International Human Migration Networks Under Regulations

Anna Nagurney[1] and Patrizia Daniele[2]

[1] Department of Operations and Information Management
Isenberg School of Management
University of Massachusetts
Amherst, Massachusetts
[2] Department of Mathematics and Computer Science
University of Catania, Italy

8TH INTERNATIONAL SYMPOSIUM ON TRANSPORT NETWORK RELIABILITY (INSTR)
June 16-18, 2021

Acknowledgments

Many thanks to the organizers of the conference and to KTH!

This presentation is based on the paper:

Nagurney, A., Daniele, P., 2021. International Human Migration Networks Under Regulations. *European Journal of Operational Research* (2021), **291(3)**, pp 894-905.

Outline of Presentation

- **Motivation and Some Background**
- **Literature Review with a Focus on Networks and Migration**
- **The International Human Migration Models and Variational Inequality Formulations**
- **Illustrative Examples**
- **Computation of Numerical Examples**
- **Summary and Conclusions**

Motivation and Some Background

Motivation and Some Background

• **Reasons for human migration are numerous**, from individuals seeking better economic opportunities and enhanced prosperity for themselves and their families, to those fleeing conflict, violence, and persecution. With climate change and the increasing number and severity of natural disasters, including hurricanes, floods, tornados, earthquakes, etc., some migrants are seeking locations of greater expected safety and security.

• **In 2017, the number of international migrants was an estimated 258 million persons (3.4% of the global population)**, with the total number of international migrants increasing by almost 50% since 2000 (United Nations (2017)).

• **The number of international migrants is growing faster than the global population.** In the same almost two decade period, the number of refugees and asylum seekers increased from 16 to 26 million, comprising about 10% of the international migrants.

Motivation and Some Background

Vivid depictions of people fleeing their origin locations permeate the news, whether attempting to escape the great strife and suffering in Syria; the violence in parts of Central America, the economic collapse of Venezuela, and even flooding in parts of Asia as well as droughts in parts of Africa.

Motivation and Some Background

At times, refugees will travel in extremely dangerous conditions to escape the dire circumstances at their origin nodes.

In 2015, the UN Refugee Agency reported a maritime refugee crisis with, in the first half of that year, 137,000 refugees crossing the Mediterranean Sea to Europe, via very risky transport modes, and with many more unsuccessfully attempting such a passage. 800 died in the largest refugee shipwreck on record that April.

Motivation and Some Background

Governments of various nations, hence, are increasingly being faced with multiple challenges associated with human migration flows. In response to challenges, they are adopting different regulations.

According to the United Nations (2013), **migration policies in both origin and destination countries play an important role in determining the migratory flows**. In managing international migration flows, governments usually focus on different types of migrants, of which the most salient are highly skilled workers, dependents of migrant workers, irregular migrants, and refugees and asylum seekers (cf. Karagiannis (2016)).

Between 11 March 2020, when the WHO declared COVID-19 a pandemic, and 22 February 2021, nearly 105,000 movement restrictions were implemented around the world, according to the International Organization for Migration.

Motivation and Some Background

Given that the United Nations (2013) identifies international migration as a global phenomenon that is growing in complexity, scope, and impact, it is quite relevant to revisit the modeling of international human migration networks in the context of different regulations.

That is the goal of this paper.

European Journal of Operational Research 291 (2021) 894–905

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

International human migration networks under regulations

Anna Nagurney[a,*], Patrizia Daniele[b]

[a] Department of Operations and Information Management, Isenberg School of Management, University of Massachusetts, Amherst, Massachusetts 01003, United States
[b] Department of Mathematics and Computer Science University of Catania, Italy

ARTICLE INFO

Article history:
Received 13 September 2019
Accepted 5 April 2020
Available online 12 April 2020

Keywords:
Human migration
Networks
Variational inequalities
Regulations

ABSTRACT

International human migration has transformed economies and societies. The new millennium, with climate change and its impacts, and increasing conflicts and displacements, has experienced a great increase in international migrants, with associated challenges faced by governments. In this paper, we advance the modeling, analysis, and solution of international human migration problems by developing a network model with regulations. The formalism uses the theory of variational inequalities, coupled with Lagrange analysis, in order to gain insights as to the impacts of the regulations on utilities of multiple classes of migrants, and on the equilibrium flows. Our results add to the literature on operations research for societal impact, inspired by the real world.

1. Introduction

Humans have engaged in migration since time immemorial. Reasons for human migration are numerous, from individuals seeking better economic opportunities and enhanced prosperity for themselves and their families, to those fleeing conflict, violence, and persecution. With climate change and the increasing number and severity of natural disasters, including hurricanes, floods, tornados, earthquakes, etc., some migrants are seeking locations of greater expected safety and security. In 2017, the number of international migrants was an estimated 258 million persons (3.4% of the global population), with the total number of international migrants increasing by almost 50% since 2000 (United Nations, 2017). As also reported therein, the number of international migrants is growing faster than the global population. Moreover, in the same almost two decade period, the number of refugees and asylum seekers increased from 16 to 26 million, comprising about 10% of the international migrants.

