Modeling Generator Power Plant Portfolios and Pollution Taxes in Electric Power Supply Chain Networks: A Transportation Network Equilibrium Transformation

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Electricity is Modernity

Electricity is Big Business

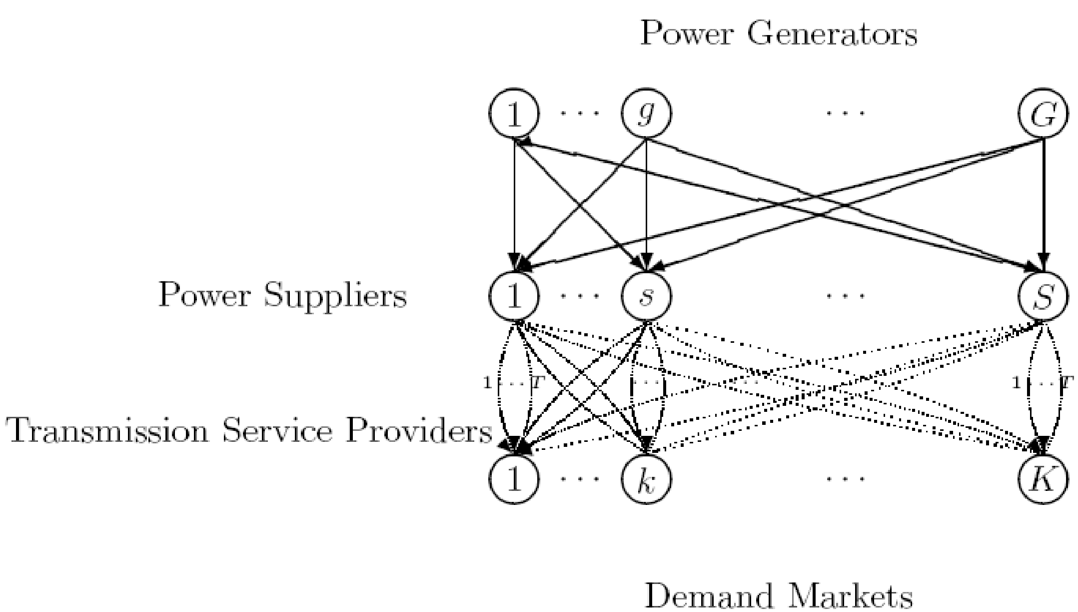
- In US: half a trillion dollars worth of net assets
- Over $220 billion of annual sales
- Consumes almost 40% of domestic primary energy
- Heavy user of fossil fuels; over a third of total CO_2, NO_x emissions
- Deregulation: from vertically integrated to competitive markets

Tax the Bad, Credit the Good

- Climate change poses immense risks
- Market failure: externalities need to be internalized
- Renewables are long-term solution
- Tax on bad emissions is one solution, e.g. C02 tax
- Credit for clean energy, e.g. Renewable Portfolio Standards, green certs

Model: Electric Power Supply Chain Network

- 2004 paper by Nagurney, Matsypura
- 3 Players: Gencos, Suppliers, Consumers



The Big VI To Solve

The utility maximization of each player in the network gives the variational inequality:

$$\sum_{g=1}^{G}\sum_{s=1}^{S}\left[\frac{\partial f_g(Q^{1*})}{\partial q_{gs}}+\frac{\partial c_{gs}(Q^{1*})}{\partial q_{gs}}+\frac{\partial c_s(Q^{1*},Q^{2*})}{\partial q_{gs}}+\frac{\partial \hat{c}_{gs}(Q^{1*})}{\partial q_{gs}}-\gamma_s^*\right]\times[q_{gs}-q_{gs}^*]$$

$$+\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{t=1}^{T}\left[\frac{\partial c_s(Q^{1*},Q^{2*})}{\partial q_{sk}^t}+\frac{\partial c_{sk}^t(Q^{2*})}{\partial q_{sk}^t}+\hat{c}_{sk}^t(Q^{2*})+\gamma_s^*-\rho_{3k}^*\right]\times[q_{sk}^t-q_{sk}^{t*}]$$

$$+\sum_{s=1}^{S}\left[\sum_{g=1}^{G}q_{gs}^*-\sum_{k=1}^{K}\sum_{t=1}^{T}q_{sk}^{t*}\right]\times[\gamma_s-\gamma_s^*]+\sum_{k=1}^{K}\left[\sum_{s=1}^{S}\sum_{t=1}^{T}q_{sk}^{t*}-d_k(\rho_3^*)\right]\times[\rho_{3k}-\rho_{3k}^*]\geq 0,$$

