Supply Chain Network Capacity Competition with Outsourcing: A Variational Equilibrium Framework

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Background and Motivation

- The logistics landscape, from warehousing to distribution, underpinning supply chains is dealing with **increased competition and tightened capacity** along with increasing consolidation.
- **29% of shippers** in a recent survey noting that they have engaged with a larger number of **third party logistics** providers to get access to **gain capacity.**
- **73% of shippers** noted that they **increased** their use of outsourced logistics services in 2015, as compared to a figure of **68%** in the previous year.

Background and Motivation

- Firms compete for **shared capacities** of third party logistics providers for both distribution center space as well as freight service provision to their demand markets.
- **UPS** has recently built several **healthcare logistics** hubs in Asia and the Pacific in order to catch up with the rapid growth in the demand for pharmaceuticals.
- **Nestle and PepsiCo**, are sharing warehouse capabilities, in the form of storage, packing operations in Belgium and Luxembourg.
- **Kimberly-Clark Corporation** has been very innovative in **sharing warehouses** as well as freight service provision with multiple different companies, including **Unilever and Kellogg**, in several European countries with results of cost reduction and improvement in customer service.

Background and Motivation

- We develop a **competitive supply chain network model** consisting of multiple firms involved in the manufacture/production of a similar, substitutable, product distinguished by each firm's brand.
- The firms have available manufacturing plants and distribution centers, and supply the same demand points, which can correspond to retailers.
- The firms may avail themselves of **external distribution centers**, to which they can **outsource** any or all of the storage of their products and also the ultimate delivery to the demand markets.
- The external distribution center storage links and freight service provision links have associated **capacities** and the firms compete for storage and freight service provision.

Generalized Nash Equilibrium

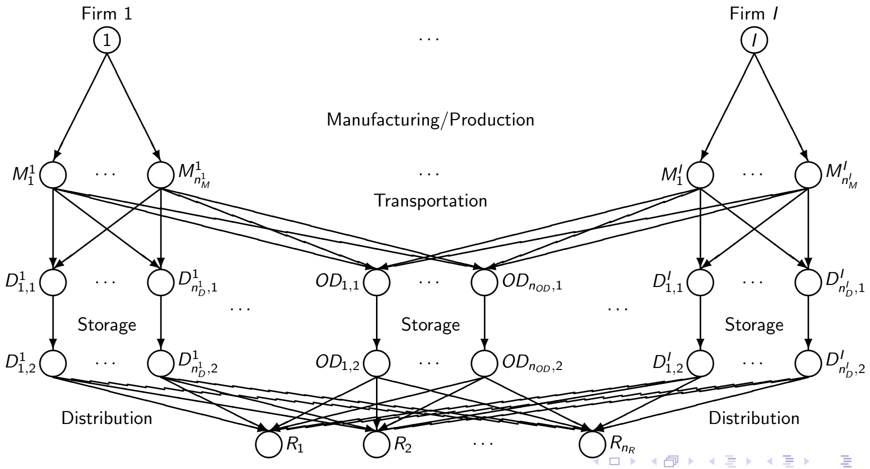
- In Nash equilibrium problems, the strategies of players, that is, decision-makers in the noncooperative game, affect the utility functions of the others, but **the feasible set of each player only depends on his/her strategies**.
- In a Generalized Nash Equilibrium game, the strategies of decision-makers depend not only on **their feasible sets**, but also depend on the strategies played by the other decision-makers.
- There are **shared or "coupling" constraints** in the model therefore, the model becomes a **Generalized Nash Equilibrium (GNE)** model.
- There has been only limited development of GNE models for supply chain networks.
- GNE problem dates to Debreu (1952) and Arrow and Debreu (1954).
- Bensoussan (1974) formulated the GNE problem as a **quasi-variational inequality**.

Some Literature on GNE

- Nagurney, Alvarez Flores, and Soylu (2016) focused on **post-disaster humanitarian relief** and constructed an integrated network model in which disaster relief NGOs compete for **financial funds from donors** while also deriving **utility from providing relief** through their supply chains to multiple points of demand.
- The shared constraints consist of lower and upper bounds for relief supplies at demand points in order to ensure that needs of the victims are met but not at the expense of material convergence and oversupply.
- The GNE model in Nagurney, Alvarez Flores, and Soylu (2016) was of a structure that **enabled its reformulation as an optimization problem**.
- Here, we make use of a *variational equilibrium* (cf. Facchinei and Kanzow (2010), Kulkarni and Shanbhag (2012)), which is a specific kind of GNE.
- GNE models have been constructed for the **energy and airline industry** (see, e.g., Contreras, Klusch, and Krawczyk (2004), Jiang and Pang (2011)).

The Network Topology

- The I firms compete noncooperatively in an oligopolistic manner.
- Firms compete for and may share space in the n_{OD} external distribution centers, and the same holds for the subsequent freight service provision for distribution to the n_R demand markets.