Vivid depictions of people fleeing their origin locations permeate the news, whether attempting to escape the great strife and suffering in Syria (United Nations Refugee Agency, 2019); the violence in parts of Central America (Bartenstein & McDonald, 2019), the economic collapse of Venezuela (Kennedy, 2019), and even flooding in parts of Asia (Dash & Paul, 2017), as well as droughts in parts of Africa (cf. Linke, Witmer, O'Loughlin, Mc Cabe, & Tir, 2018).

Interestingly, since the onset of the new millennium, the countries experiencing the largest increase in their diaspora populations are the Syrian Arab Republic (872%), India (108%), and the Philippines (85%) (cf. United Nations, 2017). At times, refugees will travel in extremely dangerous conditions to escape the dire circumstances at their origin nodes. For example, the United Nations Refugee Agency (2015) reported a maritime refugee crisis with, in the first half of that year, 137,000 refugees crossing the Mediterranean Sea to Europe, via very risky transport modes, and with many more unsuccessfully attempting such a passage. 800 died in the largest refugee shipwreck on record that April. Clearly, the issue of human migration is one of multiple dimensions, including economic, political, sociological, and even environmental aspects.

Governments of various nations, hence, are increasingly being faced with multiple challenges associated with human migration flows. In response to challenges, they are adopting different regulations. According to Bertossi (2008), since the 1990s, unprecedented efforts have been made to control the movement of people across state borders. As argued eloquently by Helbling and Leblang (2019), migration regulations are core to national sovereignty since they directly impact the composition of the community that makes up the country. According to the informative report by the United Nations (2013), migration policies in both origin and destination countries play an important role in determining the migratory flows. Moreover, as noted therein, in managing international migration flows, governments usually focus on different types of migrants, of which the most salient are highly skilled workers, dependents of migrant workers, irregular migrants, and refugees and asylum seekers (cf. Karagiannis, 2016).

* Corresponding author.
E-mail address: nagurney@isenberg.umass.edu (A. Nagurney).

https://doi.org/10.1016/j.ejor.2020.04.008
0377-2217/© 2020 Elsevier B.V. All rights reserved.

Literature Review with a Focus on Networks and Migration

Literature Review with a Focus on Networks and Migration

• Nagurney (1989) introduced a multiclass migration equilibrium model, which did not include migration/movement costs, and **was isomorphic to a traffic network equilibrium with special structure**. The model was then extended to include flow-dependent migration costs and an expanded set of equilibrium conditions in Nagurney (1990).

• Nagurney, Pan, and Zhao (1992a) proposed **a multiclass human migration model**, which further generalized to include class transformations in Nagurney, Pan, and Zhao (1992b).

• Pan and Nagurney (1994), in turn, considered **chain migration (unlike the earlier work) and introduced a multi-stage (but single class) Markov chain model**. The authors established a connection between a sequence of variational inequalities and a non-homogeneous Markov chain. They also proved that, under certain assumptions, the stability of the one-step transition matrix guarantees the stability of the *n*-step transition matrix.

Literature Review with a Focus on Networks and Migration

- Pan and Nagurney (2006) utilized the methodology of **evolution variational inequalities for the first time to model the dynamic adjustment of a socio-economic process in the context of human migration**. The question of convergence of algorithms in this framework, which is infinite-dimensional, was also addressed (see also Daniele (2006)).

- Interestingly, many of the network equilibrium models of human migration, as above, have also **found application to the migration of animals in ecology with a focus on fish and maritime ecosystems** (see Mullon and Nagurney (2012), Mullon (2014), Mariani et al. (2016)).

- Kalashnikov et al. (2008) constructed a human migration model with a **conjectural variations equilibrium (CVE)**.

- Capello and Daniele (2019) developed **a Nash equilibrium model of human migration** with features of conjectural variations. The authors also provided a numerical example with sensitivity analysis focusing on the flow of migrants from Africa through the Mediterranean sea to Italy in 2018.

The International Human Migration Models and Variational Inequality Formulations

The International Human Migration Models

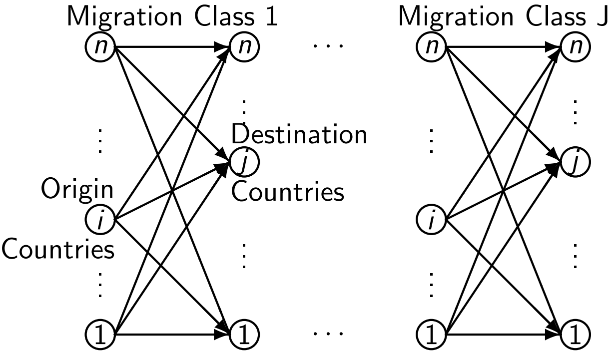

Figure: The Network Structure of International Human Migration

Table: Common Notation for the International Human Migration Models

Notation	Definition
f_{ij}^k	flow of migrants of class k from country i to country j. The $\{f_{ij}^k\}$ elements for all i and j and fixed k are grouped into the vector $f^k \in R_+^{nn}$. We group the f^k vectors; $k = 1, \ldots, J$, into vector $f \in R_+^{Jnn}$.
p_i^k	nonnegative population of migrant class k in country i. We group the populations of class k; $k = 1, \ldots, J$, into vector $p^k \in R_+^n$. We group all such vectors into vector $p \in R_+^{Jn}$.
$\bar{p}_i^k$	initial fixed population of class k in country i; $i = 1, \ldots, n$; $k = 1, \ldots, J$.
$u_i^k(p)$	utility perceived by class k in country i; $\forall i, k$.
$c_{ij}^k(f)$	cost of international migration, which includes economic, psychological, and social costs encumbered by class k in migrating from i to j; $\forall i, j, k$.

