Modeling Generator Power Plant Portfolios and Pollution Taxes in Electric Power Supply Chain Networks: A Transportation Network Equilibrium Transformation

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Electricity is Modernity
Electricity is Big Business

• In US: half a trillion dollars worth of net assets
• Over $220 billion of annual sales
• Consumes almost 40% of domestic primary energy
• Heavy user of fossil fuels; over a third of total CO$_2$, NO$_x$ emissions
• Deregulation: from vertically integrated to competitive markets
Tax the Bad, Credit the Good

- Climate change poses immense risks
- Market failure: externalities need to be internalized
- Renewables are long-term solution
- Tax on bad emissions is one solution, e.g. CO2 tax
- Credit for clean energy, e.g. Renewable Portfolio Standards, green certs
Model: Electric Power Supply Chain Network

- 2004 paper by Nagurney, Matsypura
- 3 Players: Gencos, Suppliers, Consumers
The Big VI To Solve

The utility maximization of each player in the network gives the variational inequality:

\[
\sum_{g=1}^{G} \sum_{s=1}^{S} \left[ \frac{\partial f_g(Q_1^{1*})}{\partial q_{gs}} + \frac{\partial c_{gs}(Q_1^{1*})}{\partial q_{gs}} + \frac{\partial c_s(Q_1^{1*}, Q_2^{2*})}{\partial q_{gs}} + \frac{\partial c_{gs}(Q_1^{1*})}{\partial q_{gs}} - \gamma_s^* \right] \times [q_{gs} - q_{gs}^*]
\]

\[
+ \sum_{s=1}^{S} \sum_{k=1}^{K} \sum_{t=1}^{T} \left[ \frac{\partial c_s(Q_1^{1*}, Q_2^{2*})}{\partial q_{sk}^t} + \frac{\partial c_{sk}^t(Q_2^{2*})}{\partial q_{sk}^t} + c_{sk}^t(Q_2^{2*}) + \gamma_s^* - \rho_{3k}^* \right] \times [q_{sk}^t - q_{sk}^t]
\]

\[
+ \sum_{s=1}^{S} \left[ \sum_{g=1}^{G} q_{gs}^* - \sum_{k=1}^{K} \sum_{t=1}^{T} q_{sk}^t \right] \times [\gamma_s - \gamma_s^*] + \sum_{k=1}^{K} \left[ \sum_{s=1}^{S} \sum_{t=1}^{T} q_{sk}^t - d_k(\rho_3^*) \right] \times [\rho_{3k} - \rho_{3k}^*] \geq 0,
\]

\forall(Q^1, Q^2, \gamma, \rho_3) \in \mathcal{K},

where \(\mathcal{K} \equiv \{(Q^1, Q^2, \gamma, \rho_3) | (Q^1, Q^2, \gamma, \rho_3) \in R_+^{GS+TSK+S+K}\}.

Modeling Energy Taxes and Credits: The Genco’s Choice

- Each Genco has a portfolio of power plants
- Each power plant can have different supply costs and transaction costs
- Supply costs can reflect capital costs, O&M, fuel costs
- Transaction costs reflect possible taxes or credits
The Extended Model
Variational Inequality for Extended Model

The equilibrium conditions governing the electric power supply chain network according to Definition 1 coincide with the solution of the variational inequality given by: determine 

\((q^*, h^*, Q_1^*, Q_2^*, d^*) \in \mathcal{K}^5\) satisfying:

\[
\sum_{g=1}^{G} \sum_{m=1}^{M} \left[ \frac{\partial f_{gm}(q_m^*)}{\partial q_{gm}} + \tau_{gm} \right] \times [q_{gm} - q_{gm}^*] + \sum_{s=1}^{S} \frac{\partial c_s(h^*)}{\partial h_s} \times [h_s - h_s^*] \\
+ \sum_{g=1}^{G} \sum_{m=1}^{M} \sum_{s=1}^{S} \left[ \frac{\partial c_{gms}(q_{gms}^*)}{\partial q_{gms}} + \frac{\partial c_{gms}(q_{gms}^*)}{\partial q_{gms}} \right] \times [q_{gms} - q_{gms}^*] \\
+ \sum_{s=1}^{S} \sum_{k=1}^{K} \sum_{t=1}^{T} \left[ \frac{\partial c_{sk}^t(q_{sk}^t)^*}{\partial q_{sk}^t} + c_{sk}^t(Q_{2s}^t) \right] \times [d_{sk}^t - d_{sk}^t] - \sum_{k=1}^{K} \rho_{3k}(d^*) \times [d_k - d_k^*] \geq 0,
\]

\(\forall (q, h, Q_1, Q_2, d) \in \mathcal{K}^5, \quad (18)\)

where

\[\mathcal{K}^5 \equiv \{(q, h, Q_1, Q_2, d)|(q, h, Q_1, Q_2, d) \in R_{+}^{GM+S+GMS+TSK+K}\]

and (2), (5), (11), and (15) hold.
Transportation Network Equilibrium Isomorphism

• Why do we need the transportation network equilibrium isomorphism?

