A Game Theory Model for Freight Service Provision Security Investments for High-Value Cargo

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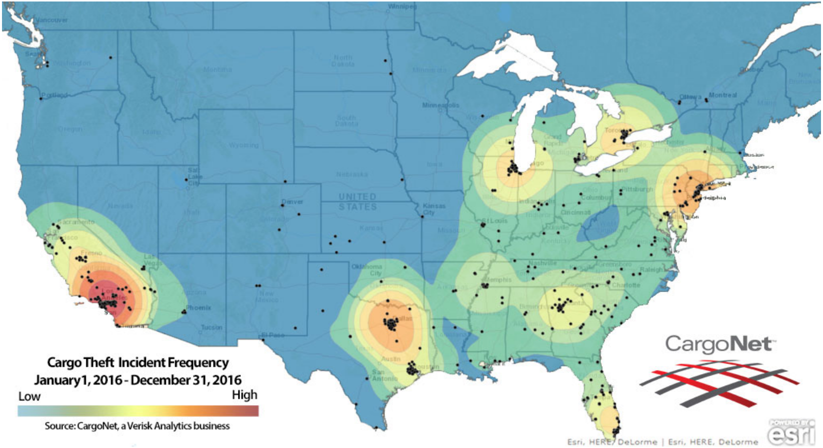
Outline

Introduction

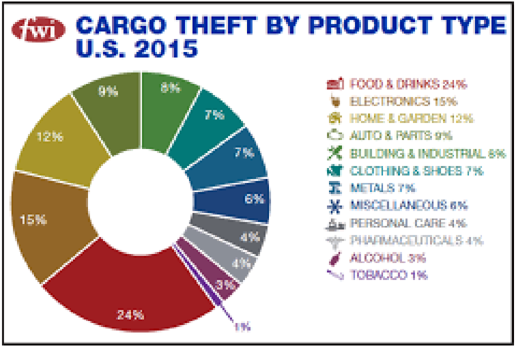
Networks and Security

- In a constantly and intricately connected world, security is imperative to not just the success but also the **survival** of businesses.
- Cargo theft is estimated to **cost shippers and trucking companies** at least **$30 billion a year** in the US, according to the FBI.
- There is an average of 63 cargo thefts per month. The **average loss value** per incident in **2015** was almost **$190,000**.
- In **2016**, CargoNet reported an average loss value of **$206,837**.
- Cargo's value continues to increase and thieves are getting sophisticated.

Incident Heat Map

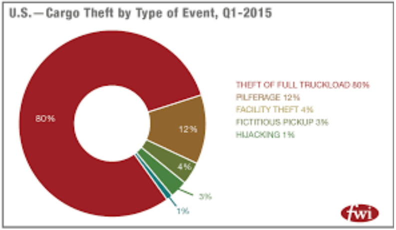


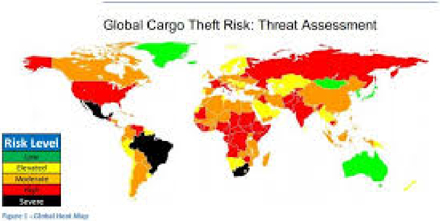
Cargo Theft by Product Type



High Value Cargo Security

- The average loss-value ceiling is rising.
- Cargo thieves continue to target high value freight.





Approach

- We develop a **game theory model** consisting of Freight Service Providers **(FSPs) who compete** with each other in terms of **quantity of the high-value product**.
- Shippers reflect their **preferences through willingness to pay** depending on the quantity and level of security provided by the FSPs.
- **FSPs** encumber the **security investment costs**.
- We also include **probability of a successful attack** on the logistics/transportation links, along with **associated damages**.
- FSPs try to **maximize their utilities** associated with quantities and security levels which **may differ for different links**.

Literature Review

- Nagurney A., Daniele P., Shukla S., 2017. A supply chain network game theory model of cybersecurity investments with nonlinear budget constraints. Annals of Operations Research, 248(1), 405-427.
- Nagurney A., Nagurney L.S., Shukla S., 2015. A supply chain game theory framework for cybersecurity investments under network vulnerability. In: Computation, Cryptography, and Network Security. Daras N.J., Rassias, M.T., Editors, Springer International Publishing, Switzerland, 381-398.
- Nagurney, A., Saberi, S., Shukla, S., Floden, J., 2015. Supply chain network competition in price and quality with multiple manufacturers and freight service providers. Transportation Research E, 77, 248-267.

