$TC_{111}^1 = .5d_{111}^1 + (q_{111}^1)^2.$$

The utility of freight service provider C_1 is:

$$U_{C_1} = p_{111}^1 d_{111}^1 - TC_{111}^1,$$

with the following limitations on his price and quality:

$$0 \leq p_{111}^1 \leq 70, \quad 9 \leq q_{111}^1 \leq 100.$$

Example 1

The equilibrium result, after 60 iterations, is:

$$p^{1*}_{111} = 16.63, \quad p^*_1 = 19.57, \quad q^{1*}_{111} = 12.90, \quad q^*_1 = 10.00.$$

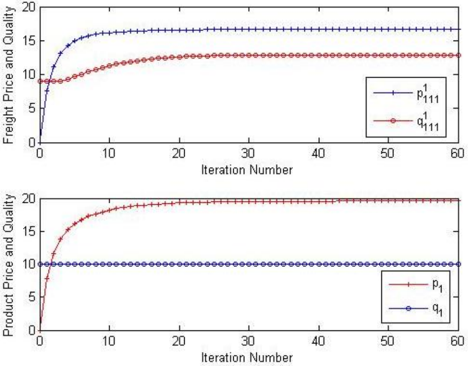


Figure : Prices and Quality Levels for the Product and Freight of Example 1

Example 2

The supply chain network topology is depicted as here:

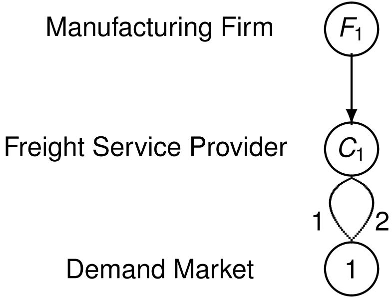


Figure : The Supply Chain Network Topology

The demand functions are:

$$d_{111}^1 = 43 - 1.62p_{111}^1 + 1.6q_{111}^1 - 1.45p_1 + 1.78q_1 + .03p_{111}^2 - .2q_{111}^2,$$
$$d_{111}^2 = 52 - 1.75p_{111}^2 + 1.21q_{111}^2 - 1.45p_1 + 1.78q_1 + .03p_{111}^1 - .2q_{111}^1.$$

Example 2

The supply of manufacturing firm F_1 is :

$$s_1 = d^1_{111} + d^2_{111}$$

The transportation costs of the freight service provider C_1 for modes 1 and 2 are:

$$TC^1_{111} = .5d^1_{111} + (q^1_{111})^2,$$

$$TC^2_{111} = .45d^2_{111} + .54(q^2_{111})^2 + .0035d^2_{111}q^2_{111}.$$

The utility of freight service provider C_1 is:

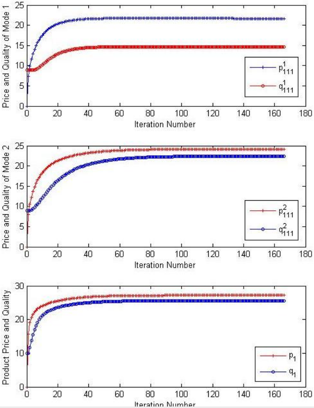
$$U_{C_1} = p^1_{111}d^1_{111} + p^2_{111}d^2_{111} - TC^1_{111} - TC^2_{111},$$

$$0 \leq p^2_{111} \leq 70, \quad 9 \leq q^2_{111} \leq 100.$$

Example 2

The equilibrium solution, after 166 iterations, is:

$$p^{1*}_{111} = 21.68, \; p^{2*}_{111} = 24.16, \; p^*_1 = 27.18,$$
$$q^{1*}_{111} = 14.58, \; q^{2*}_{111} = 22.43, \; q^*_1 = 25.59.$$



Example 3 and Variant

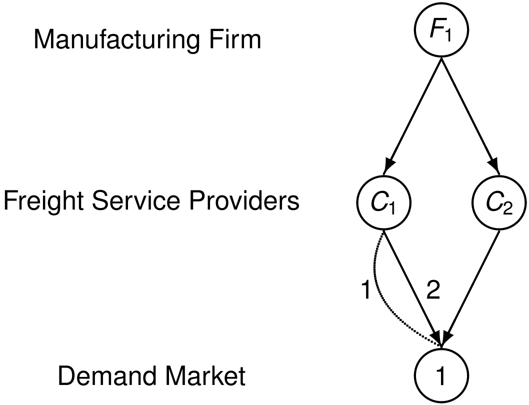


Figure : The Supply Chain Network Topology for Example 3 and Variant

Example 3

The demand functions are:

$$d^1_{111} = 43 - 1.62p^1_{111} + 1.6q^1_{111} - 1.45p_1 + 1.78q_1 + .03p^2_{111} - .2q^2_{111} + .04p^1_{121} - .1q^1_{121},$$
$$d^2_{111} = 52 - 1.75p^2_{111} + 1.21q^2_{111} - 1.45p_1 + 1.78q_1 + .03p^1_{111} - .2q^1_{111} + .04p^1_{121} - .1q^1_{121},$$
$$d^1_{121} = 47 - 1.79p^1_{121} + 1.41q^1_{121} - 1.45p_1 + 1.78q_1 + .03p^1_{111} - .2q^1_{111} + .04p^2_{111} - .1q^2_{111}.$$

