A Supply Chain Network Game Theoretic Framework for Time-Based Competition with Transportation Costs and Product Differentiation

Anna Nagurney[1] Min Yu[2]

[1]Department of Operations and Information Management
Isenberg School of Management
University of Massachusetts
Amherst, Massachusetts 01003

[2]Pamplin School of Business Administration
University of Portland
Portland, Oregon 97203

The 10th International Conference on Computational Management Science
May 1-3, 2013, HEC Montréal, Quebec, Canada

Acknowledgments

This research was supported, in part, by the National Science Foundation (NSF) grant CISE #1111276, for the NeTS: Large: Collaborative Research: Network Innovation Through Choice project awarded to the University of Massachusetts Amherst. This support is gratefully acknowledged.

The research was also supported, in part, by the School of Business, Economics and Law at the University of Gothenburg through its Visiting Professor Program.

Outline

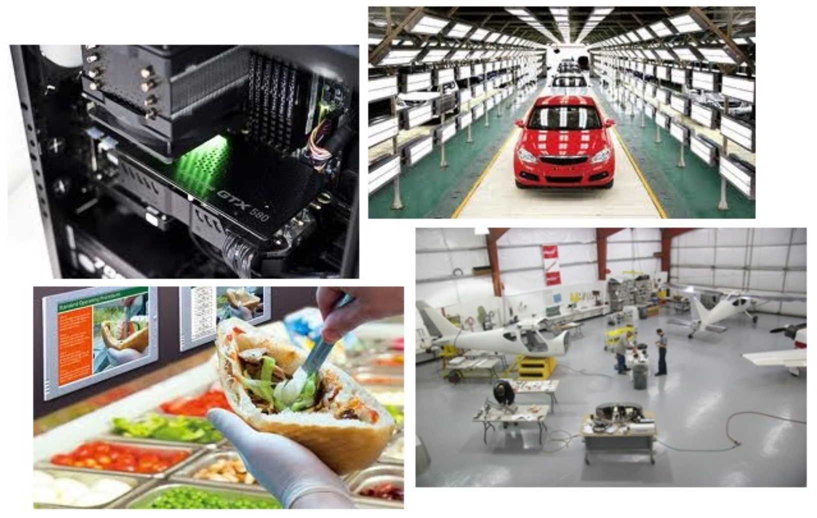
- Motivation and Background
- The Supply Chain Network Game Theoretic Model with Product Differentiation and Delivery Times
- Numerical Examples
- Summary

Motivation

Timely delivery is becoming a strategy, as important as productivity, quality, and even innovation

Today, time-based competition has emerged as a paradigm for strategizing about and operationalizing supply chain networks in which efficiency and timeliness matter

Build-to-Order / Made-on-Demand



Digitally-Based Productin and Delivery

Background

- Hum and Sim (1996) provided an extensive literature review of time-based competition.
  - They concluded that much of the time-based focus in modeling was limited to the areas of transportation modeling, lead time and inventory modeling, and set-up time reduction analysis.
  - They argued that the literature emphasized cost minimization but what was needed was the explicit incorporation of time as a significant variable in modeling.

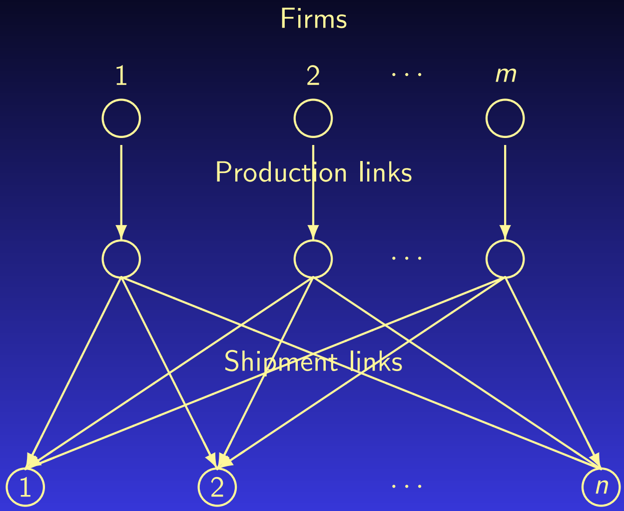
Background

- Hum and Sim (1996) provided an extensive literature review of time-based competition.
  - They concluded that much of the time-based focus in modeling was limited to the areas of transportation modeling, lead time and inventory modeling, and set-up time reduction analysis.
  - They argued that the literature emphasized cost minimization but what was needed was the explicit incorporation of time as a significant variable in modeling.
- Gunasekaran and Ngai (2005) further emphasized this shortcoming and the relevance of analyzing the trade-offs between operational costs and delivery time in supply chain management.

