

# Labor and Supply Chain Networks

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**Montreal Operations Research Student Chapter  
(MORSC)**

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# Acknowledgments

**Many thanks to the organizers of the MORSC Speaker Series for the opportunity to speak to you today!**

I remember fondly being in Montreal to speak to your Chapter on May 15, 2019, when we celebrated the 3rd anniversary of your Chapter. Congratulations on your upcoming 6th anniversary!



# Acknowledgments

**This talk is dedicated to essential workers, whose selflessness, expertise, and dedication have helped to sustain us in the Covid-19 pandemic.**



**I would also like to acknowledge all the freedom-loving people on the planet, including those fighting for their freedom in Ukraine.**

# Outline of Presentation

- **Background and Motivation** - Some of Our Relevant Research Pre-Pandemic
- **Optimization and Supply Chain Network Models Inspired by the COVID-19 Pandemic**
  - Food
  - Medical Supplies
- **Methodology - The Variational Inequality Problem**
- **Game Theory and Supply Chain Network Models Inspired by the COVID-19 Pandemic**
  - Food and Labor Disruptions
- **Dealing with the Media and Impacting Policy**



# Background and Motivation

# I Work on the Modeling of Network Systems



## Much of My Recent Research Has Been on Supply Chains

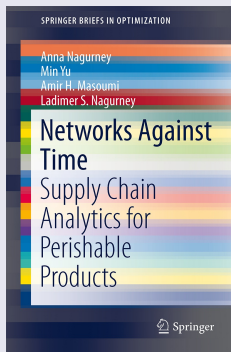


# Some of My Books



# A Multidisciplinary Approach

In our research on perishable and time-sensitive product supply chains, we utilize results from physics, chemistry, biology, and medicine in order to capture the perishability of various products over time from healthcare products such as blood, medical nucleotides, and pharmaceuticals to food.



# It's All About People

A major research theme of ours in the COVID-19 pandemic is the inclusion of labor in supply chains, using optimization and game theory. The theme continues, as does its relevance, as the war on Ukraine continues to rage.



# Food Supply Chains

Food is essential to our health and well-being. During the COVID-19 pandemic, declared on March 11, 2020 by the World Health Organization, the associated supply chains have suffered major disruptions.



# Fresh Produce Food Supply Chains

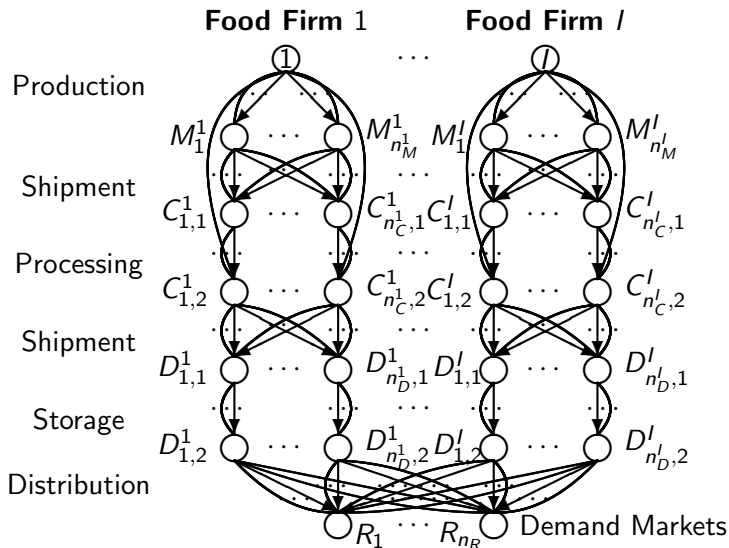
## Our fresh produce supply chain network oligopoly model:

- ① captures the deterioration of fresh food along the entire supply chain from a network perspective;
- ② handles the time decay through the introduction of arc multipliers;
- ③ formulates oligopolistic competition with product differentiation;
- ④ includes the disposal of the spoiled food products, along with the associated costs;
- ⑤ allows for the assessment of alternative technologies involved in each supply chain activity.

**M. Yu and A. Nagurney, “Competitive Food Supply Chain Networks with Application to Fresh Produce,”** *European Journal of Operational Research* 224(2) (2013), pp 273-282.



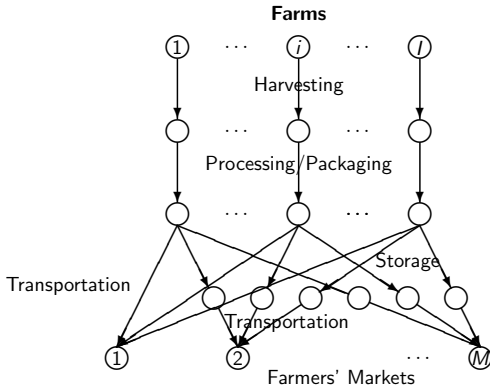
# Fresh Produce Food Supply Chains



The Fresh Produce Supply Chain Network Topology

# Farmers' Markets and Fresh Produce Supply Chains

- The  $I$  farms compete **noncooperatively** in an **oligopolistic** manner.
- Products are differentiated based on **quality** at the farmers' markets.



D. Besik and A. Nagurney, "Quality in Competitive Fresh Produce Supply Chains with Application to Farmers' Markets," *Socio-Economic Planning Sciences* 60 (2017), pp 62-76.

# Pharmaceutical Supply Chains

**The supply chain generalized network oligopoly model has the following novel features:**

- 1 it handles the perishability of the pharmaceutical product through the introduction of arc multipliers;
- 2 it allows each firm to minimize the discarding cost of waste / perished medicine;
- 3 it captures product differentiation under oligopolistic competition through the branding of drugs, which can also include generics as distinct brands.

**A.H. Masoumi, M. Yu, and A. Nagurney, “A Supply Chain Generalized Network Oligopoly Model for Pharmaceuticals Under Brand Differentiation and Perishability,” *Transportation Research E* 48 (2012), pp 762-780.**

# Pharmaceutical Firm 1

# Pharmaceutical Firm /

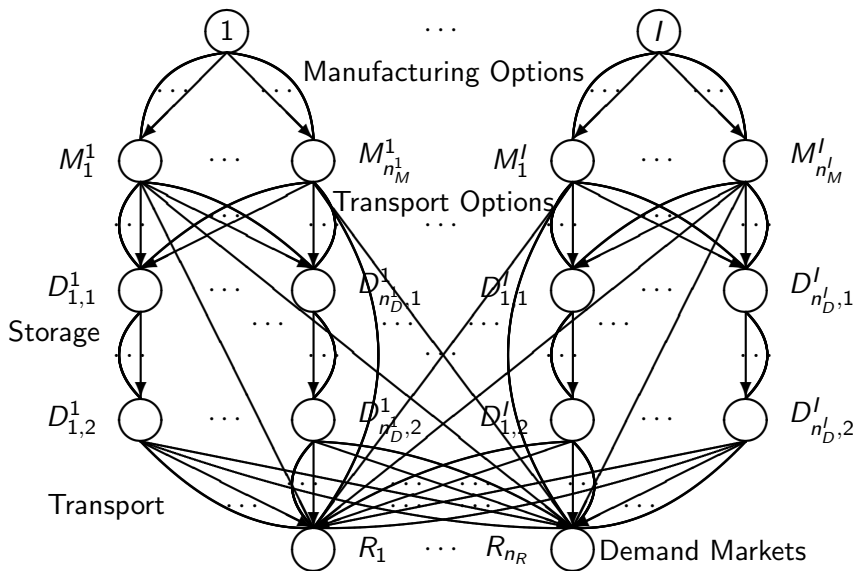


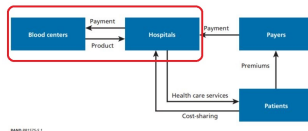
Figure: The Pharmaceutical Supply Chain Network Topology

# Blood Supply Chains

Even prior to the pandemic the blood services sector was facing many challenges. This supply chain is unique in that the product cannot be produced but must be donated.

