A Multicountry, Multicommodity Stochastic Game Theory Network Model of Competition for Medical Supplies Inspired by the Covid-19 Pandemic

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Outline

Background

- On March 11, 2020, the World Health Organization declared the novel coronavirus outbreak a pandemic.
- The Covid-19 pandemic plunged almost all countries into crisis quickly.
- One of the most effective ways in decreasing the spread of the virus is to use Personal Protective Equipment (PPE).
- Very soon, many national governments began clamoring for such (and other) medical supplies.
- This meant the beginning of an intense competition among different countries over items that had never before experienced such demand.

Background

- In today's world, every product has a path from production to demand points. In some cases, it goes through several countries. PPEs and their production process depends not only on the capacity of the factories, but also on raw materials.
- The United States Strategic National Stockpile had approximately 12 million N95 masks and 30 million surgical masks while the Department of Health and Human Services had projected that the country would need 3.5 billion face masks in the event of a year-long pandemic.

▲ Officials said logistical challenges continue seven months after the coronavirus reached the United States, and with flu season approaching. Photograph: Mario Tama/Getty Images

(The Guardian)

Background

- China historically has produced half of the world's face masks.
- The **fierce competition** over PPEs led to a **sharp rise in the price** of these items, with some prices rising by more than 1,000 percent.
- To **increase production capacity** and to respond to the sudden increase in demand for ventilators, several companies such as Ford, General Motors, and Tesla announced that they would dedicate their manufacturing facilities and capability to the production of these devices.

Competition among state, local governments creates bidding war for medical equipment

Gov. Cuomo has called for the creation of a 'nationwide buying consortium.'

By ABC News
April 3, 2020, 12:33 PM • 8 min read

CORONAVIRUS

Coronavirus USA: Federal fix sought for 'Wild West' COVID-19 PPE competition

By Chuck Goudie and Barb Markoff, Christine Tressel and Ross Weidner
Thursday, April 2, 2020

Research Questions

- What are the characteristics of competition among countries for essential goods; specifically, medical supplies, in preparation and response to the pandemic?
- What factors affect the competition?
- What are the optimal strategies of governments in achieving their objectives, and can these be determined quantitatively through rigorous modeling and computations?

Our Contributions

We construct a **stochastic game theory network model** that captures the **competition among countries** for **multiple medical supplies** in the setting of the **Covid-19 pandemic**.

We consider **governmental decisions and strategies** made both **before and after** the declaration of a pandemic.

Contributions:

- Competition among multiple countries under uncertainty
- Multiple medical items
- Limited supplies of medical items
- Penalties on the shortages of medical items

Literature Review

- Queiroz et al. (2020) mapped out a research agenda by providing a structured literature review of current Covid-19 related research and supply chain research on previous epidemics.
- Ivanov (2020) conducted simulation-based research to investigate the possible impact of the Covid-19 pandemic on global supply chains.
- Nagurney et al. (2021) constructed a GNE model with stochastic demands in which healthcare organizations competed for medical supplies in the pandemic but that model, in contrast to the model in this paper, is a single-stage model.

Literature Review

- Anparasan and Lejeune (2019) developed a resource allocation model based on the 2010 cholera outbreak in Haiti to support emergency response to an epidemic outbreak in resource-limited countries.

- Mete and Zabinsky (2010) introduced a two-stage stochastic optimization model for storage and distribution of medical supplies but considered a single decision-maker.

- Nagurney et al. (2020) addressed the issue of uncertainty but with the consideration of multiple, competing decision-makers in the first Stochastic Generalized Nash Equilibrium model for disaster relief. In that model.

Table 1: Notation for the Pandemic Stochastic Game Theory Network Model

Notation	**Parameters**
$\omega \in \Omega$	the disaster scenarios.
p_ω	the probability of disaster scenario ω in stage 2; $\forall \omega \in \Omega$.
$S^1_{j,k}$	the supply of medical item k in country j in stage 1; $j = 1, \ldots, I$; $k = 1, \ldots, K$.
$S^{2,\omega}_{j,k}$	the supply of medical item k in country j when scenario ω occurs in stage 2; $j = 1, \ldots, I$; $k = 1, \ldots, K$; $\omega \in \Omega$.
$d^{2,\omega}_{i,k}$	the demand for medical item k in country i when scenario ω occurs in stage 2; $i = 1, \ldots, I$; $k = 1, \ldots, K$; $\omega \in \Omega$.
$\beta_{i,k}$	the unit penalty encumbered by country i on the unmet demand of medical item k; $i = 1, \ldots, I$; $k = 1, \ldots, K$.
$\rho_{j,k}$	the unit price of medical item k at country j before the pandemic; $j = 1, \ldots, I$; $k = 1, \ldots, K$.
$\rho^\omega_{j,k}$	the unit price of medical item k at country j when the scenario ω occurs in stage 2; $j = 1, \ldots, I$; $k = 1, \ldots, K$; $\omega \in \Omega$.
Notation	**Variables**
$q^1_{ij,k}$	the amount of medical item k purchased by country i from country j in stage 1. We group all the j and k elements into the vector q^1_i and then group such vectors for all i into the vector q^1.
$q^{2,\omega}_{ij,k}$	the amount of medical item k purchased by country i from country j when the scenario ω occurs in stage 2. We group all the j and k elements into the vector $q^{2,\omega}_i$ and then group such vectors for all i into the vector $q^{2,\omega}$. Finally, we group these vectors for all ω into the vector q^2. We group the q^1 and q^2 vectors into the vector $q \in R^{IIK+\|\Omega\|(IIK)}_+$.
Notation	**Cost Functions**
$c^1_{ji,k}(q^1)$	the total transportation cost that country i pays to have the medical items k delivered from country j where the items are purchased before the pandemic hits the country.
$c^{2,\omega}_{ji,k}(q^{2,\omega})$	the total transportation cost that country i pays to have medical items k delivered from country j when the scenario ω occurs in stage 2.