Novelty of Work

- The **shippers respond to the security investments** of the FSPs, who compete for business, through the prices that they are willing to pay
- We capture **risk in that the level of security affects the probability of attack** and the expected damages.
- The **security levels in our model are continuous and have upper bounds**.
- Our work is focused on **high-value goods**.

The Game Theory Model for FSPs' Security Investments

Network Topology:

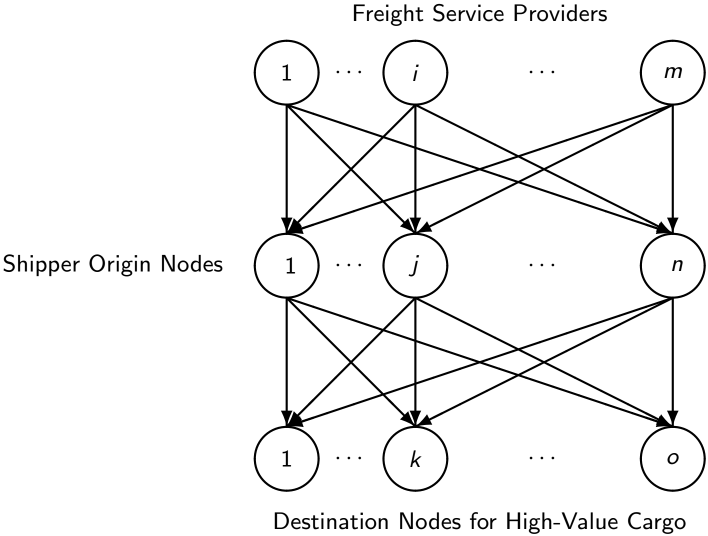


Figure: The Network Structure of the Freight Security Investment Game Theory Model

The Game Theory Model Features

Quantity of High-Value Cargo from Shipper j through FSP i to Node k:

$$0 \leq q_{ijk} \leq \bar{q}_{ijk}, \forall j, k.$$

The above can be grouped into $q \in R_{+}^{mno}$.

Security Level of FSP i for Shipping from j to k:

$$0 \leq s_{ijk} \leq \bar{s}_{ijk}, \forall j, k.$$

The above can be grouped into $s \in R_{+}^{mno}$.

Investment Cost Function h_{ijk}:

$$h_{ijk}(s_{ijk}) = \alpha_{ijk}(\frac{1}{\sqrt{(1 - s_{ijk})}} - 1), \alpha_{ijk} > 0, \forall i, j, k.$$

α_{ijk} allows FSPs to have different investments based on needs and expert knowledge about any OD pair. $h_{ijk}(1) = \infty, h_{ijk}(0) = 0$.

The Game Theory Model Features

Probability of a Successful Attack from i going from j to k:

$$p_{ijk} = (1 - s_{ijk}), \forall i, j, k.$$

If there is no security on by i along (j, k), that is, $s_{ijk} = 0$, probability of an attack is equal to 1.

Price FSP i Charges the Shipper j to Ship to Node k:

$$\rho_{ijk} = \rho_{ijk}(q, s), \forall j, k.$$

Prices are continuously differentiable, increasing in quantities but decreasing in security levels.

Total Cost Faced by FSP i in Transporting High-value Goods from j to k:

$$\hat{c}_{ijk} = \hat{c}_{ijk}(q), \forall j, k.$$

We assume that the total costs are continuously differentiable and convex. FSPs are affected by the quantities of other FSPs as well.