A. Nagurney and P. Dutta, “Supply Chain Network Competition Among Blood Service Organizations: A Generalized Nash Equilibrium Framework,” *Annals of Operations Research* 275(2) (2019), pp 551-586.

Operational challenges faced by blood service organizations.



A. Nagurney and P. Dutta, “Competition for Blood Donations,” *Omega* 212 (2019), pp 103-114.

# Optimization and Supply Chain Network Models Inspired by the COVID-19 Pandemic

# Food Supply Chain Disruptions Due to COVID-19

**The COVID-19 pandemic impacted food supply chains in a dramatic and sustained manner.**

- Infections at three of the nation's largest meat processors were significant in 2020. At Tysons Foods, the largest meat processor in the US, **the number of Tyson employees with the coronavirus exploded from less than 1,600 in April 2020 to more than 7,000 by May 25, 2020.**
- **Millions of farm animals had to be culled** because of the shutdown of several big meat processing plants. Enhanced cleaning, redesign, and emphasis on social distancing was slowing down the processing, causing additional delays.
- **Shortages of many types of meats, even organic chicken, were experienced, with price increases.**

# Food Supply Chain Disruptions Due to COVID-19

- **Fresh produce (oranges, potatoes, strawberries, etc.) on some farms, had to be discarded** because of lack of timely processing capabilities at food processing plants.
- **Labor needed to pick ripened produce was less available due to migrant labor restrictions, illnesses, etc.**
- With the closures of schools, restaurants, businesses, etc., during part of the pandemic outlets for perishable food changed dramatically. **Distribution channels were being reinvisioned and redesigned.**
- **Food insecurity was rising globally.**



# Food Supply Chain Disruptions Due to COVID-19



AMERICA'S FOOD CHAIN

## As coronavirus pandemic spikes orange juice sales, a Florida citrus grower gets squeezed

Janine Zeitlin, USA TODAY Network - Florida  
Updated 8:07 p.m. EDT May 14, 2020



An Idaho farm is giving away 2 million potatoes because coronavirus has hurt demand



By Alisha Ebrahimji, CNN

Updated 1:33 PM ET, Thu April 16, 2020



## Lacking seasonal workers, Italy elevates its long-shunned migrants

THE CHRISTIAN SCIENCE  
MONITOR



## Farms encountering guest worker shortage amid new coronavirus restrictions



### Piglets aborted, chickens gassed as pandemic slams meat sector

The Washington Post

*Democracy Dies in Darkness*

The meat industry is trying to get back to normal. But workers are still getting sick – and shortages may get worse.

There are now more than 11,000 coronavirus cases tied to Tyson Foods, Smithfield Foods and JBS

## Germany Struggles To Fill Its Farm Labor Shortage After Closing Its Borders

May 20, 2020 10:58 AM ET



ROB SCHMITZ



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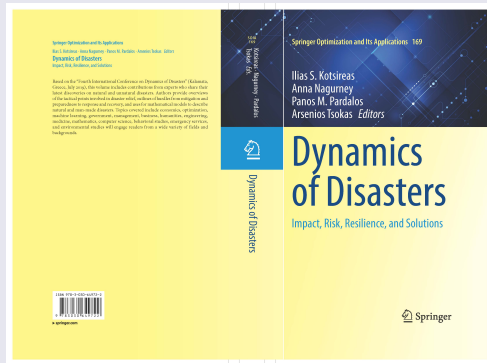
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ROB SCHMITZ



**“Perishable Food Supply Chain Networks with Labor in the Covid-19 Pandemic,” A. Nagurney, in: *Dynamics of Disasters - Impact, Risk, Resilience, and Solutions*, I.S. Kotsireas, A. Nagurney, P.M. Pardalos, and A. Tsokas, Editors, Springer Nature Switzerland AG, 2021, pp 173-193.**



# Perishable Food Supply Chain Network Model with Labor

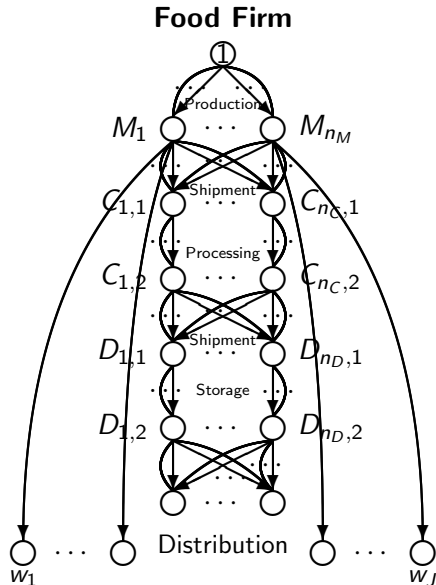
- **With lack of availability of labor being one of the drivers of supply chain disruptions**, the model considers labor in all the supply chain network economic activities of production, transportation, processing, storage, and distribution, while retaining perishability.
- **There are bounds on labor availability on each link as well as a productivity factor relating product flow to labor.**
- **Impacts of the reduction of labor (capacities) on supply chain network links** can then be quantitatively evaluated on the perishable product flows, the prices that the consumers pay, and profits of the firm.
- The framework enables a variety of sensitivity analysis exercises.

# This Research Is Highly Relevant to the War On Ukraine

**On February 24, 2022, Russia began a major invasion of Ukraine, a sovereign, democratic nation.**

- Ukraine is **the breadbasket of the world** with major exports of wheat, corn, barley, and sunflower oil. Many countries, including MENA ones (Middle Eastern and North African countries) depend on agricultural products from Ukraine.
- The war has **disrupted trade due to blockages of ports on the Black Sea and the Azov Sea and the mining of seas and has affected shipping options.**
- **The price of seeds has risen, plus the prices of fuel and fertilizer.**
- Having enough labor to plant and harvest will also be challenging and getting the products to market and points of demand.
- **The World Food Programme used to buy 50% of its wheat from Ukraine.**

# Perishable Food Supply Chain Network Model with Labor



**Figure:** The Perishable Food Supply Chain Network Topology

## Our findings include:

- ① The lack of labor on a single link, even a freight one, may significantly negatively impact a food firm.
- ② Preserving productivity in all utilized supply chain network economic activities is critical since the impact of a drastic reduction can severely reduce profits.
- ③ Adding more direct sales, whether at farmers' markets or nearby farm stands, may help a food firm in a pandemic.
- ④ Also, if a firm enhances its marketing so as to have consumers be willing to pay a higher price for its fresh produce, major profit increases can occur.

