Dynamics of Global Supply Chain and Electric Power Networks: Models, Pricing Analysis, and Computations

A Dissertation Presented
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Outline

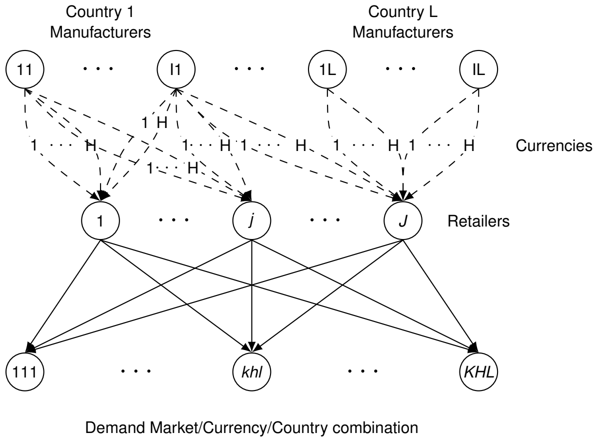
Motivation

- Challenging intellectual questions
- Importance of supply chain decision-making
- Increasing globalization
- E-commerce

Global Supply Chain

- Network equilibrium
- Several classes of decision-makers
- Optimization problem for every decision-maker is unique

Global Supply Chain Network



Optimizing Behavior of Manufacturer i in country l

Maximize Profit

$$\text{Maximize} \quad \sum_{j=1}^{J}\sum_{h=1}^{H}(\rho_{1jh}^{il*} \times e_h)q_{jh}^{il} - \sum_{j=1}^{J}\sum_{h=1}^{H} c_{jh}^{il}(q_{jh}^{il}) - f^{il}(Q^1)$$

The Optimality Conditions of All Manufacturers

Variational Inequality Formulation

Determine $Q^{1*} \in R_+^{IJHL}$ satisfying

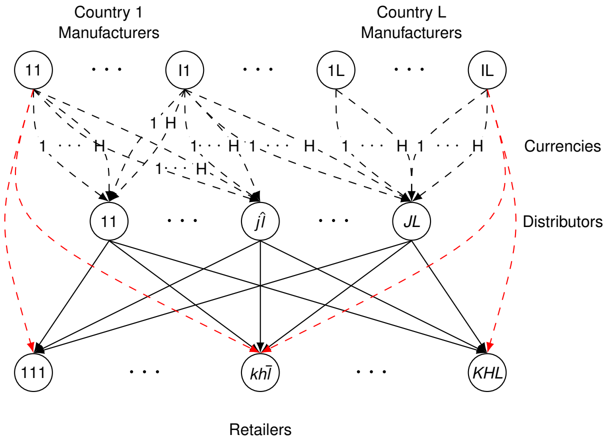
$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\left[\frac{\partial f^{il}(Q^{1*})}{\partial q_{jh}^{il}} + \frac{\partial c_{jh}^{il}(q_{jh}^{il*})}{\partial q_{jh}^{il}} - \rho_{1jh}^{il*} \times e_h\right] \times \left[q_{jh}^{il} - q_{jh}^{il*}\right] \geq 0$$

$$\forall Q^1 \in R_+^{IJHL}$$

Characteristics of the Model

- E-commerce
- Risk
- Time

Supernetwork Structure



Multicriteria Decision-Making Behavior of Manufacturer i in country l

Maximize Profit

$$\text{Maximize} \quad \sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}(\rho^{il}_{1jh\hat{l}} \times e_h)q^{il}_{jh\hat{l}} + \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\bar{l}=1}^{L}(\rho^{il}_{1kh\bar{l}} \times e_h)q^{il}_{kh\bar{l}}$$

$$- f^{il}(Q^1, Q^2) - \sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L} c^{il}_{jh\hat{l}}(q^{il}_{jh\hat{l}}) - \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\bar{l}=1}^{L} c^{il}_{kh\bar{l}}(q^{il}_{kh\bar{l}})$$