The Supply Chain Problem with Capacity Competition

Demand

The following conservation of flow equations must hold for each firm i: $i = 1, \dots, I$:

$$\sum_{p \in P_k^i} x_p^i = d_{ik}, \quad \forall k, \tag{1}$$

Nonnegativity constraint of the path flows

The path flows must be nonnegative; that is, for each firm i; $i = 1, \dots, I$:

$$x_p^i \geq 0, \quad \forall p \in P^i. \tag{2}$$

Link flows

The expression that relates the link flows of each firm i; $i = 1, \dots, I$, to the path flows is given by:

$$f_a^i = \sum_{p \in P} x_p^i \delta_{ap}, \quad \forall a \in L, \tag{3}$$

where $\delta_{ap} = 1$, if link a is contained in path p, and 0, otherwise.

The Supply Chain Problem with Capacity Competition

Capacity of the individual links of the firms:

For links corresponding to the individual firm networks L^i; $i = 1, \ldots, I$, we must have that:

$$f_a^i \leq u_a^i, \quad \forall a \in L^i. \tag{4}$$

Capacity of the outsourced links:

For the links corresponding to the outsourced storage and distribution, the following capacity constraints must be satisfied:

$$\sum_{i=1}^{I} f_a^i \leq u_a, \quad \forall a \in L^S. \tag{5}$$

Demand Price

We may express the demand price function, $\rho_{ik}(d)$, as:

$$\hat{\rho}_{ik} = \hat{\rho}_{ik}(x) \equiv \rho_{ik}(d), \quad \forall i, \forall k. \tag{6}$$

The Supply Chain Problem with Capacity Competition

Link cost

The total operational cost associated with link a, $\forall a \in L$ and all firms i;i=1,...,I, that is:

$$\hat{c}_a^i = \hat{c}_a^i(f), \quad \forall a \in L, \tag{6}$$

Profit/Utility

The profit/utility function of farm i, denoted by U_i, is given by:

$$U_i = \sum_{k=1}^{n_R} \rho_{ik}(d) d_{ik} - \sum_{a \in L^i \cup L^S} \hat{c}_a^i(f), \tag{7}$$

Vector of Strategies

X_i is the vector of path flows associated with firm i, that is:

$$X_i \equiv \{\{x_p\} | p \in P^i\} \in R_+^{n_{P^i}}. \tag{8}$$

Profits of all the firms

We group the profits of all the firms into an I-dimensional vector $\hat{U}$, where

$$\hat{U} = \hat{U}(X). \tag{9}$$

The Supply Chain Problem with Capacity Competition

Capacity in path flows

In view of the conservation of flow equations (3), we may rewrite the individual firms' capacity constraints (4) in terms of path flows as:

$$\sum_{p \in P} x_p^i \delta_{ap} \leq u_a^i, \quad \forall a \in L^i, \forall i. \tag{10}$$

Capacity of outsourced in path flows

We may rewrite the shared capacity constraints (5) in terms of path flows such that:

$$\sum_{i=1}^{I} \sum_{p \in P} x_p^i \delta_{ap} \leq u_a, \quad \forall a \in L^S. \tag{11}$$

Feasible sets of the different constraints

Each firm i has individual feasible set K_i for $i = 1, \ldots, I$, as:

$$K_i \equiv \{x_p^i \geq 0, \forall p \in P^i \text{ and } (10) \text{ holds}\}. \tag{12}$$

The feasible set consisting of the shared constraints, $\mathcal{S}$, can be defined as:

$$\mathcal{S} \equiv \{x | (11) \text{ holds}\}. \tag{13}$$

The Supply Chain Problem with Capacity Competition

Definition 1: Supply Chain Network Generalized Nash Equilibrium with Capacity Competition and Outsourcing
A path flow pattern $X^* \in K = \prod_{i=1}^{I} K_i$, $X^* \in \mathcal{S}$, constitutes a supply chain network Generalized Nash Equilibrium if for each firm i; $i = 1, \ldots, I$:

$$\hat{U}_i(X_i^*, \hat{X}_i^*) \geq \hat{U}_i(X_i, \hat{X}_i^*), \quad \forall X_i \in K_i, \forall X \in \mathcal{S}, \tag{14}$$

where $\hat{X}_i^* \equiv (X_1^*, \ldots, X_{i-1}^*, X_{i+1}^*, \ldots, X_I^*)$.

- An equilibrium is established if no firm can unilaterally improve its profit by changing its product flows in the supply chain network, given the product flow decisions of the other firms, and subject to the capacity constraints, both individual and shared/coupling ones.

The Supply Chain Problem with Capacity Competition

- If there are no coupling, that is, shared, constraints in the above model, then X and X^* in Definition 1 need only lie in the set K.

The solution to what would then be a Nash equilibrium problem (see Nash (1950, 1951)) would coincide with the solution of the following variational inequality problem: determine $X^* \in K$, such that

$$-\sum_{i=1}^{I} \langle \nabla_{X_i} \hat{U}_i(X^*), X_i - X_i^* \rangle \geq 0, \quad \forall X \in K. \tag{15}$$

- The strategies of the "players," affect not only the values of the others' objective functions, but also the strategies of the firms affect the other firms' strategies because of the shared constraints.
- Generalized Nash Equilibrium problems can no longer directly be formulated as variational inequality problems, but, instead, are formulated as quasi-variational inequalities.

Variational Equilibrium

In a GNE defined by a variational equilibrium, the Lagrange multipliers associated with the coupling constraints are all the same.

