International Financial Networks with Intermediation and Electronic Transactions

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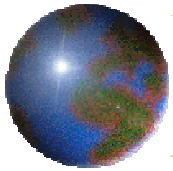
Eugene M. Isenberg School of Management

Virtual Center for Supernetworks



Background

Advances in telecommunications, including the adoption of the Internet by businesses, consumers, and financial institutions have had an enormous effect on financial services and options available for financial transactions.

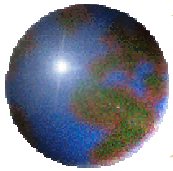
Distribution channels have been transformed, new types of services and products introduced.

Electronic commerce (e-commerce) through the Internet has allowed for new connections not previously possible.

Motivation

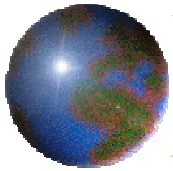
Electronic financial transactions, in 2001:

- 15 million Americans paid their bills online with up to 46 million expected by 2005.
- $160 billion in mortgages were taken out online in the US (see Mullaney and Little (2002)).



Applications of Networks in Finance

- Portfolio optimization
- Asset allocation
- Currency translation
- Risk management
  - Refinancing risk
  - Liquidity risk
  - Foreign exchange risk
  - Interest rate risk
  - Credit risk
  - Derivative instruments