# Shortages of Medical Supplies, Including PPEs

- In early March 2020, it was reported that by the Department of Health and Human Services **that the national stockpile had about 12 million N95 respirators and 30 million surgical masks - 1% of the estimated 3.5 billion masks the nation would need in a severe pandemic. Another 5 million N95 masks in the stockpile were expired.**
- **Prior to the coronavirus outbreak, China made half the world's face masks.** When the outbreak took off there, China started to use its supply and hoard what remained. This problem has only spread since, as more countries hoarded medical supplies, with some even banning most PPE exports. So as demand increased due to COVID-19 there was less supply to go around.
- **“We are out of everything, wrote a staffer at a large hospital in Tennessee in mid April. “Providers using one mask for 3+ weeks. Many COVID patients. Zero gowns.”**



# Where Are the PPEs?

The Press Democrat

Face masks in the national stockpile have not been substantially replenished since 2009



The New York Times

***F.D.A. Bans Faulty Masks, 3 Weeks After Failed Tests***



**Why America ran out of protective masks — and what can be done about it**

FierceHealthcare

A physician exec was trying to secure PPE for his hospital. Then the feds showed up

TIME

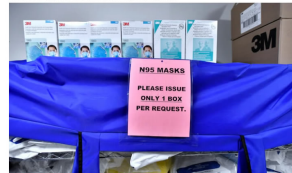
Begging for Thermometers, Body Bags, and Gowns: U.S. Health Care Workers Are Dangerously Ill-Equipped to Fight COVID-19



Why don't hospitals have enough masks? Because coronavirus broke the market.

The Washington Post  
*Democracy Dies in Darkness*

Healthcare supply chains are made for efficiency, not pandemics.



# Recurring Shortages of PPEs

Dr. Susan R. Bailey, President of the American Medical Association, wrote on August 26, 2020:

- **“It is hard to believe that our nation finds itself dealing with the same shortfalls in PPE witnessed during the first few weeks that SARS-CoV-2 began its unrelenting spread ...”**
- **“But that same situation exists today, and in many ways things have only gotten worse.”**
- **“The lack of a coordinated national strategy to acquire and distribute PPE has certainly played a role forcing state governments to compete with each other – and with the federal government as well as foreign nations – to secure masks, gowns, gloves and other gear.”**

**A. Nagurney, "Optimization of Supply Chain Networks with Inclusion of Labor: Applications to Covid-19 Pandemic Disruptions," *International Journal of Production Economics* 235 (2021), 108074.**



# Supply Chain Model with Different Labor Constraints

**The modeling framework considers first elastic demands for a product and then fixed demands**, coupled with distinct types of labor capacities in order to capture the availability of this valuable resource in a pandemic, as well as possible flexibility.

**The supply chain network framework includes electronic commerce and is relevant to many different supply chain applications including protective personal and medical equipment.**

# Supply Chain Model with Different Labor Constraints

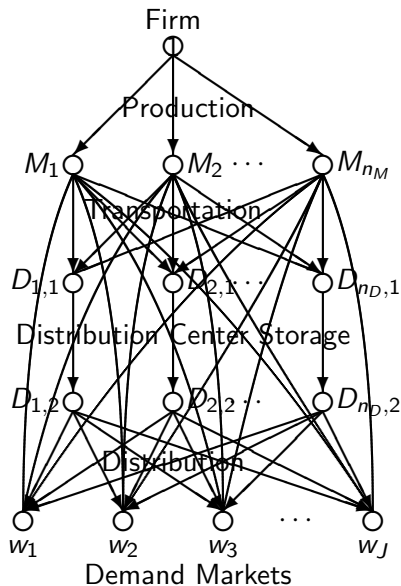


Figure: The Supply Chain Network Topology

# Supply Chain Model with Different Labor Constraints

The model considers three sets of labor constraints, of increasing flexibility of movement.

- ❶ In the first set, **each supply chain link has an upper bound of available labor**. Labor is not free to move to other production sites, nor to other distribution centers, or assist in freight service provision.
- ❷ In the second set, **labor is free to move across a supply chain set of network economic activities (such as production, or transportation, or storage, and, finally, distribution)**. There is a capacity of labor associated with each such “tier” of supply chain links. Those who have skills in production, or in distribution, etc., may be reallocated. This has been happening in freight service provision, for example, during the Covid-19 pandemic.
- ❸ In the third set, **labor is free to move across all the supply chain network economic activities, and there is a single capacity**. McKinsey & Company noted this is a means towards resilience and returning the supply chain to effectiveness while reenvisioning and reforming.

# Supply Chain Model with Different Labor Constraints

## Our findings include:

- ① **Having appropriate healthcare pandemic mitigation processes and procedures in place is essential to continuing operations.** With even one of the two manufacturing plants closed, the can prices rise at the demand markets.
- ② **Reduction in labor availability can result in a significant increase in product prices at the consumer level.**
- ③ **Even in the case of reduced labor availability, electronic commerce can result in increased profits.**
- ④ **Having the flexibility of labor being able to be reallocated across supply chain network activities can enable enhanced profits.**

# Methodology - The VI Problem



# Methodology - The Variational Inequality Problem

We utilize the theory of variational inequalities for the formulation, analysis, and solution of both centralized and decentralized supply chain network problems.

## Definition: The Variational Inequality Problem

*The finite-dimensional variational inequality problem,  $VI(F, \mathcal{K})$ , is to determine a vector  $X^* \in \mathcal{K}$ , such that:*

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

*where  $F$  is a given continuous function from  $\mathcal{K}$  to  $R^N$ ,  $\mathcal{K}$  is a given closed convex set, and  $\langle \cdot, \cdot \rangle$  denotes the inner product in  $R^N$ .*

# Methodology - The Variational Inequality Problem

The vector  $X$  consists of **the decision variables** – typically, the flows (products, prices, etc.).

$\mathcal{K}$  is the **feasible set representing how the decision variables are constrained** – for example, the flows may have to be nonnegative; budget constraints may have to be satisfied; similarly, quality and/or time constraints may have to be satisfied.

The function  $F$  that enters the variational inequality represents **functions that capture the behavior in the form of the functions such as costs, profits, risk, etc.**

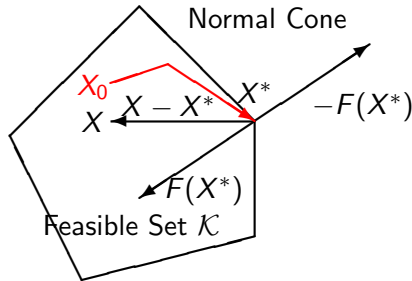
**The variational inequality problem contains, as special cases, such mathematical programming problems as:**

- systems of equations,
- optimization problems,
- complementarity problems,
- game theory problems, operating under Nash equilibrium,
- and is related to the fixed point problem.

**Hence, it is a natural methodology for a spectrum of supply chain network problems from centralized to decentralized ones.**

# Geometric Interpretation of $\text{VI}(F, \mathcal{K})$ and a Projected Dynamical System (Dupuis and Nagurney, Nagurney and Zhang)

In particular,  $F(X^*)$  is “orthogonal” to the feasible set  $\mathcal{K}$  at the point  $X^*$ .



