A General Multitiered Supply Chain Network Model of Quality Competition with Suppliers

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Outline

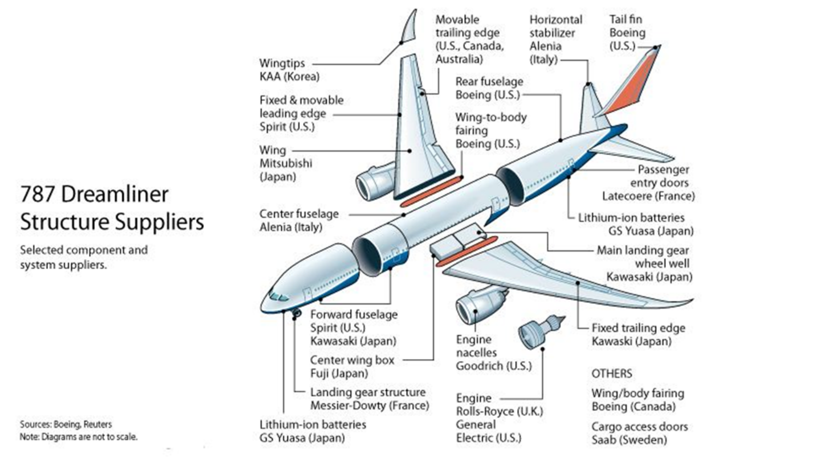
- Background and Motivation
- A Multitiered Supply Chain Network Game Theory Model with Suppliers and Quality Competition
- Qualitative Properties
- The Algorithm
- Numerical Examples
- Summary and Conclusions

Background and Motivation

Indeed, products are made of materials and components, and the components and materials are produced and supplied not by the firms that process them into products but by suppliers in globalized supply chain networks.

The quality of a product depends not only on the quality of the firm that produces and delivers it, but also on the quality of the components provided by the firm's suppliers (Robinson and Malhotra (2005) and Foster (2008)).

The number of components comprising a finished product may be small or immense as in aircraft manufacturing and other complex high-tech products.



In recent years, a series of recalls caused by suppliers' poor quality components/materials has received intensive attention.

- In 2007, the toy giant Mattel recalled 19 million toy cars because of a supplier's lead paint and small, poorly designed magnets, which could harm children if ingested (Story and Barboza (2007)).

- In 2010, four Japanese car-makers, including Toyota and Nissan, recalled 3.6 million vehicles sold around the globe, because the airbags supplied by Takata Corp., were at risk of catching fire (Kubota and Klayman (2013)). The recalls are still ongoing and have expanded to other companies as well (Tabuchi and Jensen (2014)).

- In 2013, in the food industry, Taylor Farms, a large vegetable supplier, was under investigation in connection with an illness outbreak affecting hundreds of people in the US (Strom (2013)).

Furthermore, since suppliers, which may be located on-shore or off-shore, supply chain networks of firms may be more vulnerable to disruptions than ever before.

Overview

- The firms are responsible for assembling the products under their brand names using the components from their suppliers, and delivering the products to multiple demand markets.
- Firms also have the option of producing their own components, if necessary.
- The firms compete in product quantity, the quality preservation level of its assembly process, the contracted component quantities produced by the suppliers, and in in-house component quantities and quality level.
- The potential suppliers may either provide distinct components to the firms, or provide the same component in which case they compete non-cooperatively with one another in terms of quality and prices.
- In this model, the value of each supplier to each firm is identified. This information is crucial in facilitating strategy design and development in supplier management especially in response to supplier disruptions.

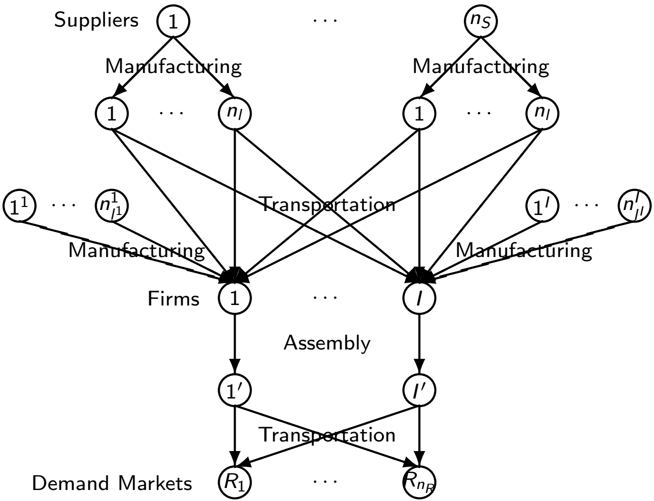


Figure: The Multitiered Supply Chain Network Topology with Suppliers

The Model

Conservation of flow equation

$$Q_{ik} = d_{ik}, \quad i = 1, \ldots, I; k = 1, \ldots, n_R. \tag{1}$$

Nonnegative shipment volumes

$$Q_{ik} \geq 0, \quad i = 1, \ldots, I; k = 1, \ldots, n_R. \tag{2}$$

Quality levels

$$q^U_{il} \geq q^S_{jil} \geq 0, \quad j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i}, \tag{3}$$

$$q^U_{il} \geq q^F_{il} \geq 0, \quad i = 1, \ldots, I; l = 1, \ldots, n_{l^i}. \tag{4}$$

The average quality level of product i's component l

$$q_{il} = \frac{q^F_{il} Q^F_{il} + \sum_{j=1}^{n_S} Q^S_{jil} q^S_{jil}}{Q^F_{il} + \sum_{j=1}^{n_S} Q^S_{jil}}, \quad i = 1, \ldots, I; l = 1, \ldots, n_{l^i}. \tag{5}$$

The Model

The quality level of a finished product

$$q_i = \alpha_i^F (\sum_{l=1}^{n_{l_i}} \omega_{il} q_{il}), \quad i = 1, \ldots, I; l = 1. \qquad (6)$$

α_i^F captures the percentage of the quality preservation of product i in the assembly process.

$$0 \leq \alpha_i^F \leq 1, \quad i = 1, \ldots, I. \qquad (7)$$

ω_{il} is the ratio of the importance of the quality of firm i's component l in one unit product i to the quality associated with one unit product i (i.e., q_i).

$$\sum_{l=1}^{n_{l^i}} \omega_{il} = 1, \quad i = 1, \ldots, I. \qquad (8)$$

The Model - The Behavior of the Firms

The total utility maximization objective of firm i

$$\text{Maximize}_{Q_i, Q_i^F, Q_i^S, q_i^F, \alpha_i^F} \quad U_i^F = \sum_{k=1}^{n_R} \hat{\rho}_{ik}(Q, Q^F, Q^S, q^F, q^{S^*}, \alpha^F) d_{ik} - f_i(Q, \alpha^F)$$

$$- \sum_{l=1}^{n_{l^i}} f_{il}^F(Q^F, q^F) - \sum_{k=1}^{n_R} \hat{tc}_{ik}^F(Q, Q^F, Q^S, q^F, q^{S^*}, \alpha^F) - \sum_{j=1}^{n_S} \sum_{l=1}^{n_{l^i}} c_{ijl}(Q^S) - \sum_{j=1}^{n_S} \sum_{l=1}^{n_{l^i}} \pi_{jil}^* Q_{jil}^S \tag{11}$$

subject to:

$$\sum_{k=1}^{n_R} Q_{ik}\theta_{il} \leq \sum_{j=1}^{n_S} Q_{jil}^S + Q_{il}^F, \quad i = 1, \ldots, I; l = 1, \ldots, n_{l^i}, \tag{12}$$

$$CAP_{jil}^S \geq Q_{jil}^S \geq 0, \quad j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i}, \tag{13}$$

$$CAP_{il}^F \geq Q_{il}^F \geq 0, \quad i = 1, \ldots, I; l = 1, \ldots, n_{l^i}, \tag{14}$$

and (1), (2), (4), and (7).