The Supply Chain Network Game Theoretic Model

- Firms compete in an oligopolistic manner and, hence, influence the prices;
- Firms are assumed to be spatially separated and can compete both on the production side and on the demand side;
- Consumers at the demand markets for the substitutable, but differentiated, products respond to both the quantities of the products and to their delivery times, as reflected in the prices of the products.

The Network Structure



The products $i = 1, \ldots, m$ may be consumed at any demand market

Let s_i denote the nonnegative product output produced by firm i and let d_{ij} denote the demand for the product of firm i at demand market j. Let Q_{ij} denote the nonnegative shipment of firm i's product to demand market j.

Conservation of Flow Equations

$$s_i = \sum_{j=1}^{n} Q_{ij}, \quad i = 1, \ldots, m, \tag{1}$$

$$d_{ij} = Q_{ij}, \quad i = 1, \ldots, m; \, j = 1, \ldots, n, \tag{2}$$

$$Q_{ij} \geq 0, \quad i = 1, \ldots, m; \, j = 1, \ldots, n. \tag{3}$$

The firms are also competing with time.

Let T_{ij} denote the delivery time bound determined by firm i. The parameters t_i and h_i reflect the time consumption associated with producing product i; and t_{ij} and h_{ij} reflect the time consumption associated with delivering product i to demand market j

Time Constraints

$$t_i s_i + h_i + t_{ij} Q_{ij} + h_{ij} \leq T_{ij}, \quad i = 1, \ldots, m;\ j = 1, \ldots, n, \qquad (4a)$$

$(4a)$ may be rewritten in product shipment variables, that is,

$$t_i \sum_{j=1}^{n} Q_{ij} + h_i + t_{ij} Q_{ij} + h_{ij} \leq T_{ij}, \quad i = 1, \ldots, m;\ j = 1, \ldots, n. \quad (4b)$$

Cost Functions

- A firm's production cost may depend not only on its production output but also on that of the other firms.

$$\hat{f}_i = \hat{f}_i(Q) \equiv f_i(s), \quad i = 1, \ldots, m. \tag{5}$$

- The total transportation cost associated with shipping firm i's product to demand market j is a functions of the product shipment on that link.

$$\hat{c}_{ij} = \hat{c}_{ij}(Q_{ij}), \quad i = 1, \ldots, m; \, j = 1, \ldots, n. \tag{6}$$

Cost Functions

- It is important to recognize that faster delivery may be more costly, since it may require additional capacity and may be dependent on the operational efficiency. The total cost associated with delivery times of each firm, is a function of its delivery times.

$$g_i = g_i(T_i), \quad i = 1, \ldots, m, \tag{7}$$

where $T_i = (T_{i1}, \ldots, T_{in})$.

The total cost functions are assumed to be convex and continuously differentiable.

Demand Price Functions

Consumers, located at the demand markets, respond not only to the quantities available of the products but also to their delivery times.

$$p_{ij} = p_{ij}(d, T), \quad i = 1, \ldots, m; \; j = 1, \ldots, n. \tag{8a}$$

The demand price functions are, typically, assumed to be monotonically decreasing in product quantity and in delivery time for the specific firm and demand market pair.

The demand price functions may be rewritten in terms of the product shipments, that is:

$$\hat{p}_{ij} = \hat{p}_{ij}(Q, T) \equiv p_{ij}(d, T), \quad i = 1, \ldots, m; \; j = 1, \ldots, n. \tag{8b}$$

Profit Maximization

The strategic variables of firm i are its product shipments $\{Q_i\}$ where $Q_i = (Q_{i1}, \ldots, Q_{in})$ and its delivery times $\{T_i\}$, note that $T_i = (T_{i1}, \ldots, T_{in})$.

The Profit or Utility U_i of Firm i

$$U_i = \sum_{j=1}^{n} \hat{p}_{ij} Q_{ij} - \hat{f}_i - g_i - \sum_{j=1}^{n} \hat{c}_{ij}, \tag{9}$$

which is the difference between its total revenue and its total costs.