Definition 2: Variational Equilibrium

A strategy vector X^* is said to be a variational equilibrium of the above Generalized Nash Equilibrium game if $X^* \in K, X^* \in \mathcal{S}$ is a solution of the variational inequality:

$$-\sum_{i=1}^{I} \langle \nabla_{X_i} \hat{U}_i(X^*), X_i - X_i^* \rangle \geq 0, \quad \forall X \in K, \forall X \in \mathcal{S}. \qquad (16)$$

Expansion of Variational Inequality

Specifically, by definition, we have that

$$-\nabla_{X_i}\hat{U}_i(X)=\left[-\frac{\partial \hat{U}_i}{\partial x_p^i};\, p\in P_k^i;\, k=1,\ldots,n_R\right]. \tag{17}$$

We also know that, in view of (1) and (7), for paths $p\in P_k^i$:

$$-\frac{\partial \hat{U}_i}{\partial x_p^i}=-\frac{\partial\left(\sum_{l=1}^{n_R}\rho_{il}(d)\sum_{q\in P_l^i}x_q^i-\sum_{b\in L^i\cup L^s}\hat{c}_b^i(f)\right)}{\partial x_p^i}. \tag{18}$$

Making use of (1) and (3) and the expressions:

$$\frac{\partial \hat{C}_p^i(x)}{\partial x_p^i}\equiv\sum_{a\in L^i\cup L^s}\sum_{b\in L^i\cup L^s}\frac{\partial \hat{c}_b^i(f)}{\partial f_a^i}\delta_{ap}, \tag{19a}$$

$$\frac{\partial \hat{\rho}_{il}(x)}{\partial x_p^i}\equiv\frac{\partial \rho_{il}(d)}{\partial d_{ik}}. \tag{19b}$$

we obtain:

$$-\frac{\partial \hat{U}_i}{\partial x_p^i}=\left[\frac{\partial \hat{C}_p^i(x)}{\partial x_p^i}-\hat{\rho}_{ik}(x)-\sum_{l=1}^{n_R}\frac{\partial \hat{\rho}_{il}(x)}{\partial x_p^i}\sum_{q\in P_l^i}x_q^i\right]. \tag{20}$$

Expansion of Variational Inequality

In view of (20), it is clear that variational inequality (16) is equivalent to the variational inequality that determines the vector of equilibrium path flows $x^* \in K$, $x^* \in \mathcal{S}$ such that:

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\sum_{p\in P_k^i}\left[\frac{\partial \hat{C}_p^i(x^*)}{\partial x_p^i} - \hat{\rho}_{ik}(x^*) - \sum_{l=1}^{n_R}\frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p^i}\sum_{q\in P_l^i} x_q^{i*}\right]\times[x_p^i - x_p^{i*}] \geq 0, \quad \forall x \in K,$$

$$\forall x \in \mathcal{S}. \tag{21}$$

Variational inequality (16) can also be expressed in terms of link flows as follows: determine the vector of equilibrium link flows and the vector of demands $(f^*, d^*) \in \mathcal{K}^0$, such that:

$$\sum_{i=1}^{I}\sum_{a\in L^i\cup L^s}\left[\sum_{b\in L^i\cup L^s}\frac{\partial \hat{c}_b^i(f^*)}{\partial f_a^i}\right]\times[f_a^i - f_a^{i*}]$$

$$+\sum_{i=1}^{I}\sum_{k=1}^{n_R}\left[-\rho_{ik}(d^*) - \sum_{l=1}^{n_R}\frac{\partial \rho_{il}(d^*)}{\partial d_{ik}}d_{il}^*\right]\times[d_{ik} - d_{ik}^*] \geq 0, \quad \forall (f,d) \in \mathcal{K}^0 \tag{22}$$

where $\mathcal{K}^0 \equiv \{(f,d) | \exists x \geq 0, (1), (3), (4), \text{and } (5) \text{ hold}\}$.

Alternative Variational Inequality Formulations

Theorem 1: Alternative Variational Inequality Formulations of the Variational Equilibrium in Path Flows and in Link Flows
The variational equilibrium (16) is equivalent to the variational inequality: determine the vector of equilibrium path flows, and the vector of optimal Lagrange multipliers, $(x^*, \lambda^*, \eta^*) \in \mathcal{K}$, *such that:*

$$\sum_{i=1}^{I} \sum_{k=1}^{n_R} \sum_{p \in P_k^i} \left[\frac{\partial \hat{C}_p^i(x^*)}{\partial x_p^i} + \sum_{a \in L^i} \lambda_a^* \delta_{ap} + \sum_{a \in L^S} \eta_a^* \delta_{ap} - \hat{\rho}_{ik}(x^*) - \sum_{l=1}^{n_R} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p^i} \sum_{q \in P_l^i} x_q^{i*} \right]$$

$$\times [x_p^i - x_p^{i*}] + \sum_{i=1}^{I} \sum_{a \in L^i} \left[u_a^i - \sum_{p \in P} x_p^{i*} \delta_{ap} \right] \times [\lambda_a - \lambda_a^*]$$

$$+ \sum_{a \in L^S} \left[u_a - \sum_{i=1}^{I} \sum_{p \in P} x_p^{i*} \delta_{ap} \right] \times [\eta_a - \eta_a^*] \geq 0, \qquad (x, \lambda, \eta) \in \mathcal{K}, \qquad (23)$$

where $\mathcal{K} \equiv \{(x, \lambda, \eta) | x \in R_+^{n_P}, \ \lambda \in R_+^{\sum_{i=1}^{I} n_{L^i}}, \ \eta \in R_+^{n_{L^S}} \}$.