The Model - The Behavior of the Firms

We define the feasible set $\overline{K}_i^F$ as $\overline{K}_i^F \equiv \{(Q_i, Q_i^F, Q_i^S, q_i^F, \alpha_i^F)|(1), (2), (4), (7), \text{ and } (12) - (14) \text{are satisfied}\}$. All $\overline{K}_i^F$; $i = 1, \ldots, I$, are closed and convex. We also define the feasible set $\overline{\mathcal{K}}^F \equiv \Pi_{i=1}^I \overline{K}_i^F$.

Definition 1: A Cournot-Nash Equilibrium

A product shipment, in-house component production, contracted component production, in-house component quality, and assembly quality preservation pattern $(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}) \in \overline{\mathcal{K}}^F$ is said to constitute a Cournot-Nash equilibrium if for each firm i; $i = 1, \ldots, I$,

$$U_i^F(Q_i^*, \hat{Q}_i^*, Q_i^{F^*}, \hat{Q}_i^{F^*}, Q_i^{S^*}, \hat{Q}_i^{S^*}, q_i^{F^*}, \hat{q}_i^{F^*}, \alpha_i^{F^*}, \hat{\alpha}_i^{F^*}, \pi_i^*, q^{S^*}) \geq$$

$$U_i^F(Q_i, \hat{Q}_i^*, Q_i^F, \hat{Q}_i^{F^*}, Q_i^S, \hat{Q}_i^{S^*}, q_i^F, \hat{q}_i^{F^*}, \alpha_i^F, \hat{\alpha}_i^{F^*}, \pi_i^*, q^{S^*}),$$

$$\forall (Q_i, Q_i^F, Q_i^S, q_i^F, \alpha_i^F) \in \overline{K}_i^F, \quad (15)$$

where

$$\hat{Q}_i^* \equiv (Q_1^*, \ldots, Q_{i-1}^*, Q_{i+1}^*, \ldots, Q_I^*),$$

$$\hat{Q}_i^{F^*} \equiv (Q_1^{F^*}, \ldots, Q_{i-1}^{F^*}, Q_{i+1}^{F^*}, \ldots, Q_I^{F^*}),$$

$$\hat{Q}_i^{S^*} \equiv (Q_1^{S^*}, \ldots, Q_{i-1}^{S^*}, Q_{i+1}^{S^*}, \ldots, Q_I^{S^*}),$$

$$\hat{q}_i^{F^*} \equiv (q_1^{F^*}, \ldots, q_{i-1}^{F^*}, q_{i+1}^{F^*}, \ldots, q_I^{F^*}),$$

and

$$\hat{\alpha}_i^{F^*} \equiv (\alpha_1^{F^*}, \ldots, \alpha_{i-1}^{F^*}, \alpha_{i+1}^{F^*}, \ldots, \alpha_I^{F^*}).$$

Variational Inequality Formulation

Theorem 1

$(Q^*, Q^{F*}, Q^{S*}, q^{F*}, \alpha^{F*}) \in \overline{\mathcal{K}}^F$ is a Counot-Nash equilibrium if and only if it satisfies the variational inequality:

$$
-\sum_{i=1}^{I}\sum_{k=1}^{n_R} \frac{\partial U_i^F(Q^*, Q^{F*}, Q^{S*}, q^{F*}, \alpha^{F*}, \pi_i^*, q^{S*})}{\partial Q_{ik}} \times (Q_{ik} - Q_{ik}^*)
$$

$$
-\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}} \frac{\partial U_i^F(Q^*, Q^{F*}, Q^{S*}, q^{F*}, \alpha^{F*}, \pi_i^*, q^{S*})}{\partial Q_{il}^F} \times (Q_{il}^F - Q_{il}^{F*})
$$

$$
-\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}} \frac{\partial U_i^F(Q^*, Q^{F*}, Q^{S*}, q^{F*}, \alpha^{F*}, \pi_i^*, q^{S*})}{\partial Q_{jil}^S} \times (Q_{jil}^S - Q_{jil}^{S*})
$$

$$
-\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}} \frac{\partial U_i^F(Q^*, Q^{F*}, Q^{S*}, q^{F*}, \bar{q}^{F*}, \pi_i^*, q^{S*})}{\partial q_{il}^F} \times (q_{il}^F - q_{il}^{F*})
$$

$$
-\sum_{i=1}^{I} \frac{\partial U_i^F(Q^*, Q^{F*}, Q^{S*}, q^{F*}, \alpha^{F*}, \pi_i^*, q^{S*})}{\partial \alpha_i^F} \times (\alpha_i^F - \alpha_i^{F*}) \geq 0, \quad \forall (Q, Q^F, Q^S, q^F, \alpha^F) \in \overline{\mathcal{K}}^F, \tag{16}
$$

equivalently, $(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \lambda^*) \in \mathcal{K}^F$ *is a Counot-Nash equilibrium if and only if it satisfies the variational inequality:*

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\left[-\frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi_i^*, q^{S^*})}{\partial Q_{ik}} + \sum_{l=1}^{n_{I^i}} \lambda_{il}^* \theta_{il}\right] \times (Q_{ik} - Q_{ik}^*)$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{n_{I^i}}\left[-\frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi_i^*, q^{S^*})}{\partial Q_{il}^F} - \lambda_{il}^*\right] \times (Q_{il}^F - Q_{il}^{F^*})$$

$$+\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{I^i}}\left[-\frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi_i^*, q^{S^*})}{\partial Q_{jil}^S} - \lambda_{il}^*\right] \times (Q_{jil}^S - Q_{jil}^{S^*})$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{n_{I^i}}\left[-\frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi_i^*, q^{S^*})}{\partial q_{il}^F}\right] \times (q_{il}^F - q_{il}^{F^*})$$

$$+\sum_{i=1}^{I}\left[-\frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi_i^*, q^{S^*})}{\partial \alpha_i^F}\right] \times (\alpha_i^F - \alpha_i^{F^*})$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{n_{I^i}}\left[\sum_{j=1}^{n_S} Q_{jil}^{S^*} + Q_{il}^{F^*} - \sum_{k=1}^{n_R} Q_{ik}^* \theta_{il}\right] \times (\lambda_{il} - \lambda_{il}^*) \geq 0, \quad \forall (Q, Q^F, Q^S, q^F, \alpha^F, \lambda) \in \mathcal{K}^F. \quad (17)$$

The Model - The Behavior of the Suppliers

The total utility maximization objective of supplier j

$$\text{Maximize}_{\pi_j, q_j^S} \quad U_j^S = \sum_{i=1}^{I} \sum_{l=1}^{n_{l^i}} \pi_{jil} Q_{jil}^{S^*} - \sum_{l=1}^{n_l} f_{jl}^S(Q^{S^*}, q^S) - \sum_{i=1}^{I} \sum_{l=1}^{n_{l^i}} tc_{jil}^S(Q^{S^*}, q^S)$$

$$- \sum_{i=1}^{I} \sum_{l=1}^{n_{l^i}} oc_{jil}(\pi) \tag{18}$$

subject to:

$$\pi_{jil} \geq 0, \quad j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i}, \tag{19}$$

and (3).