Minimize Risk

$$\text{Minimize} \quad r^{il}(Q^1, Q^2)$$

Dynamics of Product Transactions between Manufacturer i in country l and Distributor j in country $\hat{l}$

$$\dot{q}^{il}_{j\hat{h}\hat{l}} = \begin{cases} \phi^{il}_{j\hat{h}\hat{l}} \Bigg[\gamma_{j\hat{l}} - \frac{\partial f^{il}(Q^1,Q^2)}{\partial q^{il}_{j\hat{h}\hat{l}}} - \frac{\partial c^{il}_{j\hat{h}\hat{l}}(q^{il}_{j\hat{h}\hat{l}})}{\partial q^{il}_{j\hat{h}\hat{l}}} - \frac{\partial c_{j\hat{l}}(Q^1,Q^3)}{\partial q^{il}_{j\hat{h}\hat{l}}} \\ \quad -\alpha^{il} \frac{\partial r^{il}(Q^1,Q^2)}{\partial q^{il}_{j\hat{h}\hat{l}}} - \beta^{j\hat{l}} \frac{\partial r^{j\hat{l}}(Q^1,Q^3)}{\partial q^{il}_{j\hat{h}\hat{l}}} \Bigg], & \text{if} \quad q^{il}_{j\hat{h}\hat{l}} > 0 \\ \max\Bigg\{ 0, \phi^{il}_{j\hat{h}\hat{l}} \Bigg[\gamma_{j\hat{l}} - \frac{\partial f^{il}(Q^1,Q^2)}{\partial q^{il}_{j\hat{h}\hat{l}}} - \frac{\partial c^{il}_{j\hat{h}\hat{l}}(q^{il}_{j\hat{h}\hat{l}})}{\partial q^{il}_{j\hat{h}\hat{l}}} - \frac{\partial c_{j\hat{l}}(Q^1,Q^3)}{\partial q^{il}_{j\hat{h}\hat{l}}} \\ \quad -\alpha^{il} \frac{\partial r^{il}(Q^1,Q^2)}{\partial q^{il}_{j\hat{h}\hat{l}}} - \beta^{j\hat{l}} \frac{\partial r^{j\hat{l}}(Q^1,Q^3)}{\partial q^{il}_{j\hat{h}\hat{l}}} \Bigg] \Bigg\}, & \text{if} \quad q^{il}_{j\hat{h}\hat{l}} = 0 \end{cases}$$

Dynamics of the Price at Demand Market $kh\bar{l}$

Assumption

The rate of change of the price is proportional to the difference between the *expected* demand and the total amount transacted with the retailer.

$$\dot{\rho}_{3kh\bar{l}} = \begin{cases} \phi_{kh\bar{l}} \left[d_{kh\bar{l}} - \sum_{i=1}^{I} \sum_{l=1}^{L} q^{il}_{kh\bar{l}} - \sum_{j=1}^{J} \sum_{\hat{l}=1}^{L} q^{j\hat{l}}_{kh\bar{l}} \right], & \text{if} \quad \rho_{3kh\bar{l}} > 0 \\ \max\left\{ 0, \phi_{kh\bar{l}} \left[d_{kh\bar{l}} - \sum_{i=1}^{I} \sum_{l=1}^{L} q^{il}_{kh\bar{l}} - \sum_{j=1}^{J} \sum_{\hat{l}=1}^{L} q^{j\hat{l}}_{kh\bar{l}} \right] \right\}, & \text{if} \quad \rho_{3kh\bar{l}} = 0 \end{cases}$$

Projected Dynamical System (PDS)

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \quad X(0) = X_0$$

Stationary Points

Theorem

Since the feasible set is a convex polyhedron, the set of stationary points of the described PDS coincides with the set of solutions to the appropriately constructed VI problem: determine $X^* \in \mathcal{K}$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}$$

The Euler Method (Dupuis and Nagurney (1993))