Conservation of Flow

In Nagurney, Pan, and Zhao (1992a), the following conservation of flow equations were proposed:

$$p_i^k = \bar{p}_i^k + \sum_{l \neq i} f_{li}^k - \sum_{l \neq i} f_{il}^k, \quad \forall i, \forall k, \tag{1}$$

and

$$\sum_{l \neq i} f_{il}^k \leq \bar{p}_i^k, \quad \forall i, \forall k, \tag{2}$$

where $f_{il}^k \geq 0$, for all $k = 1, \ldots, J$; $\forall l$.

According to (1), the population of a class in a country is equal to the initial population plus the inflow minus the outflow of that class. Equation (2), on the other hand, states that the flow out of country i by class k cannot exceed the initial population of class k at i, since no chain migration is allowed.

Conservation of Flow

Here, we utilize, instead, the following conservation of flow equations, using also the network structure in Figure 1 as a guideline:

$$\bar{p}_i^k = \sum_l f_{il}^k, \quad \forall i, \forall k, \tag{3}$$

and

$$p_i^k = \sum_l f_{li}^k, \quad \forall i, \forall k, \tag{4}$$

with the nonnegativity assumption on the international migratory flows, following (2).

Clearly, using (3) and (4), we obtain:

$$p_i^k - \bar{p}_i^k = \sum_l f_{li}^k - \sum_l f_{il}^k, \quad \forall i, \forall k, \tag{5}$$

which is equivalent to (1), since $f_{ii}^k - f_{ii}^k = 0$. Similarly, (3) is equivalent to (2).

We define the feasible set $K^1 \equiv \{(p, f) | f \geq 0 \text{ and (3) and (4) hold}\}$.

Equilibrium Conditions

Definition 1: International Human Migration Equilibrium without Regulations

A vector of populations and international migration flows $(p^*, f^*) \in K^1$ *is in equilibrium if it satisfies the equilibrium conditions: For each class* k*;* $k = 1, \ldots, J$ *and each pair of countries* i, j*;* $i = 1, \ldots, n$*;* $j = 1, \ldots, n$*:*

$$u_i^k(p^*) + c_{ij}^k(f^*) \begin{cases} = u_j^k(p^*) - \lambda_i^{k*}, & \textit{if} \quad f_{ij}^{k*} > 0, \\ \geq u_j^k(p^*) - \lambda_i^{k*}, & \textit{if} \quad f_{ij}^{k*} = 0, \end{cases} \tag{6}$$

and

$$\lambda_i^{k*} \begin{cases} \geq 0, & \textit{if} \quad \sum_{l \neq i} f_{il}^{k*} = \bar{p}_i^k, \\ = 0, & \textit{if} \quad \sum_{l \neq i} f_{il}^{k*} < \bar{p}_i^k. \end{cases} \tag{7}$$

Variational Inequality Formulation

Theorem 1: Variational Inequality Formulation of the International Human Migration Model without Regulations

A population and migration flow pattern $(p^*, f^*) \in K^1$ is an international human migration equilibrium without regulations according to Definition 1, if and only if it satisfies the variational inequality problem

$$-\langle u(p^*), p - p^* \rangle + \langle c(f^*), f - f^* \rangle \geq 0, \quad \forall (p, f) \in K^1. \tag{8}$$

International Human Migration Model with Regulations

We now construct the constraint set that captures a plethora of international migration regulations. For definiteness, we consider regulations imposed by a single country $\bar{j}$. We define the set C^1 consisting of classes $\{k\}$ and countries $\{i\}$, with $i \neq \bar{j}$, subject to an upper bound on the international migration flows into country $\bar{j}$, denoted by $U_{\bar{j}}$.
The constraint can then be stated as follows:

$$\sum_{i \in C^1} \sum_{k \in C^1} f^k_{i\bar{j}} \leq U_{\bar{j}}. \tag{9}$$

We now highlight the different types of regulations that (9) captures. For example, the set C^1 can be defined to restrict the migratory flow from a specific country $\bar{i}$ and specific class of migrant $\bar{k}$, in which case, (9) collapses to:

$$f^{\bar{k}}_{\bar{i}\bar{j}} \leq U_{\bar{j}}. \tag{10}$$

On the other hand, an upper bound on all incoming migrants from a specific country $\bar{i}$, irrespective of class, reduces constraint (9) to:

$$\sum_{k} f_{\bar{i}\bar{j}}^{k} \leq U_{\bar{j}}. \tag{11}$$

Also, a regulation restricting the number of all incoming migrants of class $\bar{k}$ from a group of countries, reduces constraint (9) to:

$$\sum_{i \in C^1} f_{i\bar{j}}^{\bar{k}} \leq U_{\bar{j}}. \tag{12}$$

For the international human migration model with regulations, the equilibrium conditions (6) and (7) are still relevant but with a new feasible set K^2 defined as below to include the constraint (9):

$$K^2 \equiv \{(p, f) | f \geq 0 \text{ and } (3), (4), \text{ and } (9) \text{ hold}\}. \tag{13}$$