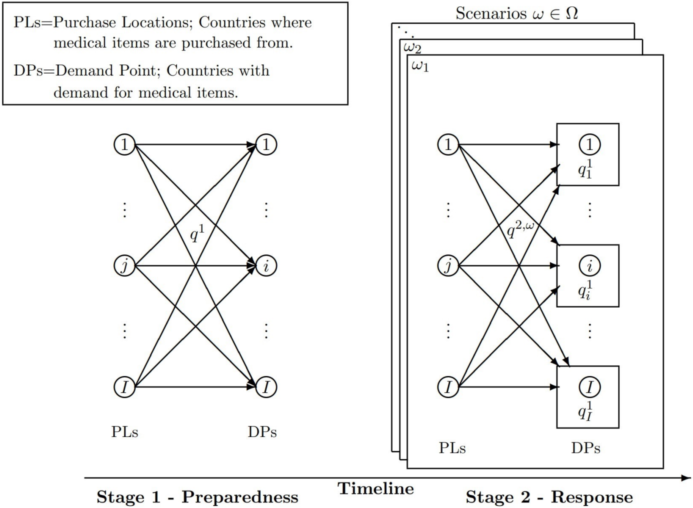

Figure 1: The Timeline of the Pandemic Disaster Preparedness and Response

There are I national governments of countries with a typical one denoted by i. We have I possible countries where the items are purchased from with a typical one denoted by j. There are K different medical items with a typical one denoted by k.

The Countries' Two-stage Stochastic Optimization Problems

$$\text{Minimize} \quad \sum_{j=1}^{I}\sum_{k=1}^{K} \rho_{j,k} q^1_{ij,k} + \sum_{j=1}^{I}\sum_{k=1}^{K} c^1_{ji,k}(q^1) + E_\Omega\left[Q^2_i(q^2,\omega)\right] \tag{1}$$

subject to:

$$\sum_{i=1}^{I} q^1_{ij,k} \leq S^1_{j,k}, \quad j=1,\ldots,I; \quad k=1,\ldots,K, \tag{2}$$

$$q^1_{ij,k} \geq 0, \quad j=1,\ldots,I; \quad k=1,\ldots,K. \tag{3}$$

.

The last term in the objective function (1) is the expected value of the loss to country i in stage 2.

The expected value of the loss to country i in stage 2, including the procurement costs and the consequences of unmet demand:
$E_\Omega\left[Q_i^2(q^2,\omega)\right] = \sum_{\omega\in\Omega} p_\omega \left[Q_i^2(q^2,\omega)\right]$, where the loss in scenario ω, for each country i, is obtained by solving the following second stage optimization problem:

The Second Stage Optimization Problem

$$\text{Minimize} \;\; Q_i^2(q^2,\omega) \equiv \sum_{j=1}^{I}\sum_{k=1}^{K} \rho_{j,k}^{\omega} q_{ij,k}^{2,\omega} + \sum_{j=1}^{I}\sum_{k=1}^{K} c_{ji,k}^{2,\omega}(q^{2,\omega})$$

$$+\sum_{k=1}^{K} \beta_{i,k}[d_{i,k}^{2,\omega} - \sum_{j=1}^{I}(q_{ij,k}^{1} + q_{ij,k}^{2,\omega})] \tag{4}$$

subject to:

$$\sum_{i=1}^{I} q_{ij,k}^{2,\omega} \leq S_{j,k}^{2,\omega}, \quad j=1,\ldots,I; \quad k=1,\ldots,K, \tag{5}$$

$$q_{ij,k}^{2,\omega} \geq 0, \quad j=1,\ldots,I; \quad k=1,\ldots,K. \tag{6}$$

The first- and second-stage problems together form the following minimization problem for each country i:

$$\text{Minimize} \quad \sum_{j=1}^{I}\sum_{k=1}^{K}\rho_{j,k}q^1_{ij,k}+\sum_{j=1}^{I}\sum_{k=1}^{K}c^1_{ji,k}(q^1)$$

$$+\sum_{\omega\in\Omega}p_\omega\left[\sum_{j=1}^{I}\sum_{k=1}^{K}\rho^\omega_{j,k}q^{2,\omega}_{ij,k}+\sum_{j=1}^{I}\sum_{k=1}^{K}c^{2,\omega}_{ji,k}(q^{2,\omega})+\sum_{k=1}^{K}\beta_{i,k}[d^{2,\omega}_{i,k}-\sum_{j=1}^{I}(q^1_{ij,k}+q^{2,\omega}_{ij,k})]\right] \quad (7)$$

subject to:

$$\sum_{i=1}^{I}q^1_{ij,k}\leq S^1_{j,k}, \quad j=1,\ldots,I; \quad k=1,\ldots,K, \quad (8)$$

$$\sum_{i=1}^{I}q^{2,\omega}_{ij,k}\leq S^{2,\omega}_{j,k}, \quad j=1,\ldots,I; \quad k=1,\ldots,K; \forall\omega\in\Omega, \quad (9)$$

$$q^1_{ij,k}\geq 0, \quad j=1,\ldots,I; \quad k=1,\ldots,K, \quad (10)$$

$$q^{2,\omega}_{ij,k}\geq 0, \quad j=1,\ldots,I; \quad k=1,\ldots,K; \forall\omega\in\Omega. \quad (11)$$

Feasible Set

Let feasible set $\mathcal{K}_i$ correspond to country i. It depends only on the strategy vector of country i, where $\mathcal{K}_i \equiv \{q_i \text{ such that (10) and (11) hold}\}$. Define $\mathcal{K}^1 \equiv \prod_{i=1}^{I} \mathcal{K}_i$. Also, let $\mathcal{S}$ denote the feasible set of shared constraints: $\mathcal{S} \equiv \{q | \text{(8) and (9) hold}\}$, and the feasible set $\mathcal{K}^2 \equiv \mathcal{K}^1 \cap \mathcal{S}$.