• We have proved the supernetwork equivalence of properly configured transportation networks with
  – Supply Chain Networks (A. Nagurney)
  – Financial Networks (Z. Liu and A. Nagurney)
  – Electrical Power Networks (A. Nagurney and Z. Liu)
Supernetwork Equivalence of the Electrical Power Network with the Transportation Network
A Numerical Example
The Cost Functions

• Generating cost functions

\[ f_{11}(q_1) = 2.5q_{11}^2 + q_{11}q_{21} + 2q_{11}, \quad f_{12}(q_2) = 2.5q_{12}^2 + q_{11}q_{12} + 2q_{22}, \quad f_{21}(q_1) = .5q_{21}^2 + .5q_{11}q_{21} + 2q_{21}, \]

\[ f_{22}(q_2) = .5q_{22}^2 + q_{12} + 2. \]

• Transaction cost between power generators and the suppliers

\[ c_{111}(q_{111}) = .5q_{111}^2 + 3.5q_{111}, \quad c_{112}(q_{112}) = .5q_{112}^2 + 3.5q_{112}, \quad c_{121}(q_{121}) = .5q_{121}^2 + 3.5q_{121}, \]

\[ c_{122}(q_{122}) = .5q_{122}^2 + 3.5q_{122}, \]

\[ c_{211}(q_{211}) = .5q_{211}^2 + 2q_{211}, \quad c_{212}(q_{212}) = .5q_{212}^2 + 2q_{212}, \quad c_{221}(q_{221}) = .5q_{221}^2 + 2q_{221}, \]

\[ c_{222}(q_{222}) = .5q_{222}^2 + 2q_{222}. \]
The Cost Functions and Price Functions

• The operating cost of the power generators

\[ c_1(Q^1) = .5\left(\sum_{i=1}^{2} q_{i1}\right)^2, \quad c_2(Q^1) = .5\left(\sum_{i=1}^{2} q_{i2}\right)^2. \]

• The demand market price function

\[ \rho_{31}(d) = -1.33d_1 + 366.6, \quad \rho_{32} = -1.33d_2 + 366.6, \]

• The transaction cost between the suppliers and the customers

\[ c_{sk}^1(q_{sk}) = q_{sk}^1 + 5, \quad s = 1, 2; \ k = 1, 2. \]
The Solution

• Equilibrium pattern for the transportation network

\[ f_{a_1}^* = 32.53, \quad f_{a_2}^* = 115.22, \]
\[ f_{a_{11}}^* = 22.57, \quad f_{a_{12}}^* = 9.96, \quad f_{a_{21}}^* = 22.90, \quad f_{a_{22}}^* = 92.32, \]
\[ f_{a_{11}'}^* = f_{a_{22}'}^* = 73.87, \]
\[ f_{a_{111}}^* = 11.29, \quad f_{a_{112}}^* = 11.29, \quad f_{a_{121}}^* = 4.98, \quad f_{a_{222}}^* = 4.98, \]
\[ f_{a_{211}}^* = 11.45, \quad f_{a_{212}}^* = 11.45, \quad f_{a_{221}}^* = 46.16, \quad f_{a_{222}}^* = 46.16, \]
\[ f_{a_{1'1}}^* = f_{a_{1'2}}^* = f_{a_{2'1}}^* = f_{a_{2'2}}^* = 36.94, \]
The Solution (cont)

- The equilibrium pattern for the electrical power network

\[ q_1^* = 32.53, \quad q_2^* = 115.22, \]
\[ q_{11}^* = 22.57, \quad q_{12}^* = 9.96, \quad q_{21}^* = 22.90, \quad q_{22}^* = 9.96, \]
\[ h_1^* = h_2^* = 73.87, \]
\[ q_{111}^* = 11.29, \quad q_{112}^* = 11.29, \quad q_{121}^* = 4.98, \quad q_{222}^* = 4.98, \]
\[ q_{211}^* = 11.45, \quad q_{212}^* = 11.45, \quad q_{221}^* = 46.16, \quad q_{222}^* = 46.16, \]
\[ q_{1/1}^* = q_{1/2}^* = q_{2/1}^* = q_{2/2}^* = 36.94. \]
Solution of Example 1
Example 2

- Same network configuration except that we added tax 133 to the first plant of genco 1 which is highly polluting.

- Solution:

\[ q_1^* = 10.77, \quad q_2^* = 128.71, \]

\[ q_{11}^* = 0.00, \quad q_{12}^* = 10.77, \quad q_{21}^* = 29.14, \quad q_{22}^* = 99.58, \]

\[ h_1^* = h_2^* = 69.74, \]

\[ q_{111}^* = 0.00, \quad q_{112}^* = 0.00, \quad q_{121}^* = 5.38, \quad q_{222}^* = 5.38, \]

\[ q_{211}^* = 11.45, \quad q_{212}^* = 14.57, \quad q_{221}^* = 49.79, \quad q_{222}^* = 49.79, \]

\[ q_{111}^{1*} = q_{112}^{1*} = q_{211}^{1*} = q_{212}^{1*} = 34.87. \]
Solution to Example 2
Thank You!

Questions?