Variational Inequality Formulation

Theorem 2: Variational Inequality Formulation of the International Human Migration Model with Regulations

A population and migration flow pattern $(p^*, f^*) \in K^2$ *is an international human migration equilibrium with regulations, if and only if it satisfies the variational inequality problem*

$$-\langle u(p^*), p - p^* \rangle + \langle c(f^*), f - f^* \rangle \geq 0, \quad \forall (p, f) \in K^2. \tag{14}$$

Illustrative Examples

Additional Theoretical Results and Illustrative Examples

Our EJOR paper contains additional theoretical results including existence results as well as the interpretations of Lagrange analysis.

Illustrative Examples

We now proceed to illustrate the above results in several simple examples. The migration network consists of the topology depicted in the Figure.

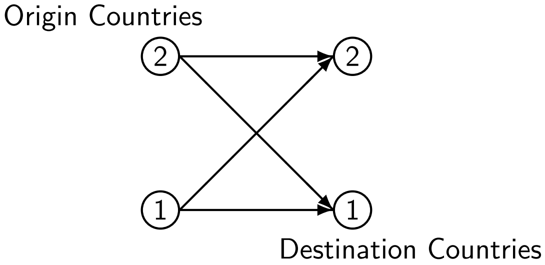

Figure: International Migration Network for Illustrative Examples

Illustrative Examples

We consider a single class of migrant. We, hence, suppress the superscript 1 in the notation.

The data are: $\bar{p}_1 = 50$ and $\bar{p}_2 = 0$ with the utility functions given by: $u_1(p) = -p_1 + 100$ and $u_2(p) = -p_2 + 120$. The migration cost functions are: $c_{11}(f) = c_{22}(f) = 0$, $c_{12}(f) = .1f_{12} + 7$, $c_{21}(f) = f_{21} + 10$.

We first consider the case without regulations. It is easy to compute the equilibrium solution, using simple algebra. Indeed, we find that:

$$f^*_{12} = 30, \quad f^*_{11} = 20, \quad f^*_{21} = 0, \quad f^*_{22} = 0;$$

hence,

$$p^*_1 = 20, \quad p^*_2 = 30,$$

with associated utilities being:

$$u_1(p^*) = 80, \quad u_2(p^*) = 90,$$

and the migration costs: $c_{11}(f^*) = c_{22}(f^*) = 0$, $c_{12}(f^*) = 10$, and $c_{21}(f^*) = 10$.

Illustrative Examples

We verify that the equilibrium conditions hold. Indeed, for the node pair $(1,2)$: $f^*_{12} > 0$, and

$$u_1(p^*) + c_{12}(f^*) = u_2(p^*),$$

since $80 + 10 = 90$. Moreover, for node pair $(1,1)$ and node pair $(2,2)$:

$$u_1(p^*) + c_{11}(f^*) = u_1(p^*), \quad u_2(p^*) + c_{22}(f^*) = u_2(p^*),$$

since the $c_{ii}(f)$s are equal to 0 for $i = 1, 2$. Finally, since $f^*_{21} = 0$:

$$u_2(p^*) + c_{21}(f^*) \geq u_1(p^*),$$

with notice that, indeed, $90 + 10 \geq 80$.

It is easy to verify that VI (8) is satisfied.

Illustrative Examples

We now suppose that a regulation is imposed on Country 2, such that

$$f_{1\bar{2}} \leq U_{\bar{2}} = 20.$$

The new equilibrium solution is:

$$f_{11}^* = 30, \quad f_{1\bar{2}}^* = 20, \quad f_{21}^* = 0, \quad f_{22}^* = 0,$$

with $p_1^* = 30$ and $p_{\bar{2}}^* = 20$, and associated utilities:

$$u_1(p^*) = 70, \quad u_{\bar{2}}(p^*) = 100,$$

and incurred migration costs: $c_{11}(f^*) = c_{22}(f^*) = 0$; $c_{1\bar{2}}(f^*) = 9$, $c_{21}(f^*) = 10$.

Under this regulation, those who manage to migrate enjoy a higher utility than before, but those who are left behind in Country 1 experience a lower utility than in the case without regulations (a utility of 70, as compared to 80).