$$s_1 = d^1_{111} + d^2_{111} + d^1_{121}.$$

The transportation costs of freight service provider C_1 are:

$$TC^1_{111} = .5d^1_{111} + (q^1_{111})^2 + .045d^1_{121},$$
$$TC^2_{111} = .45d^2_{111} + .54(q^2_{111})^2 + .005d^2_{111}q^2_{111},$$

and that of freight service provider C_2 is:

$$TC^1_{121} = .64d^1_{121} + .76(q^1_{121})^2.$$

The utility of C_2 is:

$$U_{C_2} = p^1_{121}d^1_{121} - TC^1_{121}.$$

$$0 \le p^1_{121} \le 65, \qquad 12 \le q^1_{121} \le 100.$$

Example 3

The new equilibrium solution, computed after 218 iterations, is:

$$p_{111}^{1*} = 45.69, \quad p_{111}^{2*} = 45.32, \quad p_{121}^{1*} = 44.82, \quad p_1^* = 53.91,$$

$$q_{111}^{1*} = 31.69, \quad q_{111}^{2*} = 41.32, \quad q_{121}^{1*} = 41.24, \quad q_1^* = 78.43.$$

Add trajectories.

Variant of Example 3

$$d^1_{111} = 43 - 1.44p^1_{111} + 1.53q^1_{111} - 1.82p_1 + 1.21q_1 + .03p^2_{111} - .2q^2_{111} + .04p^1_{121} - .1q^1_{121},$$

$$d^2_{111} = 52 - 1.49p^2_{111} + 1.65q^2_{111} - 1.82p_1 + 1.21q_1 + .03p^1_{111} - .2q^1_{111} + .04p^1_{121} - .1q^1_{121},$$

$$d^1_{121} = 47 - 1.57p^1_{121} + 1.64q^1_{121} - 1.82p_1 + 1.21q_1 + .03p^1_{111} - .2q^1_{111} + .04p^2_{111} - .1q^2_{111}.$$

The equilibrium solution, computed after 553 iterations, is:

$$p^{1*}_{111} = 8.71, \quad p^{2*}_{111} = 63.17, \quad p^{1*}_{121} = 16.22, \quad p^*_1 = 24.80,$$

$$q^{1*}_{111} = 9.00, \quad q^{2*}_{111} = 93.15, \quad q^{1*}_{121} = 16.92, \quad q^*_1 = 23.67.$$

Example 4

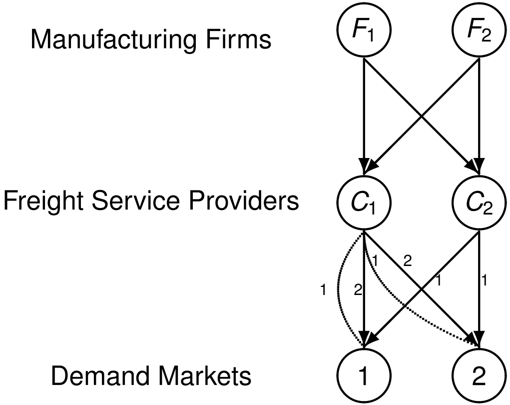


Figure : The Supply Chain Network Topology for Example 4 and Variant

Example 4: Result

The equilibrium solution, after 254 iterations, is:

$$p^{1*}_{111} = 56.79, \quad p^{2*}_{111} = 55.45, \quad p^{1*}_{112} = 72.96, \quad p^{2*}_{112} = 36.93,$$

$$p^{1*}_{121} = 55.19, \quad p^{1*}_{122} = 53.55, \quad p^{1*}_{211} = 62.77, \quad p^{2*}_{211} = 53.28,$$

$$p^{1*}_{212} = 72.94, \quad p^{2*}_{212} = 65.91, \quad p^{1*}_{221} = 76.15, \quad p^{1*}_{222} = 83.73,$$

$$p^{*}_{1} = 63.76, \quad p^{*}_{2} = 64.90, \quad q^{*}_{1} = 100.00, \quad q^{*}_{2} = 100.00,$$

$$q^{1*}_{111} = 39.53, \quad q^{2*}_{111} = 51.20, \quad q^{1*}_{112} = 74.61, \quad q^{2*}_{112} = 23.54,$$

$$q^{1*}_{121} = 50.93, \quad q^{1*}_{122} = 51.05, \quad q^{1*}_{211} = 46.25, \quad q^{2*}_{211} = 36.72,$$

$$q^{1*}_{212} = 76.89, \quad q^{2*}_{212} = 69.56, \quad q^{1*}_{221} = 61.18, \quad q^{1*}_{222} = 94.70.$$

The price and quality levels have gone up as well as utilities for both manufacturers and carriers as compared to Example 6.4 since there are two demand markets to be satisfied now as opposed to one.

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- we then provided solutions to a series of numerical examples - small to large scenarios and their variants.

THANK YOU!

For more information, see: http://supernet.isenberg.umass.edu