**Associated with a VI is a Projected Dynamical System, which provides the natural underlying dynamics.**

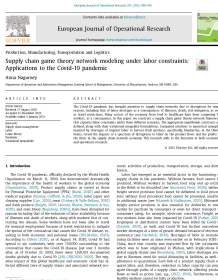
To model the **dynamic behavior of complex networks**, including supply chains, we utilize *projected dynamical systems* (PDSs) advanced by Dupuis and Nagurney (1993) in *Annals of Operations Research* and by Nagurney and Zhang (1996) in our book *Projected Dynamical Systems and Variational Inequalities with Applications*.

Such nonclassical dynamical systems are now being used in:  
**evolutionary games** (Sandholm (2005, 2011)),  
**ecological predator-prey networks** (Nagurney and Nagurney (2011a, b)),  
even **neuroscience** (Girard et al. (2008)),  
**dynamic spectrum model for cognitive radio networks** (Setoodeh, Haykin, and Moghadam (2012)),  
**Future Internet Architectures** (Saber, Nagurney, Wolf (2014); see also Nagurney et al. (2015), Marentes et al. (2016)).

# Game Theory and Supply Chain Network Models Inspired by the COVID-19 Pandemic

# Game Theory Supply Chain Network Model with Labor

This part of the presentation is based on the paper, **“Supply Chain Game Theory Network Modeling Under Labor Constraints: Applications to the Covid-19 Pandemic,”** A. Nagurney, *European Journal of Operational Research* **293(3)** (2021), pp 880-891, in which a game theory model for supply chains with labor was constructed, under three different sets of constraints, building on our previous work.

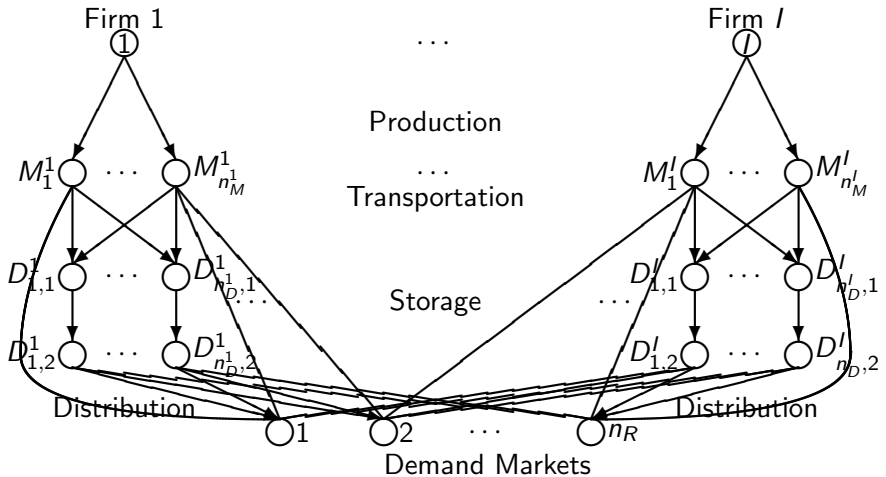


Two sets of constraints have labor being shared among the competing supply chain networks of firms/organizations, in which case the governing concept is that of a **Generalized Nash Equilibrium** (rather than a Nash Equilibrium).

**The research adds to modeling methodology as well as applications.**



# Game Theory Supply Chain Network Model with Labor



**Figure:** The Supply Chain Network Topology of the Game Theory Model with Labor

# Game Theory Supply Chain Network Model Notation

**Table:** Game Theory Supply Chain Network Model Notation

Notation	Definition
$L^i$	The set of links in firm $i$ 's supply chain network, with $L$ being all the links.
$G = [N, L]$	the graph of the supply chain network consisting of all nodes $N$ and all links $L$ .
$P_k^i$	set of paths in firm $i$ 's supply chain network terminating in demand market $k$ ; $\forall i, k$ .
$P^i$	set of all $n_{Pi}$ paths of firm $i$ ; $i = 1, \dots, I$ .
$P$	set of all $n_P$ paths in the supply chain network economy.
$x_p; p \in P_k^i$	nonnegative flow on path $p$ originating at firm node $i$ and terminating at $k$ ; $\forall i, k$ . Group firm $i$ 's path flows into vector $x^i \in R_+^{n_{Pi}}$ . Then group all firms' path flows into vector $x \in R_+^{n_P}$ .
$f_a$	nonnegative flow of the product on link $a$ , $\forall a \in L$ . Group all link flows into vector $f \in R_+^{n_L}$ .
$l_a$	labor on link $a$ (usually denoted in person hours).
$\alpha_a$	positive factor relating input of labor to output of product flow on link $a$ , $\forall a \in L$ .
$\bar{l}_a$	bound on the availability of labor on link $a$ under Scenario 1, $\forall a \in L$
$\bar{l}^t$	bound on labor availability for tier $t$ activities under Scenario 2. $T+1$ is electronic commerce tier.
$\bar{l}$	bound on labor availability under Scenario 3.
$d_{ik}$	demand for the product of firm $i$ at demand market $k$ ; $\forall i, k$ . Group $\{d_{ik}\}$ elements for firm $i$ into vector $d^i \in R_+^{n_R}$ and all demands into vector $d \in R_+^{I \times n_R}$ .
$\hat{c}_a(f)$	total operational cost associated with link $a$ , $\forall a \in L$ .
$\pi_a$	cost of a unit of labor on link $a$ , $\forall a$ .
$\rho_{ik}(d)$	demand price function for the product of firm $i$ at demand market $k$ ; $\forall i, k$ .

# Game Theory Supply Chain Network Model with Labor

For each firm  $i$ ;  $i = 1, \dots, I$ , we must have that:

$$\sum_{p \in P_k^i} x_p = d_{ik}, \quad k = 1, \dots, n_R. \quad (1)$$

The path flows must be nonnegative; that is, for each firm  $i$ ;  $i = 1, \dots, I$ :

$$x_p \geq 0, \quad \forall p \in P^i. \quad (2)$$

The link flows of each firm  $i$ ;  $i = 1, \dots, I$ , are related to the path flows as:

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L^i, \quad (3)$$

where  $\delta_{ap} = 1$ , if link  $a$  is contained in path  $p$ , and 0, otherwise.  
We now discuss how labor is related to product flow.

$$f_a = \alpha_a l_a, \quad \forall a \in L^i, \quad i = 1, \dots, I. \quad (4)$$

# Game Theory Supply Chain Network Model with Labor

The utility function of firm  $i$ ,  $U^i$ ;  $i = 1, \dots, I$ , is the profit, given by the difference between its revenue and its total costs:

$$U^i = \sum_{k=1}^{n_R} \rho_{ik}(d) d_{ik} - \sum_{a \in L^i} \hat{c}_a(f) - \sum_{a \in L^i} \pi_a l_a. \quad (5a)$$

The functions  $U_i$ ;  $i = 1, \dots, I$ , are assumed to be concave, with the demand price functions being monotone decreasing and continuously differentiable and the total link cost functions being convex and also continuously differentiable.

## The Optimization Problem of Each Firm

The optimization problem of each firm  $i$ ;  $i = 1, \dots, I$ , is:

$$\text{Maximize} \quad \sum_{k=1}^{n_R} \rho_{ik}(d) d_{ik} - \sum_{a \in L^i} \hat{c}_a(f) - \sum_{a \in L^i} \pi_a l_a, \quad (5b)$$

subject to: (1), (2), (3), and (4).