Computation of Numerical Examples

An Alternative Variational Inequality

We define $K^3 \equiv \{(f, \delta, \mu_{\bar{j}}) | f \in R_+^{Jnn}, \delta \in R^{Jn}, \mu_{\bar{j}} \in R_+\}$ and we construct an alternative VI to (14): determine $(f^*, \delta^*, \mu_{\bar{j}}^*) \in K^3$ such that

$$\sum_{i;i\notin C^1} \sum_{j\neq \bar{j}} \sum_{k;k\notin C^1} (-\hat{u}_j^k(f^*) + c_{ij}^k(f^*) + \delta_{ik}^*) \times (f_{ij}^k - f_{ij}^{k*})$$

$$+ \sum_{i\in C^1} \sum_{k\in C^1} (-\hat{u}_{\bar{j}}^k(f^*) + c_{i\bar{j}}^k(f^*) + \delta_{ik}^* + \mu_{\bar{j}}^*) \times (f_{i\bar{j}}^k - f_{i\bar{j}}^{k*})$$

$$+ \sum_i \sum_k (\bar{p}_i^k - \sum_j f_{ij}^{k*}) \times (\delta_{ik} - \delta_{ik}^*)$$

$$+(U_{\bar{j}} - \sum_{i\in C^1} \sum_{k\in C^1} f_{i\bar{j}}^{k*}) \times (\mu_{\bar{j}} - \mu_{\bar{j}}^*) \geq 0, \quad \forall (f, \delta, \mu_{\bar{j}}) \in K^3. \tag{15}$$

An Alternative Variational Inequality

We can put variational inequality (15) into standard form (cf. Nagurney (1999)): determine $X^* \in \mathcal{K}$ such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{16}$$

where F is a given continuous function from $\mathcal{K}$ to R^N, $\mathcal{K}$ is a given closed convex set, and $\langle \cdot, \cdot \rangle$ denotes the inner product in N-dimensional Euclidean space.

We set $\mathcal{K} \equiv K^3$, $X \equiv (f, \delta, \mu_{\bar{j}})$, and $N = Jnn + Jn + 1$. Also, we define the vector $F \equiv (F_1, F_2, F_3, F_4)$, where the components of F_1 consist of the elements: $-\hat{u}_j^k(f) + c_{ij}^k(f) + \delta_{ik}$, for $i; i \notin C^1$ and $j \neq \bar{j}$, and $k; k \notin C^1$; the components of F_2 consist of the elements: $-\hat{u}_{\bar{j}}^k(f) + c_{i\bar{j}}^k(f) + \delta_{ik} + \mu_{\bar{j}}$, for $i \in C^1$ and $k \in C^1$; F_3 consists of the elements: $\bar{p}_i^k - \sum_j f_{ij}^k$, $\forall i, k$, and, finally, F_4 consists of the single element: $U_{\bar{j}} - \sum_{i \in C^1} \sum_{k \in C^1} f_{i\bar{j}}^k$.

Computation of Numerical Examples

The modified projection method was implemented in FORTRAN and the computer system used was a Linux system at the University of Massachusetts Amherst.

The algorithm was initialized as follows: the initial migration flow of a class from a given origin country to a destination country was set equal to the initial population of the class in the origin country divided by the number of countries, resulting in equal flows. The Lagrange multipliers were all initialized to be equal to 1.

The convergence tolerance ϵ was set to 10^{-3}; in other words, the algorithm was deemed to have converged if the absolute value of successive flow and Lagrange multiplier iterates differed by no more than this value.

Computation of Numerical Examples

Single Class Example without and with a Regulation

The first example consists of three countries and no regulations.

Since there is only a single class of migrant we suppress the superscript 1 in the notation. The initial populations are given, respectively, by $\bar{p}_1 = 10,000$, $\bar{p}_2 = 5,000$, and $\bar{p}_3 = 1,000$. The utility functions associated with the countries are:

$$u_1(p) = -p_1 - .5p_2 + 30,000, \quad u_2(p) = -2p_2 - p_1 + 20,000,$$

$$u_3(p) = -3p_3 + .5p_2 + 10,000.$$

Computation of Numerical Examples

The migration cost functions are: $c_{ii}(f) = 0, \quad i = 1, 2, 3;$

$$c_{12}(f) = 2f_{12} + 20, \quad c_{13}(f) = f_{13} + 30,$$

$$c_{21}(f) = 5f_{21} + 40, \quad c_{23}(f) = 4f_{23} + 20,$$

$$c_{31}(f) = 6f_{31} + 80, \quad c_{32}(f) = 4f_{32} + 60.$$

The algorithm converges, yielding the following equilibrium migration flow pattern:

$$f^*_{11} = 10,000.00, \quad f^*_{12} = 0.00, \quad f^*_{13} = 0.00,$$

$$f^*_{21} = 2,447.90, \quad f^*_{22} = 1,519.66, \quad f^*_{23} = 1,032.44,$$

$$f^*_{31} = 1,000.00, \quad f^*_{32} = 0.00, \quad f^*_{33} = 0.00.$$

The incurred migration costs at equilibrium are:

$$c_{11}(f^*) = 0.00, \quad c_{12}(f^*) = 20.00, \quad c_{13}(f^*) = 30.00,$$

$$c_{21}(f^*) = 12,279.50, \quad c_{22}(f^*) = 0.00, \quad c_{23}(f^*) = 4,149.76,$$

$$c_{31}(f^*) = 6,080.09, \quad c_{32}(f^*) = 60.00, \quad c_{33}(f^*) = 0.00.$$

Computation of Numerical Examples

The above equilibrium flow pattern corresponds to the following equilibrium population distribution:

$$p_1^* = 13,447.92, \quad p_2^* = 1,519.66, \quad p_3^* = 1,032.44,$$

and associated utilities at the equilibrium given by:

$$u_1(p^*) = 15,792.25, \quad u_2(p^*) = 3,512.75, \quad u_3(p^*) = 7,662.51.$$

The computed equilibrium Lagrange multipliers are:

$$\delta_1^* = 15,792.25, \quad \delta_2^* = 3,512.75, \quad \delta_3^* = 9,712.17.$$

