Consumer Learning of Product Quality with Time Delay: Insights from Spatial Price Equilibrium Models with Differentiated Products

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Outline

- Background and Motivation
- Spatial Price Equilibrium with Quality Information and Product Differentiation
- Numerical Examples
- Summary and Conclusions

Background and Motivation

Spatial price equilibrium models have served as the foundation for the study of numerous perfectly competitive markets with network structures in regional science, economics, and operations research/management science.

Especially in the case of agricultural products, there have been numerous cases of serious shortcoming in terms of quality of food (Strom (2013) and McDonald (2014)), which have resulted in illnesses and even death.

Overview

- We present two distinct, novel models in this paper.
- The first model is a static spatial price equilibrium model with perfect information on product quality and with differentiated products.
- The second model is an adaptive spatial price equilibrium model under quality information asymmetry, in which consumers at the demand markets learn the quality of the product with a time delay.
- We provide measures to quantify the consumer welfare under the two scenarios along with the value of perfect information for consumers.
- The impacts of quality information asymmetry and consumer learning of quality are studied.
- We theoretically establish that, as time approaches infinity, the equilibrium of the adaptive model under information asymmetry converges to the that of the corresponding static model under perfect information.

Literature Review

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Literature Review

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"Integrating learning models of demand with supply side models remains under-explored and should be another important area for future research."

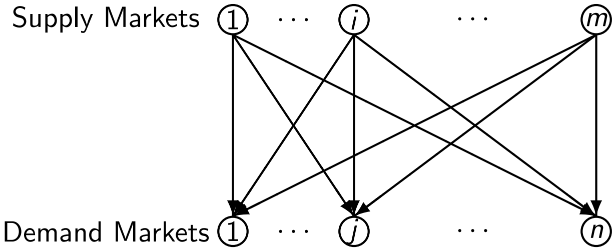

Figure: The Bipartite Network Structure of the Spatial Price Equilibrium Problems with Quality Information and Product Differentiation

Supply Functions

$$s_i = s_i(\pi), \quad i = 1, \ldots, m. \tag{1}$$

Transportation Cost Functions

$$c_{ij} = c_{ij}(Q), \quad i = 1, \ldots, m; j = 1, \ldots, n. \tag{2}$$

Quality levels

$$q_i = q_i(\pi_i), \quad i = 1, \ldots, m. \tag{3}$$

Perception of Quality

$$\hat{q}_{ij} = q_i = q_i(\pi_i), \quad i = 1, \ldots, m; j = 1, \ldots, n. \tag{4}$$

The Model: Spatial Price Equilibrium with Product Differentiation Under Perfect Quality Information

Demand Functions

$$d_{ij} = d_{ij}(\rho, \hat{q}), \quad i = 1, \ldots, m; j = 1, \ldots, n. \tag{5a}$$

$$d_{ij} = d_{ij}(\rho, q(\pi)), \quad i = 1, \ldots, m; j = 1, \ldots, n. \tag{5b}$$

Feasible Set

$$Q_{ij} \geq 0, \quad i = 1, \ldots, m; j = 1, \ldots, n, \tag{6}$$

$$\rho_{ij} \geq 0, \quad i = 1, \ldots, m; j = 1, \ldots, n, \tag{7}$$

$$\pi_i \geq 0, \quad i = 1, \ldots, m, \tag{8}$$

and we define the feasible set $K^1 \equiv \{(Q, \rho, \pi) \in R_+^{2mn+m}\}$.

Equilibrium Conditions

Definition 1: Spatial Price Equilibrium Conditions with Product Differentiation Under Perfect Quality Information

A product shipment, demand price, and supply price pattern $(Q^*, \rho^*, \pi^*) \in K^1$ *is a spatial equilibrium with product differentiation under perfect quality information if it satisfies the following conditions: for each pair of supply and demand markets* (i, j); $i = 1, \ldots, m$; $j = 1, \ldots, n$:

$$\pi_i^* + c_{ij}(Q^*) \begin{cases} = \rho_{ij}^*, & \text{if } Q_{ij}^* > 0, \\ \geq \rho_{ij}^*, & \text{if } Q_{ij}^* = 0, \end{cases} \tag{9}$$

and

$$d_{ij}(\rho^*, q(\pi^*)) \begin{cases} = Q_{ij}^*, & \text{if } \rho_{ij}^* > 0, \\ \leq Q_{ij}^*, & \text{if } \rho_{ij}^* = 0, \end{cases} \tag{10}$$

and for each supply market i; $i = 1, \ldots, m$:

$$s_i(\pi^*) \begin{cases} = \sum_{j=1}^n Q_{ij}^*, & \text{if } \pi_i^* > 0, \\ \geq \sum_{j=1}^n Q_{ij}^*, & \text{if } \pi_i^* = 0. \end{cases} \tag{11}$$

Variational Inequality Formulation

Theorem 1: Variational Inequality Formulation of Spatial Price Equilibrium with Product Differentiation Under Perfect Quality Information

A product shipment, demand price, and supply price pattern $(Q^*, \rho^*, \pi^*) \in K^1$ *is a spatial price equilibrium with product differentiation under perfect quality information according to Definition 1 if and only if it satisfies the variational inequality problem:*

$$\sum_{i=1}^{m}\sum_{j=1}^{n}(\pi_i^* + c_{ij}(Q^*) - \rho_{ij}^*) \times (Q_{ij} - Q_{ij}^*) + \sum_{i=1}^{m}\sum_{j=1}^{n}(Q_{ij}^* - d_{ij}(\rho^*, q(\pi^*))) \times (\rho_{ij} - \rho_{ij}^*)$$

$$+ \sum_{i=1}^{m}(s_i(\pi^*) - \sum_{j=1}^{n} Q_{ij}^*) \times (\pi_i - \pi_i^*) \geq 0, \quad \forall (Q, \rho, \pi) \in K^1. \qquad (12)$$

Variational Inequality Formulation - Standard Form

Variational Inequality Formulation - Standard Form

Determine $X^* \in \mathcal{K} \subset R^N$*, such that*

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{13}$$

where $\mathcal{K}$ *is the feasible set, which must be closed and convex.* $\langle \cdot, \cdot \rangle$ *denotes the inner product in N-dimensional Euclidean space.*

$X \equiv (Q, \rho, \pi)$.

$F(X) \equiv (F^1(X), F^2(X), F^3(X))$, *where*
$F^1_{ij}(X) = \pi_i + c_{ij}(Q) - \rho_{ij}$; $i = 1, \ldots, m$; $j = 1, \ldots, n$,
$F^2_{ij}(X) = Q_{ij} - d_{ij}(\rho, q(\pi))$; $i = 1, \ldots, m$; $j = 1, \ldots, n$, *and*
$F^3_i(X) = s_i(\pi) - \sum_{j=1}^{n} Q_{ij}$; $i = 1, \ldots, m$.

Also, we define the feasible set $\mathcal{K} \equiv K^1$*, and let* $N = 2mn + m$.