A Supply Chain Network Equilibrium with Product Differentiation and Delivery Times

A product shipment and delivery time pattern $(Q^*, T^*) \in K$ *is said to constitute a network equilibrium if for each firm* i; $i = 1, \ldots, m$,

$$U_i(Q_i^*, T_i^*, \hat{Q}_i^*, \hat{T}_i^*) \geq U_i(Q_i, T_i, \hat{Q}_i^*, \hat{T}_i^*), \quad \forall (Q_i, T_i) \in K^i, \quad (10)$$

where

$$\hat{Q}_i^* \equiv (Q_1^*, \ldots, Q_{i-1}^*, Q_{i+1}^*, \ldots, Q_m^*),$$

$$\hat{T}_i^* \equiv (T_1^*, \ldots, T_{i-1}^*, T_{i+1}^*, \ldots, T_m^*),$$

and $K \equiv \prod_{i=1}^m K^i$, *where*

$$K^i \equiv \{(Q_i, T_i) | Q_i \geq 0, \text{ and } (4b) \text{ is satisfied for } i\}.$$

Variational Inequality Formulations

Assume that for each firm i the profit function $U_i(Q,T)$ is concave with respect to the variables $\{Q_{i1},\dots,Q_{in}\}$, and $\{T_{i1},\dots,T_{in}\}$, and is continuous and continuously differentiable. Then $(Q^*,T^*)\in K$ is a supply chain network equilibrium if and only if it satisfies the variational inequality

$$-\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{\partial U_i(Q^*,T^*)}{\partial Q_{ij}}\times(Q_{ij}-Q_{ij}^*)-\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{\partial U_i(Q^*,T^*)}{\partial T_{ij}}\times(T_{ij}-T_{ij}^*)\geq 0,$$

$$\forall (Q,T)\in K. \tag{11}$$

Variational Inequality Formulations

Equivalently, $(Q^*, T^*, \gamma^*) \in K^1$ is an equilibrium product shipment and delivery time pattern if and only if it satisfies the variational inequality

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\partial \hat{f}_i(Q^*)}{\partial Q_{ij}} + \frac{\partial \hat{c}_{ij}(Q_{ij}^*)}{\partial Q_{ij}} - \sum_{l=1}^{n}\frac{\partial \hat{p}_{il}(Q^*, T^*)}{\partial Q_{ij}} \times Q_{il}^* - \hat{p}_{ij}(Q^*, T^*) + \sum_{l=1}^{n}\gamma_{il}^* t_i + \gamma_{ij}^* t_{ij}\right] \times (Q_{ij} - Q_{ij}^*)$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\partial g_i(T_i^*)}{\partial T_{ij}} - \sum_{l=1}^{n}\frac{\partial \hat{p}_{il}(Q^*, T^*)}{\partial T_{ij}} \times Q_{il}^* - \gamma_{ij}^*\right] \times (T_{ij} - T_{ij}^*)$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\left[T_{ij}^* - t_i\sum_{l=1}^{n}Q_{il}^* - t_{ij}Q_{ij}^* - h_i - h_{ij}\right] \times \left[\gamma_{ij} - \gamma_{ij}^*\right] \geq 0, \quad \forall (Q, T, \gamma) \in K^1, \tag{12}$$

where $K^1 \equiv \{(Q, T, \gamma) |\, Q \geq 0,\ T \geq 0,\ \gamma \geq 0\}$ with γ being the vector of the Lagrange multipliers associated with constraint (4b).

Algorithm

Explicit Formulae for the Euler Method Applied to the Supply Chain Network Model

$$Q_{ij}^{\tau+1} = \max\{0, Q_{ij}^{\tau} + a_\tau(-F_{ij}^1(X^\tau))\}, \tag{13}$$

$$T_{ij}^{\tau+1} = \max\{0, T_{ij}^{\tau} + a_\tau(-F_{ij}^2(X^\tau))\} \tag{14}$$

$$\gamma_{ij}^{\tau+1} = \max\{0, \gamma_{ij}^{\tau} + a_\tau(-F_{ij}^3(X^\tau))\}; \quad i = 1, \ldots, m; j = 1, \ldots, n, \tag{15}$$

