Multicommodity Trade Network Equilibrium with Mixed Demand and Food Security

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RESEARCH ARTICLE

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ABSTRACT

Rising food insecurity is a pressing global concern. Many developing countries rely on a combination of international trade and food aid to satisfy the nutritional and caloric demands of their populations. Trade openness has been shown to have positive long-term effects on food security; however, it can cause short-term crises in food availability and affordability. Strategic grain reserves are among the tools governments use to control food prices and ensure availability while maintaining trade openness. This paper constructs a multicommodity trade network equilibrium model with mixed demand to examine the implications of strategic grain reserves for food security. The model incorporates fixed and elastic demand components, representing commodities held in reserves and those entering markets through trade, respectively. The equilibrium conditions are stated, and alternative variational inequality formulations are presented. Several simple examples are solved analytically for illustration, and an algorithmic framework for computing equilibrium solutions to larger cases is provided. A set of numerical examples is presented, inspired by wheat exports from the Black Sea region to North and Sub-Saharan Africa, as well as purchases by the World Food Program. The impacts of the Russo-Ukrainian war on trade patterns and the effects of varying strategic reserve levels are assessed. We observe that each additional one million metric tons of fixed demand raises the corresponding market's equilibrium demand price by \$0.830–\$0.886 per metric ton, with the cumulative increase reaching approximately \$11 per metric ton over the reserve levels considered and substantial cross-market spillovers.

International Agricultural Trade and Food Security

- Grains are essential to global food security.
- Many developing countries rely on international agricultural trade and food aid to satisfy the nutritional demands of their populations.
- Trade openness can improve food security in the long term, while exposing vulnerable populations to short-term disruptions in supply and prices.





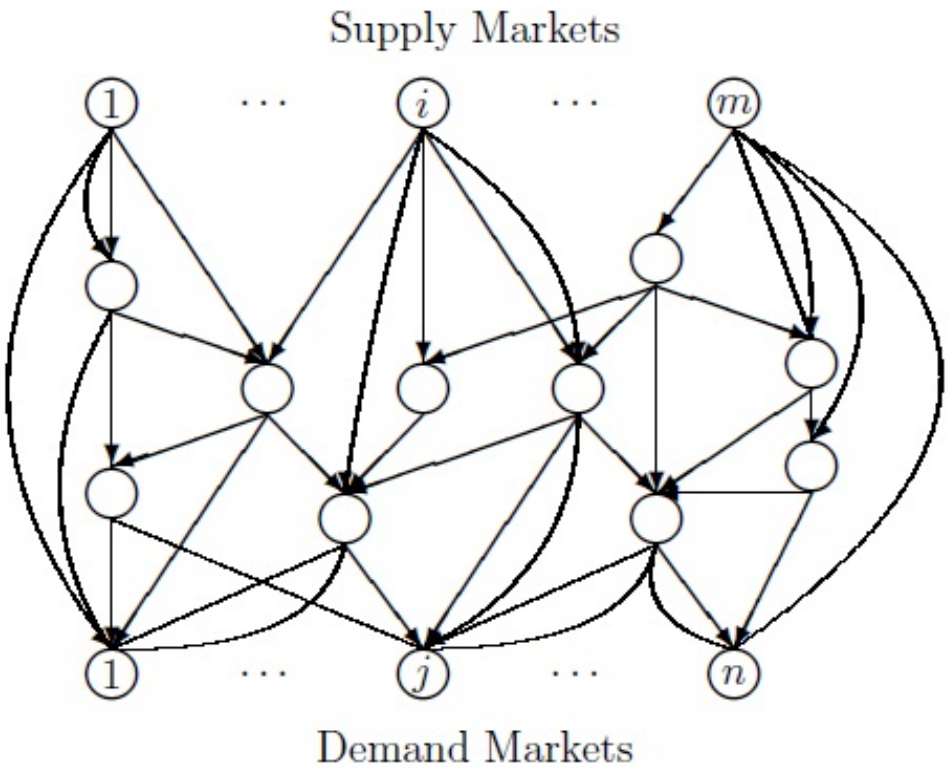
Strategic Grain Reserves and Food Security

- Governments purchase and store grains in strategic reserves to reduce the risks of shortages and sharp price increases.
- Humanitarian organizations, such as the World Food Programme (WFP), procure commodities both in advance to support their food aid operations and after disasters strike.
- **How to capture strategic grain reserves in a multicommodity, international trade network equilibrium setting? How do different levels of strategic grain reserves affect international trade flows, commodity prices, and food security?**