Consumer Welfare

Consumer Welfare Under Perfect Quality Information

$$CW_{ij}^P = \int_0^{d_{ij}(\rho^*, q(\pi^*))} \rho_{ij}(\hat{d}_{ij}^*, d_{ij}, q(\pi^*))\,\mathrm{d}(d_{ij}) - \rho_{ij}^* d_{ij}(\rho^*, q(\pi^*)),$$

$$i = 1, \ldots, m; j = 1, \ldots, n, \qquad (15)$$

Supply Functions

$$s_i^t = s_i^t(\pi^t), \quad i = 1, \ldots, m, \tag{16}$$

Transportation Cost Functions

$$c_{ij}^t = c_{ij}^t(Q^t), \quad i = 1, \ldots, m; j = 1, \ldots, n, \tag{17}$$

Quality levels

$$q_i^t = q_i^t(\pi_i^t), \quad i = 1, \ldots, m, \tag{18}$$

Perception of Quality

$$\hat{q}_{ij}^t = \begin{cases} \hat{q}_{ij}^1, & \text{if} \quad t = 1, \\ \hat{q}_{ij}^t(q_i^{t-1}, \hat{q}_{ij}^{t-1}), & \text{if} \quad t \geq 2. \end{cases} \tag{19}$$

Demand Functions

$$d_{ij}^t = \begin{cases} d_{ij}^1(\rho^1, \hat{q}^1), & \text{if} \quad t = 1, \\ d_{ij}^t(\rho^t, \hat{q}^t) = d_{ij}^t(\rho^t, \hat{q}^t(q^{t-1}, \hat{q}^{t-1})), & \text{if} \quad t \geq 2. \end{cases} \tag{20}$$

Equilibrium Conditions

Definition 2: Adaptive Spatial Price Equilibrium Conditions with Product Differentiation Under Information Asymmetry in Quality

A product shipment, demand price, and supply price pattern $(Q^{t*}, \rho^{t*}, \pi^{t*}) \in K^{2^t}$*, is a spatial equilibrium with product differentiation under quality information asymmetry in period* t*;* $t = 1, 2, \ldots,$ *if it satisfies the following conditions: for each pair of supply and demand markets* (i, j)*;* $i = 1, \ldots, m$*;* $j = 1, \ldots, n$*:*

$$\pi_i^{t*} + c_{ij}^t(Q^{t*}) \begin{cases} = \rho_{ij}^{t*}, & \text{if } Q_{ij}^{t^*} > 0, \\ \geq \rho_{ij}^{t*}, & \text{if } Q_{ij}^{t^*} = 0, \end{cases} \tag{21}$$

and,
if $t = 1$,

$$d_{ij}^1(\rho^{1*}, \hat{q}^1) \begin{cases} = Q_{ij}^{1^*}, & \text{if } \rho_{ij}^{1^*} > 0, \\ \leq Q_{ij}^{1^*}, & \text{if } \rho_{ij}^{1^*} = 0; \end{cases} \tag{22a}$$

Equilibrium Conditions

Definition 2: Adaptive Spatial Price Equilibrium Conditions with Product Differentiation Under Information Asymmetry in Quality

if $t \geq 2$,

$$d_{ij}^t(\rho^{t*}, \hat{q}^t(q^{t-1}, \hat{q}^{t-1})) \begin{cases} = Q_{ij}^{t^*}, & \text{if} \quad \rho_{ij}^{t^*} > 0, \\ \leq Q_{ij}^{t^*}, & \text{if} \quad \rho_{ij}^{t^*} = 0, \end{cases} \tag{22b}$$

and for each supply market i; $i = 1, \ldots, m$:

$$s_i^t(\pi^{t*}) \begin{cases} = \sum_{j=1}^n Q_{ij}^{t^*}, & \text{if} \quad \pi_i^{t*} > 0, \\ \geq \sum_{j=1}^n Q_{ij}^{t^*}, & \text{if} \quad \pi_i^{t*} = 0. \end{cases} \tag{23}$$

Variational Inequality Formulation

Theorem 2: Variational Inequality Formulation of the Adaptive Spatial Price Equilibrium with Product Differentiation Under Information Asymmetry in Quality

A product shipment, demand price, and supply price pattern $(Q^{t*}, \rho^{t*}, \pi^{t*}) \in K^{2^t}$ *is a spatial price equilibrium with product differentiation under information asymmetry in quality in period t;* $t = 1, 2, \ldots,$ *according to Definition 2 if and only if it satisfies the variational inequality problem:*
if $t = 1$*:*

$$\sum_{i=1}^{m}\sum_{j=1}^{n}(\pi_i^{1^*} + c_{ij}^1(Q^{1*}) - \rho_{ij}^{1^*}) \times (Q_{ij}^1 - Q_{ij}^{1^*}) + \sum_{i=1}^{m}\sum_{j=1}^{n}(Q_{ij}^{1^*} - d_{ij}^1(\rho^{1^*}, \hat{q}^1)) \times (\rho_{ij}^1 - \rho_{ij}^{1^*})$$

$$+ \sum_{i=1}^{m}(s_i^1(\pi^{1*}) - \sum_{j=1}^{n} Q_{ij}^{1^*}) \times (\pi_i^1 - \pi_i^{1^*}) \geq 0, \quad \forall (Q^1, \rho^1, \pi^1) \in K^{2^1}, \qquad (24a)$$

Variational Inequality Formulation

Theorem 2: Variational Inequality Formulation of the Adaptive Spatial Price Equilibrium with Product Differentiation Under Information Asymmetry in Quality

if $t \geq 2$:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}(\pi_i^{t^*}+c_{ij}^t(Q^{t^*})-\rho_{ij}^{t^*})\times(Q_{ij}^t-Q_{ij}^{t^*})+\sum_{i=1}^{m}\sum_{j=1}^{n}(Q_{ij}^{t^*}-d_{ij}^t(\rho^{t^*},\hat{q}^t(q^{t-1}(\pi^{t-1^*}),\hat{q}^{t-1}))$$

$$\times(\rho_{ij}^t-\rho_{ij}^{t^*})+\sum_{i=1}^{m}(s_i^t(\pi^{t^*})-\sum_{j=1}^{n}Q_{ij}^{t^*})\times(\pi_i^t-\pi_i^{t^*})\geq 0,$$

$$\forall (Q^t,\rho^t,\pi^t)\in K^{2^t}. \qquad (24b)$$

Variational Inequality Formulation - Standard Form

Variational Inequality Formulation - Standard Form

Determine $X^{t^*} \in \mathcal{L}^t \subset R^N$ *for time period* t*, such that*

$$\langle G^t(X^{t^*}), X^t - X^{t^*} \rangle \geq 0, \quad \forall X^t \in \mathcal{L}^t, \tag{25}$$

where $\mathcal{L}^t$ *is the closed and convex feasible set.*

$X^t \equiv (Q^t, \rho^t, \pi^t)$.