Definition 1: Stochastic Generalized Nash Equilibrium for the Countries

A strategy vector $q^* \in \mathcal{K}^2$ is a Stochastic Generalized Nash Equilibrium if for each country i; $i = 1, \ldots, I$:

$$E(DU_i(q_i^*, \hat{q}_i^*)) \leq E(DU_i(q_i, \hat{q}_i^*)), \quad \forall q_i \in \mathcal{K}_i \cap \mathcal{S}, \tag{12}$$

where $\hat{q}_i^* \equiv (q_1^*, \ldots, q_{i-1}^*, q_{i+1}^*, \ldots, q_I^*)$.

- Each government wishes to minimize its expected disutility.
- The above definition states that no government, given the circumstances and the strategies of the other national governments, at equilibrium, is willing unilaterally to change its vector of strategies, because it may end up with a higher expected disutility.
- The expected disutility of each country depends not only on the decisions of its government, but also on the strategies of other countries. Also, their feasible sets are interconnected because of the shared constraints. The latter condition makes the problem a Generalized Nash Equilibrium model (Debreu (1952)).

Definition 2: Variational Equilibrium

A medical item flow vector q^* *is a Variational Equilibrium of the above Stochastic Generalized Nash Equilibrium problem if* $q^* \in \mathcal{K}^2$ *is a solution to the following variational inequality:*

$$\sum_{i=1}^{I} \langle \nabla_{q_i} E[DU_i(q^*)], q_i - q_i^* \rangle \geq 0, \quad \forall q \in \mathcal{K}^2, \tag{13}$$

where $\langle \cdot, \cdot \rangle$ *denotes the inner product in* $IIK + |\Omega|(IIK)$*-dimensional Euclidean space.*

Expanding variational inequality (13), we have:

$$\sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{k=1}^{K}\left[\rho_{j,k} + \sum_{r=1}^{I}\sum_{s=1}^{K}\frac{\partial c^1_{ri,s}(q^{1*})}{\partial q^1_{ij,k}} - \beta_{i,k}\right] \times \left[q^1_{ij,k} - q^{1*}_{ij,k}\right]$$

$$+\sum_{\omega\in\Omega} p_\omega \sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{k=1}^{K}\left[\rho^\omega_{j,k} + \sum_{r=1}^{I}\sum_{s=1}^{K}\frac{\partial c^{2,\omega}_{ri,s}(q^{2,\omega*})}{\partial q^{2,\omega}_{ij,k}} - \beta_{i,k}\right] \times \left[q^{2,\omega}_{ij,k} - q^{2,\omega*}_{ij,k}\right] \geq 0, \quad \forall q \in \mathcal{K}^2. \tag{14}$$

Standard Form

We now put variational inequality (13) into standard variational inequality form (see Nagurney (1999)), that is: determine vector $X^* \in \mathcal{K}$, such that:

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{15}$$

where $\mathcal{K}$ is a closed, convex set. In order to put VI (13) into the standard form, we define $X \equiv q$, $F(X) \equiv (F^1(X), F^2(X))$, and $\mathcal{K} \equiv \mathcal{K}^2$ where:

$$F^1_{ij,k}(X) \equiv \left[\rho_{j,k} + \sum_{r=1}^{I} \sum_{s=1}^{K} \frac{\partial c^1_{ri,s}(q^1)}{\partial q^1_{ij,k}} - \beta_{i,k} \right], \quad \forall i, j, k,$$

$$F^{2,\omega}_{ij,k}(X) \equiv p_\omega \left[\rho^\omega_{j,k} + \sum_{r=1}^{I} \sum_{s=1}^{K} \frac{\partial c^{2,\omega}_{ri,s}(q^{2,\omega})}{\partial q^{2,\omega}_{ij,k}} - \beta_{i,k} \right], \quad \forall i, j, k, \omega. \tag{16}$$

Illustrative Examples

Data for Illustrative Example 1

$$\omega_1 = 1, \quad p_{\omega_1} = 1, \quad \rho_{1,1} = 2, \quad \rho^1_{1,1} = 25, \quad \beta_{1,1} = 3000$$

$$c^1_{11,1} = (q^1_{11,1})^2, \quad c^{2,1}_{11,1} = 2(q^{2,1}_{11,1})^2, \quad S^1_{1,1} = 2000, \quad S^{2,1}_{1,1} = 500, \quad d^{2,1}_{1,1} = 3000$$