The Multicommodity Trade Network: An Illustration



Notation for the Multicommodity Trade Model

Variables	Definition
s_i^k	the supply of commodity k at supply market i. We group all the commodity supplies into the vector $s \in R^{Km}$.
Q_p^k	the flow of commodity k on path p. We group the commodity path flows into the vector $Q \in R_+^{Knp}$.
f_a^k	the flow of commodity k on link a. We group the commodity link flows into the vector $f \in R_+^{Kn_L}$.
d_j^k	the total demand for commodity k at demand market j. We group all the commodity total demands into the vector $d \in R^{Kn}$.
$d_j^{k,e}$	the elastic component of the total demand for commodity k at demand market j. We group all the elastic commodity demands into the vector $d^e \in R^{Kn}$.
μ_j^k	the Lagrange multiplier associated with the minimum demand constraint for commodity k at demand market j. We group all the Lagrange multipliers into the vector $\mu \in R_+^{Kn}$.
Functions	**Definition**
$\pi_i^k(s)$	the supply price function for commodity k at supply market i. We group all the supply price functions into the vector $\pi(s) \in R^{Km}$.
$c_a^k(f)$	the unit cost of transportation of commodity k on link a, $\forall k$, $\forall a \in L$.
$C_p^k(Q)$	the unit cost of transportation of commodity k on path p. We group all the path costs into the vector $C \in R^{Knp}$.
$\rho_j^k(d^e)$	the demand price function for commodity k at demand market j that depends on the volume of commodity elastic demands. We group all the demand price functions into the vector $\rho(d^e) \in R^{Kn}$.
Parameters	**Definition**
$d_j^{k,f}$	the fixed demand for the purchase of commodity k at demand market j, $\forall k$, $\forall j$.

Mixed Demand

Two Components of Commodity Demand

At each demand market j, for each commodity k, total demand is the sum of its fixed and elastic components:

$$d_j^k = d_j^{k,f} + d_j^{k,e}. \tag{1}$$

- **Fixed demand, $d_j^{k,f}$:** Purchases by governments or humanitarian organizations that must be satisfied, including acquisitions for strategic grain reserves.
- **Elastic demand, $d_j^{k,e}$:** Demand in the market that depends on commodity prices.
- The demand price functions depend on *elastic demand*, rather than total demand.

The Equilibrium Conditions

Definition 1: Multicommodity Trade Network Equilibrium with Mixed Demand

A path flow and multiplier pattern $(Q^*, \mu^*) \in \mathcal{K} = \mathbb{R}_+^{K(n_P+n)}$ is an equilibrium if, for each k and $p \in P_j^i$,

$$\widetilde{\pi}_i^k(Q^*) + C_p^k(Q^*) - \mu_j^{k*} \begin{cases} = \widetilde{\rho}_j^k(Q^*), & Q_p^{k*} > 0, \\ \geq \widetilde{\rho}_j^k(Q^*), & Q_p^{k*} = 0, \end{cases} \tag{2}$$

$$\sum_{p \in P_j} Q_p^{k*} \begin{cases} = d_j^{k,f}, & \mu_j^{k*} > 0, \\ \geq d_j^{k,f}, & \mu_j^{k*} = 0. \end{cases} \tag{3}$$

Variational Inequality Formulation

Theorem 1

A path flow and Lagrange multiplier pattern $(Q^*, \mu^*) \in \mathcal{K}$ satisfies the equilibrium conditions if and only if it solves:

$$\sum_{k=1}^{K} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{p \in P_j^i} \left[\widetilde{\pi}_i^k(Q^*) + C_p^k(Q^*) - \mu_j^{k*} - \widetilde{\rho}_j^k(Q^*) \right] \left[Q_p^k - Q_p^{k*} \right]$$

$$+ \sum_{k=1}^{K} \sum_{j=1}^{n} \left[\sum_{p \in P_j} Q_p^{k*} - d_j^{k,f} \right] \left[\mu_j^k - \mu_j^{k*} \right] \geq 0, \tag{4}$$

$$\forall (Q, \mu) \in \mathcal{K}.$$

The formulation is suitable for numerical solution by variational inequality algorithms.