# Game Theory Supply Chain Network Model with Labor

## Labor Scenario 1 – A Bound on Labor on Each Supply Chain Network Link

In Scenario 1, the additional constraints on the fundamental model are:

$$l_a \leq \bar{l}_a, \quad \forall a \in L. \quad (6)$$

## Labor Scenario 2 – A Bound on Labor on Each Tier of Links in the Supply Chain Network

In Scenario 2, firms are faced with the ff. additional constraints:

$$\sum_{a \in L^1} l_a \leq \bar{l}^1, \quad (7, 1)$$

$$\sum_{a \in L^2} l_a \leq \bar{l}^2, \quad (7, 2)$$

and so on, until

$$\sum_{a \in L^{T+1}} l_a \leq \bar{l}^{T+1}. \quad (7, T + 1)$$

## Labor Scenario 3 – A Single Labor Bound on Labor for All the Links in the Supply Chain Network

Scenario 3 may be interpreted as being the least restrictive of the scenarios considered here in that labor can be transferable across different activities of production, transportation, storage, and distribution. In Scenario 3, in addition to constraints (1) through (4), the firms are now faced with the following single constraint:

$$\sum_{a \in L} l_a \leq \bar{l}. \quad (8)$$

# Game Theory Supply Chain Network Model with Labor

Recall that  $x^i$  denotes the vector of strategies, which are the path flows, for each firm  $i$ ;  $i = 1, \dots, I$ . We can redefine the utility/profit functions  $\tilde{U}^i(x) \equiv U^i$ ;  $i = 1 \dots, I$  and group the profits of all the firms into an  $I$ -dimensional vector  $\tilde{U}$ , such that

$$\tilde{U} = \tilde{U}(x). \quad (9)$$

Objective function (5b), in lieu of the above, can now be expressed as:

$$\text{Maximize } \tilde{U}^i(x) = \sum_{k=1}^{n_R} \tilde{\rho}_{ik}(x) \sum_{p \in P_k^i} x_p - \sum_{a \in L^i} \tilde{c}_a(x) - \sum_{a \in L^i} \frac{\pi_a}{\alpha_a} \sum_{p \in P} x_p \delta_{ap}. \quad (10)$$

# Governing Equilibrium Conditions

## Scenario 1 Nash Equilibrium Conditions

We define the feasible set  $K_i$  for firm  $i$ :

$$K_i \equiv \{x^i | x^i \in R_+^{n_{pi}}, \frac{\sum_{p \in P^i} x_p \delta_{ap}}{\alpha_a} \leq \bar{l}_a, \forall a \in L^i\}, \text{ for } i = 1, \dots, I.$$

Also, we define  $K \equiv \prod_{i=1}^I K_i$ .

**In Scenario 1, each firm competes noncooperatively until the following equilibrium is achieved.**

**Definition: Supply Chain Network Nash Equilibrium for Scenario 1**

*A path flow pattern  $x^* \in K$  is a supply chain network Nash Equilibrium if for each firm  $i$ ;  $i = 1, \dots, I$ :*

$$\tilde{U}^i(x^{i*}, \hat{x}^{i*}) \geq \tilde{U}^i(x^i, \hat{x}^{i*}), \quad \forall x^i \in K_i, \quad (11)$$

*where  $\hat{x}^{i*} \equiv (x^{1*}, \dots, x^{i-1*}, x^{i+1*}, \dots, x^{I*})$ .*



# Variational Inequality Formulations

Applying the classical theory of Nash equilibria and variational inequalities, under our imposed assumptions on the underlying functions, it follows that (cf. Gabay and Moulin (1980) and Nagurney (1999)) the solution to the above Nash Equilibrium problem (see Nash (1950, 1951)) coincides with the solution of the variational inequality problem: determine  $x^* \in K$ , such that

$$-\sum_{i=1}^I \langle \nabla_{x^i} \tilde{U}^i(x^*), x^i - x^{i*} \rangle \geq 0, \quad \forall x \in K, \quad (12)$$

where  $\langle \cdot, \cdot \rangle$  represents the inner product in the corresponding Euclidean space, which here is of dimension  $n_P$ , and  $\nabla_{x^i} \tilde{U}^i(x)$  is the gradient of  $\tilde{U}^i(x)$  with respect to  $x^i$ .

We introduce Lagrange multipliers  $\lambda_a$  associated with constraint (6),  $\forall a \in L$  and group the Lagrange multipliers for each firm  $i$ 's network  $L^i$  into the vector  $\lambda^i$ . Group all such vectors for firms into vector  $\lambda \in R_+^{n_L}$ . Define feasible sets:  $K_i^1 \equiv \{(x^i, \lambda^i) | (x^i, \lambda^i) \in R_+^{n_{P^i} + n_{L^i}}\}; i = 1, \dots, I$ , and  $K^1 \equiv \prod_{i=1}^I K_i^1$ .

# Variational Inequality Formulations

## Theorem: Alternative VI of Nash Equilibrium for Scenario 1

The supply chain network Nash Equilibrium satisfying the Definition 3.1 is equivalent to the solution of the variational inequality: determine vectors of path flows and Lagrange multipliers,  $(x^*, \lambda^*) \in K^1$ , where:

$$\sum_{i=1}^m \sum_{k=1}^{n_R} \sum_{p \in P_k^i} \left[ \frac{\partial \tilde{C}_p(x^*)}{\partial x_p} + \sum_{a \in L^i} \frac{\lambda_a^*}{\alpha_a} \delta_{ap} + \sum_{a \in L^i} \frac{\pi_a}{\alpha_a} \delta_{ap} - \tilde{\rho}_{ik}(x^*) - \sum_{l=1}^{n_R} \frac{\partial \tilde{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^j} x_q^* \right] \times [x_p - x_p^*] \\ + \sum_{a \in L} \left[ \bar{l}_a - \frac{\sum_{p \in P} x_p^* \delta_{ap}}{\alpha_a} \right] \times [\lambda_a - \lambda_a^*] \geq 0, \quad \forall (x, \lambda) \in K^1; \quad (13)$$

where for each path  $p$ ;  $p \in P_k^i$ ;  $i = 1, \dots, m$ ;  $k = 1, \dots, n_R$ :

$$\frac{\partial \tilde{C}_p(x)}{\partial x_p} \equiv \sum_{a \in L^i} \sum_{b \in L^i} \frac{\partial \tilde{c}_b(f)}{\partial f_a} \delta_{ap}, \quad (14)$$

$$\frac{\partial \tilde{\rho}_{ik}(x)}{\partial x_p} \equiv \frac{\partial \rho_{il}(d)}{\partial d_{ik}}. \quad (15)$$

For both Scenarios 2 and 3, we use a refinement of the Generalized Nash Equilibrium, known as a *Variational Equilibrium* to construct variational inequality formulations.

Hence, the labor supply chain network equilibrium models, under three different scenarios of constraints, can be uniformly qualitatively studied and solution to numerical problems, quantitatively computed using rigorous algorithms!