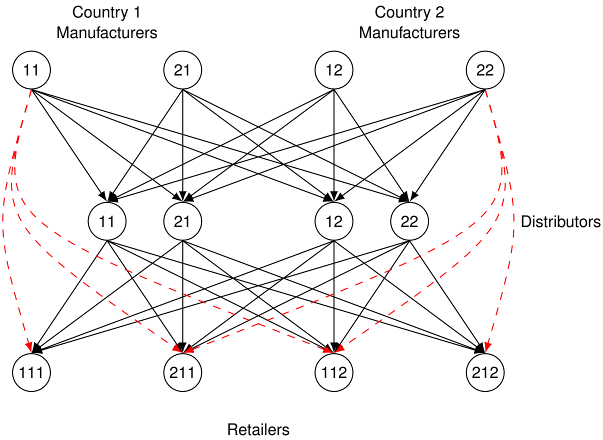
Step 0: Initialization Set $X^0 \in \mathcal{K}$. Let $\mathcal{T}$ denote an iteration counter. Let $\mathcal{T} = 1$ and set the sequence $\{\alpha_\mathcal{T}\}$ so that $\sum_{\mathcal{T}=1}^{\infty} \alpha_\mathcal{T} = \infty$, $\alpha_\mathcal{T} > 0$, $\alpha_\mathcal{T} \to 0$, as $\mathcal{T} \to \infty$.

Step 1: Computation Compute $X^\mathcal{T} \in \mathcal{K}$ by solving the variational inequality subproblem:

$$\langle X^\mathcal{T} + \alpha_\mathcal{T} F(X^{\mathcal{T}-1}) - X^{\mathcal{T}-1}, X - X^\mathcal{T} \rangle \geq 0, \qquad \forall X \in \mathcal{K}.$$

Step 2: Convergence Verification If $|X^\mathcal{T} - X^{\mathcal{T}-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\mathcal{T} := \mathcal{T} + 1$, and go to Step 1.

Numerical Example: Network Structure



Numerical Example: Data

Production cost functions

$$f^{11}(q) = 2.5(q^{11})^2 + q^{11}q^{21} + 2q^{11}, \quad f^{21}(q) = 2.5(q^{21})^2 + q^{11}q^{21} + 2q^{21},$$
$$f^{12}(q) = 2.5(q^{12})^2 + q^{12}q^{22} + 2q^{12}, \quad f^{22}(q) = 2.5(q^{12})^2 + q^{12}q^{22} + 2q^{22}.$$

Transaction cost functions

$$c^{il}_{jh\hat{l}} = .5(q^{il}_{jh\hat{l}})^2 + 3.5q^{il}_{jh\hat{l}}, \quad \forall i, l, j, h, \hat{l}$$
$$c^{il}_{kh\bar{l}} = .5(q^{il}_{kh\bar{l}})^2 + 5q^{il}_{kh\bar{l}}, \quad \forall i, l, k, h, \bar{l}$$

Numerical Example: Data

Handling cost functions

$$c_{j\hat{l}} = .5(\sum_{i=1}^{2}\sum_{l=1}^{2} q_{jh\hat{l}}^{il})^2, \quad \forall j, \hat{l}$$
$$c_{kh\bar{l}} = .5(\sum_{j=1}^{2}\sum_{\hat{l}=1}^{2} q_{kh\bar{l}}^{j\hat{l}})^2, \quad \forall k, h, \bar{l}$$

Demand

Uniformly distributed in $\left[0, \frac{100}{\rho_{3kh\bar{l}}}\right]$ $\forall k, h, \bar{l}$

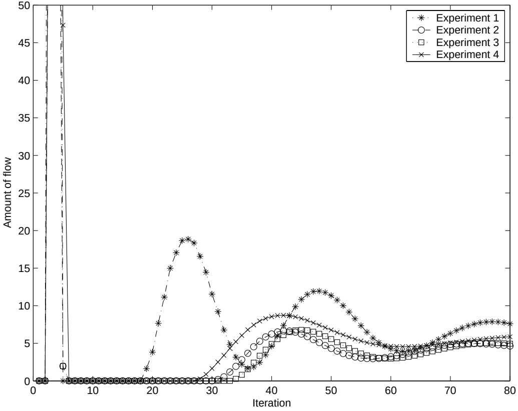
Risk function

$$r^{11} = (\sum_{kh\bar{l}} q_{kh\bar{l}}^{11} - 2)^2$$

Numerical Example: Results

	Example 1	Example 2	Example 3
Electronic Transactions			
$q^{11*}_{kh\bar{l}}$	0.175	1.07	0.632
$q^{21*}_{kh\bar{l}}$	0.175	1.07	1.185
$q^{12*}_{kh\bar{l}}$	0.175	1.07	1.182
$q^{22*}_{kh\bar{l}}$	0.175	1.07	1.182
Physical Transactions			
$q^{11*}_{jh\hat{l}}$	0.186	0.286	0.700
$q^{21*}_{jh\hat{l}}$	0.186	0.286	0.185
$q^{12*}_{jh\hat{l}}$	0.186	0.286	0.182
$q^{22*}_{jh\hat{l}}$	0.186	0.286	0.182
$q^{\hat{j}\hat{l}*}_{kh\bar{l}}$	0.186	0.286	0.314
Prices			
$\gamma^*_{\hat{j}\hat{l}}$	15.097	39.46	39.50
$\rho^*_{3kh\bar{l}}$	32.88	90.31	90.53