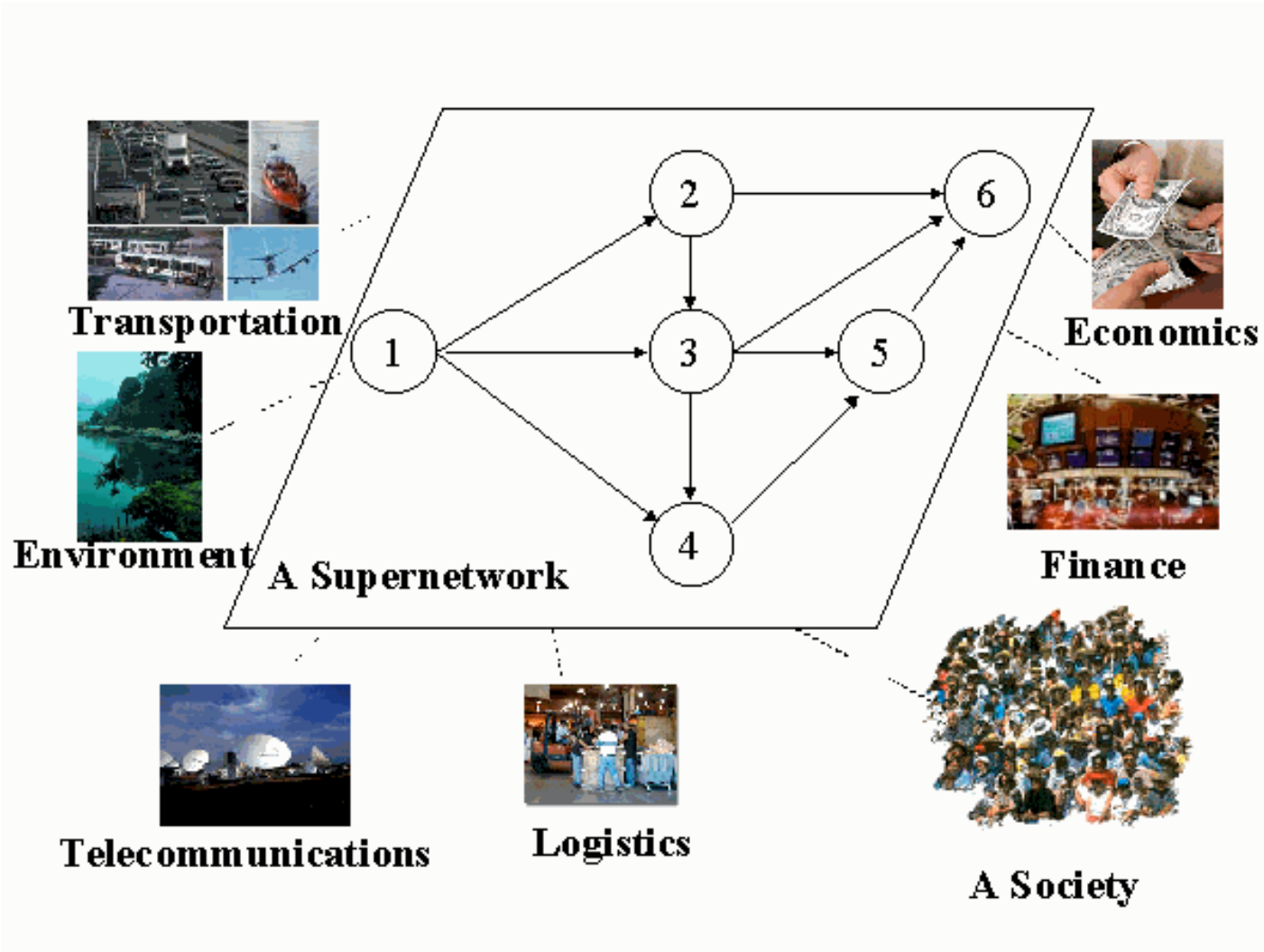


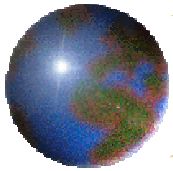
Background

Financial networks have been utilized to develop general models with multiple tiers of decision-makers by Nagurney and Ke (2001, 2002) and Nagurney and Cruz (2002).

The approach has utilized the concept of ***Supernetworks***; see the book by Nagurney and Dong (2002).

Supernetworks: A New Paradigm



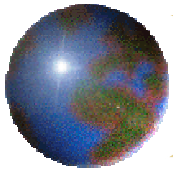


Supernetworks

Supernetworks may be comprised of such networks as transportation, telecommunication, logistical, and/or financial networks.

They may be *multilevel* as when they formalize the study of supply chain networks or *multitiered* as in the case of financial networks with intermediation.

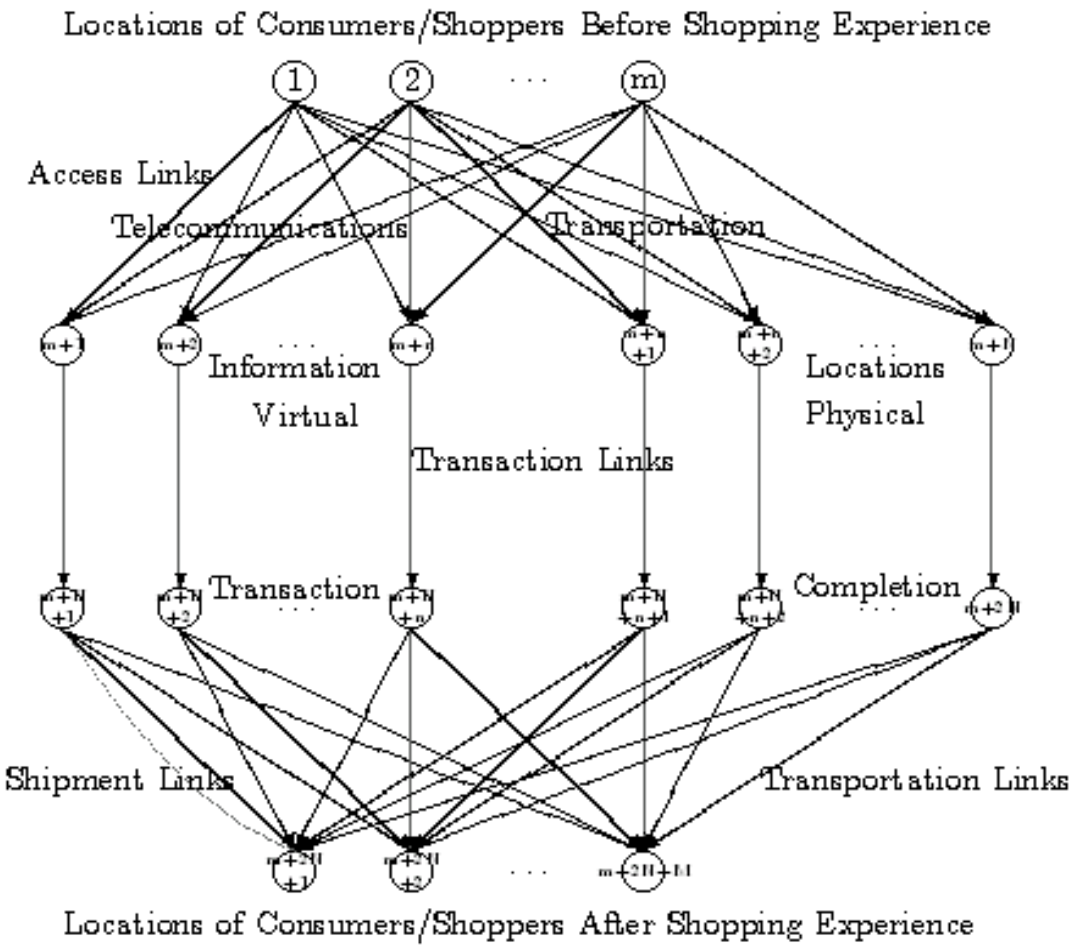
Decision-makers may be faced with multiple criteria; thus, the study of supernetworks also includes the study of *multicriteria decision-making*.