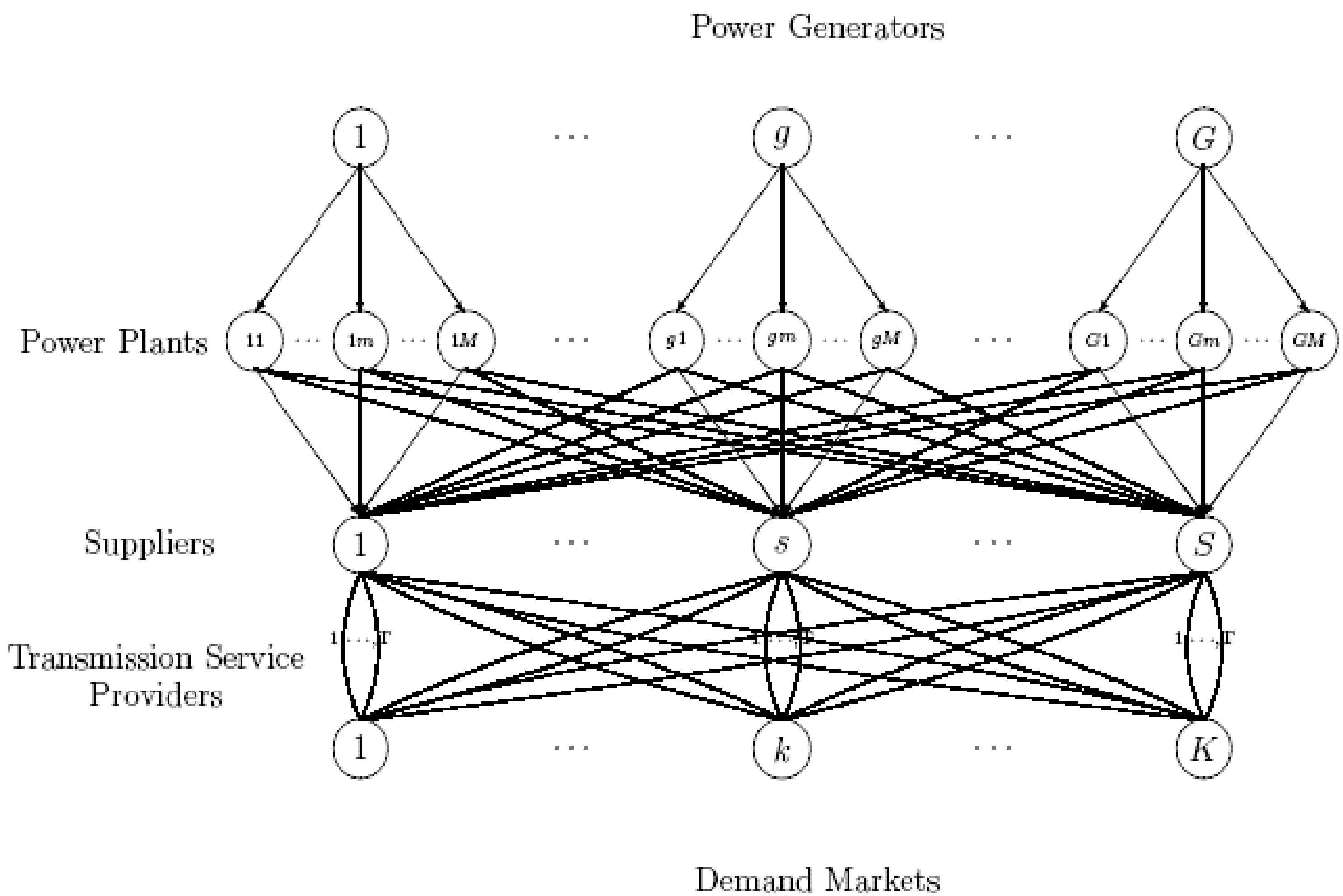
$$\forall (Q^1,Q^2,\gamma,\rho_3)\in\mathcal{K},$$