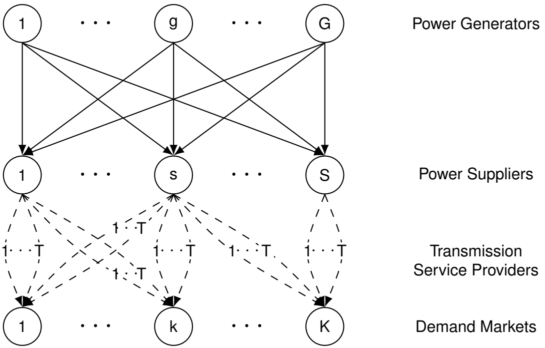
Dynamic Trajectory: Flow



Electric Power Market

- Size
- Structure
- Complexity

Electric Power Supply Chain Network



Decision-Making Behavior of Power Generator g

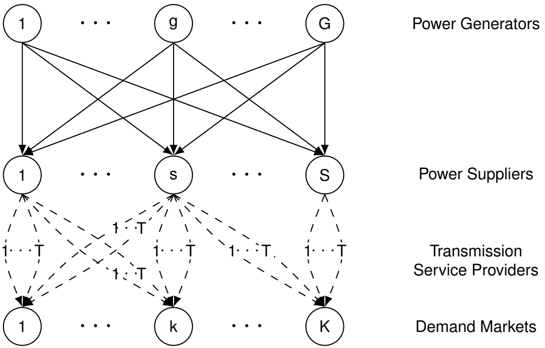
Maximize Profit

$$\text{Maximize} \quad \sum_{s=1}^{S} \rho_{1gs}^{*} q_{gs} - f_g(Q^1) - \sum_{s=1}^{S} c_{gs}(q_{gs})$$

Additional Motivation

- Weather conditions
- Demands for various goods and services
- Prices variability introduces risk
- Supply and/or demand sensitivity

Electric Power Supply Chain Network



Multicriteria Decision-Making Behavior of Power Generator g

Maximize Profit

$$\text{Maximize} \quad \sum_{s=1}^{S} \rho_{1gs} q_{gs} - f_g(Q^1) - \sum_{s=1}^{S} c_{gs}(q_{gs})$$

Minimize Risk

$$\text{Minimize} \quad r_g(Q^1)$$

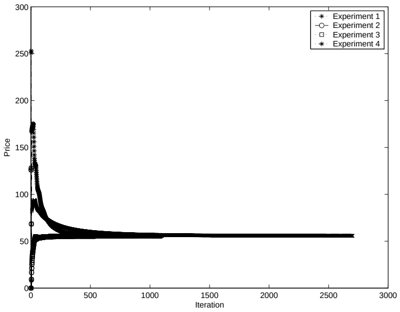
Dynamics of the Price at Demand Market k

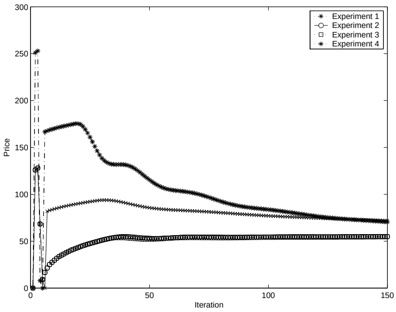
Assumption

The rate of change of the price is proportional to the difference between the *expected* demand and the total amount transacted with the demand market.