The Model - The Behavior of the Suppliers

Definition 2: A Bertrand-Nash Equilibrium

A price and contracted component quality pattern $(\pi^*, q^{S^*}) \in \mathcal{K}^S$ *is said to constitute a Bertrand-Nash equilibrium if for each supplier* j; $j = 1, \ldots, n_S$,

$$U_j^S(Q^{S^*}, \pi_j^*, \hat{\pi}_j^*, q_j^{S^*}, \hat{q}_j^{S^*}) \geq U_j^S(Q^{S^*}, \pi_j, \hat{\pi}_j^*, q_j^S, \hat{q}_j^{S^*}), \quad \forall (\pi_j, q_j^S) \in K_j^S, \qquad (20)$$

where

$$\hat{\pi}_j^* \equiv (\pi_1^*, \ldots, \pi_{j-1}^*, \pi_{j+1}^*, \ldots, \pi_{n_S}^*)$$

and

$$\hat{q}_j^{S^*} \equiv (q_1^{S^*}, \ldots, q_{j-1}^{S^*}, q_{j+1}^{S^*}, \ldots, q_{n_S}^{S^*}).$$

Variational Inequality Formulation

Theorem 2

$(\pi^*, q^{S^*}) \in \mathcal{K}^S$ *is a Bertrand-Nash equilibrium if and only if it satisfies the variational inequality:*

$$-\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}} \frac{\partial U_j^S(Q^{S^*}, \pi^*, q^{S^*})}{\partial \pi_{jil}} \times (\pi_{jil} - \pi_{jil}^*)$$

$$-\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}} \frac{\partial U_j^S(Q^{S^*}, \pi^*, q^{S^*})}{\partial q_{jil}^S} \times (q_{jil}^S - q_{jil}^{S^*}) \geq 0, \quad \forall (\pi, q^S) \in \mathcal{K}^S. \qquad (21)$$

Definition 3

The equilibrium state of the multitiered supply chain network with suppliers is one where both variational inequalities (16), or, equivalently, (17), and (21) hold simultaneously.

Theorem 3

Determine $(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi^*, q^{S^*}) \in \overline{\mathcal{K}}$, *such that:*

$$-\sum_{i=1}^{I}\sum_{k=1}^{n_R} \frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi_i^*, q^{S^*})}{\partial Q_{ik}} \times (Q_{ik} - Q_{ik}^*)$$

$$-\sum_{i=1}^{I}\sum_{l=1}^{n_{li}} \frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi_i^*, q^{S^*})}{\partial Q_{il}^F} \times (Q_{il}^F - Q_{il}^{F^*})$$

$$-\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{li}} \frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi_i^*, q^{S^*})}{\partial Q_{jil}^S} \times (Q_{jil}^S - Q_{jil}^{S^*})$$

$$-\sum_{i=1}^{I}\sum_{l=1}^{n_{li}} \frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi_i^*, q^{S^*})}{\partial q_{il}^F} \times (q_{il}^F - q_{il}^{F^*})$$

$$-\sum_{i=1}^{I} \frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi_i^*, q^{S^*})}{\partial \alpha_i^F} \times (\alpha_i^F - \alpha_i^{F^*})$$

$$-\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{li}} \frac{\partial U_j^S(Q^{S^*}, \pi^*, q^{S^*})}{\partial \pi_{jil}} \times (\pi_{jil} - \pi_{jil}^*)$$

$$-\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{li}} \frac{\partial U_j^S(Q^{S^*}, \pi^*, q^{S^*})}{\partial q_{jil}^S} \times (q_{jil}^S - q_{jil}^{S^*}) \geq 0, \quad \forall (Q, Q^S, Q^F, q^F, \alpha^F, \pi, q^S) \in \overline{\mathcal{K}}. \quad (22)$$

Determine $(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \lambda^*, \pi^*, q^{S^*}) \in \mathcal{K}$, *such that:*

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\left[-\frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi_i^*, q^{S^*})}{\partial Q_{ik}} + \sum_{l=1}^{n_{l^i}} \lambda_{il}^* \theta_{il}\right] \times (Q_{ik} - Q_{ik}^*)$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[-\frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi_i^*, q^{S^*})}{\partial Q_{il}^F} - \lambda_{il}^*\right] \times (Q_{il}^F - Q_{il}^{F^*})$$

$$+\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[-\frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi_i^*, q^{S^*})}{\partial Q_{jil}^S} - \lambda_{il}^*\right] \times (Q_{jil}^S - Q_{jil}^{S^*})$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[-\frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi_i^*, q^{S^*})}{\partial q_{il}^F}\right] \times (q_{il}^F - q_{il}^{F^*})$$

$$+\sum_{i=1}^{I}\left[-\frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi_i^*, q^{S^*})}{\partial \alpha_i^F}\right] \times (\alpha_i^F - \alpha_i^{F^*})$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[\sum_{j=1}^{n_S} Q_{jil}^{S^*} + Q_{il}^{F^*} - \sum_{k=1}^{n_R} Q_{ik}^*\theta_{il}\right] \times (\lambda_{il} - \lambda_{il}^*) + \sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[-\frac{\partial U_j^S(Q^{S^*}, \pi^*, q^{S^*})}{\partial \pi_{jil}}\right] \times (\pi_{jil} - \pi_{jil}^*)$$

$$+\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[-\frac{\partial U_j^S(Q^{S^*}, \pi^*, q^{S^*})}{\partial q_{jil}^S}\right] \times (q_{jil}^S - q_{jil}^{S^*}) \geq 0, \quad \forall (Q, Q^F, Q^S, q^F, \alpha^F, \lambda, \pi, q^S) \in \mathcal{K}. \quad (23)$$

Standard form VI

We now put variational inequality (23) into standard form: Determine $X^* \in \mathcal{K}$ *where* X *is a vector in* R^N, $F(X)$ *is a continuous function such that* $F(X): X \mapsto \mathcal{K} \subset R^N$, *and*

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{24}$$

where $\langle \cdot, \cdot \rangle$ *is the inner product in the* N*-dimensional Euclidean space,* $N = In_R + 3\sum_{i=1}^{I} n_{l^i} + 3n_S \sum_{i=1}^{I} n_{l^i} + I$, *and* $\mathcal{K}$ *is closed and convex. Define the vector* $X \equiv (Q, Q^F, Q^S, q^F, \alpha^F, \lambda, \pi, q^S)$.

Standard form VI

We also put variational inequality (22) into standard form: Determine $Y^* \in \overline{\mathcal{K}}$ *where* Y *is a vector in* R^M, $G(Y)$ *is a continuous function such that* $G(Y): Y \mapsto \overline{\mathcal{K}} \subset R^M$, *and*

$$\langle G(Y^*), Y - Y^* \rangle \geq 0, \quad \forall Y \in \overline{\mathcal{K}}, \tag{26}$$

where $M = In_R + 2\sum_{i=1}^{I} n_{l^i} + 3n_S \sum_{i=1}^{I} n_{l^i} + I$, *and* $\overline{\mathcal{K}}$ *is closed and convex.*

Qualitative Properties

Assumption 1

Suppose that in our multitiered supply chain network model with suppliers and quality competition, there exist a sufficiently large Π, *such that,*

$$\pi_{jil} \leq \Pi, \quad j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i}. \tag{27}$$

Theorem 4: Existence

With Assumption 1 satisfied, there exists at least one solution to variational inequality (24) and (26), equivalently, (23) and (22).