The Game Theory Model Features

Damages on i Traveling from j to k:

$$\sum_{j=1}^{n}\sum_{k=1}^{o} p_{ijk} D_{ijk}.$$

Each FSP i Seeks to Maximize Profit $E(U_i)$:

$$E(U_i) = \sum_{j=1}^{n}\sum_{k=1}^{o}(1 - p_{ijk})(\rho_{ijk}(q,s)q_{ijk} - \hat{c}_{ijk}(q))$$

$$+ \sum_{j=1}^{n}\sum_{k=1}^{o} p_{ijk}(\rho_{ijk}(q,s)q_{ijk} - \hat{c}_{ijk}(q) - D_{ijk}) - \sum_{j=1}^{n}\sum_{k=1}^{o} h_{ijk}(s_{ijk}), \forall i.$$

Let K_i denote the feasible set corresponding to FSP i, where $K_i \equiv \{(q_i, s_i) | 0 \leq q_{ijk} \leq \bar{q}_{ijk}, \forall j, k \text{ and } 0 \leq s_{ijk} \leq \bar{s}_{ijk}, \forall j, k\}$. We also dene the feasible set corresponding to all FSPs: $K \equiv \prod_{i=1}^{m} K^i$.

The Game Theory Model

Denition 1: A Nash Equilibrium in High-Value Product Shipments and Security Levels

A high-value product shipment and security level pattern $(q^*, s^*) \in K$ is said to constitute a Nash equilibrium if for each FSP i:

$$E(U_i(q_i^*, s_i^*, \hat{q}_i^*, \hat{s}_i^*)) \geq E(U_i(q_i, s_i, \hat{q}_i^*, \hat{s}_i^*)), \quad \forall (q_i, s_i) \in K^i,$$

where

$$\hat{q}_i^* \equiv (q_1^*, \ldots, q_{i-1}^*, q_{i+1}^*, \ldots, q_m^*); \quad \text{and} \quad \hat{s}_i^* \equiv (s_1^*, \ldots, s_{i-1}^*, s_{i+1}^*, \ldots, s_m^*).$$

An equilibrium is established if no FSP can unilaterally improve upon his expected profits by selecting an alternative vector of high-value product shipments and security levels.

Variational Inequality Formulations

Variational Inequality Formulations

Theorem 1

Assume that for each FSP, i, the expected profit function $E(U_i(q,s))$ is concave with respect to the variables $\{q_{i11}, ..., q_{ino}\}$ and $\{s_{i11}, ..., s_{ino}\}$, and is continuously differentiable. Then $(q^*, s^*) \in K$ is a Nash Equilibrium according to Definition 1 if and only if it satisfies the variational inequality

$$-\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\frac{\partial E(U_i(q^*,s^*))}{\partial q_{ijk}} \times (q_{ijk} - q^*_{ijk})$$

$$-\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\frac{\partial E(U_i(q^*,s^*))}{\partial s_{ijk}} \times (s_{ijk} - s^*_{ijk}) \geq 0, \forall (q,s) \in K,$$

Variational Inequality Formulations

or, equivalently, $(q^*, s^*) \in K$ is a Nash Equilibrium high-value product shipment and security level pattern if and only if it satisfies the variational inequality

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}[\sum_{h=1}^{n}\sum_{l=1}^{o}\frac{\partial \hat{c}_{ihl}(q^*)}{\partial q_{ijk}} - \rho_{ijk}(q^*, s^*) - \sum_{h=1}^{n}\sum_{l=1}^{o}\frac{\partial \rho_{ihl}(q^*, s^*)}{\partial q_{ijk}} q^*_{ihl}] \times (q_{ijk} - q^*_{ijk})$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}[-\sum_{h=1}^{n}\sum_{l=1}^{o}\frac{\partial \rho_{ihl}(q^*, s^*)}{\partial s_{ijk}} q^*_{ihl} - D_{ijk} + \frac{\partial h_{ijk}(s^*_{ijk})}{\partial s_{ijk}}] \times (s_{ijk} - s^*_{ijk}) \geq 0, \forall (q, s) \in K.$$

The Algorithm and Results

The Euler Method

Explicit Formulae for the Euler Method Applied to the Freight Service Provision Game Theory Model with Security Investments