$$where\ \mathcal{K}\equiv\{(Q^1,Q^2,\gamma,\rho_3)|(Q^1,Q^2,\gamma,\rho_3)\in R_+^{GS+TSK+S+K}\}.$$

Modeling Energy Taxes and Credits: The Genco's Choice

- Each Genco has a portfolio of power plants
- Each power plant can have different supply costs and transaction costs
- Supply costs can reflect capital costs, O&M, fuel costs
- Transaction costs reflect possible taxes or credits

The Extended Model



Variational Inequality for Extended Model

The equilibrium conditions governing the electric power supply chain network according to Definition 1 coincide with the solution of the variational inequality given by: determine $(q^*, h^*, Q^{1*}, Q^{2*}, d^*) \in \mathcal{K}^5$ *satisfying:*

$$\sum_{g=1}^{G}\sum_{m=1}^{M}\left[\frac{\partial f_{gm}(q_m^*)}{\partial q_{gm}} + \tau_{gm}\right] \times [q_{gm} - q_{gm}^*] + \sum_{s=1}^{S}\frac{\partial c_s(h^*)}{\partial h_s} \times [h_s - h_s^*]$$

$$+\sum_{g=1}^{G}\sum_{m=1}^{M}\sum_{s=1}^{S}\left[\frac{\partial c_{gms}(q_{gms}^*)}{\partial q_{gms}} + \frac{\partial \hat{c}_{gms}(q_{gms}^*)}{\partial q_{gms}}\right] \times [q_{gms} - q_{gms}^*]$$

$$+\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{t=1}^{T}\left[\frac{\partial c_{sk}^t(q_{sk}^{t*})}{\partial q_{sk}^t} + \hat{c}_{sk}^t(Q^{2*})\right] \times [q_{sk}^t - q_{sk}^{t*}] - \sum_{k=1}^{K}\rho_{3k}(d^*) \times [d_k - d_k^*] \geq 0,$$

$$\forall (q, h, Q^1, Q^2, d) \in \mathcal{K}^5, \qquad (18)$$

where

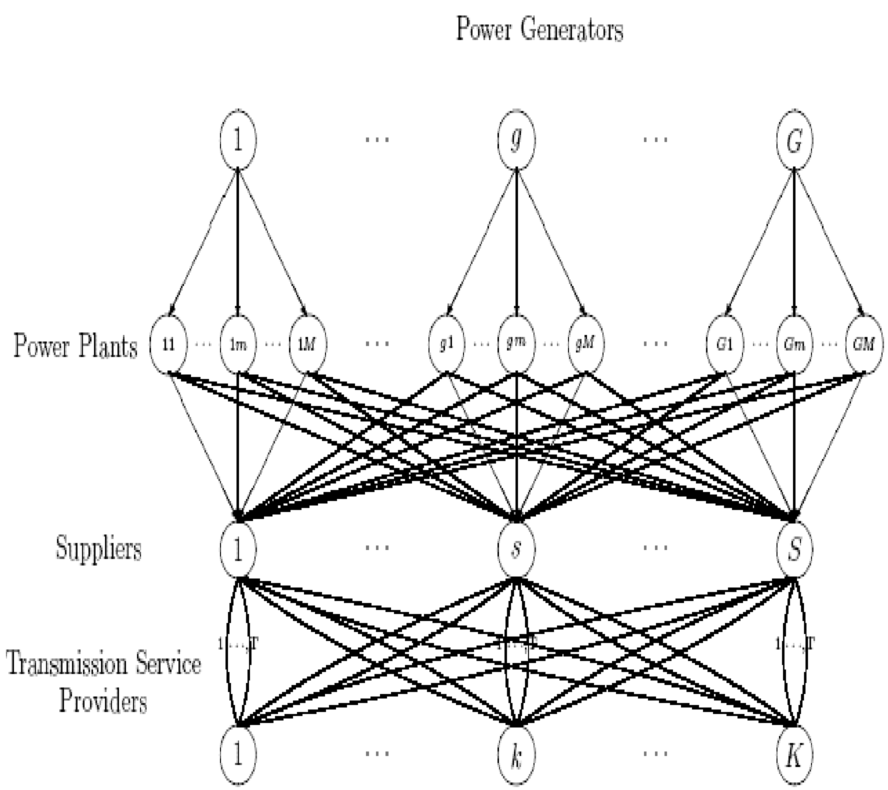
$$\mathcal{K}^5 \equiv \{(q, h, Q^1, Q^2, d) | (q, h, Q^1, Q^2, d) \in R_+^{GM+S+GMS+TSK+K}$$