Alternative Variational Inequality Formulations

The variational inequality (23), in turn, can be rewritten in terms of link flows as: determine the vector of equilibrium link flows, the vector of demands, and the vector of optimal Lagrange multipliers, $(f^*, d^*, \lambda^*, \eta^*) \in \mathcal{K}^1$*, such that:*

$$\sum_{i=1}^{I} \sum_{a \in L^i} \left[\sum_{b \in L^i} \frac{\partial \hat{c}_b^i(f^*)}{\partial f_a^i} + \lambda_a^* \right] \times [f_a^i - f_a^{i*}] + \sum_{i=1}^{I} \sum_{a \in L^S} \left[\sum_{b \in L^S} \frac{\partial \hat{c}_b^i(f^*)}{\partial f_a^i} + \eta_a^* \right] \times [f_a^i - f_a^{i*}]$$

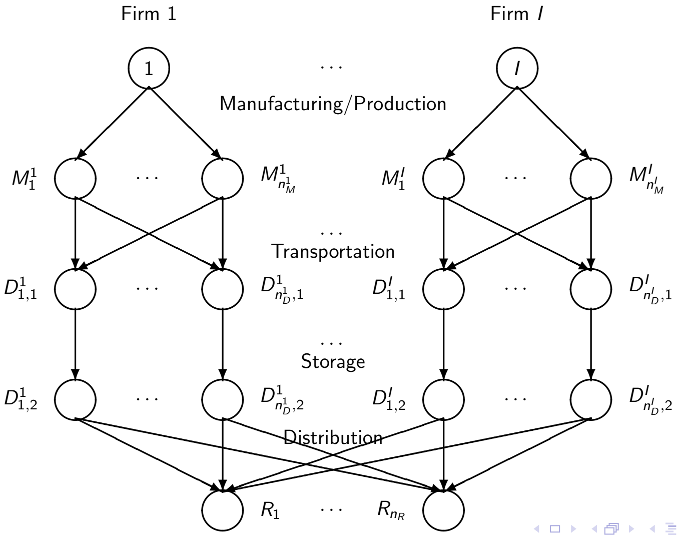
$$+ \sum_{i=1}^{I} \sum_{k=1}^{n_R} \left[-\rho_{ik}(d^*) - \sum_{l=1}^{n_R} \frac{\partial \rho_{il}(d^*)}{\partial d_{ik}} d_{il}^* \right] \times [d_{ik} - d_{ik}^*]$$

$$+ \sum_{i=1}^{I} \sum_{a \in L^i} \left[u_a^i - f_a^{i*} \right] \times [\lambda_a - \lambda_a^*] + \sum_{a \in L^S} \left[u_a - \sum_{i=1}^{I} f_a^{i*} \right] \times [\eta_a - \eta_a^*] \geq 0, \qquad (f, d, \lambda, \eta) \in \mathcal{K}^1, \tag{24}$$

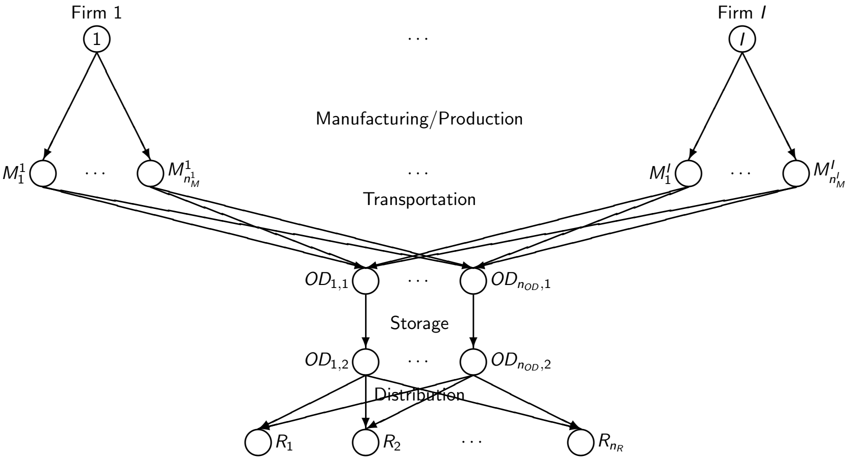
where $\mathcal{K}^1 \equiv \{(f, d, \lambda, \eta) | \exists x \geq 0,\ (1)$ *and* (3) *hold, and* $\lambda \geq 0,\ \eta \geq 0\}$.

Proof: Variational inequality (23) follows from the Karush Kuhn Tucker conditions (see also Lemma 1.2 in Yashtini and Malek (2007)). Variational inequality (24) then follows from variational inequality (23) by making use of the conservation of flow equations. □

Special Cases of the Supply Chain Network Oligopoly Model: No Shared Constraints



Special Cases of the Supply Chain Network Oligopoly Model: Outsource Storage and Freight Services



The Algorithm - Euler Method

Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993) is a solution methodology of the Variational Inequality Problem. Specifically, iteration τ of the Euler method is given by:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau} F(X^{\tau})), \tag{25}$$

The Euler method, the sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \to 0$, as $\tau \to \infty$.