# Application of the Modified Projection Method

## Realization of the Modified Projection Method Computation Step for VI (13)

Specifically, at iteration  $\tau$ , we compute each of the path flows  $\bar{x}_p^\tau$ ,  $\forall P_k^i$ ,  $\forall i$ ,  $\forall k$ , according to:

$$\begin{aligned} \bar{x}_p^\tau = \max\{0, x_p^{\tau-1} - \beta(\frac{\partial \tilde{C}_p(x^{\tau-1})}{\partial x_p} + \sum_{a \in L^i} \frac{\lambda_a^{\tau-1}}{\alpha_a} \delta_{ap} + \sum_{a \in L^i} \frac{\pi_a}{\alpha_a} \delta_{ap} \\ - \tilde{\rho}_{ik}(x^{\tau-1}) - \sum_{l=1}^{n_R} \frac{\partial \tilde{\rho}_{il}(x^{\tau-1})}{\partial x_p} \sum_{q \in P_l^j} x_q^{\tau-1})\} \end{aligned} \quad (16)$$

and each of the Lagrange multipliers  $\bar{\lambda}_a^\tau$ ,  $\forall a \in L$ , according to:

$$\bar{\lambda}_a^\tau = \max\{0, \lambda_a^{\tau-1} - \beta(\bar{l}_a - \frac{\sum_{p \in P} x_p^{\tau-1} \delta_{ap}}{\alpha_a})\}. \quad (17)$$

# Application of the Modified Projection Method

## Realization of the Modified Projection Method Computation Step for VI (13)

At iteration  $\tau$ , we compute each of the path flows  $x_p^\tau$ ,  $\forall P_k^i$ ,  $\forall i$ ,  $\forall k$ , according to:

$$x_p^\tau = \max\{0, x_p^{\tau-1} - \beta\left(\frac{\partial \tilde{C}_p(\bar{x}^\tau)}{\partial x_p} + \sum_{a \in L^i} \frac{\bar{\lambda}_a^\tau}{\alpha_a} \delta_{ap} + \sum_{a \in L^i} \frac{\pi_a}{\alpha_a} \delta_{ap} - \tilde{\rho}_{ik}(\bar{x}^\tau) - \sum_{l=1}^{n_R} \frac{\partial \tilde{\rho}_{il}(\bar{x}^\tau)}{\partial x_p} \sum_{q \in P_l^i} \bar{x}_q^\tau\right)\} \quad (18)$$

and each of the Lagrange multipliers  $\lambda_a^\tau$ ,  $\forall a \in L$ , according to:

$$\lambda_a^\tau = \max\{0, \lambda_a^{\tau-1} - \beta(\bar{l}_a - \frac{\sum_{p \in P} \bar{x}_p^\tau \delta_{ap}}{\alpha_a})\}. \quad (19)$$

# Numerical Experiments

**Our numerical examples are based on disruptions in migrant labor in the blueberry supply chain in the Northeast of the US in the summer of 2020.**



**The numerical examples investigate:**

- Modifications in demand price functions;
- Disruptions in labor on a supply chain network link, with additional numerical examples presented in the EJOR paper.

# Numerical Examples

Examples 1, 2, and 3 have the supply chain network topology given below. There are two competing food firms (blueberry farms), each with two production locations, and with a single distribution center. There are two demand markets. We consider Scenario 1.

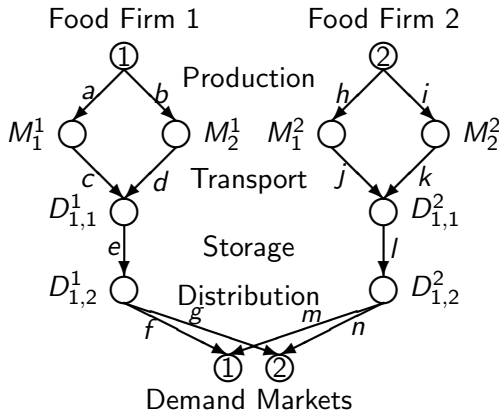


Figure: The Supply Chain Network Topology for the Numerical Examples

# Example 1 - Baseline Example

The total operational cost functions for Food Firm 1 on its supply chain network  $L^1$  are:

$$\hat{c}_a(f) = .0006f_a^2, \quad \hat{c}_b(f) = .0007f_b^2, \quad \hat{c}_c(f) = .001f_c^2, \quad \hat{c}_d(f) = .001f_d^2, \\ \hat{c}_e(f) = .002f_e^2, \quad \hat{c}_f(f) = .005f_f^2, \quad \hat{c}_g(f) = .005f_g^2.$$

Also, the total operational costs associated with Food Firm 2's supply chain network  $L^2$  are:

$$\hat{c}_h(f) = .00075f_h^2, \quad \hat{c}_i(f) = .0008f_i^2, \quad \hat{c}_j(f) = .0005f_j^2, \quad \hat{c}_k(f) = .0005f_k^2, \\ \hat{c}_l(f) = .0015f_l^2, \quad \hat{c}_m(f) = .01f_m^2, \quad \hat{c}_n(f) = .01f_n^2.$$

The costs for labor (wages) for Food Firm 1 are:

$$\pi_a = 10, \quad \pi_b = 10, \quad \pi_c = 15, \quad \pi_d = 15, \quad \pi_e = 20, \quad \pi_f = 17, \quad \pi_g = 18,$$

and for Food Firm 2:

$$\pi_h = 11, \quad \pi_i = 22, \quad \pi_j = 15, \quad \pi_k = 15, \quad \pi_l = 18, \quad \pi_m = 18, \quad \pi_n = 18.$$



# Example 1 - Baseline Example

The link labor productivity factors for the first firm are:

$$\alpha_a = 24, \alpha_b = 25, \alpha_c = 100, \alpha_d = 100, \alpha_e = 50, \alpha_f = 100, \alpha_g = 100,$$

and for the second firm:

$$\alpha_h = 23, \alpha_i = 24, \alpha_j = 100, \alpha_k = 100, \alpha_l = 70, \alpha_m = 100, \alpha_n = 100.$$

The bounds on labor for the first firm are:

$$\bar{l}_a = 10, \quad \bar{l}_b = 200, \quad \bar{l}_c = 300, \quad \bar{l}_d = 300, \quad \bar{l}_e = 100, \quad \bar{l}_f = 120, \quad \bar{l}_g = 120,$$

and for the second firm:

$$\bar{l}_h = 800, \quad \bar{l}_i = 90, \quad \bar{l}_j = 200, \quad \bar{l}_k = 200, \quad \bar{l}_l = 300, \quad \bar{l}_m = 100, \quad \bar{l}_n = 100.$$

**Observe that the labor availability on link  $a$  is low. This is done in order to capture a disruption to labor in the pandemic.**

# Example 1 - Baseline Example

The demand price functions for Food Firm 1 are:

$$\rho_{11}(d) = -.0001d_{11} - .00005d_{21} + 6, \quad \rho_{12}(d) = -.0002d_{12} - .0001d_{22} + 8.$$

The demand price functions for Food Firm 2 are:

$$\rho_{21}(d) = -.0003d_{21} + 7, \quad \rho_{22}(d) = -.0002d_{22} + 7.$$

The paths are:  $p_1 = (a, c, e, f)$ ,  $p_2 = (b, d, e, f)$ ,  $p_3 = (a, c, e, g)$ ,  
path  $p_4 = (b, d, e, g)$ ,  $p_5 = (h, j, l, m)$ ,  $p_6 = (i, k, l, m)$ ,  
 $p_7 = (h, j, l, n)$ , and  $p_8 = (i, k, l, n)$ .