Theorem 5: Monotonicity

Under the assumptions in Theorems 1 and 2, the $F(X)$ that enters variational inequality (24), is monotone, that is,

$$\langle F(X') - F(X''), X' - X'' \rangle \geq 0, \quad \forall X', X'' \in \mathcal{K}, \tag{28}$$

and the $G(Y)$ that enters variational inequality (27) is also monotone,

$$\langle G(Y') - G(Y''), Y' - Y'' \rangle \geq 0, \quad \forall Y', Y'' \in \overline{\mathcal{K}}. \tag{29}$$

Theorem 6: Uniqueness

Assume that the function $G(Y)$ in variational inequality (26) is strictly monotone on $\bar{\mathcal{K}}$. Then, if variational inequality (26) admits a solution, $(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi^*, q^{S^*})$, that is the only solution.

The Algorithm - The Modified Projection Method

The modified projection method

Step 0: Initialization

Start with $X^0 \in \mathcal{K}$. Set $\mathcal{T} := 1$ and select a, such that $0 < a \leq \frac{1}{L}$, where L is the Lipschitz continuity constant for $F(X)$.

Step 1: Construction and Computation

Compute $\overline{X}^{\mathcal{T}-1}$ by solving the variational inequality subproblem:

$$\langle \overline{X}^{\mathcal{T}-1} + (aF(X^{\mathcal{T}-1}) - X^{\mathcal{T}-1})^T, X - \overline{X}^{\mathcal{T}-1} \rangle \geq 0, \quad \forall X \in \mathcal{K}.$$

Step 2: Adaptation

Compute $X^{\mathcal{T}}$ by solving the variational inequality subproblem:

$$\langle X^{\mathcal{T}} + (aF(\overline{X}^{\mathcal{T}-1}) - X^{\mathcal{T}-1})^T, X - X^{\mathcal{T}} \rangle \geq 0, \quad \forall X \in \mathcal{K}.$$

Step 3: Convergence Verification

If $| X_l^{\mathcal{T}} - X_l^{\mathcal{T}-1} | \leq \epsilon$, for all l, with $\epsilon > 0$, a prespecified tolerance, then stop; else set $\mathcal{T} := \mathcal{T} + 1$, and go to step 1.

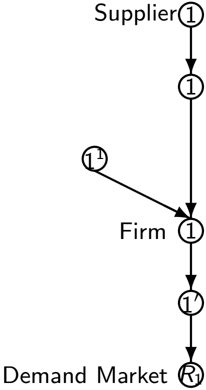


Figure: Supply Chain Network Topology for Example 1

The product of firm 1 requires only one component 1^1. 2 units of 1^1 are needed for producing one unit of firm 1's product. Thus,

$$\theta_{11} = 2.$$

The capacity of the supplier is:

$$CAP^S_{111} = 120.$$

The firm's capacity for producing its component is:

$$CAP^F_{11} = 80.$$

The value that represents the perfect component quality is:

$$q^U_{11} = 75.$$

The supplier's production cost is:

$$f^S_{11}(Q^S_{111}, q^S_{111}) = 5Q^S_{111} + 0.8(q^S_{111} - 62.5)^2.$$

The supplier's transportation cost is:

$$tc^S_{111}(Q^S_{111}, q^S_{111}) = 0.5Q^S_{111} + 0.2(q^S_{111} - 125)^2 + 0.3Q^S_{111}q^S_{111},$$

and its opportunity cost is:

$$oc_{111}(\pi_{111}) = 0.7(\pi_{111} - 100)^2.$$

The firm's assembly cost is:

$$f_1(Q_{11}, \alpha_1^F) = 0.75Q_{11}^2 + 200\alpha_1^{F^2} + 200\alpha_1^F + 25Q_{11}\alpha_1^F.$$

The firm's production cost for producing its component is:

$$f_{11}^F(Q_{11}^F, q_{11}^F) = 2.5Q_{11}^{F^2} + 0.5(q_{11}^F - 60)^2 + 0.1Q_{11}^F q_{11}^F,$$

and its transaction cost is:

$$c_{111}(Q_{111}^S) = 0.5Q_{111}^{S^2} + Q_{111}^S + 100.$$

The firm's transportation cost for shipping its product to the demand market is:

$$tc_{11}^F(Q_{11}, q_1) = 0.5Q_{11}^2 + 0.02q_1^2 + 0.1Q_{11}q_1,$$

and the demand price function at demand market R_1 is:

$$\rho_{11}(d_{11}, q_1) = -d_{11} + 0.7q_1 + 1000,$$

where $q_1 = \alpha_1^F \omega_{11} \frac{Q_{11}^F q_{11}^F + Q_{111}^S q_{111}^S}{Q_{11}^F + Q_{111}^S}$ and $\omega_{11} = 1$.

The equilibrium solution obtained using the modified projection method is:

$$Q_{11}^{*} = 89.26, \quad Q_{11}^{F^*} = 60.16, \quad Q_{111}^{S^*} = 118.38, \quad q_{11}^{F^*} = 71.17,$$

$$q_{111}^{S^*} = 57.25, \quad \pi_{11}^{*} = 184.53, \quad \alpha_{1}^{F^*} = 1.00, \quad \lambda_{11}^{*} = 305.25.$$

with the induced demand, demand price, and product quality being

$$d_{11} = 89.26, \quad \rho_{11} = 954.10, \quad q_1 = 61.94.$$

The profit of the firm is 33,331.69, and the profit of the supplier is 13,218.67.

For this example, the eigenvalues of the symmetric part of the Jacobian matrix of $G(Y)$ are all positive. Therefore, $\nabla G(Y)$ is positive-definite, and $G(Y)$ is strictly monotone. The uniqueness of the solution $(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi^*, q^{S^*})$ and the convergence of the modified projection method are then guaranteed.

Example 1 - Sensitivity Analysis

We maintain the capacity of the firm at 80, and vary the capacity of the supplier from 0 to 20, 40, 60, 80, 100, and 120.

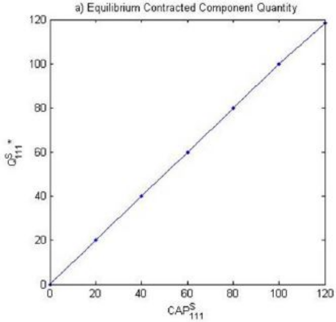


Figure: Equilibrium Component Quantities as the Capacity of the Supplier Varies

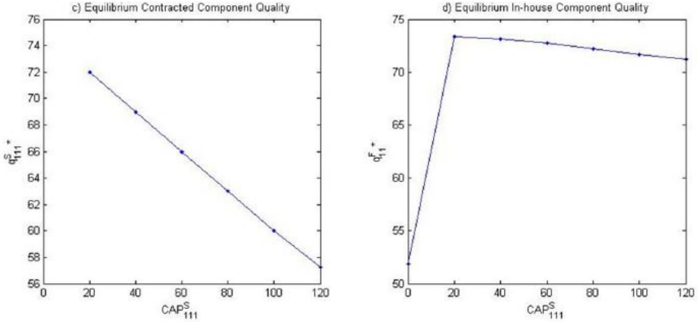


Figure: Equilibrium Component Quality Levels as the Capacity of the Supplier Varies

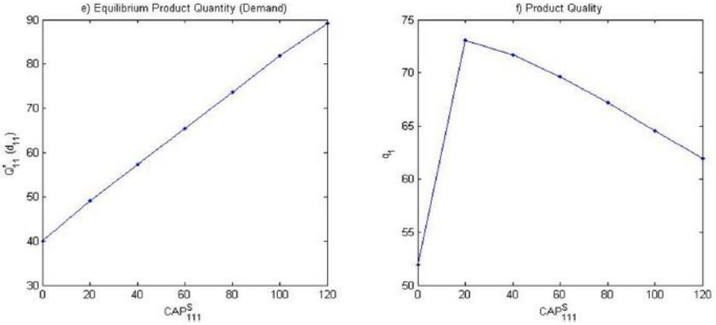


Figure: Equilibrium Product Quantity (Demand) and Product Quality as the Capacity of the Supplier Varies