The Euler Method Explicit Formulae

For each path $p \in P_j^i$, $\forall i, j$, compute:

$$x_p^{i^{\tau+1}} = \max\{0, x_p^{i^{\tau}} + a_\tau(\hat{\rho}_{ik}(x^\tau) + \sum_{l=1}^{n_R} \frac{\partial \hat{\rho}_{il}(x^\tau)}{\partial x_p^i} \sum_{q \in P_l^i} x_q^{i^{\tau}} - \frac{\partial \hat{C}_p^i(x^\tau)}{\partial x_p^i} - \sum_{a \in L^i} \lambda_a^\tau \delta_{ap} - \sum_{a \in L^S} \eta_a^\tau \delta_{ap})\},$$

$$\forall p \in P_k^i;\ i = 1, \ldots, I;\ k = 1, \ldots, n_R. \qquad (26)$$

The Lagrange multipliers for each link $a \in L^i; i = 1, \ldots, I$, compute:

$$\lambda_a^{\tau+1} = \max\{0, \lambda_a^\tau + a_\tau(\sum_{p \in P} x_p^{i^{\tau}} \delta_{ap} - u_a^i)\},\ \forall a \in L^i; i = 1, \ldots, I. \qquad (27)$$

The computation process for the Lagrange multipliers for the shared link $a \in L^S$, can be given as:

$$\eta_a^{\tau+1} = \max\{0, \eta_a^\tau + a_\tau(\sum_{i=1}^{I} \sum_{p \in P} x_p^{i^{\tau}} \delta_{ap} - u_a)\},\ \forall a \in L^S. \qquad (28)$$

Case Study - 4 Examples

- We focus on apple growers in western Massachusetts.
- We consider two farmers that grow the apples, which, because of their quality, are represented by brands.
- Each farmer has two areas in which he grows his apples and each farm supplies its produce to two major retailers in the form of supermarkets in western Massachusetts.
- The Euler method was implemented in FORTRAN and a Linux system at the University of Massachusetts Amherst was used.
- The convergence tolerance $\epsilon = 10^{-6}$, the Lagrange multipliers were all initialized to 0.00 and the sequence $\{a_\tau\} = .1\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \ldots\}$.

Total Link Cost Functions for the Numerical Examples

Link a	From Node	To Node	$\hat{c}^1_a(f^1_a)$	$\hat{c}^2_a(f^2_a)$
1	1	M^1_1	$0.03(f^1_1)^2 + 3f^1_1$	–
2	1	M^1_2	$0.02(f^1_2)^2 + 2f^1_2$	–
3	M^1_1	$D^1_{1,1}$	$0.01(f^1_3)^2 + 4f^1_3$	–
4	M^1_2	$D^1_{1,1}$	$0.025(f^1_4)^2 + 3f^1_4$	–
5	$D^1_{1,1}$	$D^1_{1,2}$	$0.035(f^1_5)^2 + 5f^1_5$	–
6	$D^1_{1,2}$	R_1	$0.02(f^1_6)^2 + 2f^1_6$	–
7	$D^1_{1,2}$	R_2	$0.03(f^1_7)^2 + 5f^1_7$	–
8	2	M^2_1	–	$0.01(f^2_8)^2 + 6f^2_8$
9	2	M^2_2	–	$0.01(f^2_9)^2 + 6f^2_9$
10	M^2_1	$D^2_{1,1}$	–	$0.02(f^2_{10})^2 + 4f^2_{10}$
11	M^2_2	$D^2_{1,1}$	–	$0.02(f^2_{11})^2 + 4f^2_{11}$
12	$D^2_{1,1}$	$D^2_{1,2}$	–	$0.03(f^2_{12})^2 + 5f^2_{12}$
13	$D^2_{1,2}$	R_1	–	$0.02(f^2_{13})^2 + 8f^2_{13}$
14	$D^2_{1,2}$	R_2	–	$0.035(f^2_{14})^2 + 5f^2_{14}$
15	M^1_1	$OD_{1,1}$	$0.01(f^1_{15})^2 + 6f^1_{15}$	–
16	M^1_2	$OD_{1,1}$	$0.02(f^1_{16})^2 + 5f^1_{16}$	–
17	M^2_1	$OD_{1,1}$	–	$0.02(f^2_{17})^2 + 5f^2_{17}$
18	M^2_2	$OD_{1,1}$	–	$0.02(f^2_{18})^2 + 6f^2_{18}$
19	$OD_{1,1}$	$OD_{1,2}$	$0.01(f^1_{19})^2 + f^1_{19}$	$0.01(f^2_{19})^2 + f^2_{19}$
20	$OD_{1,2}$	R_1	$0.012(f^1_{20})^2 + 2f^1_{20}$	$0.012(f^2_{20})^2 + 2f^2_{20}$
21	$OD_{1,2}$	R_2	$0.01(f^1_{21})^2 + f^1_{21}$	$0.01(f^2_{21})^2 + f^2_{21}$

- Cost functions are constructed according to the information gathered from Berkett (1994) and CISA (2016).
- Farm 1 has **200 acres** and Farm 2 has **100 acres** of land, therefore the **labor and machinery costs** of Farm 1 are expected to be higher.
- Farm 1's second production facility is smaller than first one, whereas Farm 2 has **identical production** facilities.
- External distribution center charges both farmers the same price,since the two supermarkets are in proximity to one another.