$G^t(X^t) \equiv (G^{t^1}(X^t), G^{t^2}(X^t), G^{t^3}(X^t))$ *for time period* t*, where*
$G_{ij}^{t^1}(X^t) = \pi_i^t + c_{ij}^t(Q^t) - \rho_{ij}^t$; $i = 1, \ldots, m$; $j = 1, \ldots, n$,
$G_{ij}^{t^2}(X^t) = Q_{ij}^1 - d_{ij}^1(\rho^1, \hat{q}^1))$, *if* $t = 1$*, and*
$G_{ij}^{t^2}(X^t) = Q_{ij}^t - d_{ij}^t(\rho^t, \hat{q}^t(q^{t-1}(\pi^{t-1}), \hat{q}^{t-1}))$, *if* $t \geq 2$; $i = 1, \ldots, m$;
$j = 1, \ldots, n$, *and*
$G_i^{t^3}(X^t) = s_i^t(\pi^t) - \sum_{j=1}^n Q_{ij}^t$; $i = 1, \ldots, m$.

Also, we define the feasible set $\mathcal{L}^t \equiv K^{2^t}$.

Consumer Welfare

Consumer Welfare Under Information Asymmetry in Quality

$$CW^{I^t}_{ij} = \begin{cases} \int_0^{d^1_{ij}(\rho^{1^*},\hat{q}^1)} \rho^1_{ij}(\tilde{d}^{1^*}_{ij}, d^1_{ij}, \hat{q}^1)\,\mathrm{d}(d^1_{ij}) - \rho^{1^*}_{ij} d^1_{ij}(\rho^{1^*}, \hat{q}^1), & \text{if } t = 1, \\ \int_0^{d^t_{ij}(\rho^{t*},\hat{q}^t(q^{t-1}(\pi^{t-1^*}),\hat{q}^{t-1}))} \rho^t_{ij}(\hat{d}^{t^*}_{ij}, d^t_{ij}, \hat{q}^t(q^{t-1}(\pi^{t-1^*}), \hat{q}^{t-1}))\,\mathrm{d}(d^t_{ij}) \\ -\rho^{t^*}_{ij} d^t_{ij}(\rho^{t*}, \hat{q}^t(q^{t-1}(\pi^{t-1^*}), \hat{q}^{t-1})), & \text{if } t \geq 2, \end{cases} \tag{27}$$

Value of Perfect Quality Information for Consumers

Value of Perfect Quality Information for Consumers

$$CVPI_{ij}^{t} = CW_{ij}^{P} - CW_{ij}^{I^t}, \quad i = 1, \dots, m; j = 1, \dots, n. \qquad (28)$$

Quantitative Properties

Assumption 1

Suppose that for our spatial price equilibrium problems with quality information and product differentiation, there exists a sufficiently large B and a sufficiently large $\bar{B}$, such that, for any supply and demand market pair (i, j):

$$F_{ij}^1(X) = \pi_i + c_{ij}(Q) - \rho_{ij} > 0, \tag{32}$$

$$F_{ij}^2(X) = Q_{ij} - d_{ij}(\rho, q(\pi)) > 0, \tag{33}$$

$$G_{ij}^{t^1}(X^t) = \pi_i^t + c_{ij}^t(Q^t) - \rho_{ij}^t > 0, \quad \forall t, \tag{34}$$

$$G_{ij}^{t^2}(X^t) = Q_{ij}^1 - d_{ij}^1(\rho^1, \hat{q}^1)) > 0, \quad t = 1, \tag{35}$$

$$G_{ij}^{t^2}(X^t) = Q_{ij}^t - d_{ij}^t(\rho^t, \hat{q}^t(q^{t-1}(\pi^{t-1}), \hat{q}^{t-1})) > 0, \quad t \geq 2, \tag{36}$$

for all shipment patterns Q with $Q_{ij} \geq B$ and Q^t with $Q_{ij}^t \geq B$ and for all demand price patterns ρ with $\rho_{ij} \geq \bar{B}$ and ρ^t with $\rho_{ij}^t \geq \bar{B}$. In addition, suppose that there exists a sufficiently large $\hat{B}$, such that, for any supply market i:

$$F_i^3(X) = s_i(\pi) - \sum_{j=1}^{n} Q_{ij} > 0, \tag{37}$$

$$G_i^{t^3}(X^t) = s_i^t(\pi^t) - \sum_{j=1}^{n} Q_{ij}^t > 0, \quad \forall t, \tag{38}$$

for all supply price patterns π with $\pi_i \geq \hat{B}$ and π^t with $\pi_i^t \geq \hat{B}$.

Quantitative Properties - Existence and Uniqueness

Theorem 3: Existence

Any spatial price equilibrium problem with quality information and product differentiation that satisfies Assumption 1 possesses at least one equilibrium shipment, demand price, and supply price pattern.

Theorem 4: Uniqueness

Suppose that $F(X)$ in (13) is strictly monotone on $\mathcal{K}$. Then the solution X^* to variational inequality (13) is unique, if one exists.

Similarly, suppose that G^t in (25) is strictly monotone on $\mathcal{L}^t$. Then the solution X^{t^*} to variational inequality (25) is unique, if one exists.

Theorem 5: Existence and Uniqueness

Suppose that F is strongly monotone. Then there exists a unique solution to variational inequality (13).

Similarly, suppose that G^t is strongly monotone. Then there exists a unique solution to variational inequality (25).

Quantitative Properties

For purpose of discussion, we define the following notation:

$$g(x,y) \equiv G^t(X^t, X^{t-1}), \quad \forall t. \tag{39}$$

$$K \equiv K^1 = K^{2t}, \quad \forall t, \tag{40}$$

Theorem 6: Convergence of Variational Inequality (24b) Under Information Asymmetry in Quality

Assume that there is a constant $\theta > 0$ such that

$$|||\nabla_x g^{-\frac{1}{2}}(x^1,y^1)\nabla_x g(x^2,y^2)\nabla_x g^{-\frac{1}{2}}(x^3,y^3)||| \leq \theta < 1, \tag{41}$$

for all $(x^1,y^1),(x^2,y^2),(x^3,y^3) \in K$, where $|||\cdot|||$ denotes the standard norm of a matrix; and that infimum over $K \times K$ of the minimum eigenvalue of $\nabla_x g(x,y)$ is positive. Then as $t \to \infty$, the solution X^{t^*} to variational inequality (24b) of the problem under information asymmetry converges to the solution X^* to the corresponding variational inequality (12) under perfect quality information.

Numerical Examples

We implemented the Euler method using Matlab on an OS X 10.10.5 system. The convergence tolerance is 10^{-6}, so that the algorithm is deemed to have converged when the absolute value of the difference between each successive product shipment, demand price, and supply price is less than or equal to 10^{-6}. The sequence $\{a_\tau\}$ is set to: $\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \ldots\}$. We initialize the algorithm by setting the demand of each product at 10 and equally distributed the demand among the demand markets; the demand and supply prices are set to 0 initially.