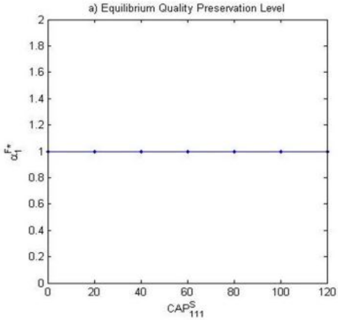


Figure: Equilibrium Quality Preservation Level and Equilibrium Lagrange Multiplier as the Capacity of the Supplier Varies

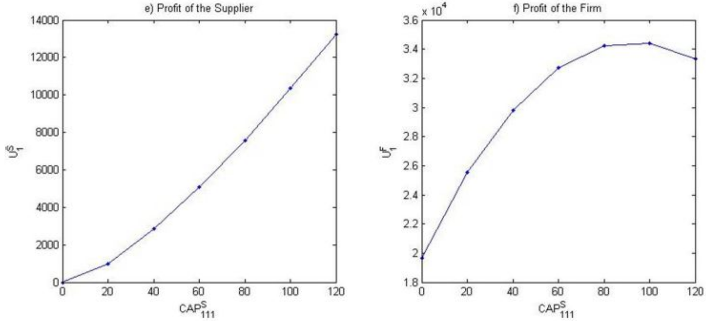


Figure: The Supplier's Profit and the Firm's Profit as the Capacity of the Supplier Varies

We then maintain the capacity of the supplier at 120, and vary the capacity of the firm from 0 to 20, 40, 60, and 80.

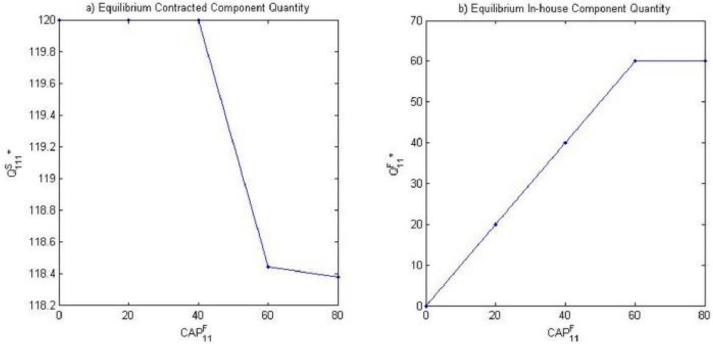


Figure: Equilibrium Component Quantities as the Capacity of the Firm Varies

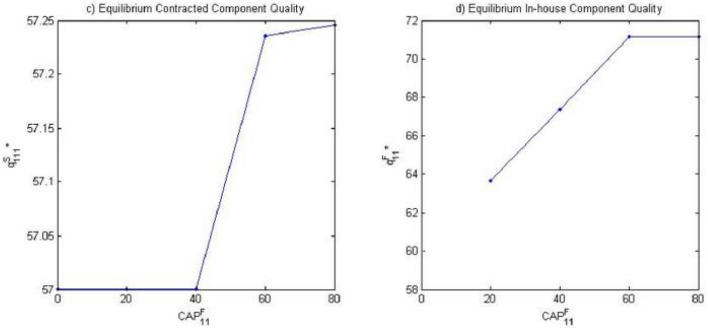


Figure: Equilibrium Component Quality Levels as the Capacity of the Firm Varies

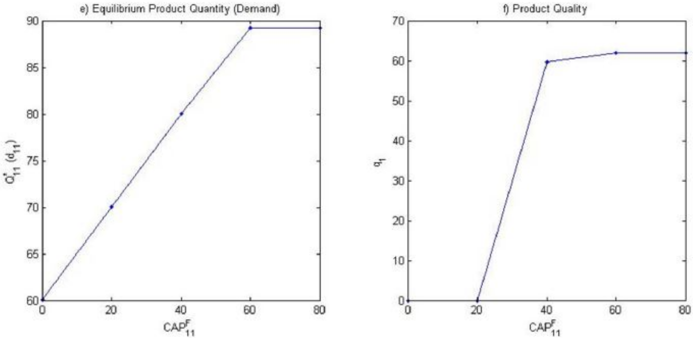


Figure: Equilibrium Product Quantity (Demand) and Product Quality as the Capacity of the Firm Varies

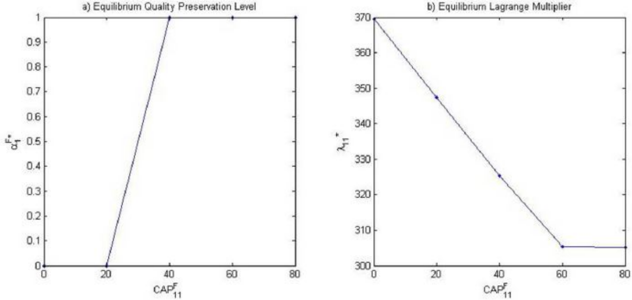


Figure: Equilibrium Quality Preservation Level and Equilibrium Lagrange Multiplier as the Capacity of the Firm Varies

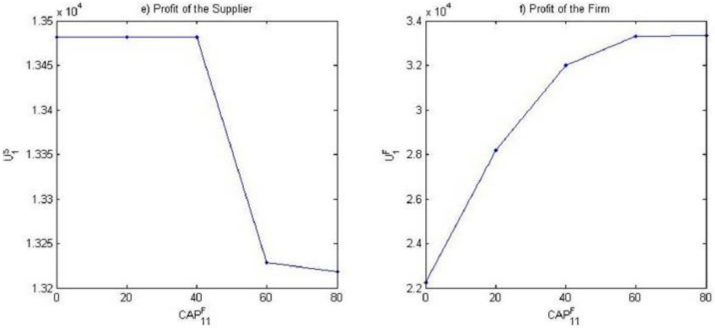


Figure: The Supplier's Profit and the Firm's Profit as the Capacity of the Firm Varies

From \ To	CAP^S_{111}=0	20	40	60	80	100	120
CAP^S_{111}=0	–	0.97, 5.89	2.86, 10.17	5.08, 13.09	7.57, 14.62	10.37, 14.77	13.22, 13.69
20	-0.97, -5.89	–	1.90, 4.28	4.09, 7.20	6.60, 8.73	9.40, 8.88	12.25, 7.80
40	-2.86, -10.17	-1.90, -4.28	–	2.20, 2.92	4.70, 4.45	7.51, 4.60	10.36, 3.52
60	-5.06, -13.09	-4.09, -7.20	-2.20, -2.92	–	2.50, 1.53	5.31, 1.68	8.16, 0.60
80	-7.57, -14.62	-6.60, -8.73	-4.70, -4.45	-2.50, -1.53	–	2.81, 0.15	*5.66, -0.93*
100	-10.37, -14.77	-9.40, -8.88	-7.51, -4.60	-5.31, -1.68	-2.81, -0.15	–	*2.85, -1.08*
120	-13.22, -13.69	-12.25, -7.80	-10.36, -3.52	-8.16, -0.60	*-5.65, 0.93*	*-2.85, 1.08*	–

Figure: Maximum Acceptable Investments ($\times 10^3$) for Capacity Changing when the Capacity of the Firm Maintains 80 but that of the Supplier Varies