Capacity on the Links

- The capacities on the links associated with the farms are constructed based on size of land, the available manpower, machinery, and vehicles.
- Farm 1 is larger in size, however, as expected, the capacities of the external distribution centers and freight services are as high or higher than those associated with the individual farms.

Link capacities of Farm 1 in bushels of apples:

$$\mathbf{u_1^1 = 3000}, \quad u_2^1 = 1000, \quad u_3^1 = 2000, \quad u_4^1 = 1000, \quad u_5^1 = 10000,$$
$$u_6^1 = 500, \quad u_7^1 = 300, \quad u_{15}^1 = 2000, \quad u_{16}^1 = 500.$$

Link capacities of Farm 2 in bushels of apples:

$$\mathbf{u_8^2 = 1500}, \quad u_9^2 = 500, \quad u_{10}^2 = 1000, \quad u_{11}^2 = 500, \quad u_{12}^2 = 5000,$$
$$u_{13}^2 = 400, \quad u_{14}^2 = 200, \quad u_{17}^2 = 1500, \quad u_{18}^2 = 400.$$

Link capacities the External Distribution Center and Freight Service Provider in bushels of apples:

$$\mathbf{u_{19} = 10000}, \quad \mathbf{u_{20} = 1000}, \quad \mathbf{u_{21} = 1000}.$$

Demand Price Functions

Farm 1:

$$\rho_{11}(d) = -0.002d_{11} - 0.001d_{21} + 90,$$

$$\rho_{12}(d) = -0.003d_{12} - 0.001d_{22} + 100.$$

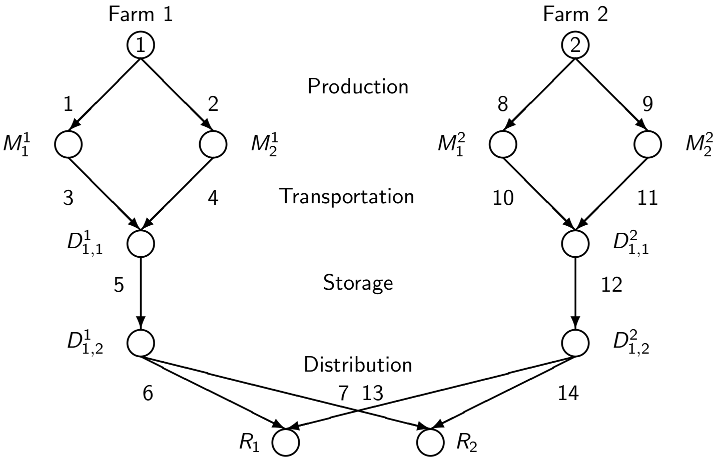
Farm 2:

$$\rho_{21}(d) = -0.002d_{21} - 0.001d_{11} + 80,$$

$$\rho_{22}(d) = -0.0025d_{22} - 0.001d_{12} + 100.$$

Example 1: Only Farmers' Storage Facilities Are Available

- There are **no available external distribution centers**.
- This example serves as a **baseline**.



Equilibrium Link Flow and Lagrange Multiplier Pattern for Example 1

Link a	f_a^{1*}	f_a^{2*}	λ_a^*
1	291.33	–	0.00
2	281.54	–	0.00
3	291.33	–	0.00
4	281.54	–	0.00
5	572.86	–	0.00
6	279.24	–	0.00
7	293.62	–	0.00
8	–	244.48	0.00
9	–	244.48	0.00
10	–	244.48	0.00
11	–	244.48	0.00
12	–	488.96	0.00
13	–	288.96	0.00
14	–	**200.00**	**19.68**

Equilibrium Path Flows, Prices, Demands and Profits

Equilibrium product path flows:

$$x_{p_1}^{1*} = 142.07, \quad x_{p_2}^{1*} = 137.17, \quad x_{p_3}^{1*} = 149.26, \quad x_{p_4}^{1*} = 144.37,$$

$$x_{p_5}^{2*} = 144.48, \quad x_{p_6}^{2*} = 144.48, \quad x_{p_7}^{2*} = 100.00, \quad x_{p_8}^{2*} = 100.00.$$

Equilibrium prices at the demand markets:

$$\rho_{11} = 89.15, \rho_{12} = 98.92, \rho_{21} = 79.14, \rho_{22} = 98.71.$$

Equilibrium demands:

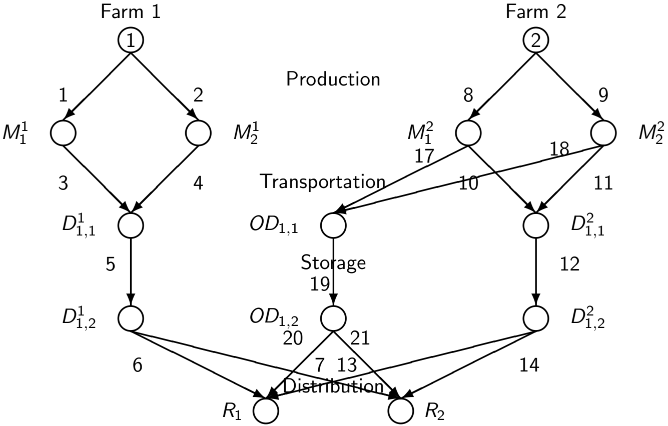
$$d_{11}^* = 279.24, d_{12}^* = 293.62, d_{21}^* = 288.96, d_{22}^* = 200.00$$

Profits:

$$U_1 = 23,008.39, \quad U_2 = 18,135.58.$$

Example 2: An External Distribution Center is Made Available But Only Farmer 2 Is Considering It

- External distribution center has become available **but only the second farmer** is interested in considering it.



