Dynamics of Global Supply Chain and Electric Power Networks: Models, Pricing Analysis, and Computations

A Dissertation Presented
by
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Outline

1. Global Supply Chain Networks
2. Electric Power Networks
3. Future Research
Motivation

- Challenging intellectual questions
- Importance of supply chain decision-making
- Increasing globalization
- E-commerce
Global Supply Chain

- Network equilibrium
- Several classes of decision-makers
- Optimization problem for every decision-maker is unique
Global Supply Chain Network

Country 1 Manufacturers

11 → 1H → 1 → j → jH → j → J → JH → IL

Country L Manufacturers

1L → 1H → 1 → j → jH → j → J → JH → IL

Currencies

Retailers

Demand Market/Currency/Country combination

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Maximize Profit

\[
\text{Maximize } \sum_{j=1}^{J} \sum_{h=1}^{H} (\rho_{1jh}^{il} \times e_h) q_{jh}^{il} - \sum_{j=1}^{J} \sum_{h=1}^{H} c_{jh}^{il}(q_{jh}^{il}) - f^{il}(Q^1)
\]
The Optimality Conditions of All Manufacturers

Variational Inequality Formulation

Determine $Q^1 \in R_{+}^{IJHL}$ satisfying

$$\sum_{i=1}^{I} \sum_{l=1}^{L} \sum_{j=1}^{J} \sum_{h=1}^{H} \left[ \frac{\partial f_{il}(Q_{1}^*)}{\partial q_{jh}^l} + \frac{\partial c_{jh}^l(q_{jh}^*)}{\partial q_{jh}^l} - \rho_{1jh}^i \times e_h \right] \times \left[ q_{jh}^l - q_{jh}^{l*} \right] \geq 0$$

$\forall Q^1 \in R_{+}^{IJHL}$
Characteristics of the Model

- E-commerce
- Risk
- Time
Supernetwork Structure
Multicriteria Decision-Making Behavior of Manufacturer $i$ in country $l$

Maximize Profit

$$\max \sum_{j=1}^{J} \sum_{h=1}^{H} \sum_{l=1}^{L} (\rho_{1jhl}^i \times e_h) q_{jh\tilde{l}}^i + \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{l=1}^{L} (\rho_{1kh\tilde{l}}^i \times e_h) q_{kh\tilde{l}}^i - f^{ii}(Q^1, Q^2) - \sum_{j=1}^{J} \sum_{h=1}^{H} \sum_{l=1}^{L} c_{jh\tilde{l}}^{ii}(q_{jh\tilde{l}}^i) - \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{l=1}^{L} c_{kh\tilde{l}}^{ii}(q_{kh\tilde{l}}^i)$$

Minimize Risk

$$\min r^{ii}(Q^1, Q^2)$$
Dynamics of Product Transactions between Manufacturer $i$ in country $l$ and Distributor $j$ in country $\hat{l}$

\[
\dot{q}_{j\hat{l}}^{il} = \begin{cases} 
\phi_{j\hat{l}}^{il} \left[ \gamma_{\hat{l}} - \frac{\partial f_{j\hat{l}}(Q^1, Q^2)}{\partial q_{j\hat{l}}^{il}} - \frac{\partial c_{j\hat{l}}^{il}(q_{j\hat{l}}^{il})}{\partial q_{j\hat{l}}^{il}} - \frac{\partial c_{j\hat{l}}(Q^1, Q^3)}{\partial q_{j\hat{l}}^{il}} \right], & \text{if } q_{j\hat{l}}^{il} > 0 \\
\max\left\{ 0, \phi_{j\hat{l}}^{il} \left[ \gamma_{\hat{l}} - \frac{\partial f_{j\hat{l}}(Q^1, Q^2)}{\partial q_{j\hat{l}}^{il}} - \frac{\partial c_{j\hat{l}}^{il}(q_{j\hat{l}}^{il})}{\partial q_{j\hat{l}}^{il}} - \frac{\partial c_{j\hat{l}}(Q^1, Q^3)}{\partial q_{j\hat{l}}^{il}} \right] \right\}, & \text{if } q_{j\hat{l}}^{il} = 0
\end{cases}
\]
Assumption

The rate of change of the price is proportional to the difference between the expected demand and the total amount transacted with the retailer.