# Example 1 - Baseline Example

All the Lagrange multipliers are equal to 0.00 except for  $\lambda_a^* = 4.925$  with the labor equilibrium value on link  $a$  equal to its upper bound of 10.00.

The product prices at equilibrium are:

$$\rho_{11} = 5.97, \quad \rho_{12} = 7.91, \quad \rho_{21} = 6.94, \quad \rho_{22} = 6.96,$$

with equilibrium demands of:

$$d_{11}^* = 172.07, \quad d_{12}^* = 359.15, \quad \rho_{21} = 195.94, \quad \rho_{22} = 197.86.$$

The profit of Food Firm 1 is: 1,671.80 and the profit of Food Firm 2 is: 1,145.06.

## Example 2 – Modification of Demand Price Functions

Example 2 has the same data as Example 1 except that we modify the demand price functions for the second firm to include a cross term, so that:

$$\rho_{21}(d) = -.0003d_{21} - .0001d_{11} + 6, \quad \rho_{22}(d) = -.0002d_{22} - .0001d_{12} + 7.$$

The Lagrange multipliers are all equal to 0.00 except for  $\lambda_a^* = 4.93$ .

## Example 2 – Modification of Demand Price Functions

The product prices at equilibrium are now:

$$\rho_{11} = 5.97, \quad \rho_{12} = 7.91, \quad \rho_{21} = 6.92, \quad \rho_{22} = 6.92,$$

with the equilibrium demands:

$$d_{11}^* = 172.07, \quad d_{12}^* = 359.16, \quad d_{21}^* = 195.48, \quad d_{22}^* = 196.48.$$

**The profit for Food Firm 1 is: 1,671.86 and the profit for Food Firm 2 is: 1,134.61. The profit for Food Firm 1 rises ever so slightly, whereas that for Food Firm 2 decreases.**

## Example 3 – Disruptions in Storage Facilities

Example 3 has the same data as Example 2 except that we now consider a sizable disruption in terms of the spread of COVID-19 at the distribution centers of both food firms with the bounds on labor corresponding to the associated respective links being reduced to:

$$\bar{l}_e = 5, \quad \bar{l}_l = 5.$$

All computed equilibrium Lagrange multipliers are now equal to 0 except for those associated with the distribution center links, since the equilibrium labor values attain the imposed upper bounds on links  $e$  and  $l$ , with the respective equilibrium Lagrange multiplier values being:

$$\lambda_e^* = 157.2138, \quad \lambda_l^* = 43.6537.$$

## Example 3 – Disruptions of Labor in Storage Facilities

The product prices at equilibrium are now:

$$\rho_{11} = 5.99, \quad \rho_{12} = 7.94, \quad \rho_{21} = 6.94, \quad \rho_{22} = 6.94,$$

with the equilibrium demands:

$$d_{11}^* = 30.03, \quad d_{12}^* = 219.96, \quad d_{21}^* = 174.61, \quad d_{22}^* = 175.39.$$

**The profit for Food Firm 1 is now dramatically reduced to 1,218.74 and the profit for Food Firm 2 also declines, but by a much smaller amount, to 1,126.73.**

# Equilibrium Path Flows

Table: Equilibrium Product Path Flows for Examples 1 Through 3

Equilibrium Product Path Flows	Ex. 1	Ex. 2	Ex. 3
$x_{p1}^*$	73.23	73.22	15.65
$x_{p2}^*$	98.85	98.85	14.38
$x_{p3}^*$	166.77	166.78	110.60
$x_{p4}^*$	192.38	192.38	109.35
$x_{p5}^*$	142.85	142.62	131.97
$x_{p6}^*$	53.08	52.86	42.63
$x_{p7}^*$	143.81	143.12	132.36
$x_{p8}^*$	54.04	53.36	43.02



# Equilibrium Link Labor Values

Table: Equilibrium Link Labor Values for Examples 1 Through 3

Equilibrium Link Labor Values	Ex. 1	Ex. 2	Ex. 3
$l_a^*$	10.00	10.00	5.26
$l_b^*$	11.65	11.65	4.95
$l_c^*$	2.40	2.40	1.26
$l_d^*$	2.91	2.91	1.24
$l_e^*$	10.62	10.62	5.00
$l_f^*$	1.72	1.72	0.30
$l_g^*$	3.59	3.59	2.20
$l_h^*$	12.46	12.42	11.49
$l_i^*$	4.46	4.43	3.57
$l_j^*$	2.87	2.86	2.64
$l_k^*$	1.07	1.06	0.86
$l_l^*$	5.63	5.60	5.00
$l_m^*$	1.96	1.95	1.75
$l_n^*$	1.98	1.96	1.75

Farmers should do everything possible to secure the health of workers at their production/harvesting and other facilities, so that the blueberries can be harvested in a timely manner and so that profits do not suffer. Keeping workers healthy, through appropriate measures, impacts the bottom line!

# Some Additional Research

**A. Nagurney, “Attracting International Migrant Labor: Investment Optimization to Alleviate Supply Chain Labor Shortages,”** *Operations Research Perspectives* 9 (2022), 100233.

**A. Nagurney, “Optimization of Investments in Labor Productivity in Supply Chain Networks,”** *International Transactions in Operational Research* 29(4), (2022), pp 2116-2144. **This article was recognized with an Editor’s Choice Award.**

**A. Nagurney, “Supply Chain Networks, Wages, and Labor Productivity: Insights from Lagrange Analysis and Computations,”** in press in *Journal of Global Optimization*.

# Some Additional Research

**The numerical results in our papers clearly reveal the importance of a holistic approach to supply chain network modeling since decisions made by a specific firm can have unexpected impacts on other competing firms in the supply chain network economy.**

**Our results also strongly suggest that having wages and labor equilibrate without any wage ceilings can be beneficial for an individual firm and also for firms engaged in competition.**

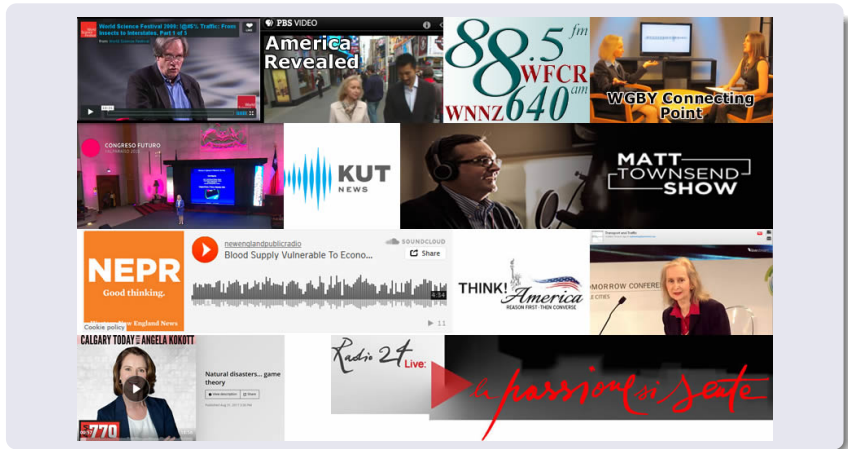
**And, most importantly, taking care of workers is critical in times of peace and war!**

# Dealing with the Media and Impacting Policy

# Writing OpEds



# Coverage by the Media



On March 11, 2020 the WHO declared the pandemic. On March 12 my article on blood supply chains in *The Conversation* appeared and, on March 24 my article in *INFORMS Analytics Coronavirus Chronicles*.