Numerical Examples

The perceived quality in time period t; $t \geq 2$, is measured as

$$\hat{q}_{ij}^t = \alpha_j^t q_i^{t-1}(\pi_i^{t-1}) + (1 - \alpha_j^t)\hat{q}_{ij}^{t-1}, \quad i = 1, \ldots, m; j = 1, \ldots, n, \qquad (42)$$

where $0 \leq \alpha_j^t \leq 1$; $j = 1, \ldots, n$.

The higher the α_j^t, in period t, the greater the impact of actual product quality on consumers' quality perception, and the less the impact of consumers' previous knowledge of quality.

Numerical Examples - Example 1

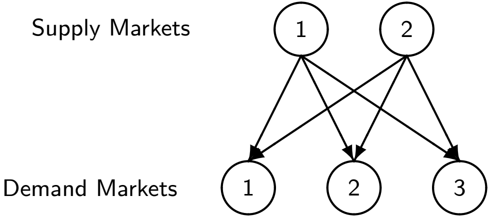

Figure: Example 1 Network Topology

Under perfect quality information, the data are as follows.

The supply functions are:

$$s_1(\pi_1, \pi_2) = 2\pi_1 - 0.5\pi_2 - 2, \quad s_2(\pi_1, \pi_2) = 2\pi_2 - 0.5\pi_1 - 2.$$

The unit transportation cost functions are:

$$c_{11}(Q_{11}) = Q_{11} + 6, \quad c_{12}(Q_{12}) = 2Q_{12} + 7, \quad c_{13}(Q_{13}) = 4Q_{13} + 5,$$

$$c_{21}(Q_{21}) = 2Q_{21} + 7, \quad c_{22}(Q_{22}) = Q_{22} + 5, \quad c_{23}(Q_{23}) = 4Q_{23} + 6.$$

Numerical Examples - Example 1

The quality functions are:

$$q_1(\pi_1) = 2\pi_1 - 3, \quad q_2(\pi_2) = 2\pi_2 - 3,$$

and the demand functions are:

$$d_{11}(\rho_{11}, \rho_{21}, \hat{q}_{11}, \hat{q}_{21}) = -\rho_{11} + 0.4\hat{q}_{11} + 0.1\rho_{21} - 0.05\hat{q}_{21} + 35,$$

$$d_{12}(\rho_{12}, \rho_{22}, \hat{q}_{12}, \hat{q}_{22}) = -\rho_{12} + 0.4\hat{q}_{12} + 0.1\rho_{22} - 0.05\hat{q}_{22} + 35,$$

$$d_{13}(\rho_{13}, \rho_{23}, \hat{q}_{13}, \hat{q}_{23}) = -\rho_{13} + 0.4\hat{q}_{13} + 0.1\rho_{23} - 0.05\hat{q}_{23} + 35,$$

$$d_{21}(\rho_{11}, \rho_{21}, \hat{q}_{11}, \hat{q}_{21}) = -\rho_{21} + 0.4\hat{q}_{21} + 0.1\rho_{11} - 0.05\hat{q}_{11} + 35,$$

$$d_{22}(\rho_{12}, \rho_{22}, \hat{q}_{12}, \hat{q}_{22}) = -\rho_{22} + 0.4\hat{q}_{22} + 0.1\rho_{12} - 0.05\hat{q}_{12} + 35,$$

$$d_{23}(\rho_{13}, \rho_{23}, \hat{q}_{13}, \hat{q}_{23}) = -\rho_{23} + 0.4\hat{q}_{23} + 0.1\rho_{13} - 0.05\hat{q}_{13} + 35.$$

Numerical Examples - Example 1

Table: Equilibrium and Induced Solutions to Examples 1, 2, 3, 4, and 5 Under Perfect Quality Information

	Example 1	Example 2	Example 3	Example 4	Example 5
Q_{11}^* (i.e., d_{11})	13.21	16.51	10.83	13.62	12.78
Q_{12}^* (i.e., d_{12})	8.31	10.51	6.74	3.02	14.21
Q_{13}^* (i.e., d_{13})	5.56	6.88	4.59	5.73	5.38
Q_{21}^* (i.e., d_{21})	8.32	8.01	6.74	8.69	7.95
Q_{22}^* (i.e., d_{22})	13.70	13.29	11.29	4.17	23.13
Q_{23}^* (i.e., d_{23})	5.35	5.20	4.37	5.58	5.12
π_1^*	19.43	26.48	16.14	15.72	23.42
π_2^*	19.55	20.87	16.24	14.16	24.96
ρ_{11}^*	38.64	48.99	32.97	35.34	42.20
ρ_{12}^*	43.05	54.51	36.63	46.51	58.83
ρ_{13}^*	46.67	58.99	39.50	43.63	49.95
ρ_{21}^*	43.19	43.88	36.72	38.54	47.85
ρ_{22}^*	38.25	39.15	32.53	44.18	53.09
ρ_{23}^*	46.96	47.68	39.74	42.48	51.45
s_1	27.09	33.91	22.16	22.37	32.37
s_2	27.38	26.50	22.41	18.45	36.20
q_1	35.86	70.11	14.63	28.44	43.84
q_2	36.09	38.74	14.74	25.31	46.91
CW_{11}	88.19	137.63	59.19	93.72	82.46
CW_{12}	34.89	55.84	22.97	4.60	101.92
CW_{13}	15.62	23.90	10.64	16.57	14.63
CW_{21}	34.97	32.38	22.95	38.17	31.88
CW_{22}	94.82	89.15	64.42	8.80	270.25
CW_{23}	14.47	13.68	9.67	15.74	13.25

Numerical Examples - Example 1

Under quality information asymmetry, the functional forms of the functions are the same as those under perfect information, but with different variables.

Here, we assume consumers' perceived quality levels in period 1 are:

$$\hat{q}^1_{11} = 18, \quad \hat{q}^1_{12} = 22, \quad \hat{q}^1_{13} = 25,$$

$$\hat{q}^1_{21} = 18, \quad \hat{q}^1_{22} = 22, \quad \hat{q}^1_{23} = 25.$$

Consumers in city 3 are most attracted by the initial extrinsic attributes of the two milk products when they just enter the market, and consumers in city 1 are least attracted.

In addition, from period 2 onwards, (42) is used to measure the perception of quality with α^t_1, α^t_2, α^t_3=0.7, $\forall t \geq 2$.