Computation of Numerical Examples

Country 1 is clearly a very attractive country. No-one migrates out of the country. Moreover, about half of those in Country 2 migrate to Country 1, and the entire original population of Country 3 migrates to Country 1. Less than a third of the original population of Country 2 remains, with others migrating also to Country 3, in addition to Country 1.

Furthermore, it is easy to verify that the equilibrium conditions (cf. (6) and (7)) hold for all migration outflows associated with Country 1 and with Country 2, and, clearly, $\lambda_1^* = 0$ and $\lambda_2^* = 0$. Furthermore, it follows that $\lambda_3^* = 2,049.65$, with notice that $u_3(p^*) + c_{31}(f^*) = u_1(p^*) - \lambda_3^*$, since: $7,622.51 + 6,080.09 = 15,792.25 - 2,049.65$.

Computation of Numerical Examples

We now proceed to impose a regulation on the above example. Specifically, Country 1 is concerned about overpopulation and imposes the following regulation:

$$f_{2\bar{1}} + f_{3\bar{1}} \leq 2,000.$$

The modified projection method converges, yielding the following equilibrium migration flow pattern:

$$f^*_{11} = 10,000.00, \quad f^*_{12} = 0.00, \quad f^*_{13} = 0.00,$$

$$f^*_{2\bar{1}} = 1,458.42, \quad f^*_{22} = 2,545.93, \quad f^*_{23} = 995.65,$$

$$f^*_{3\bar{1}} = 541.58, \quad f^*_{32} = 0.00, \quad f^*_{33} = 458.42.$$

The incurred migration costs at equilibrium are:

$$c_{11}(f^*) = 0.00, \quad c_{12}(f^*) = 20.00, \quad c_{13}(f^*) = 30.00,$$

$$c_{2\bar{1}}(f^*) = 7,332.10, \quad c_{22}(f^*) = 0.00, \quad c_{23}(f^*) = 4,002.61,$$

$$c_{3\bar{1}}(f^*) = 3,329.50, \quad c_{32}(f^*) = 60.00, \quad c_{33}(f^*) = 0.00.$$

Computation of Numerical Examples

The above equilibrium flow pattern corresponds to the following equilibrium population distribution:

$$p_1^* = 12,000.00, \quad p_2^* = 2,545.93, \quad p_3^* = 1,454.07,$$

and associated utilities at the equilibrium given by:

$$u_{\bar{1}}(p^*) = 16,727.04, \quad u_2(p^*) = 2,908.15, \quad u_3(p^*) = 6,910.75.$$

The computed equilibrium Lagrange multipliers are:

$$\delta_1^* = 10,240.25, \quad \delta_2^* = 2,908.15, \quad \delta_3^* = 6,910.75,$$

and

$$\mu_{\bar{1}}^* = 6,486.79.$$

Observe that the migration flows into Country 1 are precisely equal to the regulatory upper bound of 2,000 and, hence, the associated Lagrange multiplier $\mu_{\bar{1}}^* > 0$.

Computation of Numerical Examples

Note, also, that, under the regulation, denizens of Country 1 now enjoy a high utility of $16,727.04$, as opposed to a utility of $15,792.25$, without the regulation.

However, migrants who move to either Country 2 or to Country 3 now experience a lower utility; for Country 2: $2,908.15$ **versus** $3,512.75$ **and, for Country 3:** $6,910.76$ **as opposed to** $7,662.51$**. Both Countries 2 and 3, under the regulation, have a higher final population than that under no regulation.**

The equilibrium conditions hold with excellent accuracy for this example with the regulation, as well.

Multiclass Example without a Regulation

We now present examples consisting of two classes of migrants. There are, again, three countries.

The initial populations of Class 1 in the countries are as in the preceding example as are the migration costs associated with the three countries. The utility functions, on the other hand, now capture interactions among classes. The superscripts 1 and 2 in the notation below refer to the respective class.

The utility functions are:

$$u_1^1(p) = -p_1^1 - .5p_2^1 - .5p_1^2 + 30,000, \; u_2^1(p) = -2p_2^1 - p_1^1 - p_2^2 + 20,000,$$
$$u_3^1(p) = -3p_3^1 + .5p_2^1 - p_3^2 + 10,000,$$
$$u_1^2(p) = -2p_1^2 - p_1^1 + 25,000, \; u_2^2(p) = -3p_2^2 - p_2^1 + 15,000,$$
$$u_3^2(p) = -p_3 - .5p_1^1 + 20,000.$$

The initial populations of Class 2 in the countries are: $\bar{p}_1^2 = 5,000$, $\bar{p}_2^2 = 3,000$, and $\bar{p}_3^2 = 500$.