From \ To	CAP_{11}^F=0	20	40	60	80
CAP_{11}^F=0	–	0.00, 5.94	0.00, 9.77	*-0.25, 11.10*	*-0.26, 11.10*
20	0.00, -5.94	–	0.00, 3.83	*-0.25, 5.16*	*-0.26, 5.16*
40	0.00, -9.77	0.00, -3.83	–	*-0.25, 1.33*	*-0.26, 1.33*
60	*0.25, -11.10*	*0.25, -5.16*	*0.25, -1.33*	–	*-0.01, 0.004*
80	*0.26, -11.10*	*0.26, -5.16*	*0.26, -1.33*	*0.01, -0.004*	–

Figure: Maximum Acceptable Investments ($\times 10^3$) for Capacity Changing when the Capacity of the Supplier Maintains 120 but that of the Firm Varies

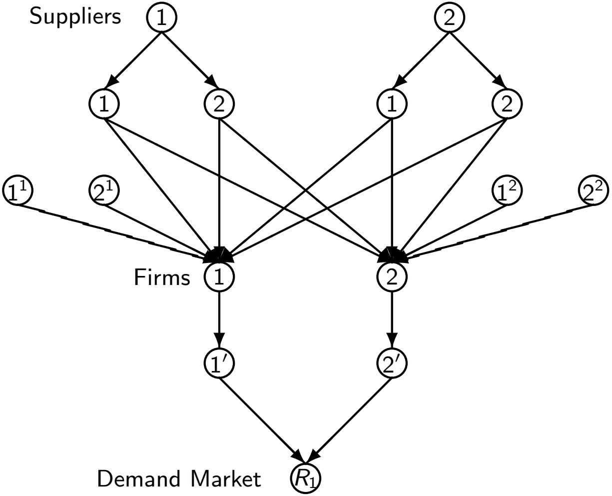


Figure: Supply Chain Network Topology for Example 2

$$\theta_{11} = 1, \quad \theta_{12} = 2, \quad \theta_{21} = 2, \quad \theta_{22} = 1.$$

The ratio of the importance of the quality of the components to the quality of one unit product is:

$$\omega_{11} = 0.2, \quad \omega_{12} = 0.8, \quad \omega_{21} = 0.4, \quad \omega_{22} = 0.6.$$

The capacities of the suppliers are:

$$CAP^S_{111} = 80, \quad CAP^S_{112} = 100, \quad CAP^S_{121} = 100, \quad CAP^S_{122} = 60,$$

$$CAP^S_{211} = 60, \quad CAP^S_{212} = 100, \quad CAP^S_{221} = 100, \quad CAP^S_{222} = 50.$$

The firms' capacities for in-house component production are:

$$CAP^F_{11} = 30, \quad CAP^F_{12} = 30, \quad CAP^F_{21} = 30, \quad CAP^F_{22} = 30.$$

The values representing the perfect component quality are:

$$q^U_{11} = 60, \quad q^U_{12} = 75, \quad q^U_{21} = 60, \quad q^U_{22} = 75.$$

The suppliers' production costs are:

$$f_{11}^S = 0.4(Q_{111}^S + Q_{121}^S) + 1.5(q_{111}^S - 50)^2 + 1.5(q_{121}^S - 50)^2 + q_{211}^S + q_{221}^S,$$

$$f_{12}^S = 0.4(Q_{112}^S + Q_{122}^S) + 2(q_{112}^S - 45)^2 + 2(q_{122}^S - 45)^2 + q_{212}^S + q_{222}^S,$$

$$f_{21}^S = Q_{211}^S + Q_{221}^S + 2(q_{211}^S - 31.25)^2 + 2(q_{221}^S - 31.25)^2 + q_{111}^S + q_{121}^S,$$

$$f_{12}^S = Q_{212}^S + Q_{222}^S + (q_{212}^S - 85)^2 + (q_{222}^S - 85)^2 + q_{112}^S + q_{122}^S.$$

Their transportation costs are:

$$tc_{111}^S = 0.2Q_{111}^S + 1.2(q_{111}^S - 41.67)^2, \quad tc_{112}^S = 0.1Q_{112}^S + 1.2(q_{112}^S - 37.5)^2,$$

$$tc_{121}^S = 0.2Q_{121}^S + 1.4(q_{121}^S - 39.29)^2, \quad tc_{122}^S = 0.1Q_{122}^S + 1.1(q_{122}^S - 36.36)^2,$$

$$tc_{211}^S = 0.3Q_{211}^S + 1.3(q_{211}^S - 30.77)^2, \quad tc_{212}^S = 0.4Q_{212}^S + 1.7(q_{212}^S - 32.35)^2,$$

$$tc_{221}^S = 0.2Q_{221}^S + 1.3(q_{221}^S - 30.77)^2, \quad tc_{222}^S = 0.1Q_{222}^S + 1.5(q_{222}^S - 30)^2.$$

The opportunity costs of the suppliers are:

$$oc_{111} = 5(\pi_{111} - 80)^2 + 0.5\pi_{211}, \quad oc_{112} = 9(\pi_{112} - 80)^2 + \pi_{212},$$

$$oc_{121} = 5(\pi_{121} - 100)^2 + \pi_{221}, \quad oc_{122} = 7.5(\pi_{122} - 50)^2 + 0.1\pi_{222},$$

$$oc_{211} = 5(\pi_{211} - 50)^2 + 2\pi_{111}, \quad oc_{212} = 8(\pi_{212} - 70)^2 + 0.5\pi_{112},$$

$$oc_{221} = 9(\pi_{221} - 60)^2 + \pi_{121}, \quad oc_{222} = 8(\pi_{222} - 60)^2 + 0.5\pi_{122}.$$

The firms' assembly costs are:

$$f_1(Q_{11}, \alpha_1^F) = 3Q_{11}^2 + 0.5Q_{11}\alpha_1^F + 100\alpha_1^{F^2} + 50\alpha_1^F,$$

$$f_2(Q_{21}, \alpha_2^F) = 2.75Q_{21}^2 + 0.6Q_{21}\alpha_2^F + 100\alpha_2^{F^2} + 50\alpha_2^F.$$

Their production costs for producing components are:

$$f_{11}^F(Q_{11}^F, q_{11}^F) = Q_{11}^{F^2} + 0.0001Q_{11}^F q_{11}^F + 1.1(q_{11}^F - 36.36)^2,$$

$$f_{12}^F(Q_{12}^F, q_{12}^F) = 1.25Q_{12}^{F^2} + 0.0001Q_{12}^F q_{12}^F + 1.2(q_{12}^F - 41.67)^2,$$

$$f_{21}^F(Q_{21}^F, q_{21}^F) = Q_{21}^{F^2} + 0.0001Q_{21}^F q_{21}^F + 1.5(q_{21}^F - 33.33)^2,$$

$$f_{22}^F(Q_{22}^F, q_{22}^F) = 0.75Q_{22}^{F^2} + 0.0001Q_{22}^F q_{22}^F + 1.25(q_{22}^F - 36)^2.$$

The transaction costs are:

$$c_{111}(Q_{111}^S) = 0.5Q_{111}^{S^2} + Q_{111}^S + 100, \quad c_{112}(Q_{112}^S) = 0.5Q_{112}^{S^2} + 0.5Q_{112}^S + 150,$$

$$c_{121}(Q_{211}^S) = 0.75Q_{211}^{S^2} + 0.75Q_{211}^S + 150, \quad c_{122}(Q_{212}^S) = Q_{212}^{S^2} + Q_{212}^S + 100,$$

$$c_{211}(Q_{121}^S) = 0.75Q_{121}^{S^2} + Q_{121}^S + 150, \quad c_{212}(Q_{122}^S) = 0.5Q_{122}^{S^2} + 0.75Q_{122}^S + 100,$$

$$c_{221}(Q_{221}^S) = 0.8Q_{221}^{S^2} + 0.25Q_{221}^S + 100, \quad c_{222}(Q_{222}^S) = 0.5Q_{222}^{S^2} + Q_{222}^S + 175.$$