Numerical Examples - Example 1

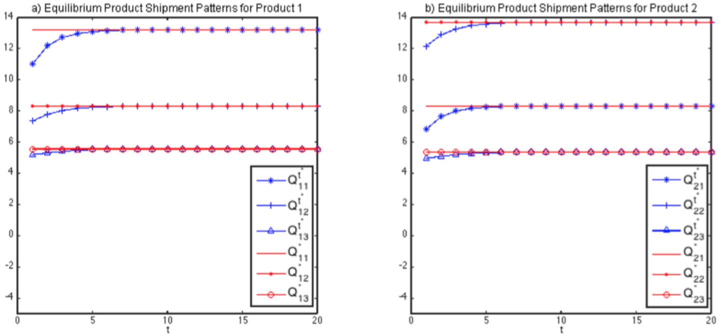

Figure: Evolution of the Equilibrium Product Shipments from Period 1 to Period 20 for Example 1, Along with Associated Results Under Perfect Quality Information

Numerical Examples - Example 1

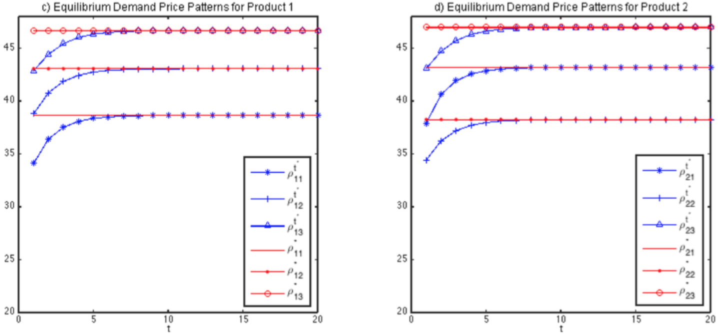

Figure: Evolution of the Equilibrium Demand Price Patterns from Period 1 to Period 20 for Example 1, Along with Associated Results Under Perfect Quality Information

Numerical Examples - Example 1

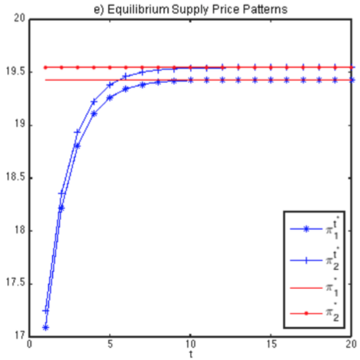

Figure: Evolution of the Equilibrium Supply Price Patterns from Period 1 to Period 20 for Example 1, Along with Associated Results Under Perfect Quality Information

Numerical Examples - Example 1

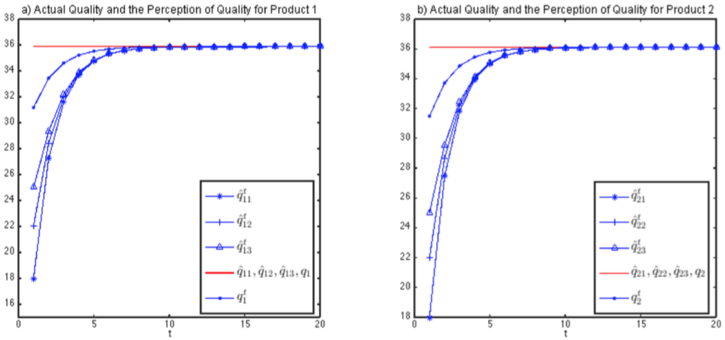

Figure: Evolution of the Actual Product Quality and Perception of Quality from Period 1 to Period 20 for Example 1, Along with Associated Results Under Perfect Quality Information

Numerical Examples - Example 1

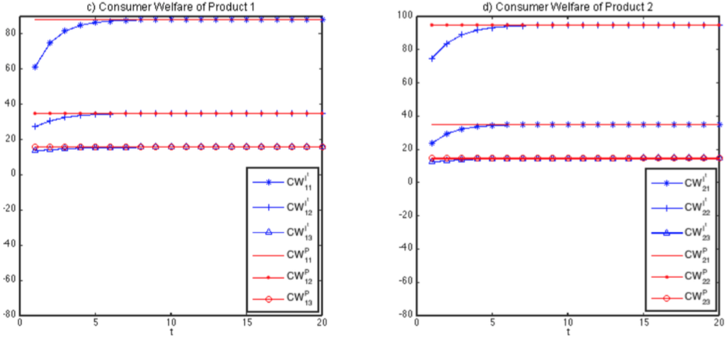

Figure: Evolution of Consumer Welfare from Period 1 to Period 20 for Example 1, Along with Associated Results Under Perfect Quality Information

Numerical Examples - Example 1

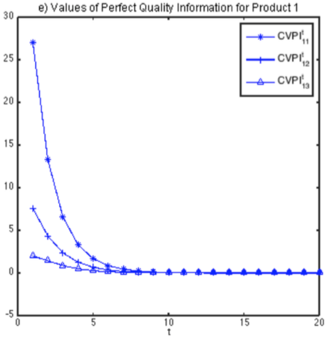

Figure: Evolution of Values of Perfect Quality Information from Period 1 to Period 20 for Example 1, Along with Associated Results Under Perfect Quality Information

Numerical Examples - Example 2

Example 2 is the same as in Example 1, but in this example, supply market 1 applies a new technology that is able to improve product quality in a more efficient way from time period 8 onwards. With this new technology, a higher supply price is charged for product 1.

From time period 8, the supply function and the quality function of supply market 1 become:

$$s_1^t(\pi_1^t, \pi_2^t) = 1.75\pi_1^t - 0.5\pi_2^t - 2, \quad q_1^t(\pi_1^t) = 0.1{\pi_1^t}^2, \quad \forall t \geq 8.$$

Numerical Examples - Example 2

Table: Equilibrium and Induced Solutions to Examples 1, 2, 3, 4, and 5 Under Perfect Quality Information

	Example 1	Example 2	Example 3	Example 4	Example 5
Q_{11}^* (i.e., d_{11})	13.21	16.51	10.83	13.62	12.78
Q_{12}^* (i.e., d_{12})	8.31	10.51	6.74	3.02	14.21
Q_{13}^* (i.e., d_{13})	5.56	6.88	4.59	5.73	5.38
Q_{21}^* (i.e., d_{21})	8.32	8.01	6.74	8.69	7.95
Q_{22}^* (i.e., d_{22})	13.70	13.29	11.29	4.17	23.13
Q_{23}^* (i.e., d_{23})	5.35	5.20	4.37	5.58	5.12
π_1^*	19.43	26.48	16.14	15.72	23.42
π_2^*	19.55	20.87	16.24	14.16	24.96
ρ_{11}^*	38.64	48.99	32.97	35.34	42.20
ρ_{12}^*	43.05	54.51	36.63	46.51	58.83
ρ_{13}^*	46.67	58.99	39.50	43.63	49.95
ρ_{21}^*	43.19	43.88	36.72	38.54	47.85
ρ_{22}^*	38.25	39.15	32.53	44.18	53.09
ρ_{23}^*	46.96	47.68	39.74	42.48	51.45
s_1	27.09	33.91	22.16	22.37	32.37
s_2	27.38	26.50	22.41	18.45	36.20
q_1	35.86	70.11	14.63	28.44	43.84
q_2	36.09	38.74	14.74	25.31	46.91
CW_{11}	88.19	137.63	59.19	93.72	82.46
CW_{12}	34.89	55.84	22.97	4.60	101.92
CW_{13}	15.62	23.90	10.64	16.57	14.63
CW_{21}	34.97	32.38	22.95	38.17	31.88
CW_{22}	94.82	89.15	64.42	8.80	270.25
CW_{23}	14.47	13.68	9.67	15.74	13.25