Multiclass Example without a Regulation

Again, we have that $c^k_{ii}(f) = 0$, $\forall i$, and for $k = 1, 2$. The additional migration cost functions for Class 2 are:

$$c^2_{12}(f) = 2f^2_{12} + 10, \quad c^2_{13}(f) = f^2_{13} + 20,$$

$$c^2_{21}(f) = 3f^2_{21} + 10, \quad c^2_{23}(f) = 2f^2_{23} + 30,$$

$$c^2_{31}(f) = f^2_{31} + 25, \quad c^2_{32}(f) = 2f^2_{32} + 15.$$

The remainder of the migration cost functions are as in the preceding example(s).

Multiclass Example without and with a Regulation

The equilibrium migration flows are now:

for Class 1:

$$f_{11}^{1*} = 10,000.00, \quad f_{12}^{1*} = 0.00, \quad f_{13}^{1*} = 0.00,$$

$$f_{21}^{1*} = 2,649.57, \quad f_{22}^{1*} = 1,547.75, \quad f_{23}^{1*} = 802.68,$$

$$f_{31}^{1*} = 1,000.00, \quad f_{32}^{1*} = 0.00, \quad f_{33}^{1*} = 0.00,$$

for Class 2:

$$f_{11}^{2*} = 2,343.67, \quad f_{12}^{2*} = 182.49, \quad f_{13}^{2*} = 2,473.85,$$

$$f_{21}^{2*} = 0.00, \quad f_{22}^{2*} = 1,955.57, \quad f_{23}^{2*} = 1,044.43,$$

$$f_{31}^{2*} = 0.00, \quad f_{32}^{2*} = 0.00, \quad f_{33}^{2*} = 500.00.$$

Multiclass Example without a Regulation

The incurred migration costs at equilibrium are:

for Class 1:

$$c^1_{11}(f^*) = 0.00, \quad c^1_{12}(f^*) = 20.00, \quad c^1_{13}(f^*) = 30.00,$$

$$c^1_{21}(f^*) = 13,287.86, \quad c^1_{22}(f^*) = 0.00, \quad c^1_{23}(f^*) = 3,230.72,$$

$$c^1_{31}(f^*) = 6,080.09, \quad c^1_{32}(f^*) = 60.00, \quad c^1_{33}(f^*) = 0.00,$$

for Class 2:

$$c^2_{11}(f^*) = 0.00, \quad c^2_{12}(f^*) = 374.98, \quad c^2_{13}(f^*) = 2,493.85,$$

$$c^2_{21}(f^*) = 10.00, \quad c^2_{22}(f^*) = 0.00, \quad c^2_{23}(f^*) = 2,118.86,$$

$$c^2_{31}(f^*) = 25.00, \quad c^2_{32}(f^*) = 15.00, \quad c^2_{33}(f^*) = 0.00.$$

Multiclass Example without a Regulation

The above equilibrium flow pattern corresponds to the following multiclass equilibrium population distribution:

$$p_1^{1*} = 13,649.59, \quad p_2^{1*} = 1,547.75, \quad p_3^{1*} = 802.68,$$

$$p_1^{2*} = 2,343.67, \quad p_2^{2*} = 2,138.06, \quad p_3^{2*} = 4,018.28,$$

and associated multiclass utilities at the equilibrium given by:

$$u_1^1(p^*) = 14,404.70, \quad u_2^1(p^*) = 1,116.84, \quad u_3^1(p^*) = 4,347.56,$$

$$u_1^2(p^*) = 6,663.08, \quad u_2^2(p^*) = 7,038.06, \quad u_3^2(p^*) = 9,156.92.$$

The computed equilibrium Lagrange multipliers are:

$$\delta_1^{1*} = 14,404.70, \quad \delta_2^{1*} = 1,116.84, \quad \delta_3^{1*} = 8,324.62,$$

$$\delta_1^{2*} = 6,663.08, \quad \delta_2^{2*} = 7,038.06, \quad \delta_3^{2*} = 9,156.92.$$

Multiclass Example with a Regulation

Country 1 remains the most popular for Class 1; whereas Country 3 has the greatest number of Class 2 residents. Class 2 prefers Country 3 whereas Class 1 prefers Country 1.