\[
\dot{\rho}_{3k\bar{l}} = \begin{cases} 
\phi_{k\bar{l}} \left[ d_{k\bar{l}} - \sum_{i=1}^{I} \sum_{l=1}^{L} q_{k\bar{l}i} - \sum_{j=1}^{J} \sum_{l=1}^{L} q_{j\bar{l}i} \right], & \text{if } \rho_{3k\bar{l}} > 0 \\
\max \left\{ 0, \phi_{k\bar{l}} \left[ d_{k\bar{l}} - \sum_{i=1}^{I} \sum_{l=1}^{L} q_{k\bar{l}i} - \sum_{j=1}^{J} \sum_{l=1}^{L} q_{j\bar{l}i} \right] \right\}, & \text{if } \rho_{3k\bar{l}} = 0 
\end{cases}
\]
Projected Dynamical System (PDS)

\[ \dot{X} = \Pi_K(X, -F(X)), \quad X(0) = X_0 \]
Stationary Points

Theorem

Since the feasible set is a convex polyhedron, the set of stationary points of the described \textit{PDS} coincides with the set of solutions to the appropriately constructed \textit{VI} problem: determine $X^* \in \mathcal{K}$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}$$
Step 0: Initialization  Set $X^0 \in \mathcal{K}$. Let $T$ denote an iteration counter. Let $T = 1$ and set the sequence $\{\alpha_T\}$ so that
\[ \sum_{T=1}^{\infty} \alpha_T = \infty, \quad \alpha_T > 0, \quad \alpha_T \to 0, \quad \text{as } T \to \infty. \]

Step 1: Computation  Compute $X^T \in \mathcal{K}$ by solving the variational inequality subproblem:
\[ \langle X^T + \alpha_T F(X^{T-1}) - X^{T-1}, X - X^T \rangle \geq 0, \quad \forall X \in \mathcal{K}. \]

Step 2: Convergence Verification  If $|X^T - X^{T-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $T := T + 1$, and go to Step 1.
Numerical Example: Network Structure

Country 1 Manufacturers

Country 2 Manufacturers

Distributors

Retailers
### Numerical Example: Data

#### Production cost functions

\[ f_{11}(q) = 2.5(q_{11}^2) + q_{11}q_{21} + 2q_{11}^2, \]
\[ f_{12}(q) = 2.5(q_{12}^2) + q_{12}q_{22} + 2q_{12}^2, \]
\[ f_{21}(q) = 2.5(q_{21}^2) + q_{11}q_{21}^2 + 2q_{21}^2, \]
\[ f_{22}(q) = 2.5(q_{12}^2) + q_{12}q_{22}^2 + 2q_{22}^2. \]

#### Transaction cost functions

\[ c_{ijh\hat{l}}^{il} = 0.5(q_{ijh\hat{l}}^{il})^2 + 3.5q_{ijh\hat{l}}^{il}, \quad \forall i, l, j, h, \hat{l} \]
\[ c_{ikh\bar{l}}^{il} = 0.5(q_{ikh\bar{l}}^{il})^2 + 5q_{ikh\bar{l}}^{il}, \quad \forall i, l, k, h, \bar{l} \]
Numerical Example: Data

Handling cost functions

\[ c_{j\hat{l}} = 0.5 \left( \sum_{i=1}^{2} \sum_{l=1}^{2} q_{j\hat{l}}^{i} \right)^{2}, \quad \forall j, \hat{l} \]
\[ c_{k\hat{h}\bar{l}} = 0.5 \left( \sum_{j=1}^{2} \sum_{l=1}^{2} q_{k\hat{l}}^{j} \right)^{2}, \quad \forall k, h, \bar{l} \]

Demand

Uniformly distributed in \([0, \frac{100}{\rho_{3kl}}] \) \( \forall k, h, \bar{l} \)

Risk function

\[ r_{11} = \left( \sum_{k\hat{l}} q_{k\hat{l}}^{11} - 2 \right)^{2} \]
### Numerical Example: Results

<table>
<thead>
<tr>
<th>Example 1</th>
<th>Example 2</th>
<th>Example 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Electronic Transactions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q^{11*}_{kh\bar{L}}$</td>
<td>0.175</td>
<td>1.07</td>
</tr>
<tr>
<td>$q^{21*}_{kh\bar{L}}$</td>
<td>0.175</td>
<td>1.07</td>
</tr>
<tr>
<td>$q^{12*}_{kh\bar{L}}$</td>
<td>0.175</td>
<td>1.07</td>
</tr>
<tr>
<td>$q^{22*}_{kh\bar{L}}$</td>
<td>0.175</td>
<td>1.07</td>
</tr>
<tr>
<td><strong>Physical Transactions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q^{11*}_{jh\hat{l}}$</td>
<td>0.186</td>
<td>0.286</td>
</tr>
<tr>
<td>$q^{21*}_{jh\hat{l}}$</td>
<td>0.186</td>
<td>0.286</td>
</tr>
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<td>$q^{12*}_{jh\hat{l}}$</td>
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<td>0.286</td>
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<tr>
<td>$q^{22*}_{jh\hat{l}}$</td>
<td>0.186</td>
<td>0.286</td>
</tr>
<tr>
<td>$q^{j\hat{l}i*}_{kh\bar{L}}$</td>
<td>0.186</td>
<td>0.286</td>
</tr>
<tr>
<td><strong>Prices</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma^{*}_{ji}$</td>
<td>15.097</td>
<td>39.46</td>
</tr>
<tr>
<td>$\rho^{*}_{3kh\bar{L}}$</td>
<td>32.88</td>
<td>90.31</td>
</tr>
</tbody>
</table>
Dynamic Trajectory: Flow
Electric Power Market