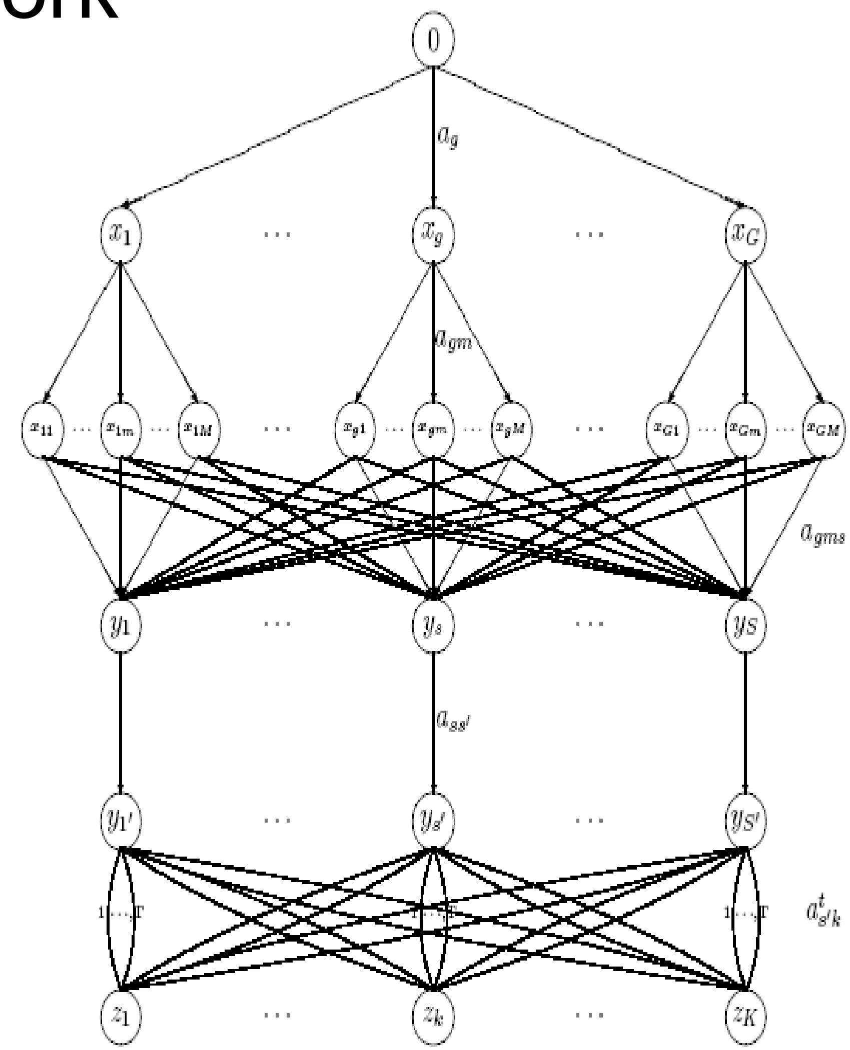
$$\text{and } (2),\ (5),\ (11),\ \text{and } (15) \text{ hold}\}.$$