Numerical Examples

- The numerical examples are inspired by agricultural commodity exports from the Black Sea region to North and Sub-Saharan Africa.
- **Supply countries:** Bulgaria, Romania, Ukraine, and Russia.
- **Demand markets:** WFP's humanitarian hub in the Port of Djibouti, Egypt, Tunisia, and Ethiopia.
- We consider annual commodity shipments in metric tons and prices in U.S. dollars per metric ton.
- The examples examine the effects of Russia's full-scale invasion of Ukraine and strategic grain reserve policies. All model data and parameters are reported in the paper.
- The modified projection method of Korpelevich is the algorithm used to solve the examples.

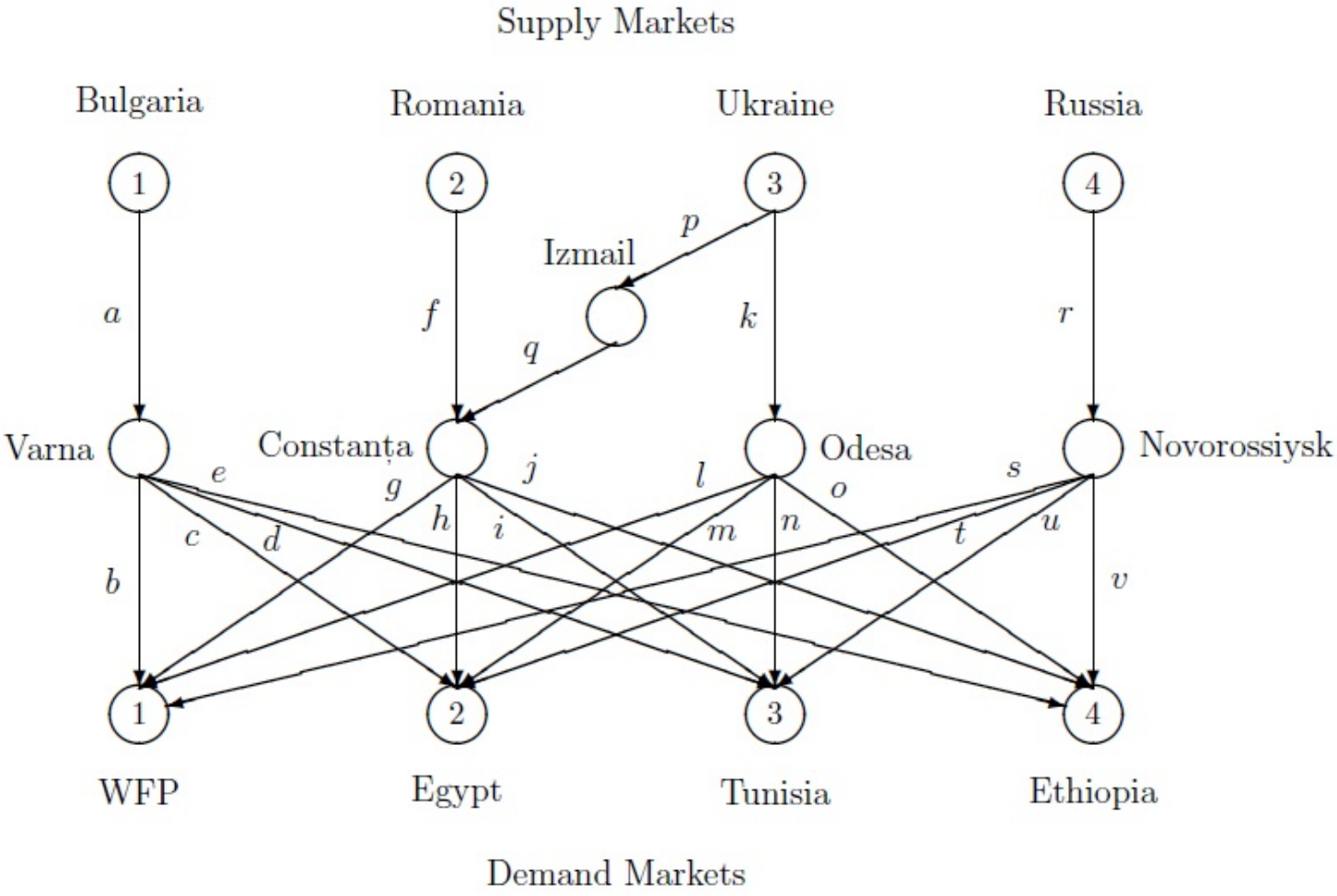
Numerical Examples



Numerical Examples



Network Topology for the Numerical Examples



Numerical Example Scenarios

Example 5: Prior to Russia's Full-Scale Invasion — 2021
Wheat trade flows and prices before the full-scale invasion.