The firms' transportation costs are:

$$tc^F_{11}(Q_{11}, q_1) = 3Q^2_{11} + 0.3Q_{11}q_1 + 0.25q_1,$$

$$tc^F_{21}(Q_{21}, q_2) = 3Q^2_{21} + 0.3Q_{21}q_2 + 0.1q_2,$$

and the demand price functions are:

$$\rho_{11}(d_{11}, d_{21}, q_1, q_2) = -3d_{11} - 1.3d_{21} + q_1 + 0.74q_2 + 2200,$$

$$\rho_{21}(d_{21}, d_{11}, q_2, q_1) = -3.5d_{21} - 1.4d_{11} + 1.1q_2 + 0.9q_1 + 1800,$$

where $q_1 = \alpha^F_1(\omega_{11}\frac{Q^F_{11}q^F_{11}+Q^S_{111}q^S_{111}+Q^S_{211}q^S_{211}}{Q^F_{11}+Q^S_{111}+Q^S_{211}} + \omega_{12}\frac{Q^F_{12}q^F_{12}+Q^S_{112}q^S_{112}+Q^S_{212}q^S_{212}}{Q^F_{12}+Q^S_{112}+Q^S_{212}})$ and $q_2 = \alpha^F_2(\omega_{21}\frac{Q^F_{21}q^F_{21}+Q^S_{121}q^S_{121}+Q^S_{221}q^S_{221}}{Q^F_{21}+Q^S_{121}+Q^S_{221}} + \omega_{22}\frac{Q^F_{22}q^F_{22}+Q^S_{122}q^S_{122}+Q^S_{222}q^S_{222}}{Q^F_{22}+Q^S_{122}+Q^S_{222}})$.

The modified projection method converges to the following equilibrium solution:

$$Q_{11}^* = 93.56, \quad Q_{21}^* = 71.34,$$

$$Q_{11}^{F^*} = 30.00, \quad Q_{12}^{F^*} = 30.00, \quad Q_{21}^{F^*} = 30.00, \quad Q_{22}^{F^*} = 30.00,$$
$$Q_{111}^{S^*} = 27.37, \quad Q_{112}^{S^*} = 100.00, \quad Q_{121}^{S^*} = 45.44, \quad Q_{122}^{S^*} = 23.35,$$
$$Q_{211}^{S^*} = 36.19, \quad Q_{212}^{S^*} = 57.12, \quad Q_{221}^{S^*} = 67.24, \quad Q_{222}^{S^*} = 17.99,$$
$$q_{11}^{F^*} = 38.26, \quad q_{12}^{F^*} = 45.15, \quad q_{21}^{F^*} = 34.93, \quad q_{22}^{F^*} = 41.71,$$
$$q_{111}^{S^*} = 46.30, \quad q_{112}^{S^*} = 42.19, \quad q_{121}^{S^*} = 44.83, \quad q_{122}^{S^*} = 41.94,$$
$$q_{211}^{S^*} = 31.06, \quad q_{212}^{S^*} = 51.85, \quad q_{221}^{S^*} = 31.06, \quad q_{222}^{S^*} = 52.00,$$
$$\pi_{111}^* = 82.74, \quad \pi_{112}^* = 85.56, \quad \pi_{121}^* = 104.54, \quad \pi_{122}^* = 51.56,$$
$$\pi_{211}^* = 53.62, \quad \pi_{212}^* = 73.57, \quad \pi_{221}^* = 63.74, \quad \pi_{222}^* = 61.12,$$
$$\alpha_1^{F^*} = 1.00, \quad \alpha_2^{F^*} = 1.00,$$
$$\lambda_{11}^* = 109.83, \quad \lambda_{12}^* = 187.06, \quad \lambda_{21}^* = 172.34, \quad \lambda_{22}^* = 76.58,$$

and the induced demands, demand prices, and product quality levels are:

$$d_{11} = 93.56, \quad d_{21} = 71.34, \quad \rho_{11} = 1,901.07, \quad \rho_{21} = 1,504.22,$$

$$q_1 = 44.06, \quad q_2 = 41.13.$$

The firms' profits are 94,610.69 and 57,787.69, respectively, and those of the suppliers are 15,671.13 and 6923.20.

The eigenvalues of the symmetric part of the Jacobian matrix of $G(Y)$ are all positive. Therefore, the uniqueness of the solution $(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi^*, q^{S^*})$ and the convergence of the modified projection method are guaranteed.

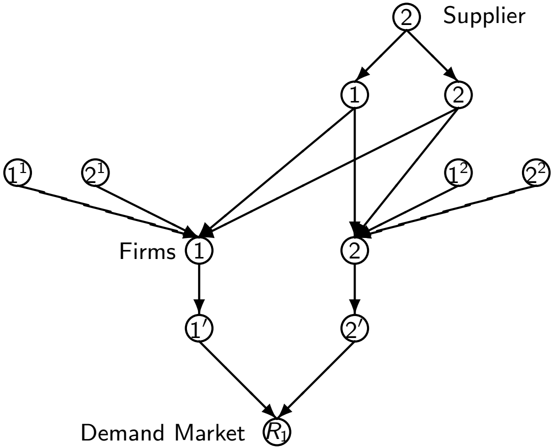


Figure: Supply Chain Network Topology With Disruption to Supplier 1

The equilibrium solution achieved by the modified projection method is:

$$Q_{11}^* = 65.00, \quad Q_{21}^* = 65.00,$$

$$Q_{11}^{F^*} = 30.00, \quad Q_{12}^{F^*} = 30.00, \quad Q_{21}^{F^*} = 30.00, \quad Q_{22}^{F^*} = 30.00,$$

$$Q_{111}^{S^*} = 0.00, \quad Q_{112}^{S^*} = 0.00, \quad Q_{121}^{S^*} = 0.00, \quad Q_{122}^{S^*} = 0.00,$$

$$Q_{211}^{S^*} = 35.00, \quad Q_{212}^{S^*} = 100.00, \quad Q_{221}^{S^*} = 100.00, \quad Q_{222}^{S^*} = 35.00,$$

$$q_{11}^{F^*} = 38.26, \quad q_{12}^{F^*} = 45.16, \quad q_{21}^{F^*} = 34.93, \quad q_{22}^{F^*} = 41.75,$$

$$q_{211}^{S^*} = 31.06, \quad q_{212}^{S^*} = 51.85, \quad q_{221}^{S^*} = 31.06, \quad q_{222}^{S^*} = 52.00,$$

$$\pi_{211}^* = 53.50, \quad \pi_{212}^* = 76.25, \quad \pi_{221}^* = 65.56, \quad \pi_{222}^* = 62.19,$$

$$\alpha_1^{F^*} = 1.00, \quad \alpha_2^{F^*} = 1.00,$$

$$\lambda_{11}^* = 107.53, \quad \lambda_{12}^* = 448.93, \quad \lambda_{21}^* = 242.02, \quad \lambda_{22}^* = 95.98,$$

and the induced demands, demand prices, and product quality levels are:

$$d_{11} = 65.00, \quad d_{21} = 65.00 \quad \rho_{11} = 1,998.07, \quad \rho_{21} = 1,569.17,$$

$$q_1 = 47.12, \quad q_2 = 41.14.$$

The firms' profits are 80,574.83 and 57,406.47, respectively, and supplier 2's profit is 13,635.49. The uniqueness of the solution $(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi^*, q^{S^*})$ is guaranteed.