Transportation Network Equilibrium Isomorphism

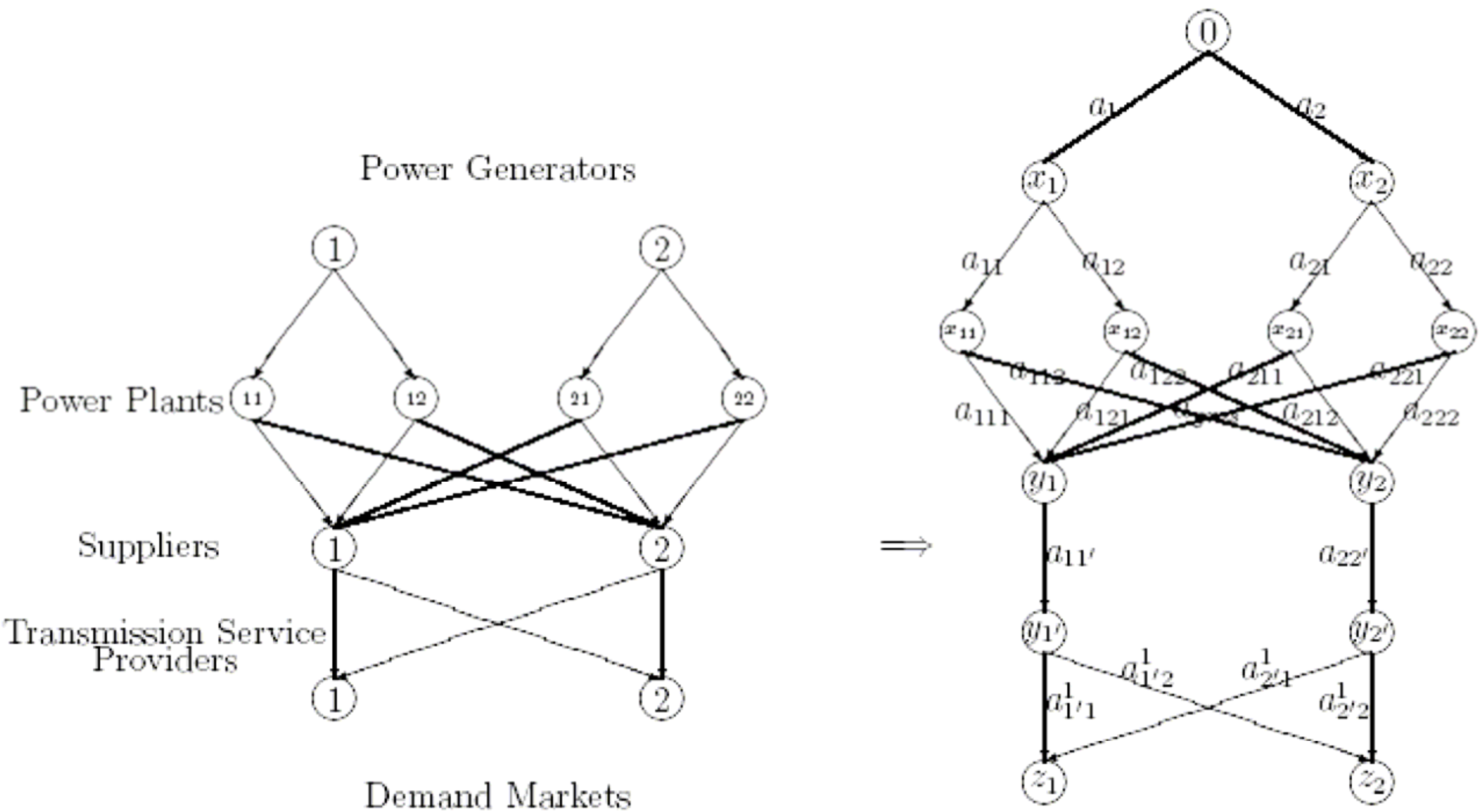
- Why do we need the transportation network equilibrium isomorphism?
- We have proved the supernetwork equivalence of properly configured transportation networks with
  - Supply Chain Networks (A. Nagurney)
  - Financial Networks (Z. Liu and A. Nagurney)
  - Electrical Power Networks (A. Nagurney and Z. Liu)