We have the following closed form expression for the high-value cargo shipments $i = 1, ..., m; j = 1, ..., n; k = 1, ..., o$:

$$q_{ijk}^{\tau+1} = \max\{0, \min\{\bar{q}_{ijk}, q_{ij}^{\tau} + a_{\tau}(-\sum_{h=1}^{n}\sum_{l=1}^{o}\frac{\partial \hat{c}_{ihl}(q^{\tau})}{\partial q_{ijk}} + \rho_{ijk}(q^{\tau}, s^{\tau})$$
$$+\sum_{h=1}^{n}\sum_{l=1}^{o}\frac{\partial \rho_{ihl}(q^{\tau}, s^{\tau})}{\partial q_{ijk}} q_{ihl}^{\tau})\}\}$$

and the following closed form expression for the security levels $i = 1, ..., m; j = 1, ..., n; k = 1, ..., o$:

$$s_{ijk}^{\tau+1} = \max\{0, \min\{\bar{s}_{ijk}, s_{ijk}^{\tau} + a_{\tau}(\sum_{h=1}^{n}\sum_{l=1}^{o}\frac{\partial \rho_{ihl}(q^{\tau}, s^{\tau})}{\partial s_{ijk}} q_{ihl}^{\tau} + D_{ijk} - \frac{\partial h_{ijk}(s_{ijk}^{\tau})}{\partial s_{ijk}})\}\}.$$

Numerical Results

We now apply the above Euler method to compute the high-value product shipments and security level investments in a series of numerical examples.

We implemented the algorithm in FORTRAN and used a LINUX system at the University of Massachusetts Amherst for the computations.

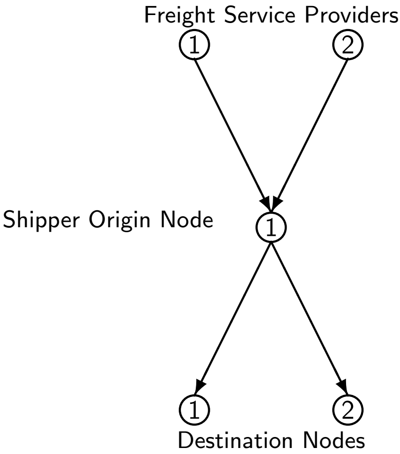
The convergence criterion was that the absolute value of the difference of the cargo shipment and security level iterates at two successive iterations was less than or equal to 10^5.

All the variables (shipments and security levels) were initialized to 0.00.

The sequence $\{\alpha_\tau\} = \{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, ...\}$.

Example: Two FSPs, One Shipper, Two Destination Nodes

The Network:



Example: Two FSPs, One Shipper, Two Destination Nodes

Total Costs:

$$\hat{c}_{111} = q_{111}^2 + 5q_{111}, \quad \hat{c}_{211} = .5q_{211}^2 + 5q_{211},$$
$$\hat{c}_{112} = 1.5q_{112}^2 + 5q_{112}, \quad \hat{c}_{212} = q_{212}^2 + 5q_{212}.$$

Demand Price Functions:

$$\rho_{111} = 2q_{111} + 10s_{111} + 100, \quad \rho_{211} = 3q_{211}2q_{111} + 10s_{211} + 110,$$
$$\rho_{112} = 3q_{112}q_{212} + 5s_{112} + 270, \quad \rho_{212} = 2q_{212}q_{112} + 5s_{212} + 200.$$

Damages:

$$D_{111} = 50,000, \quad D_{211} = 40,000, \; D_{112} = 5,600, \quad D_{212} = 10,000.$$

Alphas of the Investment Cost Functions:

$$\alpha_{111} = 10, \quad \alpha_{211} = 10, \; \alpha_{112} = 12, \quad \alpha_{212} = 10.$$

Upper Bounds on Shipments:

$$\bar{q}_{111} = 100, \quad \bar{q}_{211} = 120, \; \bar{q}_{112} = 80, \quad \bar{q}_{212} = 100.$$

Example: Two FSPs, One Shipper, Two Destination Nodes

The Euler method converges to the following equilibrium solution:

$$q^*_{111} = 15.49, q^*_{112} = 26.64, q^*_{211} = 11.99, q^*_{212} = 28.89,$$

$$s^*_{111} = .99, s^*_{112} = .46, s^*_{211} = s^*_{212} = .99.$$

The demand prices at the computed equilibrium pattern are:

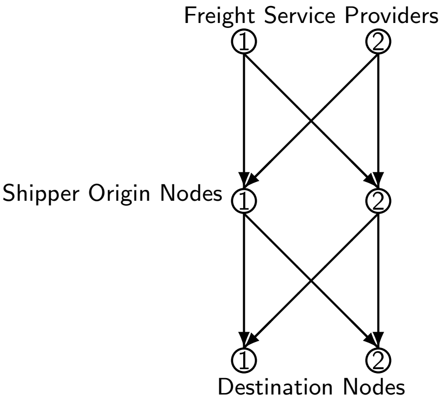
$$\rho_{111} = 66.94, \rho_{112} = 163.48, \rho_{211} = 52.96, \rho_{212} = 120.54.$$

The expected utilities of the freight service providers are now:

$$E(U_1) = 237.83, E(U_2) = 2371.25.$$

Example: Two FSPs, Two Shippers, Two Destination Nodes

The Network:



Example: Two FSPs, Two Shippers, Two Destination Nodes

Example 4 is constructed from Example 3 and has the same data except that now we have an additional shipper who wishes to explore freight service provision from the two freight service providers.

The **total cost functions** associated with the second shipper are:
$\hat{c}_{121} = q_{121}^2 + q_{121}, \hat{c}_{122} = .5q_{122}^2 + q_{122}, \hat{c}_{221} = q_{221}^2 + 2q_{221}, \hat{c}_{222} = 1.5q_{222}^2 + 3q_{222}$.
The **demand price functions** associated with transacting with the second shipper are:
$\rho_{121} = 2q_{121}q_{221} + s_{121} + 150, \rho_{122} = 3q_{122}q_{222} + 2s_{122} + 130, \rho_{221} = 4q_{221}q_{121} + 5s_{221} + 120, \rho_{222} = 5q_{222}2q_{112} + 3s_{222} + 140.$
The additional **Alpha terms** are:
$\alpha_{121} = 5, \alpha_{122} = 4, \alpha_{221} = 3, \alpha_{222} = 12.$
The additional **damage terms** are:
$D_{121} = 20000, D_{122} = 15000, D_{221} = 25000, D_{222} = 2000.$ **Upper Bounds**:
$\bar{q}_{121} = 100, \bar{q}_{122} = 80, \bar{q}_{221} = 70, \bar{q}_{222} = 60.$

Example: Two FSPs, Two Shippers, Two Destination Nodes

The Euler method converges to the following equilibrium shipment and security level pattern:

$$q^*_{111} = 15.71, q^*_{112} = 26.64, q^*_{121} = 23.34, q^*_{122} = 17.78,$$
$$q^*_{211} = 10.65, q^*_{212} = 28.89, q^*_{221} = 9.96, q^*_{222} = 6.56.$$

$$s^*_{111} = .99, s^*_{112} = .46, s^*_{121} = .99, s^*_{122} = .99, s^*_{211} = .99, s^*_{212} = .99, s^*_{221} = .99, s^*_{222} = .00.$$

The demand prices incurred at the equilibrium pattern are:

$$\rho^*_{111} = 67.83, \rho^*_{112} = 163.48, \rho^*_{121} = 94.35, \rho^*_{122} = 72.10,$$

$$\rho^*_{211} = 47.61, \rho^*_{212} = 120.54, \rho^*_{221} = 61.77, \rho^*_{222} = 53.94.$$

The expected utilities of the freight service providers are:
$E(U_1) = 2567.49, E(U_2) = 708.97.$

Conclusions

Conclusions

- We quantify security investment cost functions which **may differ for distinct FSP/shipper/destination node combinations**.
- Shippers reveal their **preferences and sensitivity to investments in security through the prices** that they are will to pay for freight service provision and these also can be **distinct for different freight service provider/shipper/destination node combinations**.
- The FSPs seek to maximize their expected utilities, which capture the **probability of an attack associated with different links and are a function of the security level associated with that link**. Hence, **risk is also captured** in the competitors objective functions.

Conclusions

- The model is **not limited to the number** of FSPs, shippers, and/or destination nodes.
- The equilibrium conditions, which correspond to a Nash Equilibrium, are formulated as a variational inequality problem for which a solution is **guaranteed to exist**.
- The model is **computable** and numerical examples reveal the **equilibrium high-value cargo shipments plus security levels** that the freight service providers deliver and invest in, respectively.

Thank You!

Please visit https://supernet.isenberg.umass.edu/ for more information.