- Size
- Structure
- Complexity
Electric Power Supply Chain Network

Power Generators

Power Suppliers

Transmission

Service Providers

Demand Markets

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Maximize Profit

Maximize \( \sum_{s=1}^{S} \rho_{1gs} q_{gs} - f_{g}(Q^{1}) - \sum_{s=1}^{S} c_{gs}(q_{gs}) \)
Additional Motivation

- Weather conditions
- Demands for various goods and services
- Prices variability introduces risk
- Supply and/or demand sensitivity
Electric Power Supply Chain Network

- Demand Markets
- Transmission
- Service Providers
- Power Suppliers
- Power Generators
Multicriteria Decision-Making Behavior of Power Generator $g$

Maximize Profit

Maximize $\sum_{s=1}^{S} \rho_{1gs}q_{gs} - f_{g}(Q^{1}) - \sum_{s=1}^{S} c_{gs}(q_{gs})$

Minimize Risk

Minimize $r_{g}(Q^{1})$
Dynamics of the Price at Demand Market $k$

Assumption

The rate of change of the price is proportional to the difference between the expected demand and the total amount transacted with the demand market.

\[
\dot{\rho}_k = \begin{cases} 
\phi_k \left[ d_k(\rho_k) - \sum_{s=1}^{S} \sum_{t=1}^{T} q_{sk}^t \right], & \text{if } \rho_k > 0 \\
\max\{0, \phi_k \left[ d_k(\rho_k) - \sum_{s=1}^{S} \sum_{t=1}^{T} q_{sk}^t \right]\}, & \text{if } \rho_k = 0
\end{cases}
\]
Dynamic Trajectory: Price

- Experiment 1
- Experiment 2
- Experiment 3
- Experiment 4
Theoretical Results

Theorem

If the feasible set is $R_+^N$ and $\phi \equiv (\phi_1, \ldots, \phi_N)^T$ is a vector of positive terms. Then:

$$
\text{VI}(F, \mathcal{K}) \equiv \text{VI}(F', \mathcal{K})
$$

where:

$$
F' \equiv (F'_1, \ldots, F'_N)^T \quad \text{and} \quad F \equiv (\phi_1 F'_1, \ldots, \phi_N F'_N)^T
$$
More Motivation

- Explicit modeling of fuel suppliers
- Spatially distributed generation plants owned by one company
- ‘Self-supply’ generation
- Inelastic demand
Maximize Profit

Maximize

\[
\sum_{n=1}^{N} \left[ \sum_{s=1}^{S} \rho_{1gns}^* q_{gns} + \sum_{k=1}^{K} \rho_{1gmk}^* q_{gmk} - \sum_{h=1}^{H} \sum_{i=1}^{I} \rho_{0hi}^* q_{hi}^* - f_{gn}(q_{gn}) - \sum_{s=1}^{S} c_{gns}(q_{gns}) - \sum_{k=1}^{K} c_{gmk}(q_{gmk}) \right] \\
+ \sum_{m=1}^{M} \left[ \sum_{s=1}^{S} \rho_{1gms}^* q_{gms} + \sum_{k=1}^{K} \rho_{1gmk}^* q_{gmk} - f_{gm}(q_{gm}) - \sum_{s=1}^{S} c_{gms}(q_{gms}) - \sum_{k=1}^{K} c_{gmk}(q_{gmk}) \right]
\]

subject to:

\[
\sum_{s=1}^{S} q_{gns} + \sum_{k=1}^{K} q_{gmk} = \sum_{h=1}^{H} \sum_{i=1}^{I} \alpha_{hi}^* q_{hi}^*, \quad \forall n
\]
Summary

Global Supply Chain
- user-optimization
- risk
- statics & dynamics
- e-commerce
- multiple currencies

Electric Power Network
- user-optimization
- risk
- statics & dynamics
- self-supply
- fuel suppliers
Contribution

- Transportation Research: Part E (2005)
Theoretical Development

- Transform the model to a transportation network equilibrium model as was done in the recent work by Nagurney, Liu, Cojocaru, and Daniele (2005)
- Reformulate the model as an evolutionary variational inequality
Empirical Testing

Data from New England ISO

- 300+ power generators
- 800+ power generating units
- 7 owners of transmission lines
- 2500 locations (6.5 million consumers)
Thank You!

- Questions?
- Comments?