Supernetwork Equivalence of the Electrical Power Network with the Transportation Network





A Numerical Example



The Cost Functions

- Generating cost functions

$$f_{11}(q_1) = 2.5q_{11}^2 + q_{11}q_{21} + 2q_{11}, \quad f_{12}(q_2) = 2.5q_{12}^2 + q_{11}q_{12} + 2q_{22}, \quad f_{21}(q_1) = .5q_{21}^2 + .5q_{11}q_{21} + 2q_{21},$$

$$f_{22}(q_2) = .5q_{22} + q_{12} + 2.$$

- Transaction cost between power generators and the suppliers

$$c_{111}(q_{111}) = .5q_{111}^2 + 3.5q_{111}, \; c_{112}(q_{112}) = .5q_{112}^2 + 3.5q_{112}, \; c_{121}(q_{121}) = .5q_{121}^2 + 3.5q_{121},$$

$$c_{122}(q_{122}) = .5q_{122}^2 + 3.5q_{122},$$

$$c_{211}(q_{211}) = .5q_{211}^2 + 2q_{211}, \; c_{212}(q_{212}) = .5q_{212}^2 + 2q_{212}, \; c_{221}(q_{221}) = .5q_{221}^2 + 2q_{221},$$

$$c_{222}(q_{222}) = .5q_{222}^2 + 2q_{222}.$$

The Cost Functions and Price Functions

- The operating cost of the power generators

$$c_1(Q^1) = .5(\sum_{i=1}^{2} q_{i1})^2, \quad c_2(Q^1) = .5(\sum_{i=1}^{2} q_{i2})^2.$$

- The demand market price function

$$\rho_{31}(d) = -1.33d_1 + 366.6, \quad \rho_{32} = -1.33d_2 + 366.6,$$

- The transaction cost between the suppliers and the customers

$$\hat{c}^1_{sk}(q^1_{sk}) = q^1_{sk} + 5, \quad s = 1, 2; k = 1, 2.$$

The Solution

- Equilibrium pattern for the transportation network

$$f^*_{a_1} = 32.53, \quad f^*_{a_2} = 115.22,$$

$$f^*_{a_{11}} = 22.57, \quad f^*_{a_{12}} = 9.96, \quad f^*_{a_{21}} = 22.90, \quad f^*_{a_{22}} = 92.32,$$

$$f^*_{a_{11'}} = f^*_{a_{22'}} = 73.87,$$

$$f^*_{a_{111}} = 11.29, \quad f^*_{a_{112}} = 11.29, \quad f^*_{a_{121}} = 4.98, \quad f^*_{a_{222}} = 4.98,$$

$$f^*_{a_{211}} = 11.45, \quad f^*_{a_{212}} = 11.45, \quad f^*_{a_{221}} = 46.16, \quad f^*_{a_{222}} = 46.16,$$

$$f^*_{a^1_{1'1}} = f^*_{a^1_{1'2}} = f^*_{a^1_{2'1}} = f^*_{a^1_{2'2}} = 36.94,$$

The Solution (cont)

- The equilibrium pattern for the electrical power network

$$q_1^* = 32.53, \quad q_2^* = 115.22,$$

$$q_{11}^* = 22.57, \quad q_{12}^* = 9.96, \quad q_{21}^* = 22.90, \quad q_{22}^* = 9.96,$$