We now present the multiclass human migration network example under a regulation. The data are as in the multiclass example above except now we consider the following scenario. The government of Country 3 is concerned about overpopulation and imposes the following regulation, which bounds the number of migrants of either class to the country:

$$f^1_{13} + f^1_{23} + f^2_{13} + f^2_{23} \leq 2,000.$$

Multiclass Example with a Regulation

The modified projection method yields the following multiclass equilibrium solution.

for Class 1:

$$f_{11}^{1*} = 10,000.00, \quad f_{12}^{1*} = 0.00, \quad f_{13}^{1*} = 0.00,$$

$$f_{21}^{1*} = 2,746.92, \quad f_{22}^{1*} = 1,788.86, \quad f_{23}^{1*} = 464.22,$$

$$f_{31}^{1*} = 1,000.00, \quad f_{32}^{1*} = 0.00, \quad f_{33}^{1*} = 0.00,$$

for Class 2:

$$f_{11}^{2*} = 3,581.93, \quad f_{12}^{2*} = 232.53, \quad f_{13}^{2*} = 1,185.54,$$

$$f_{21}^{2*} = 0.00, \quad f_{22}^{2*} = 2,649.76, \quad f_{23}^{2*} = 350.24,$$

$$f_{31}^{2*} = 0.00, \quad f_{32}^{2*} = 0.00, \quad f_{33}^{2*} = 500.00.$$

Multiclass Example with a Regulation

The incurred migration costs at equilibrium are:

for Class 1:

$$c^1_{11}(f^*) = 0.00, \quad c^1_{12}(f^*) = 20.00, \quad c^1_{1\bar{3}}(f^*) = 30.00,$$

$$c^1_{21}(f^*) = 13,774.62, \quad c^1_{22}(f^*) = 0.00, \quad c^1_{2\bar{3}}(f^*) = 1,876.90,$$

$$c^1_{31}(f^*) = 6.080.02, \quad c^1_{32}(f^*) = 60.00, \quad c^1_{33}(f^*) = 0.00,$$

for Class 2:

$$c^2_{11}(f^*) = 0.00, \quad c^2_{12}(f^*) = 475.06, \quad c^2_{1\bar{3}}(f^*) = 1,205.54,$$

$$c^2_{21}(f^*) = 10.00, \quad c^2_{22}(f^*) = 0.00, \quad c^2_{2\bar{3}}(f^*) = 730.47,$$

$$c^2_{31}(f^*) = 25.00, \quad c^2_{32}(f^*) = 15.00, \quad c^2_{33}(f^*) = 0.00.$$

Multiclass Example with a Regulation

The above equilibrium migration flow pattern corresponds to the following multiclass equilibrium population distribution:

$$p_1^{1*} = 13,746.93, \quad p_2^{1*} = 1,788.86, \quad p_3^{1*} = 464.22,$$

$$p_1^{2*} = 3,581.93, \quad p_2^{2*} = 2,882.29, \quad p_3^{2*} = 2,035.78,$$

and associated multiclass utilities at the equilibrium given by:

$$u_1^1(p^*) = 13,567.68, \quad u_2^1(p^*) = -206.93, \quad u_3^1(p^*) = 7,465.98,$$

$$u_1^2(p^*) = 4,089.21, \quad u_2^2(p^*) = 4,564.27, \quad u_3^2(p^*) = 11,090.76.$$

The computed equilibrium Lagrange multipliers are:

$$\delta_{11}^* = 13,567.69, \quad \delta_{21}^* = -206.94, \quad \delta_{31}^* = 7,487.66,$$

$$\delta_{12}^* = 4,089.20, \quad \delta_{22}^* = 4,564.27, \quad \delta_{32}^* = 11,090.75,$$

and for the regulation constraint: $\mu_3^* = 5,796.02$. Note that the regulation constraint holds tightly and, hence, the associated Lagrange multiplier is positive.

Multiclass Example with a Regulation

Both classes in Country 3, under the regulation, experience a higher utility than in the case of no regulation. However, with freedom of migration restricted, both classes in Countries 1 and 2 now experience lower utilities.

Members of Class 2, again, enjoy the highest utility in Country 3, whereas members of Class 1, again, enjoy the highest utility in Country 1.

Summary and Conclusions

Summary and Conclusions

International human migration is a subject of global concern with the number of international migrants growing faster than the world's population.

Push Forces: Challenges such as climate change and associated disruptions, along with wars, conflicts, and strife, are acting as push forces for humans to seek locations of greater safety and security.

Pull Forces: Others, on the other hand, are being pulled by the prospect of better economic conditions and enhanced prosperity in different countries.

Governments, in turn, are being forced to deal with increases in migratory flows across national boundaries. This has given rise to various regulations.

Summary and Conclusions

The paper builds on the earlier literature but with notable extensions:

- 1. We provide a new underlying network structure for problems of human migration and associated feasibility conditions.
- 2. A constraint is introduced to capture a plethora of regulations of migratory flows.
- 3. Lagrange analysis is conducted in the paper, with accompanying insights, on the associated utilities of migrants at origin and at destination nodes, along with incurred migration costs.
- 4. Alternative variational inequality formulations of the governing equilibrium conditions are constructed.
- 5. An algorithm is applied to solve a series of single class and multiclass international human migration problems, in order to illustrate the framework. **The numerical examples support the theoretical result that, under the regulation, denizens of the country imposing the migratory flow bound, incur higher utility, whereas those in other countries can experience reduced utility, due to restrictions on their movement across national boundaries for relocation.**

Summary and Conclusions

The perspective, in terms of relationships to transportation science, has been that of user-optimization.

We have also constructed models of human migration from a system-optimization perspectives and have even introduced regulations in the forms of "tolls," incentivizing migrants to move to locations, which is optimal from a system viewpoint.

Thank You!

For more information: https://supernet.isenberg.umass.edu/