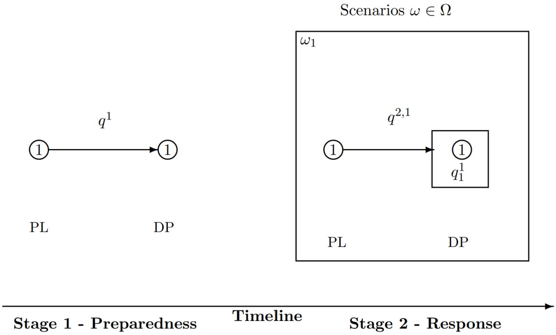

Figure 2: The Timeline of Illustrative Example 1

Illustrative Examples

We can rewrite variational inequality (13); equivalently, variational inequality (14), for this example as: determine $q^{1*}_{11,1}$ and $q^{2,1*}_{11,1}$, where $q^{1*}_{11,1} \leq 2000$ and $q^{2,1*}_{11,1} \leq 500$, such that:

$$\left[2+2(q^{1*}_{11,1})-3000\right]\times\left[q^{1}_{11,1}-q^{1*}_{11,1}\right]+(1)\left[25+4(q^{2,1*}_{11,1})-3000\right]\times\left[q^{2,1}_{11,1}-q^{2,1*}_{11,1}\right]\geq 0, \quad (17)$$

for all $q^{1}_{11,1}$ and $q^{2,1}_{11,1}$ such that

$$q^{1}_{11,1} \leq 2000, \quad q^{2,1}_{11,1} \leq 500.$$

The solution to the above variational inequality, which we obtained analytically, is:

$$q^{1*}_{11,1} = 1499, \quad q^{2,1*}_{11,1} = 500.$$

Illustrative Examples

Data for Illustrative Example 2

In addition to the scenario $\omega_1 = 1$ from Example 1, the government predicts another scenario with a greater severity in stage 2.

$$\omega_2 = 2, \quad p_{\omega_1} = 0.3, \quad p_{\omega_2} = 0.7, \quad d^{2,2}_{1,1} = 5,000, \quad \rho^2_{1,1} = 80, \quad S^{2,2}_{1,1} = 1,000$$

$$c^1_{11,1} = (q^1_{11,1})^2, \quad c^{2,1}_{11,1} = 2(q^{2,1}_{11,1})^2, \quad c^{2,2}_{11,1} = 2(q^{2,2}_{11,1})^2$$

Illustrative Examples

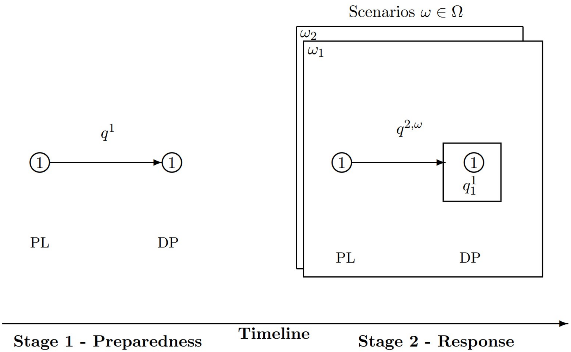

Figure 3: The Timeline of the Illustrative Example 2

Proceeding in a similar manner as that in Example 1, yields:

$$q^{1*}_{11,1} = 1,499.00, \quad q^{2,1*}_{11,1} = 500.00, \quad q^{2,2*}_{11,1} = 730.00,$$

Data for Illustrative Example 3

This example is constructed from the first example, with the difference that now there are two governments trying to procure N95 masks for their countries and competing over limited supplies. The data for the second country are:

$$\rho_{2,1} = 2, \quad \rho^1_{2,1} = 120, \quad \beta_{2,1} = 3000, \quad S^1_{2,1} = 2000, \quad S^{2,1}_{2,1} = 500, \quad d^{2,1}_{2,1} = 5000,$$

$$c^1_{22,1} = (q^1_{22,1})^2, \quad c^{2,1}_{22,1} = 2(q^{2,1}_{22,1})^2,$$

$$c^1_{21,1}(q^1) = 2(q^1_{12,1})^2 + 5q^1_{12,1}, \quad c^{2,1}_{21,1}(q^{2,1}) = 12(q^{2,1}_{12,1})^2 + 5q^{2,1}_{12,1},$$

$$c^1_{12,1}(q^1) = 2(q^1_{21,1})^2 + 5q^1_{21,1}, \quad c^{2,1}_{12,1}(q^{2,1}) = 6(q^{2,1}_{21,1})^2 + 5q^{2,1}_{21,1}.$$

Illustrative Examples

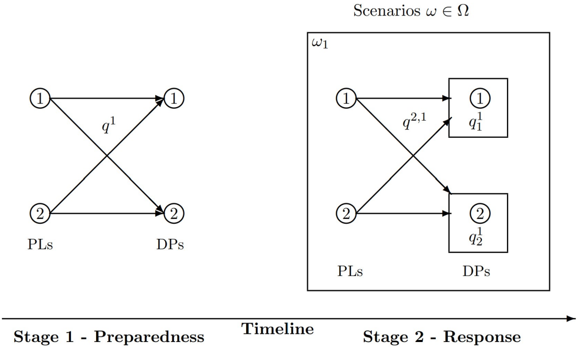

Figure 4: The Timeline of the Illustrative Example 3

The equilibrium solution for this example is:

$$q^{1*}_{11,1} = 1,334.17, \quad q^{1*}_{12,1} = 665.83, \quad q^{1*}_{21,1} = 665.83, \quad q^{1*}_{22,1} = 1,334.17,$$

$$q^{2,1*}_{11,1} = 375.31, \quad q^{2,1*}_{12,1} = 71.25, \quad q^{2,1*}_{21,1} = 124.69, \quad q^{2,1*}_{22,1} = 428.75.$$

We associate Lagrange multiplier $\alpha^1_{j,k}$ with the supply constraint (8) on the availability of medical item k in country j, for each j and each k. We also let $\gamma^{2,\omega}_{j,k}$ be the Lagrange multiplier associated with supply constraint (9) on medical item k in country j when scenario ω occurs, for each j and k. We gather these Lagrange multipliers into the respective vectors: $\alpha^1 \in R^{IK}_+$ and $\gamma^2 \in R^{|\Omega|(IK)}_+$.