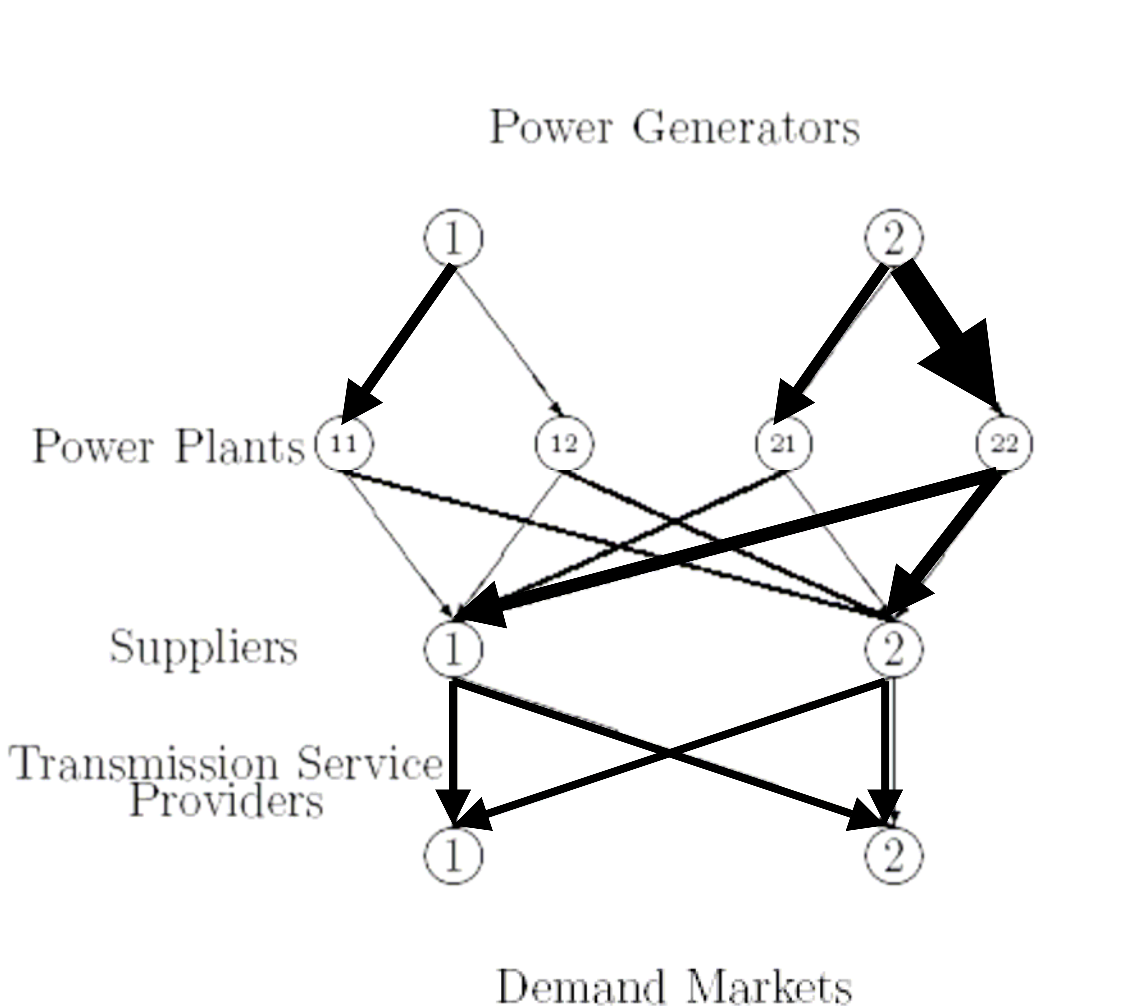
$$h_1^* = h_2^* = 73.87,$$

$$q_{111}^* = 11.29, \quad q_{112}^* = 11.29, \quad q_{121}^* = 4.98, \quad q_{222}^* = 4.98,$$

$$q_{211}^* = 11.45, \quad q_{212}^* = 11.45, \quad q_{221}^* = 46.16, \quad q_{222}^* = 46.16,$$

$$q_{1'1}^{1*} = q_{1'2}^{1*} = q_{2'1}^{1*} = q_{2'2}^{1*} = 36.94.$$

Solution of Example 1



Example 2

- Same network configuration except that we added tax 133 to the first plant of genco 1 which is highly polluting.
- Solution: $q_1^* = 10.77, \quad q_2^* = 128.71,$

$$q_{11}^* = 0.00, \quad q_{12}^* = 10.77, \quad q_{21}^* = 29.14, \quad q_{22}^* = 99.58,$$