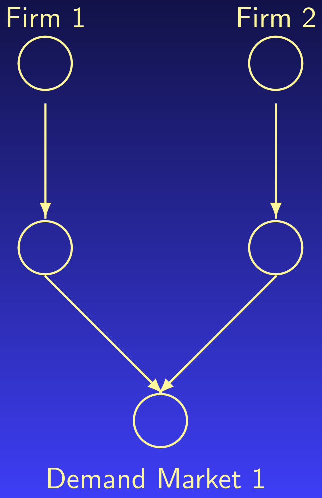
where

$$F_{ij}^1(X) \equiv \frac{\partial \hat{f}_i(Q)}{\partial Q_{ij}} + \frac{\partial \hat{c}_{ij}(Q_{ij})}{\partial Q_{ij}} - \sum_{l=1}^{n} \frac{\partial \hat{p}_{il}(Q, T)}{\partial Q_{ij}} \times Q_{il} - \hat{p}_{ij}(Q, T) + \sum_{l=1}^{n} \gamma_{il} t_i + \gamma_{ij} t_{ij},$$

$$F_{ij}^2(X) \equiv \frac{\partial g_i(T_i)}{\partial T_{ij}} - \sum_{l=1}^{n} \frac{\partial \hat{p}_{il}(Q, T)}{\partial T_{ij}} \times Q_{il} - \gamma_{ij},$$

$$F_{ij}^3(X) = T_{ij} - t_i \sum_{l=1}^{n} Q_{il} - t_{ij} Q_{ij} - h_i - h_{ij}.$$

Example 1



- These two firms are located in the same area.
- Both of them adopt similar technologies for the production and delivery of their highly substitutable products. T
- Consumers at the demand market are indifferent between products of Firms 1 and 2.

Example 1

The production cost functions are:

$$f_1(s) = 2s_1^2 + 3s_1, \quad f_2(s) = 2s_2^2 + 3s_2,$$

the total transportation cost functions are:

$$\hat{c}_{11}(Q_{11}) = Q_{11}^2 + Q_{11}, \quad \hat{c}_{21}(Q_{21}) = Q_{21}^2 + Q_{21},$$

the total cost functions associated with delivery times are:

$$g_1(T_1) = T_{11}^2 - 30T_{11} + 400, \quad g_2(T_2) = T_{21}^2 - 40T_{21} + 450.$$

Example 1

The parameters associated with the production time consumption are:

$$t_1 = 0, \quad h_1 = 1, \quad t_2 = 0, \quad h_2 = 1,$$

and the parameters associated with the transportation time consumption are:

$$t_{11} = 0, \quad h_{11} = 1, \quad t_{21} = 0, \quad h_{21} = 1,$$

The demand price functions are assumed to be:

$$p_{11}(d, T) = 300 - 2d_{11} - 0.5d_{21} - T_{11} + 0.2T_{21},$$

$$p_{21}(d, T) = 300 - 2d_{21} - 0.5d_{11} - T_{21} + 0.2T_{11}.$$

Example 1

The equilibrium product shipment and guaranteed delivery time pattern is:

$$Q_{11}^* = 28.14, \quad Q_{21}^* = 27.61, \quad T_{11}^* = 2.00, \quad T_{21}^* = 6.19,$$

and the corresponding Lagrange multipliers are:

$$\gamma_{11}^* = 2.14, \quad \gamma_{21}^* = 0.00.$$

Furthermore, the equilibrium prices associated with these two products are:

$$p_{11} = 229.15, \quad p_{21} = 224.91,$$

and the profits of the two firms are:

$$U_1 = 3,616.20, \quad U_2 = 3,571.90.$$

Example 1

Firm 2's guaranteed delivery time, which is 6.19, is longer than the actual delivery time, which is 2, mainly because the total cost associated with delivery time would increase notably if Firm 2 were to reduce its guaranteed delivery time.

Example 2

Example 2 has the same data as Example 1 except that now the actual production times and the actual transportation times of Firms 1 and 2 depend on how much is produced and how much is shipped, respectively, that is,

$$t_1 = 0.2, \quad t_2 = 0.3, \quad t_{11} = 0.1, \quad t_{21} = 0.2.$$

The new equilibrium product shipment and guaranteed delivery time pattern is:

$$Q_{11}^* = 27.06, \quad Q_{21}^* = 26.13, \quad T_{11}^* = 10.12, \quad T_{21}^* = 15.07,$$

and the corresponding Lagrange multipliers are:

$$\gamma_{11}^* = 17.30, \quad \gamma_{21}^* = 16.26.$$

Example 2

The equilibrium prices associated with these two products are:

$$p_{11} = 225.70, \quad p_{21} = 221.17,$$

and the profits of the two firms are:

$$U_1 = 3,603.89, \quad U_2 = 3,551.89.$$

This example shows that Firm 1 attracts more consumers with a notably shorter guaranteed delivery time, although the price of its product is higher than that of Firm 2's product. Due to its competitive advantage in delivery time performance, Firm 1 achieves a relatively higher profit.