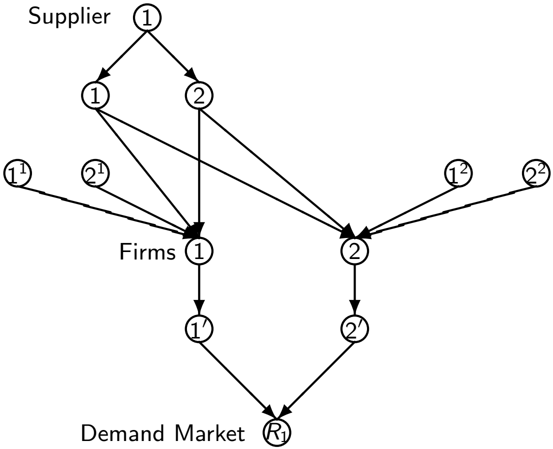


Figure: Supply Chain Network Topology With Disruption to Supplier 2

The modified projection method converges to the following equilibrium solution:

$$Q_{11}^* = 65.00, \quad Q_{21}^* = 63.79,$$

$$Q_{11}^{F^*} = 30.00, \quad Q_{12}^{F^*} = 30.00, \quad Q_{21}^{F^*} = 30.00, \quad Q_{22}^{F^*} = 30.00,$$
$$Q_{111}^{S^*} = 35.00, \quad Q_{112}^{S^*} = 100.00, \quad Q_{121}^{S^*} = 97.58, \quad Q_{122}^{S^*} = 33.79,$$
$$Q_{211}^{S^*} = 0.00, \quad Q_{212}^{S^*} = 0.00, \quad Q_{221}^{S^*} = 0.00, \quad Q_{222}^{S^*} = 0.00,$$
$$q_{11}^{F^*} = 38.26, \quad q_{12}^{F^*} = 45.16, \quad q_{21}^{F^*} = 34.93, \quad q_{22}^{F^*} = 41.75,$$
$$q_{111}^{S^*} = 46.30, \quad q_{112}^{S^*} = 42.19, \quad q_{121}^{S^*} = 44.83, \quad q_{122}^{S^*} = 41.94,$$
$$\pi_{111}^* = 83.50, \quad \pi_{112}^* = 85.56, \quad \pi_{121}^* = 109.76, \quad \pi_{122}^* = 52.25,$$
$$\alpha_1^{F^*} = 1.00, \quad \alpha_2^{F^*} = 1.00,$$
$$\lambda_{11}^* = 119.17, \quad \lambda_{12}^* = 442.79, \quad \lambda_{21}^* = 256.75, \quad \lambda_{22}^* = 86.75.$$

The induced demands, demand prices, and product quality levels are:

$$d_{11} = 65.00, \quad d_{21} = 63.79, \quad \rho_{11} = 1,996.05, \quad \rho_{21} = 1,570.59,$$

$$q_1 = 42.82, \quad q_2 = 42.11.$$

The firms' profits are 83,895.42 and 53,610.96, respectively, and supplier 1's profit is 22,729.18. The uniqueness of the solution $(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi^*, q^{S^*})$ is guaranteed.

- Without supplier 1, the profit of firm 1 decreases by 14.84%, and that of firm 2 decreases by 0.66%. Therefore, from this perspective, supplier 1 is more important to firm 1 than to firm 2. The value of supplier 1 to firm 1 is 14,035.86, and that to firm 2 is 381.22, which are measured by the associated profit declines.
- Without supplier 2, firm 1's profit declines by 11.33%, and that of firm 2 reduces by 7.23%. Thus, supplier 2 is more important to firm 1 than to firm 2 under this disruption. The value of supplier 2 to firm 1 is 10,715.27, and that to firm 2 is 4,176.73.
- In addition, according to the above results, supplier 1 is more important than supplier 2 to firm 1, and to firm 2, supplier 2 is more important.

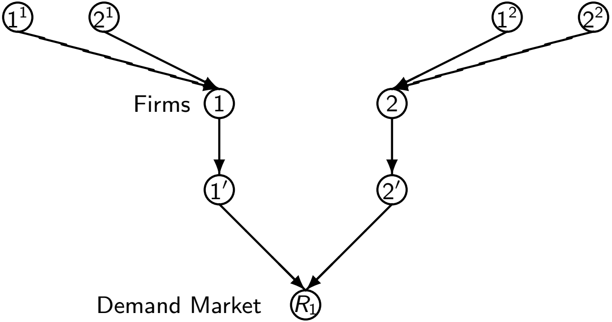


Figure: Supply Chain Network Topology With Disruption to Suppliers 1 and 2

The equilibrium solution obtained using the modified projection method is:

$$Q_{11}^* = 15.00, \quad Q_{21}^* = 15.00,$$

$$Q_{11}^{F^*} = 15.00, \quad Q_{12}^{F^*} = 30.00, \quad Q_{21}^{F^*} = 30.00, \quad Q_{22}^{F^*} = 30.00,$$

$$Q_{111}^{S^*} = 0.00, \quad Q_{112}^{S^*} = 0.00, \quad Q_{121}^{S^*} = 0.00, \quad Q_{122}^{S^*} = 0.00,$$

$$Q_{211}^{S^*} = 0.00, \quad Q_{212}^{S^*} = 0.00, \quad Q_{221}^{S^*} = 0.00, \quad Q_{222}^{S^*} = 0.00,$$

$$q_{11}^{F^*} = 37.29, \quad q_{12}^{F^*} = 45.08, \quad q_{21}^{F^*} = 35.71., \quad q_{22}^{F^*} = 37.90,$$

$$\alpha_1^{F^*} = 1.00, \quad \alpha_2^{F^*} = 1.00,$$

$$\lambda_{11}^* = 30.46, \quad \lambda_{12}^* = 967.28, \quad \lambda_{21}^* = 772.88, \quad \lambda_{22}^* = 22.63.$$

The induced demands, demand prices, and the product quality levels are:

$$d_{11} = 15.00, \quad d_{21} = 15.00, \quad \rho_{11} = 2,206.42, \quad \rho_{21} = 1,806.40,$$

$$q_1 = 43.52, \quad q_2 = 37.02.$$

The firms' profits are 30,016.91 and 24,391.32, respectively. The uniqueness of the solution $(Q^*, Q^{F^*}, Q^{S^*}, q^{F^*}, \alpha^{F^*}, \pi^*, q^{S^*})$ is guaranteed.

Compared to Example 2, without the suppliers, the demands at the demand market decrease, the firms' product quality levels decrease, and the prices at the demand market increase. Firm 1's profit deceases by 68.27%, firm 2's reduces by 57.79%. The value of the suppliers to firm 1 is 64,593.78, and that to firm 2 is 33,396.37.

Summary and Conclusions

- The novelty of this framework lies in its generality and its computability.
- It is illustrated with numerical examples, accompanied by sensitivity analysis that explores such critical issues as the impacts of capacity disruptions and the potential investments in capacity enhancements.
- We also conduct sensitivity analysis to reveal the impacts of specific supplier unavailability along with their values as reflected in the profits of the firms and in the quality of the finished products.
- With knowledge of the values of the suppliers to the firms, the firms can make more specific, targeted efforts in their supplier management strategies and in their contingency plans in the case of supplier disruptions.

Thank you!

- For more information, please visit http://supernet.isenberg.umass.edu.