Equilibrium Link Flow and Lagrange Multiplier Pattern for Example 2

Link a	f_a^{1*}	f_a^{2*}	λ_a^*	η_a^*
1	290.13	–	0.00	–
2	280.12	–	0.00	–
3	290.13	–	0.00	–
4	280.12	–	0.00	–
5	570.25	–	0.00	–
6	283.73	–	0.00	–
7	286.53	–	0.00	–
8	–	847.72	0.00	–
9	–	**500.00**	**13.40**	–
10	–	259.91	0.00	–
11	–	100.01	0.00	–
12	–	359.92	0.00	–
13	–	159.92	0.00	–
14	–	**200.00**	**6.58**	–
17	–	587.81	0.00	–
18	–	**400.00**	**0.03**	–
19	–	987.81	–	0.00
20	–	175.09	–	0.00
21	–	812.71	–	0.00

Equilibrium Path Flows, Prices, Demands and Profits

Equilibrium product path flows:

$$x_{p_1}^{1*} = 144.37, x_{p_2}^{1*} = 139.35, x_{p_3}^{1*} = 145.76,$$

$$x_{p_4}^{1*} = 140.77, x_{p_5}^{2*} = 119.98, x_{p_6}^{2*} = 39.94,$$

$$x_{p_7}^{2*} = 139.93, x_{p_8}^{2*} = 60.07, x_{p_9}^{2*} = 134.86, x_{p_{10}}^{2*} = 40.23, x_{p_{11}}^{2*} = 452.95, x_{p_{12}}^{2*} = 359.76.$$

Equilibrium prices at the demand markets:

$$\rho_{11} = 89.10, \rho_{12} = 98.13, \rho_{21} = 79.05, \rho_{22} = 94.65$$

Equilibrium demands:

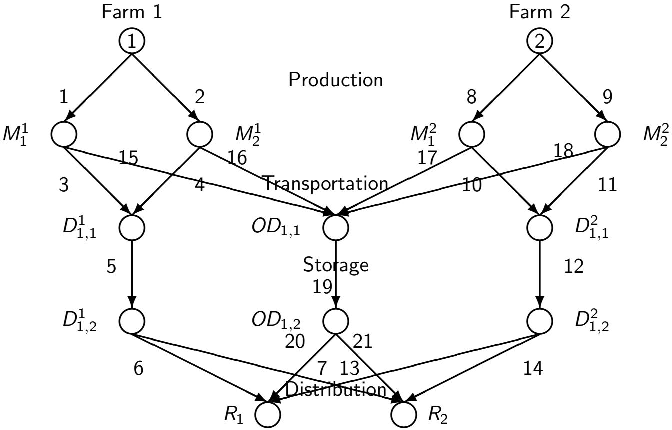
$$d_{11}^* = 283.73, d_{12}^* = 286.53, d_{21}^* = 335.01, d_{22}^* = 1012.71$$

Profits:

$$U_1 = 22,760.00, \quad \mathbf{U_2 = 57,363.86}.$$

Example 3: Both Farmers are Considering the External Distribution Center

- **Both farmers** now are considering the option of the external distribution center.



Equilibrium Link Flow and Lagrange Multiplier Pattern for Example 3

Link a	f_a^{1*}	f_a^{2*}	λ_a^*	η_a^*
1	596.67	–	0.00	–
2	708.78	–	0.00	–
3	142.33	–	0.00	–
4	245.04	–	0.00	–
5	387.37	–	0.00	–
6	177.57	–	0.00	–
7	209.80	–	0.00	–
8	–	778.03	0.00	–
9	–	**500.00**	**10.63**	–
10	–	247.45	0.00	
11	–	120.70	0.00	
12	–	368.15	0.00	
13	–	168.14	0.00	
14	–	**200.00**	**10.20**	–
15	454.34	–	0.00	–
16	463.74	–	0.00	–
17	–	530.58	0.00	–
18	–	379.29	0.00	–
19	918.08	909.88	–	0.00
20	480.99	346.97	–	0.00
21	**437.10**	**562.91**	–	**12.41**