Then, using arguments as in Nagurney, Salarpour, and Daniele (2019), an alternative variational inequality for (14) is: determine $(q^*, \alpha^{1*}, \gamma^{2*}) \in R^{IIK+|\Omega|(IIK)+IK+|\Omega|(IK)}_+$ such that

$$\sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{k=1}^{K}\left[\rho_{j,k}+\sum_{r=1}^{I}\sum_{s=1}^{K}\frac{\partial c^1_{ri,s}(q^{1*})}{\partial q^1_{ij,k}}-\beta_{i,k}+\alpha^{1*}_{j,k}\right]\times\left[q^1_{ij,k}-q^{1*}_{ij,k}\right]$$

$$+\sum_{\omega\in\Omega}p_\omega\sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{k=1}^{K}\left[\rho^\omega_{j,k}+\sum_{r=1}^{I}\sum_{s=1}^{K}\frac{\partial c^{2,\omega}_{ri,s}(q^{2,\omega*})}{\partial q^{2,\omega}_{ij,k}}-\beta_{i,k}+\gamma^{2,\omega*}_{j,k}\right]\times\left[q^{2,\omega}_{ij,k}-q^{2,\omega*}_{ij,k}\right]$$

$$+\sum_{j=1}^{I}\sum_{k=1}^{K}\left[S^1_{j,k}-\sum_{i=1}^{I}q^{1*}_{ij,k}\right]\times\left[\alpha^1_{j,k}-\alpha^{1*}_{j,k}\right]+\sum_{\omega\in\Omega}\sum_{j=1}^{I}\sum_{k=1}^{K}\left[S^{2,\omega}_{j,k}-\sum_{i=1}^{I}q^{2,\omega*}_{ij,k}\right]\times\left[\gamma^{2,\omega}_{j,k}-\gamma^{2,\omega*}_{j,k}\right]\geq 0$$

$$\forall (q, \alpha^1, \gamma^2) \in R^{IIK+|\Omega|(IIK)+IK+|\Omega|(IK)}_+. \quad (18)$$

Algorithm

Modified Projection Method (Korpelevich (1977))

Step 0: Initialization

Initialize with $X^0 \in \mathcal{K}$. Set $\tau := 1$ and select ψ, such that $0 < \psi \leq \frac{1}{L}$, where L is the Lipschitz continuity constant for $F(X)$.

Step 1: Construction and Computation

Compute $\overline{X}^{\tau-1}$ by solving the variational inequality subproblem:

$$\langle \overline{X}^{\tau-1} + (\psi F(X^{\tau-1}) - X^{\tau-1}), X - \overline{X}^{\tau-1} \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{19}$$

Step 2: Adaptation

Compute X^τ by solving the variational inequality subproblem:

$$\langle X^\tau + (\psi F(\overline{X}^{\tau-1}) - X^{\tau-1}), X - X^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{20}$$

Step 3: Convergence Verification

If $| X^\tau - X^{\tau-1} | \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise set $\tau := \tau + 1$, and go to Step 1.

Algorithm

Explicit Formulae for Step 1 for the Medical Supply Flows in Stage 1

For each i, j, k, compute

$$(\bar{q}^1_{ij,k})^\tau = \max\{0, (q^1_{ij,k})^{\tau-1} - \psi(\rho_{j,k} + \sum_{r=1}^{I}\sum_{s=1}^{K} \frac{\partial c^1_{ri,s}((q^1)^{\tau-1})}{\partial q^1_{ij,k}} - \beta_{i,k} + (\alpha^1_{j,k})^{\tau-1})\}; \quad (21)$$

Explicit Formulae for Step 1 for the Medical Supply Flows in Stage 2

For each ω, i, j, k, compute

$$(\bar{q}^{2,\omega}_{ij,k})^\tau = \max\{0, (q^{2,\omega}_{ij,k})^{\tau-1} - \psi p_\omega(\rho^\omega_{j,k} + \sum_{r=1}^{I}\sum_{s=1}^{K} \frac{\partial c^{2,\omega}_{ri,s}((q^{2,\omega})^{\tau-1})}{\partial q^{2,\omega}_{ij,k}} - \beta_{i,k} + (\gamma^{2,\omega}_{j,k})^{\tau-1})\}; \quad (22)$$

Algorithm

Explicit Formulae for Step 1 the Lagrange Multipliers Associated with the Supply Constraints in Stage 1

For each j, k, compute

$$(\bar{\alpha}^1_{j,k})^\tau = \max\{0, (\alpha^1_{j,k})^{\tau-1} - \psi(S^1_{j,k} - \sum_{i=1}^{I}(q^1_{ij,k})^{\tau-1})\}; \tag{23}$$

Explicit Formulae for Step 1 for the Lagrange Multipliers Associated with the Supply Constraints in Stage 2

For each ω, j, k, compute

$$(\bar{\gamma}^{2,\omega}_{j,k})^\tau = \max\{0, (\gamma^{2,\omega}_{j,k})^{\tau-1} - \psi(S^{2,\omega}_{j,k} - \sum_{i=1}^{I}(q^{2,\omega}_{ij,k})^{\tau-1})\}. \tag{24}$$

Numerical Examples

- The examples in this section are inspired by the Covid-19 pandemic.
- When the global outbreak occurred, the demand for PPEs was much higher than the inventory and production capacities.
- Disruptions in the supply chain also added to the shortage crisis of medical supplies.
- Although the production process has been disrupted in many areas, some governments and companies made every effort to increase production capacity.