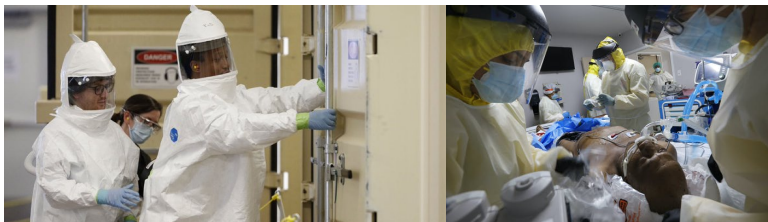
## Labor and Supply Chain Networks



# Writing OpEds in the Pandemic

On August 4, 2020, I published an article in *The Conversation*,

**“The Raging Competition for Medical Supplies is not a Game, but Game Theory Can Help.”**



On September 18, 2020, I published another article in *The Conversation*,

**“Keeping Coronavirus Vaccines at Subzero Temperatures During Distribution Will Be Hard, but Likely Key to Ending Pandemic.”**

# Writing OpEds in the Pandemic

On January 8, 2021, my article,

**“Vaccine Delays Reveal Unexpected Weak Link in Supply Chains: A Shortage of Workers,”** appeared in *The Conversation*.



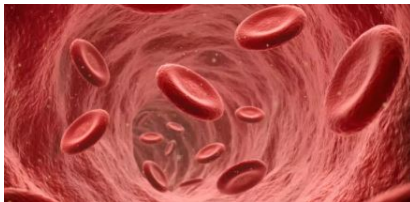
On April 5, 2021, I published the article,

**“Today’s Global Economy Runs on Standardized Containers, as the Ever Given Fiasco Illustrates,”** also in *The Conversation*.

# Writing OpEds in the Pandemic

On September 21, 2021, my article,

**“Global Shortage of Shipping Containers Highlights Their Importance in Getting Goods to Amazon Warehouses, Store Shelves and Your Door in Time for Christmas,”** appeared in *The Conversation*. It has had over 330,000 reads.



On February 3, 2022, my article,

**“Heading into the Third Year of the Pandemic, the US Blood Supply is at a 10-Year Low,”** was published in *The Conversation*.

# Some of My Media Interviews in the Pandemic



# Some of the Media Interviews on the War on Ukraine

## Economic dangers from Russia's invasion ripple across globe

By PAUL WISEMAN and DAVID McHUGH March 2, 2022



Feedstuffs went live.  
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## Russian war in world's 'breadbasket' threatens food supply

By JOSEPH WILSON, SAMY MAGDY, AYA BATRAWY and CHINEDU ASADU March 6, 2022

## 'I fear a cultural genocide'; Ukrainians in Western Mass. watch, worry and help

Published: Feb. 28, 2022, 5:55 p.m.



## Threat of Russian cyber attacks likely for not just Ukraine, but also in the US

### No Ikea Shelves, No Levis: The Retail Exodus From Russia Is On

Since the invasion of Ukraine began, the increasing financial and reputational risks of doing business in Russia are leading Western brands to halt operations.

## Russian Sanctions Snarl Shipping Even as Pandemic Pressure Eases



March 11, 2022  
Liz Alderman and Jeremy Gross

On April 22, 2020, a letter from California Attorney General Xavier Becerra to the Admiral Brett Giroir, the Assistant Secretary of the US Department of Health & Human Services, and signed by US Attorney Generals of 21 other states, requested updates, because of the pandemic blood shortages, to blood donation policies that discriminate.

**My March 2020 article in *The Conversation*, which was reprinted in LiveScience, was the first reference and was cited on the first page.**



State of California  
Office of the Attorney General

XAVIER BECERRA  
ATTORNEY GENERAL

April 22, 2020

*Via Electronic Mail*

The Honorable Admiral Brett Giroir, MD  
Assistant Secretary for Health  
U.S. Department of Health & Human Services  
Mary E. Switzer Building  
330 C Street SW, Room L600  
Washington, DC 20024  
Attn: ACHTSA-PAHPAIA Sec. 209  
[ACBTS4@hhs.gov](mailto:ACBTS4@hhs.gov)

RE: "Solicitation for Public Comments on Section 209 of the Pandemic and All-Hazards Preparedness and Advancing Innovation Act," 85 Fed. Reg. 16,372 (March 23, 2020)

Dear Assistant Secretary Giroir:

The undersigned State Attorneys General from California, Colorado, Connecticut, Delaware, the District of Columbia, Hawaii, Illinois, Iowa, Maine, Massachusetts, Michigan, Minnesota, Nevada, New Jersey, New Mexico, New York, Oregon, Pennsylvania, Vermont, and Virginia submit this letter in response to the federal government's "Solicitation for Public Comments on Section 209 of the Pandemic and All-Hazards Preparedness and Advancing Innovation Act," (85 Fed. Reg. 16,372). We support the Office of the Assistant Secretary for Health in the U.S. Department of Health and Human Services' (HHS) efforts and work in maintaining an adequate national blood supply during the COVID-19 pandemic.

An adequate blood supply is critical to the nation's healthcare. Blood transfusions and blood products are needed for major surgeries, to treat diseases such as sickle cell anemia and some cancers, and to treat victims who have injuries caused by accidents or natural disasters.<sup>1</sup> Every day, the United States needs approximately 36,000 units of red blood cells, nearly 7,000

<sup>1</sup> Anna Nagurney, How Coronavirus is Upsetting the Blood Supply Chain, Live Science (Mar. 13, 2020), <https://www.livescience.com/coronavirus-blood-supply-chain.html/>.

# Impacting Policy

Hon. Brett Owens  
April 22, 2020  
Page 7



WILLIAM TONG  
Connecticut Attorney General



KATHLEEN JENNINGS  
Delaware Attorney General



KARLA A. RACINE  
District of Columbia Attorney General



CLARE E. CONNORS  
Hawaii Attorney General



KWAME RAUL  
Illinois Attorney General



TOM MILLER  
Iowa Attorney General



AARON M. FREY  
Maine Attorney General



MAURA HEALEY  
Massachusetts Attorney General



DANA NESSEL  
Michigan Attorney General



KEITH ELLISON  
Minnesota Attorney General



AARON D. FORD  
Nevada Attorney General



ROBERT S. OREWAL  
New Jersey Attorney General



HECTOR BALDERAS  
New Mexico Attorney General



LETITIA JAMES  
New York Attorney General

**Xavier Becerra, previously California's Attorney General, has now been confirmed as President Joe Biden's Health and Human Services Secretary!**



# Thank You!



## The Virtual Center for Supernetworks



*Supernetworks for Optimal Decision-Making and Improving the Global Quality of Life*

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Virtual Center  
for Supernetworks

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