Example 6: After the Full-Scale Invasion — 2024
Wheat trade with altered transportation costs and regional trade patterns.

Example 7: The Full-Scale Invasion Stops
A hypothetical scenario with lower transportation costs.

Example 8: Increased Fixed Demands
Larger strategic grain reserves, including binding minimum demand constraints.

Example 9: No Fixed Demands
All wheat demand is elastic.

Example 10: Two Commodities — Wheat and Corn
Trade interactions through shared supply and transportation infrastructure.

Some Insights: Before and After the Full-Scale Invasion

- Russia's position in the wheat markets of North and Sub-Saharan Africa strengthened after the full-scale invasion.
- Its share of wheat supply in the modeled network increases from roughly **42% in 2021** to approximately **70% in 2024**.
- Ukraine's wheat export patterns are affected by wartime disruptions and changes in transportation costs.
- Egypt remains strongly dependent on Russia for wheat imports, exposing it to possible food security risks from disruptions to those shipments.

Some Insights: If the Full-Scale Invasion Stops

- Reductions in transportation costs improve wheat availability and affordability in the demand markets.
- Russia remains the leading wheat supplier in the modeled network, with a share of approximately **60%**.
- Demand prices decrease more markedly than supply prices change; much of the benefit of lower transportation costs accrues to consumers.
- WFP's procurement remains constrained, suggesting that reduced purchasing power also presents a substantial hurdle to its food aid operations.

Some Insights: Increased Fixed Demands

- Higher fixed demands represent increased purchases for strategic grain reserves.
- In Example 8, the demand lower bounds for WFP and Ethiopia become binding.
- The associated multipliers are $\mu_1^* = 2.23$ and $\mu_4^* = 2.01$, respectively; the other demand-market multipliers are zero.
- Increasing strategic grain reserves may improve long-term availability but can raise current market prices and reduce short-term affordability.

Some Insights: No Fixed Demands

- In Example 9, all the fixed demands are zero and the total demand in each market is elastic.
- Wheat is supplied by Ukraine and Russia, while Bulgaria and Romania have zero shipments in the equilibrium solution.
- Total elastic demand increases by approximately **17%** compared with the 2024 example, and demand prices decrease.
- However, lower purchases for strategic reserves can increase vulnerability to future disruptions, despite improving short-term affordability.

Sensitivity Analysis on Fixed Demands

- We vary the fixed demand at one demand market while keeping the other three fixed demands at their Example 6 values.
- The analysis shows how changing the level of strategic grain reserves affects equilibrium demand prices, both locally and in the other markets.
- An increase in fixed demand produces the greatest average demand price increase in the market where that increase occurs.
- **There are also substantial cross-market effects:** trade and demand markets are interconnected through the network equilibrium.

Sensitivity Analysis: Average Demand Price Increases

	WFP (ρ_1)	Egypt (ρ_2)	Tunisia (ρ_3)	Ethiopia (ρ_4)
WFP (d_1^f)	0.886	0.712	0.738	0.705
Egypt (d_2^f)	0.737	0.880	0.743	0.716
Tunisia (d_3^f)	0.733	0.712	0.830	0.686
Ethiopia (d_4^f)	0.659	0.687	0.677	0.841

- Each cell gives the average increase in the column market's equilibrium demand price (USD/metric ton) per additional **one million metric tons** of fixed demand in the row market.
- Own-market increases range from **\$0.830 to \$0.886** per metric ton; cross-market increases range from **\$0.659 to \$0.743**.

Summary

- We constructed a multicommodity trade network equilibrium model incorporating **fixed and elastic demand** to examine food security implications.
- We formulated the equilibrium conditions and derived variational inequality formulations amenable to algorithmic solution.
- Numerical examples highlighted wartime changes in Black Sea grain trade and the dependence of African demand markets on particular exporters.
- **Strategic grain reserves present a trade-off:** they can safeguard future commodity availability, while raising present market prices and affecting food affordability.

Thank You!

Any Questions?