Numerical Examples: Example 1

- There are two countries and two scenarios.
- The medical item is the N95 mask.
- In the first scenario, $\omega_1 = 1$, the pandemic is severe and the demand for such PPEs in both countries is very high.
- The second scenario, $\omega_2 = 2$, is one in which the consequences of the pandemic are less and the countries have a lower demand.
- They purchase the N95 masks in large bulks of 1000 masks each; therefore, $q^1_{ij,1}$ represents 1000 of N95 masks purchased by country i from country j in stage 1.
- Both countries have supplies of N95 masks.

Numerical Examples: Example 1

The example's data are as follows.

$$p_{\omega_1} = 0.7, \quad p_{\omega_2} = 0.3,$$

$$\rho_{1,1} = 1,000, \quad S^1_{1,1} = 20,000 \quad \rho^1_{1,1} = 5,000, \quad S^{2,1}_{1,1} = 7,000,$$

$$\rho^2_{1,1} = 2,000, \quad S^{2,2}_{1,1} = 20,000,$$

$$\rho_{2,1} = 1,000, \quad S^1_{2,1} = 25,000 \quad \rho^1_{2,1} = 2,500, \quad S^{2,1}_{2,1} = 25,000,$$

$$\rho^2_{2,1} = 2,000, \quad S^{2,2}_{2,1} = 40,000,$$

$$\beta_{1,1} = \beta_{2,1} = 100,000, \quad d^{2,1}_{1,1} = d^{2,1}_{2,1} = 80,000, \quad d^{2,2}_{1,1} = d^{2,2}_{2,1} = 55,000,$$

$$c^1_{11,1} = 2(q^1_{11,1})^2, \quad c^1_{22,1} = 2(q^1_{22,1})^2,$$

$$c^1_{21,1}(q^1) = 5(q^1_{12,1})^2 + 3q^1_{12,1}, \quad c^1_{12,1}(q^1) = 5(q^1_{21,1})^2 + 3q^1_{21,1},$$

$$c^{2,1}_{11,1} = 5(q^{2,1}_{11,1})^2, \quad c^{2,1}_{22,1} = 3(q^{2,1}_{22,1})^2,$$

$$c^{2,1}_{21,1}(q^{2,1}) = 9(q^{2,1}_{12,1})^2 + 6q^{2,1}_{12,1}, \quad c^{2,1}_{12,1}(q^{2,1}) = 9(q^{2,1}_{21,1})^2 + 6q^{2,1}_{21,1},$$

$$c^{2,2}_{11,1} = 2(q^{2,2}_{11,1})^2, \quad c^{2,2}_{22,1} = 2(q^{2,2}_{22,1})^2,$$

$$c^{2,2}_{21,1}(q^{2,2}) = 5(q^{2,2}_{12,1})^2 + 3q^{2,2}_{12,1}, \quad c^{2,2}_{12,1}(q^{2,2}) = 5(q^{2,2}_{21,1})^2 + 3q^{2,2}_{21,1}.$$

The computed equilibrium solution via the modified projection method for this example is:

$$q^{1*}_{11,1} = 14,285.92, \quad q^{1*}_{12,1} = 7,142.64, \quad q^{1*}_{21,1} = 5,714.07, \quad q^{1*}_{22,1} = 17,857.35,$$

$$q^{2,1*}_{11,1} = 4,500.22, \quad q^{2,1*}_{12,1} = 5,416.33, \quad q^{2,1*}_{21,1} = 2,499.78, \quad q^{2,1*}_{22,1} = 16,250.00,$$

$$q^{2,2*}_{11,1} = 14,285.92, \quad q^{2,2*}_{12,1} = 9,799.70, \quad q^{2,2*}_{21,1} = 5,714.07, \quad q^{2,2*}_{22,1} = 24,500.00,$$

$$\alpha^{1*}_{1,1} = 41,856.28, \quad \alpha^{1*}_{2,1} = 27,570.57,$$

$$\gamma^{2,1*}_{1,1} = 49,997.79, \quad \gamma^{2,1*}_{2,1} = 0.00, \quad \gamma^{2,2*}_{1,1} = 40,856.28, \quad \gamma^{2,2*}_{2,1} = 0.00.$$

Numerical Examples: Example 2

- In Numerical Example 2, in addition to the face masks, the countries are also trying to meet their demand for ventilators.
- The need for these devices is less than the demand for the face masks, but the value of a ventilator in saving lives in this pandemic is very high.

The additional data needed for this example are:

$$\rho_{1,2} = 10,000, \quad S^1_{1,2} = 2,000 \quad \rho^1_{1,2} = 45,000, \quad S^{2,1}_{1,2} = 2,000, \rho^2_{1,2} = 20,000, \quad S^{2,2}_{1,2} = 2,000,$$

$$\rho_{2,2} = 10,000, \quad S^1_{2,2} = 10,000 \quad \rho^1_{2,2} = 20,000, \quad S^{2,1}_{2,2} = 10,000, \rho^2_{2,2} = 15,000, \quad S^{2,2}_{2,2} = 20,000,$$

$$\beta_{1,2} = \beta_{2,2} = 1,000,000, \quad d^{2,1}_{1,2} = d^{2,1}_{2,2} = 50,000, \quad d^{2,2}_{1,2} = d^{2,2}_{2,2} = 25,000,$$

$$c^1_{11,2} = 2(q^1_{11,2})^2, \quad c^1_{22,2} = 2(q^1_{22,2})^2,$$

$$c^1_{21,2}(q^1) = 5(q^1_{12,2})^2 + 3q^1_{12,2}, \quad c^1_{12,2}(q^1) = 5(q^1_{21,2})^2 + 3q^1_{21,2},$$