Numerical Examples - Example 2

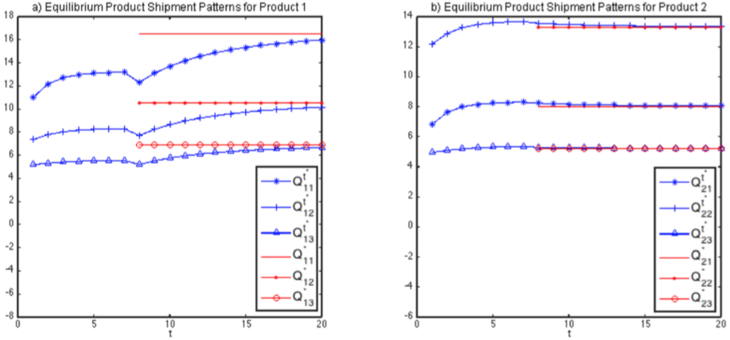

Figure: Evolution of the Equilibrium Product Shipments from Period 1 to Period 30 for Example 2, Along with Associated Results Under Perfect Quality Information

Numerical Examples - Example 2

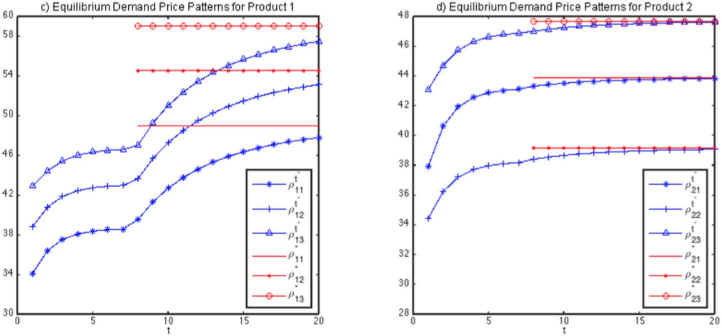

Figure: Evolution of the Equilibrium Demand Price Patterns from Period 1 to Period 30 for Example 2, Along with Associated Results Under Perfect Quality Information

Numerical Examples - Example 2

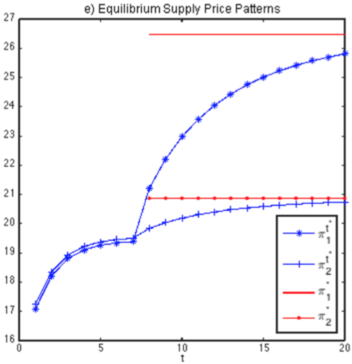

Figure: Evolution of the Equilibrium Supply Price Patterns from Period 1 to Period 30 for Example 2, Along with Associated Results Under Perfect Quality Information

Numerical Examples - Example 2

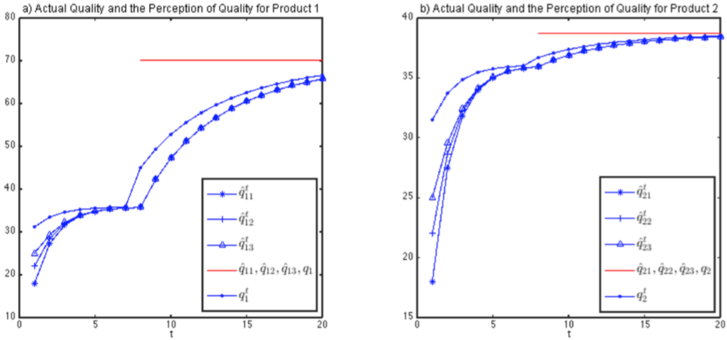

Figure: Evolution of the Actual Product Quality and Perception of Quality from Period 1 to Period 30 for Example 2, Along with Associated Results Under Perfect Quality Information

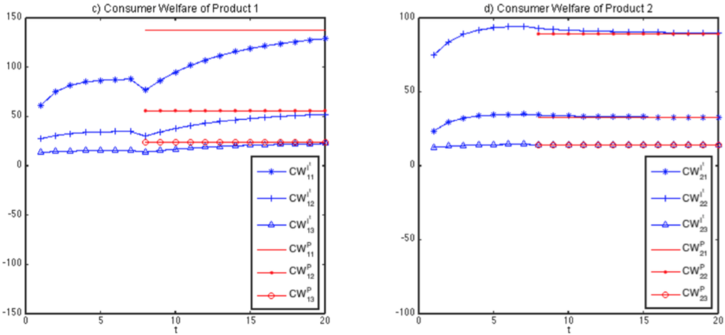

Figure: Evolution of Consumer Welfare from Period 1 to Period 30 for Example 2, Along with Associated Results Under Perfect Quality Information

Numerical Examples - Example 2

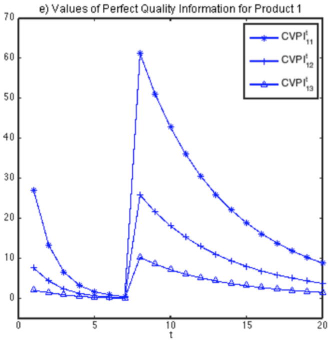

Figure: Evolution of Values of Perfect Quality Information from Period 1 to Period 30 for Example 2, Along with Associated Results Under Perfect Quality Information

Numerical Examples - Example 3

This example is the same as Example 1, except for the following. In time period 5, due to technology development in milk production, sterilization, and in quality management, a new process, and higher expectation from consumers, stricter quality requirements and higher standards for milk products are adopted. As a result, the measurement of quality changes. The two supply markets then re-evaluate the relationship between their quality and supply prices and determine new quality functions as the following:

$$q_1^t(\pi_1^t) = \pi_1^t - 1.5, \quad q_2^t(\pi_2^t) = \pi_2^t - 1.5, \quad \forall t \geq 5.$$

As in these new functions, the same supply price leads to half of the quality value as before.

Numerical Examples - Example 3

Table: Equilibrium and Induced Solutions to Examples 1, 2, 3, 4, and 5 Under Perfect Quality Information

	Example 1	Example 2	Example 3	Example 4	Example 5
Q_{11}^* (i.e., d_{11})	13.21	16.51	10.83	13.62	12.78
Q_{12}^* (i.e., d_{12})	8.31	10.51	6.74	3.02	14.21
Q_{13}^* (i.e., d_{13})	5.56	6.88	4.59	5.73	5.38
Q_{21}^* (i.e., d_{21})	8.32	8.01	6.74	8.69	7.95
Q_{22}^* (i.e., d_{22})	13.70	13.29	11.29	4.17	23.13
Q_{23}^* (i.e., d_{23})	5.35	5.20	4.37	5.58	5.12
π_1^*	19.43	26.48	16.14	15.72	23.42
π_2^*	19.55	20.87	16.24	14.16	24.96
ρ_{11}^*	38.64	48.99	32.97	35.34	42.20
ρ_{12}^*	43.05	54.51	36.63	46.51	58.83
ρ_{13}^*	46.67	58.99	39.50	43.63	49.95
ρ_{21}^*	43.19	43.88	36.72	38.54	47.85
ρ_{22}^*	38.25	39.15	32.53	44.18	53.09
ρ_{23}^*	46.96	47.68	39.74	42.48	51.45
s_1	27.09	33.91	22.16	22.37	32.37
s_2	27.38	26.50	22.41	18.45	36.20
q_1	35.86	70.11	14.63	28.44	43.84
q_2	36.09	38.74	14.74	25.31	46.91
CW_{11}	88.19	137.63	59.19	93.72	82.46
CW_{12}	34.89	55.84	22.97	4.60	101.92
CW_{13}	15.62	23.90	10.64	16.57	14.63
CW_{21}	34.97	32.38	22.95	38.17	31.88
CW_{22}	94.82	89.15	64.42	8.80	270.25
CW_{23}	14.47	13.68	9.67	15.74	13.25