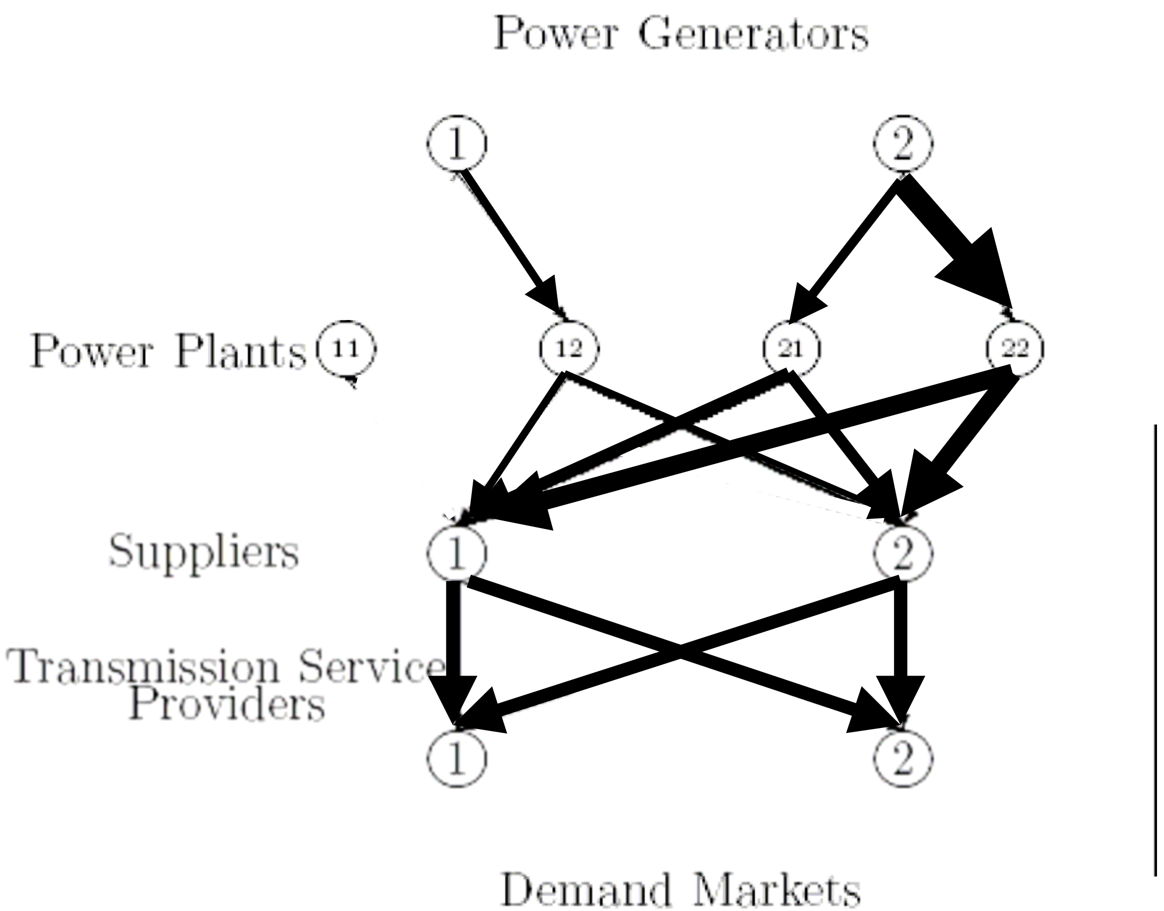
$$h_1^* = h_2^* = 69.74,$$

$$q_{111}^* = 0.00, \quad q_{112}^* = 0.00, \quad q_{121}^* = 5.38, \quad q_{222}^* = 5.38,$$

$$q_{211}^* = 11.45, \quad q_{212}^* = 14.57, \quad q_{221}^* = 49.79, \quad q_{222}^* = 49.79,$$

$$q_{1'1}^{1*} = q_{1'2}^{1*} = q_{2'1}^{1*} = q_{2'2}^{1*} = 34.87.$$

Solution to Example 2



Thank You!

Questions?