Equilibrium Path Flows, Prices, Demands and Profits

Equilibrium product path flows:

$$x_{p_1}^{1*} = 62.86, x_{p_2}^{1*} = 114.71, x_{p_3}^{1*} = 79.47,$$

$$x_{p_4}^{1*} = 130.33, x_{p_5}^{2*} = 115.78, x_{p_6}^{2*} = 52.36,$$

$$x_{p_7}^{2*} = 131.66, \quad x_{p_8}^{2*} = 68.34, \quad x_{p_9}^{2*} = 211.69,$$

$$x_{p_{10}}^{2*} = 135.28, x_{p_{11}}^{2*} = 318.89, x_{p_{12}}^{2*} = 244.01.$$

$$x_{p_{13}}^{1*} = 238.13, x_{p_{14}}^{1*} = 242.8,$$

$$x_{p_{15}}^{1*} = 216.12, x_{p_{16}}^{1*} = 220.88.$$

Equilibrium prices at the demand markets:

$$\rho_{11} = 88.17, \rho_{12} = 97.30, \rho_{21} = 78.31, \rho_{22} = 95.54$$

Equilibrium Demands and Profits

Equilibrium demands:

$$d^*_{11} = 658.56, d^*_{12} = 646.89, d^*_{21} = 515.12, d^*_{22} = 762.91$$

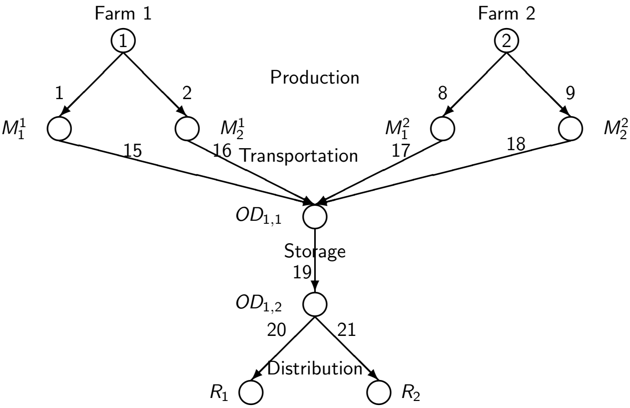
Profits:

$$\mathbf{U_1 = 56,673.31}, \quad U_2 = 42,412.05.$$

- With increased services and competition for them, Farm 2 experiences a drop in profits.
- Farm 2 should try to purchase more land in proximity to that facility since it is constraining its apple production capabilities and demand for its brand of apples.

Example 4: A Supply Chain Disruption Has Damaged Farmers' Distribution Centers

- **Both farmers** have their storage facilities made unavailable due to a **natural disaster, such as flooding**.
- Picking has not been affected.



Equilibrium Link Flow and Lagrange Multiplier Pattern for Example 4

Link a	From Node	To Node	f_a^{1*}	f_a^{2*}	λ_a^*	η_a^*
1	1	M_1^1	513.45	–	0.00	–
2	1	M_2^1	500.00	–	0.00	–
8	2	M_1^2	–	574.26	0.00	–
9	2	M_2^2	–	400.00	0.00	–
15	M_1^1	$OD_{1,1}$	513.45	–	0.00	–
16	M_2^1	$OD_{1,1}$	**500.00**	–	**3.07**	–
17	M_1^2	$OD_{1,1}$	–	574.26	0.00	–
18	M_2^2	$OD_{1,1}$	–	**400.00**	**9.38**	–
19	$OD_{1,1}$	$OD_{1,2}$	1013.45	974.26	–	0.00
20	$OD_{1,2}$	R_1	576.09	411.62	–	0.00
21	$OD_{1,2}$	R_2	**437.36**	**562.64**	–	**15.75**

Equilibrium Path Flows, Prices, Demands and Profits

Equilibrium product path flows:

$$x_{p_{13}}^{1*} = 291.20, x_{p_{14}}^{1*} = 284.90, x_{p_{15}}^{1*} = 225.25,$$

$$x_{p_{15}}^{1*} = 222.25, x_{p_{16}}^{1*} = 215.10,$$

$$x_{p_9}^{2*} = 249.52, x_{p_{10}}^{2*} = 162.10, x_{p_{11}}^{2*} = 324.75, x_{p_{12}}^{2*} = 237.90.$$

Equilibrium prices at the demand markets:

$$\rho_{11} = 88.44, \rho_{12} = 98.13, \rho_{21} = 78.60, \rho_{22} = 96.75.$$

Equilibrium demands:

$$d_{11}^* = 576.09, d_{12}^* = 437.36, d_{21}^* = 411.62, d_{22}^* = 562.64.$$

Profits:

$$\mathbf{U_1 = 46,427.75}, \quad U_2 = 29,237.16.$$

Summary of the Profits in Different Examples

Example	Farm 1 Profit	Farm 2 Profit
1	23,008.39	18,135.58
2	22,760.00	57,363.86
3	56,673.31	42,412.05
4	46,427.75	29,237.16

- Both farms benefit by utilizing the external distribution centers as revealed by the profit increase from Example 1 to Example 3.
- Using an external distribution center increases the farm profits, as it is in Example 2.

Conclusions

- We developed a **supply chain network framework** using **game theory** in which multiple manufacturers/producers have their own production facilities, distribution centers and freight services.
- We focused on **capacity competition** and considered **outsourcing** through the product storage to external distribution centers who also provide freight service provision to the demand points.
- Due to the shared constraints, we utilize the concept of **variational equilibrium**, which is a special case of a **Generalized Nash Equilibrium.**
- We then illustrate the **novel supply chain game theory framework** with a case study consisting of producers that are **apple farmers.**

THANK YOU!

For more information: https://supernet.isenberg.umass.edu/