Numerical Examples - Example 4

This example considers the same problem as in Example 1, except that city 2 becomes much more congested than cities 1 and 3 from time period 10 onwards, as sections of major highways to city 2 are under construction/maintenance. From period 10 onwards, the unit transportation cost functions to city 2 are changed to:

$$c_{12}^t(Q_{12}^t, Q_{22}^t) = 2Q_{12}^{t^2} + Q_{12}^t Q_{22}^t, \quad c_{22}^t(Q_{22}^t, Q_{12}^t) = Q_{22}^{t^2} + Q_{12}^t Q_{22}^t, \quad \forall t \geq 10.$$

Numerical Examples - Example 4

Table: Equilibrium and Induced Solutions to Examples 1, 2, 3, 4, and 5 Under Perfect Quality Information

	Example 1	Example 2	Example 3	Example 4	Example 5
Q_{11}^* (i.e., d_{11})	13.21	16.51	10.83	13.62	12.78
Q_{12}^* (i.e., d_{12})	8.31	10.51	6.74	3.02	14.21
Q_{13}^* (i.e., d_{13})	5.56	6.88	4.59	5.73	5.38
Q_{21}^* (i.e., d_{21})	8.32	8.01	6.74	8.69	7.95
Q_{22}^* (i.e., d_{22})	13.70	13.29	11.29	4.17	23.13
Q_{23}^* (i.e., d_{23})	5.35	5.20	4.37	5.58	5.12
π_1^*	19.43	26.48	16.14	15.72	23.42
π_2^*	19.55	20.87	16.24	14.16	24.96
ρ_{11}^*	38.64	48.99	32.97	35.34	42.20
ρ_{12}^*	43.05	54.51	36.63	46.51	58.83
ρ_{13}^*	46.67	58.99	39.50	43.63	49.95
ρ_{21}^*	43.19	43.88	36.72	38.54	47.85
ρ_{22}^*	38.25	39.15	32.53	44.18	53.09
ρ_{23}^*	46.96	47.68	39.74	42.48	51.45
s_1	27.09	33.91	22.16	22.37	32.37
s_2	27.38	26.50	22.41	18.45	36.20
q_1	35.86	70.11	14.63	28.44	43.84
q_2	36.09	38.74	14.74	25.31	46.91
CW_{11}	88.19	137.63	59.19	93.72	82.46
CW_{12}	34.89	55.84	22.97	4.60	101.92
CW_{13}	15.62	23.90	10.64	16.57	14.63
CW_{21}	34.97	32.38	22.95	38.17	31.88
CW_{22}	94.82	89.15	64.42	8.80	270.25
CW_{23}	14.47	13.68	9.67	15.74	13.25

Numerical Examples - Example 5

This example is the same as Example 1, except that, from time period 12 onwards, consumers in city 2 are more sensitive to product quality than before. They are willing to purchase more of higher quality products and fewer of lower quality products. The new demand functions at city 2 from time period 12 onwards become the following:

$$d_{12}^t(\rho_{12}^t, \rho_{22}^t, \hat{q}_{12}^t, \hat{q}_{22}^t) = -\rho_{12}^t + 0.8\hat{q}_{12}^t + 0.1\rho_{22}^t - 0.05\hat{q}_{22}^t + 35,$$

$$d_{22}^t(\rho_{12}^t, \rho_{22}^t, \hat{q}_{12}^t, \hat{q}_{22}^t) = -\rho_{22}^t + 0.8\hat{q}_{22}^t + 0.1\rho_{12}^t - 0.05\hat{q}_{12}^t + 35.$$

Numerical Examples - Example 5

Table: Equilibrium and Induced Solutions to Examples 1, 2, 3, 4, and 5 Under Perfect Quality Information

	Example 1	Example 2	Example 3	Example 4	Example 5
Q_{11}^* (i.e., d_{11})	13.21	16.51	10.83	13.62	12.78
Q_{12}^* (i.e., d_{12})	8.31	10.51	6.74	3.02	14.21
Q_{13}^* (i.e., d_{13})	5.56	6.88	4.59	5.73	5.38
Q_{21}^* (i.e., d_{21})	8.32	8.01	6.74	8.69	7.95
Q_{22}^* (i.e., d_{22})	13.70	13.29	11.29	4.17	23.13
Q_{23}^* (i.e., d_{23})	5.35	5.20	4.37	5.58	5.12
π_1^*	19.43	26.48	16.14	15.72	23.42
π_2^*	19.55	20.87	16.24	14.16	24.96
ρ_{11}^*	38.64	48.99	32.97	35.34	42.20
ρ_{12}^*	43.05	54.51	36.63	46.51	58.83
ρ_{13}^*	46.67	58.99	39.50	43.63	49.95
ρ_{21}^*	43.19	43.88	36.72	38.54	47.85
ρ_{22}^*	38.25	39.15	32.53	44.18	53.09
ρ_{23}^*	46.96	47.68	39.74	42.48	51.45
s_1	27.09	33.91	22.16	22.37	32.37
s_2	27.38	26.50	22.41	18.45	36.20
q_1	35.86	70.11	14.63	28.44	43.84
q_2	36.09	38.74	14.74	25.31	46.91
CW_{11}	88.19	137.63	59.19	93.72	82.46
CW_{12}	34.89	55.84	22.97	4.60	101.92
CW_{13}	15.62	23.90	10.64	16.57	14.63
CW_{21}	34.97	32.38	22.95	38.17	31.88
CW_{22}	94.82	89.15	64.42	8.80	270.25
CW_{23}	14.47	13.68	9.67	15.74	13.25

Example 6 is the same as in Example 1. Nevertheless, under quality information asymmetry, in city 3, consumers' past knowledge/memory of quality plays a more significant role in their quality perception in this example with $\alpha_3^t = 0.5$, $\forall t \geq 2$, instead of 0.7. The impact of consumers' most recent observation of quality (i.e., the latest actual product quality) is, hence, less.