$$c^{2,1}_{11,2} = 5(q^{2,1}_{11,2})^2, \quad c^{2,1}_{22,2} = 3(q^{2,1}_{22,2})^2,$$

$$c^{2,1}_{21,2}(q^{2,1}) = 9(q^{2,1}_{12,2})^2 + 6q^{2,1}_{12,2}, \quad c^{2,1}_{12,2}(q^{2,1}) = 9(q^{2,1}_{21,2})^2 + 6q^{2,1}_{21,2},$$

$$c^{2,2}_{11,2} = 2(q^{2,2}_{11,2})^2, \quad c^{2,2}_{22,2} = 2(q^{2,2}_{22,2})^2,$$

$$c^{2,2}_{21,2}(q^{2,2}) = 5(q^{2,2}_{12,2})^2 + 3q^{2,2}_{12,2}, \quad c^{2,2}_{12,2}(q^{2,2}) = 5(q^{2,2}_{21,2})^2 + 3q^{2,2}_{21,2}.$$

Numerical Examples: Example 2

The computed equilibrium solution for this example is:

$$q^{1*}_{11,1} = 14,285.92, \quad q^{1*}_{12,1} = 7,142.64, \quad q^{1*}_{21,1} = 5,714.07, \quad q^{1*}_{22,1} = 17,857.35,$$

$$q^{2,1*}_{11,1} = 4,500.22, \quad q^{2,1*}_{12,1} = 5,416.33, \quad q^{2,1*}_{21,1} = 2,499.78, \quad q^{2,1*}_{22,1} = 16,250.00,$$

$$q^{2,2*}_{11,1} = 14,285.92, \quad q^{2,2*}_{12,1} = 9,799.70, \quad q^{2,2*}_{21,1} = 5,714.07, \quad q^{2,2*}_{22,1} = 24,500.00,$$

$$q^{1*}_{11,2} = 1,428.78, \quad q^{1*}_{12,2} = 2,856.92, \quad q^{1*}_{21,2} = 571.21, \quad q^{1*}_{22,2} = 7,143.07,$$

$$q^{2,1*}_{11,2} = 1,285.93, \quad q^{2,1*}_{12,2} = 2,499.75, \quad q^{2,1*}_{21,2} = 714.07, \quad q^{2,1*}_{22,2} = 7,500.25,$$

$$q^{2,2*}_{11,2} = 1,428.78, \quad q^{2,2*}_{12,2} = 5,714.07, \quad q^{2,2*}_{21,2} = 571.21, \quad q^{2,2*}_{22,2} = 14,285.92,$$

$$\alpha^{1*}_{1,1} = 41,856.28, \quad \alpha^{1*}_{2,1} = 27,570.57, \quad \alpha^{1*}_{1,2} = 984,284.85, \quad \alpha^{1*}_{2,2} = 961,427.71,$$

$$\gamma^{2,1*}_{1,1} = 49,997.79, \quad \gamma^{2,1*}_{2,1} = 0.00, \quad \gamma^{2,2*}_{1,1} = 40,856.28, \quad \gamma^{2,2*}_{2,1} = 0.00,$$

$$\gamma^{2,1*}_{1,2} = 942,140.65, \quad \gamma^{2,1*}_{2,2} = 934,998.49, \quad \gamma^{2,2*}_{1,2} = 974,284.85, \quad \gamma^{2,2*}_{2,2} = 927,856.28.$$

Numerical Examples: Example 3

- In Numerical Example 3, we address the key issue of restrictions on the export of vital medical supplies in times of a pandemic.
- We have seen that, in some cases, the import of essential medical items from foreign countries has become very difficult or expensive for various reasons, such as the severe disruptions in international transportation and/or the enactment of laws by governments.
- In this example, we examine the effects of such restriction on countries' strategies by increasing the international transportation rates as compared to the previous example.

The data for this example are:

$$c^{2,1}_{21,1}(q^{2,1}) = 25(q^{2,1}_{12,1})^2 + 10q^{2,1}_{12,1}, \quad c^{2,1}_{12,1}(q^{2,1}) = 25(q^{2,1}_{21,1})^2 + 10q^{2,1}_{21,1},$$

$$c^{2,2}_{21,1}(q^{2,2}) = 25(q^{2,2}_{12,1})^2 + 10q^{2,2}_{12,1}, \quad c^{2,2}_{12,1}(q^{2,2}) = 25(q^{2,2}_{21,1})^2 + 10q^{2,2}_{21,1},$$

$$c^{2,1}_{21,2}(q^{2,1}) = 25(q^{2,1}_{12,2})^2 + 10q^{2,1}_{12,2}, \quad c^{2,1}_{12,2}(q^{2,1}) = 25(q^{2,1}_{21,2})^2 + 10q^{2,1}_{21,2},$$

$$c^{2,2}_{21,2}(q^{2,2}) = 25(q^{2,2}_{12,2})^2 + 10q^{2,2}_{12,2}, \quad c^{2,2}_{12,2}(q^{2,2}) = 25(q^{2,2}_{21,2})^2 + 10q^{2,2}_{21,2}.$$