Example 3

Example 3 has the same data as Example 2 except that now Firm 2 has reduced its production cost by improving its operational efficiency. The production cost function of Firm 2 is now given by:

$$f_2(s) = s_2^2 + 2s_2.$$

The equilibrium product shipment and guaranteed delivery time pattern is:

$$Q_{11}^* = 26.86, \quad Q_{21}^* = 31.75, \quad T_{11}^* = 10.06, \quad T_{21}^* = 17.87,$$

and the corresponding Lagrange multipliers are:

$$\gamma_{11}^* = 16.97, \quad \gamma_{21}^* = 27.49.$$

Example 3

The equilibrium prices associated with these two products are:

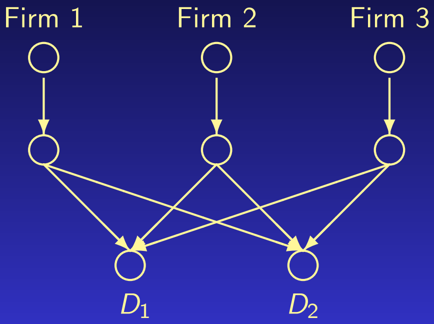
$$p_{11} = 223.93, \quad p_{21} = 207.22,$$

and the profits of the two firms are:

$$U_1 = 3,543.33, \quad U_2 = 4,413.00.$$

As a result of its lower production cost, Firm 2 is able to provide consumers with its product at an appealing price. Hence, the demand for Firm 2's product increases remarkably, even with a longer guaranteed delivery time, while there is a slight decrease in the demand for Firm 1's product. Therefore, in this example, Firm 2's profit improves significantly.

Example 4



Consumers at Demand Market 2 are more sensitive with respect to guaranteed delivery times than consumers at Demand Market 1.

Example 4

Firm 1:

$$f_1(s) = s_1^2 + 0.5s_1s_2 + 0.5s_1s_3,$$

$$g_1(T_1) = T_{11}^2 + T_{12}^2 - 30T_{11} - 40T_{12} + 650,$$

$$\hat{c}_{11}(Q_{11}) = Q_{11}^2 + 0.5Q_{11}, \quad \hat{c}_{12}(Q_{12}) = Q_{12}^2 + Q_{12},$$

$$p_{11}(d, T) = 400 - 2d_{11} - d_{21} - 0.8d_{31} - 1.2T_{11} + 0.3T_{21} + 0.2T_{31},$$

$$p_{12}(d, T) = 400 - 1.5d_{12} - 0.5d_{22} - 0.8d_{32} - 2T_{12} + 0.2T_{22} + 0.3T_{32},$$

$$t_1 = 0.8, \quad h_1 = 1.5, \quad t_{11} = 0.4, \quad h_{11} = 1.5, \quad t_{12} = 0.5, \quad h_{12} = 1.5.$$

Example 4

Firm 2:

$$f_2(s) = 1.5s_2^2 + 0.8s_1s_2 + 0.8s_2s_3,$$

$$g_2(T_2) = T_{21}^2 + T_{22}^2 - 30T_{21} - 30T_{22} + 480,$$

$$\hat{c}_{21}(Q_{21}) = Q_{21}^2 + Q_{21}, \quad \hat{c}_{22}(Q_{22}) = Q_{22}^2 + Q_{22},$$

$$p_{21}(d, T) = 400 - 2d_{21} - d_{11} - d_{31} - 1.2T_{21} + 0.2T_{11} + 0.2T_{31},$$

$$p_{22}(d, T) = 400 - 1.5d_{22} - 0.5d_{12} - 0.5d_{32} - 2T_{22} + 0.3T_{12} + 0.3T_{32},$$

$$t_2 = 0.6, \quad h_2 = 1.5, \quad t_{21} = 0.4, \quad h_{21} = 1.3, \quad t_{22} = 0.4, \quad h_{22} = 1.3.$$