$$\dot{\rho}_{3k} = \begin{cases} \phi_k \left[d_k(\rho_{3k}) - \sum_{s=1}^{S} \sum_{t=1}^{T} q_{sk}^t \right], & \text{if} \quad \rho_{3k} > 0 \\ \max\{0, \phi_k \left[d_k(\rho_{3k}) - \sum_{s=1}^{S} \sum_{t=1}^{T} q_{sk}^t \right]\}, & \text{if} \quad \rho_{3k} = 0 \end{cases}$$

Dynamic Trajectory: Price





Theoretical Results

Theorem

If the feasible set is R_+^N and $\phi \equiv (\phi_1, \ldots, \phi_N)^T$ is a vector of positive terms. Then:

$$\mathrm{VI}(F, \mathcal{K}) \equiv \mathrm{VI}(F', \mathcal{K})$$

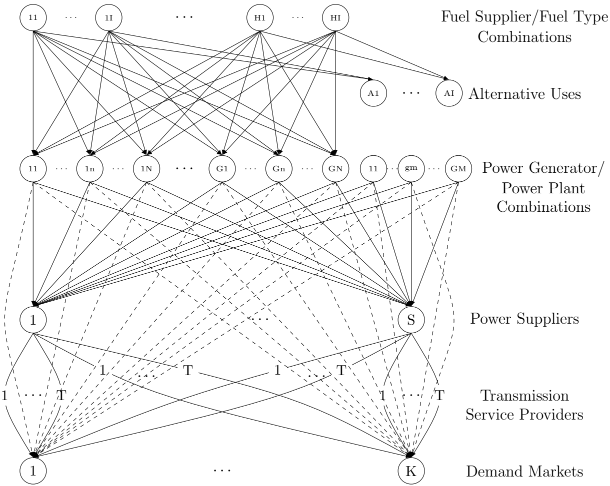
where:

$$F' \equiv (F'_1, \ldots, F'_N)^T \quad \text{and} \quad F \equiv (\phi_1 F'_1, \ldots, \phi_N F'_N)^T$$

More Motivation

- Explicit modeling of fuel suppliers
- Spatially distributed generation plants owned by one company
- 'Self-supply' generation
- Inelastic demand

Global Electric Power Supply Chain Network



Decision-Making Behavior of Power Generator g

Maximize Profit

Maximize

$$\sum_{n=1}^{N}\left[\sum_{s=1}^{S}\rho_{1gns}^{*}q_{gns}+\sum_{k=1}^{K}\rho_{1gnk}^{*}q_{gnk}-\sum_{h=1}^{H}\sum_{i=1}^{I}\rho_{0hi}^{gn*}q_{hi}^{gn}-f_{gn}(q_{gn})-\sum_{s=1}^{S}c_{gns}(q_{gns})-\sum_{k=1}^{K}c_{gnk}(q_{gnk})\right]$$

$$+\sum_{m=1}^{M}\left[\sum_{s=1}^{S}\rho_{1gms}^{*}q_{gms}+\sum_{k=1}^{K}\rho_{1gmk}^{*}q_{gmk}-f_{gm}(q_{gm})-\sum_{s=1}^{S}c_{gms}(q_{gms})-\sum_{k=1}^{K}c_{gmk}(q_{gmk})\right]$$

subject to:

$$\sum_{s=1}^{S}q_{gns}+\sum_{k=1}^{K}q_{gnk}=\sum_{h=1}^{H}\sum_{i=1}^{I}\alpha_{hi}^{gn}q_{hi}^{gn},\quad\forall n$$

Summary

Global Supply Chain

- user-optimization
- risk
- statics & dynamics
- e-commerce
- multiple currencies

Electric Power Network

- user-optimization
- risk
- statics & dynamics
- self-supply
- fuel suppliers

Contribution

- Mathematical and Computer Modelling (2003)
- Transportation Research: Part E (2005)
- **Advances in Computational Economics, Finance and Management Science. Computational Management Science**, Springer (2005)
- **Transportation Economics: Towards better Performing Transport Systems**, Routledge (2006)

Theoretical Development

- Transform the model to a transportation network equilibrium model as was done in the recent work by Nagurney, Liu, Cojocaru, and Daniele (2005)
- Reformulate the model as an evolutionary variational inequality

Empirical Testing

Data from New England ISO

- 300+ power generators
- 800+ power generating units
- 7 owners of transmission lines
- 2500 locations (6.5 million consumers)

Thank You!

- Questions?
- Comments?