Numerical Examples: Example 3

The computed equilibrium solution for this example is:

$$q^{1*}_{11,1} = 14,285.92, \quad q^{1*}_{12,1} = 7,142.64, \quad q^{1*}_{21,1} = 5,714.07, \quad q^{1*}_{22,1} = 17,857.35,$$

$$q^{2,1*}_{11,1} = 5,833.50, \quad q^{2,1*}_{12,1} = 1,949.80, \quad q^{2,1*}_{21,1} = 1,166.50, \quad q^{2,1*}_{22,1} = 16,250.00,$$

$$q^{2,2*}_{11,1} = 18,518.70, \quad q^{2,2*}_{12,1} = 1,959.80, \quad q^{2,2*}_{21,1} = 1,481.298, \quad q^{2,2*}_{22,1} = 24,500.00,$$

$$q^{1*}_{11,2} = 1,428.78, \quad q^{1*}_{12,2} = 2,856.92, \quad q^{1*}_{21,2} = 571.21, \quad q^{1*}_{22,2} = 7,143.07,$$

$$q^{2,1*}_{11,2} = 1,666.84, \quad q^{2,1*}_{12,2} = 1,071.25, \quad q^{2,1*}_{21,2} = 333.16, \quad q^{2,1*}_{22,2} = 8,928.75,$$

$$q^{2,2*}_{11,2} = 1,852.03, \quad q^{2,2*}_{12,2} = 1,481.29, \quad q^{2,2*}_{21,2} = 147.96, \quad q^{2,2*}_{22,2} = 18,518.70,$$

$$\alpha^{1*}_{1,1} = 41,856.28, \quad \alpha^{1*}_{2,1} = 27,570.57, \quad \alpha^{1*}_{1,2} = 984,284.85, \quad \alpha^{1*}_{2,2} = 961,427.71,$$

$$\gamma^{2,1*}_{1,1} = 36,664.99, \quad \gamma^{2,1*}_{2,1} = 0.00, \quad \gamma^{2,2*}_{1,1} = 23,925.18, \quad \gamma^{2,2*}_{2,1} = 0.00,$$

$$\gamma^{2,1*}_{1,2} = 938,331.50, \quad \gamma^{2,1*}_{2,2} = 926,427.49, \quad \gamma^{2,2*}_{1,2} = 972,591.85, \quad \gamma^{2,2*}_{2,2} = 910,925.18.$$

Results

- In Numerical Example 1, we observe that both countries are not prepared sufficiently to respond to the pandemic, but country 2, which has greater flexibility and resilience in terms of its country's supply, performs better in reducing the shortage of face masks post the disaster.
- In Numerical Example 2, Country 2 has a much higher supply of the ventilators than country 1 even before the pandemic declaration, and this important and significant advantage of country 2 in accessing the supply of a life-saving device leads to a much lower expected disutility for this country than that for country 1.
- In Numerical Example 3, we see that the difficulties in imports have increased the relative shortage of face masks in both countries, but still country 2 is less affected than country 1.
- Country 2, which dominates the supply of ventilators, has benefited greatly from the disruption in exchanges between the two countries. Most of the country's supply is allocated to its own demand and the country's competitor has received a very small share.

Results

- We observed that it is important to pay attention to **both disaster preparedness and to response** when we are planning for a pandemic.
- We saw that in the event of **a severe crisis** in countries, harsh global supply shortages and rising prices **significantly affect countries with low domestic production capacity**. This finding is consistent with what we have seen in reality in the efforts of various governments to address the Covid-19 pandemic crisis.
- Firstly, in many cases, it is not possible to predict the occurrence of a crisis in the long run, and, secondly, it is not easy to store and maintain many goods in large volumes; **Maintaining readiness for a sudden increase in supply** is one of the most important strategies that managers and policy-makers should pay special attention to.
- Decision-makers should consider and analyze the **effects of different types of medical products** on the amount of damage that might be incurred by the society and take the necessary actions to procure these products before and after the occurrence of a possible disaster, such as the pandemic.

Summary and Conclusions

- After the official declaration of the **Covid-19 pandemic** in March 2020, it soon became apparent that many **countries were not prepared** to deal with this healthcare disaster.
- The **sudden increase in demand** for medical supplies, including **PPEs and ventilators**, added to the crisis. The **supply levels** of these products were **much lower than the level of demand** and initiated an **intense competition** among different countries in procuring such items.
- Taking into account the specific features of the current pandemic, we examine countries' competition over the purchase of medical supplies under limited availability in our **stochastic Generalized Nash Equilibrium model**.
- Specific features of the model include: **the uncertainty of the scenarios**, **the supply capacities of the medical items**, and **the fluctuating prices before and after the pandemic declaration**, as well as **disruptions to the global supply chains**.

Summary and Conclusions

- We formulate the model as a **variational inequality problem** applying the concept of a Variational Equilibrium. Also, we utilize an alternative variational inequality formulation with Lagrange multipliers associated with the medical item supply capacities in each country.
- Our study is the first to address not only the stages of preparedness and response, along with the uncertainty, but also the critical issue of **resource competition among countries**, which was not investigated in previous studies on epidemic crises.
- Most previous research has considered the occurrence of an epidemic to be limited to a single region or, ultimately, to one country, and, as a result, the issue of global supply shortages in a pandemic has not been addressed through game theory and variational inequality theory before.

Summary and Conclusions

- We study the model both **qualitatively and quantitatively**.
- The results reveal that countries that have **more flexibility and resilience in increasing their domestic supply** after the pandemic declaration are better at dealing with the pandemic disaster and meeting the need for medical supplies.
- In times of crisis, uncertainty in many cases, including the supply chain status, plays a key role in a country's success or failure in disaster management. Hence, countries must be **ready to supply strategic medical supplies domestically**.
- This study adds to the literature on **game theory and two-stage stochastic models in disaster management** with the focus on specific features of the **Covid-19 pandemic**.
- This model has the potential to be extended in future research. For example, we are still in the midst of this pandemic, and every day we see new changes and decisions by governments. The fierce competition among the countries over vaccines requires comprehensive and careful planning.

THANK YOU!

Thank you!