Numerical Examples - Example 6

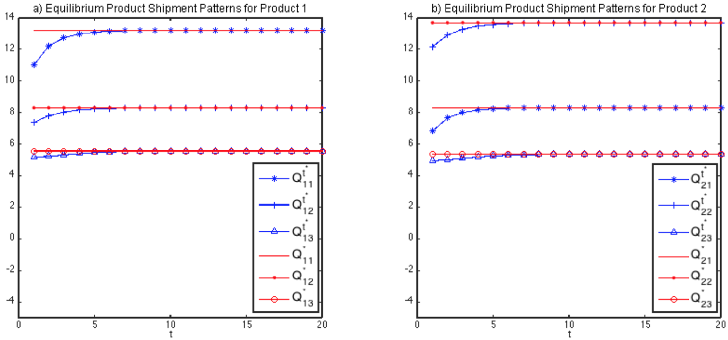

Figure: Evolution of the Equilibrium Product Shipments from Period 1 to Period 30 for Example 6, Along with Associated Results Under Perfect Quality Information

Numerical Examples - Example 6

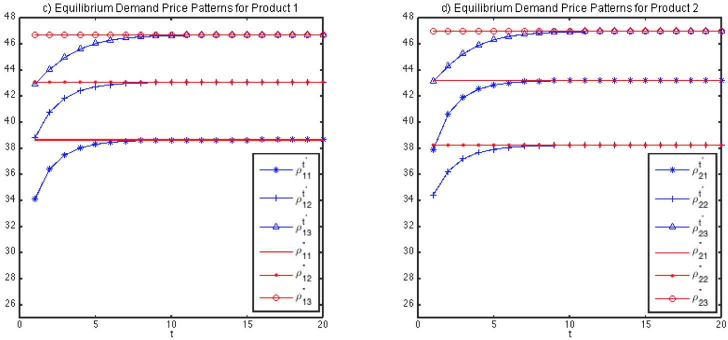

Figure: Evolution of the Equilibrium Demand Price Patterns from Period 1 to Period 30 for Example 6, Along with Associated Results Under Perfect Quality Information

Numerical Examples - Example 6

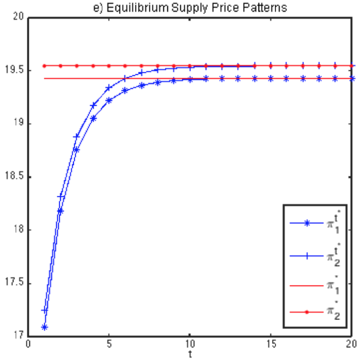

Figure: Evolution of the Equilibrium Supply Price Patterns from Period 1 to Period 30 for Example 6, Along with Associated Results Under Perfect Quality Information

Numerical Examples - Example 6

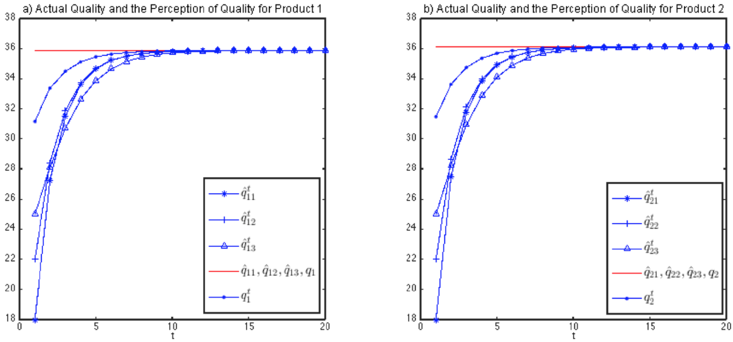

Figure: Evolution of the Actual Product Quality and Perception of Quality from Period 1 to Period 30 for Example 6, Along with Associated Results Under Perfect Quality Information

Numerical Examples - Example 6

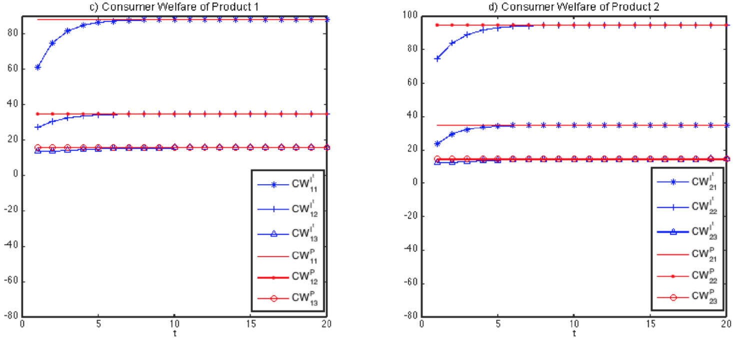

Figure: Evolution of Consumer Welfare from Period 1 to Period 30 for Example 6, Along with Associated Results Under Perfect Quality Information

Numerical Examples - Example 6

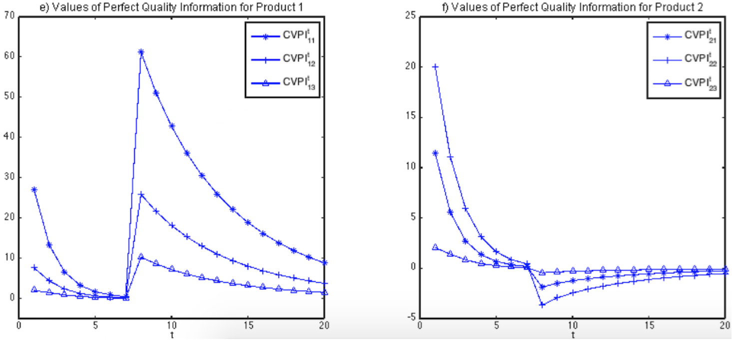

Figure: Evolution of Values of Perfect Quality Information from Period 1 to Period 30 for Example 6, Along with Associated Results Under Perfect Quality Information

Numerical Examples

The following practical insights can be drawn from Examples 1-6:

- As in the results for Example 1, consumers at the demand markets that are closer to the supply markets will receive more product shipments, lower demand prices, and more consumer welfare than those in farther demand markets.
- Comparing Examples 1 and 2, a supply market's more efficient quality technology will enhance the consumer welfare of its own consumers but may hurt that of its competitors'.
- From the results for Example 1 and Example 3, simply imposing stricter quality requirements will not improve consumer welfare, if no other effort is made.
- Comparing Examples 1 and 4, traffic congestion will harm consumer welfare; thus, efficient and reliable transportation infrastructure is important for the benefit of consumers.
- Based on the results for Examples 1 and 5, consumers who value quality more will benefit in terms of their welfare, but consumers who do not may obtain lower welfare.
- From the results for Example 6, consumers who rely more on their past memory of quality will adjust to the actual quality in a slower manner than who do not.

Summary and Conclusions

- We advance the modeling, analysis, and understanding of spatial price equilibrium network models in which the products are differentiated and consumers respond to the quality of the products through the prices that they are willing to pay with consumers in our dynamic, adaptive model, learning about the product quality over time.
- The models are especially reasonable in the case of agricultural products, since consumers typically consume such products repetitively and will learn about the brand's product quality over time.
- We also provide qualitative properties of the equilibrium supply price, demand price, product flow, and quality level patterns, in terms of existence, uniqueness, and convergence results.
- Several numerical examples are provided with practical insights.

Thank you!

- For more information, please visit http://supernet.isenberg.umass.edu.