Example 4

Firm 3:

$$f_3(s) = 2s_3^2 + 0.8s_1 s_3 + 0.8s_2 s_3,$$

$$g_3(T_3) = 0.8T_{31}^2 + 0.8T_{32}^2 - 25T_{31} - 20T_{32} + 400,$$

$$\hat{c}_{31}(Q_{31}) = 1.5Q_{31}^2 + Q_{31}, \quad \hat{c}_{32}(Q_{32}) = Q_{32}^2 + 1.5Q_{32},$$

$$p_{31}(d, T) = 400 - 2d_{31} - 0.8d_{11} - d_{21} - 1.2T_{31} + 0.2T_{11} + 0.3T_{21},$$

$$p_{32}(d, T) = 400 - 1.5d_{32} - 0.8d_{12} - 0.5d_{22} - 2T_{32} + 0.3T_{12} + 0.2T_{22},$$

$$t_3 = 0.3, \quad h_3 = 1, \quad t_{31} = 0.2, \quad h_{31} = 1, \quad t_{32} = 0.1, \quad h_{32} = 1.$$

Example 5

Example 5 has the identical data to that in Example 4, except that consumers at Demand Market 2 are becoming even more time-sensitive. The new demand price functions are now given by:

$$p_{12}(d, T) = 400 - 1.5d_{12} - 0.5d_{22} - 0.8d_{32} - 3T_{12} + 0.2T_{22} + 0.3T_{32},$$

$$p_{22}(d, T) = 400 - 1.5d_{22} - 0.5d_{12} - 0.5d_{32} - 3T_{22} + 0.3T_{12} + 0.3T_{32},$$

$$p_{32}(d, T) = 400 - 1.5d_{32} - 0.8d_{12} - 0.5d_{22} - 3T_{32} + 0.3T_{12} + 0.2T_{22}.$$

The Equilibrium Product Shipment and Guaranteed Delivery Time Patterns, the Lagrange Multipliers, and the Prices for Examples 4 and 5

		Example 4				Example 5			
Firm	Demand Market	Q^*	T^*	γ^*	p	Q^*	T^*	γ^*	p
1	1	18.05	36.44	64.54	302.76	19.30	35.34	63.84	299.96
	2	14.73	36.59	62.65	288.15	11.48	33.36	61.15	268.47
2	1	15.96	29.10	47.36	308.78	17.01	28.32	47.04	305.76
	2	17.23	29.61	63.67	311.74	14.19	27.19	66.94	295.34
3	1	17.14	17.63	23.78	330.18	17.47	17.24	23.55	327.49
	2	23.55	16.56	53.60	328.05	21.69	15.91	70.54	318.91

The Profits of Firms 1, 2, and 3 in Examples 4 and 5

	Firm 1	Firm 2	Firm 3
Example 4	$6,097.14$	$5,669.63$	$6,782.11$
Example 5	$5,697.97$	$5,3072.64$	$6,560.58$

Examples 4 and 5

- In Example 4,
  - Firm 1 has a slight advantage over its competitors in Demand Market 1, despite the longer guaranteed delivery time, perhaps as a consequence of the lower price.
  - Firm 3 captures the majority of the market share at Demand Market 2, due to consumers' preference for timely delivery.
  - Firm 2 attains the lowest profit, as compared to its rivals, since Firm 2 is neither cost-effective enough nor sufficiently time-efficient.

Examples 4 and 5

- In Example 5,
  - Consumers' increasing time sensitivity at Demand Market 2 has forced all these three firms to shorten their guaranteed delivery times.
  - Firms 1 and 3 still dominate Demand Markets 1 and 2, respectively.
  - The decrease in Firm 3's profit is negligible, while the profits of Firms 1 and 2 shrink notably.
- The results in Examples 4 and 5 suggest that delivery times, as a strategy, are particularly influential in time-based competition.

Summary

We developed a rigorous modeling and computational framework for time-based competition in supply chain networks, in which,

- firms are assumed to be spatially separated and can compete both on the production side and on the demand side;
- firms compete in an oligopolistic manner and, hence, influence the prices;
- the time consumption of both production and transportation/shipment supply chain activities is made explicit;
- the strategic variables of the firms are quantity variables and guaranteed delivery time variables;
- consumers at the demand markets for the substitutable, but differentiated, products respond to both the quantities of the products and to their guaranteed delivery times, as reflected in the prices of the products.

Summary

- The modeling and analytical framework can be used as the foundation for the investigation of supply chain networks in the case of build to order and made on demand products.
- It can also be extended in several directions through the inclusion of multiple options of transportation and multiple technologies for production.
- One may also incorporate additional tiers of suppliers.

Thank You!

For more information, see: http://supernet.isenberg.umass.edu