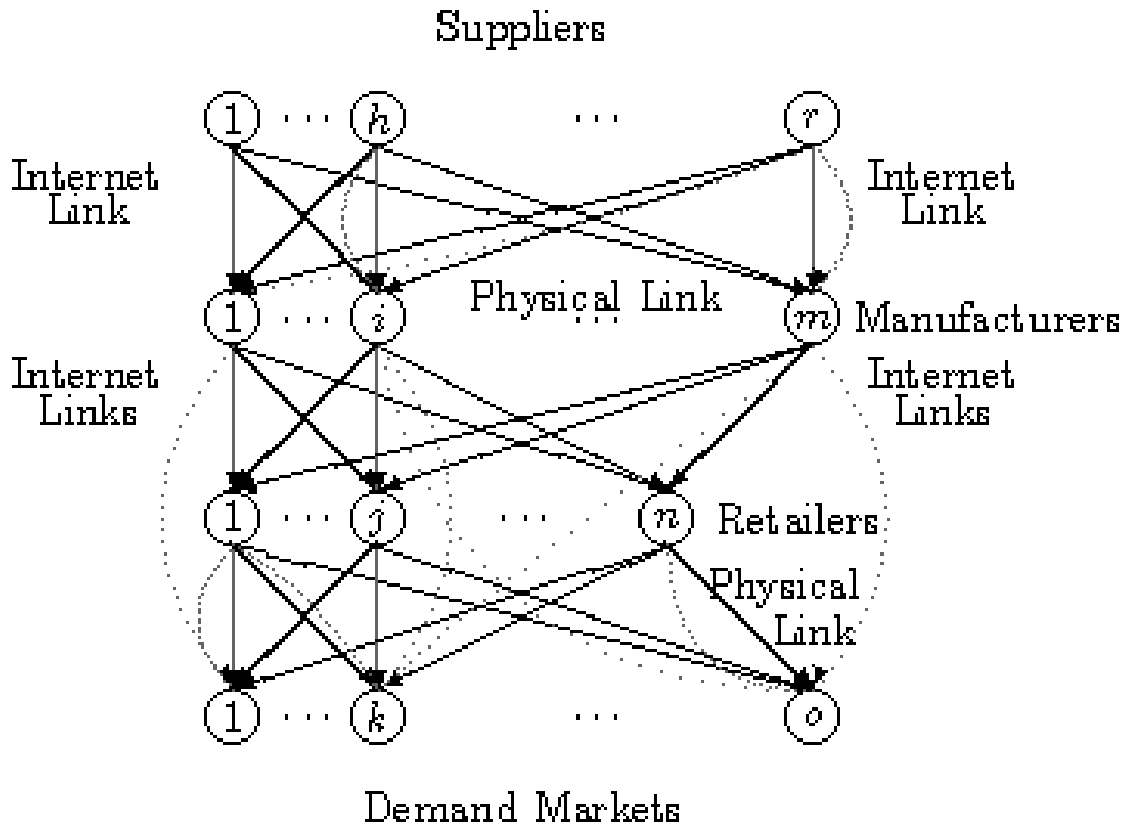
Applications of Supernetworks

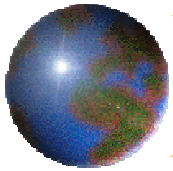
- Telecommuting/Commuting Decision-Making
- Teleshopping/Shopping Decision-Making
- Supply Chain Networks with Electronic Commerce
- Financial Networks with Electronic Transactions.

A Supernetwork Framework for Teleshopping versus Shopping



The Supernetwork Structure of a Supply Chain Network





Decision-Making Setting

- International financial networks
- Three distinct types of decision-makers
- Optimizing agents
- Multiple countries
- Multiple currencies
- Multiple instruments
- Non-investment
- Competition within a tier
- Cooperation between tiers

The International Financial Network with Electronic Transactions

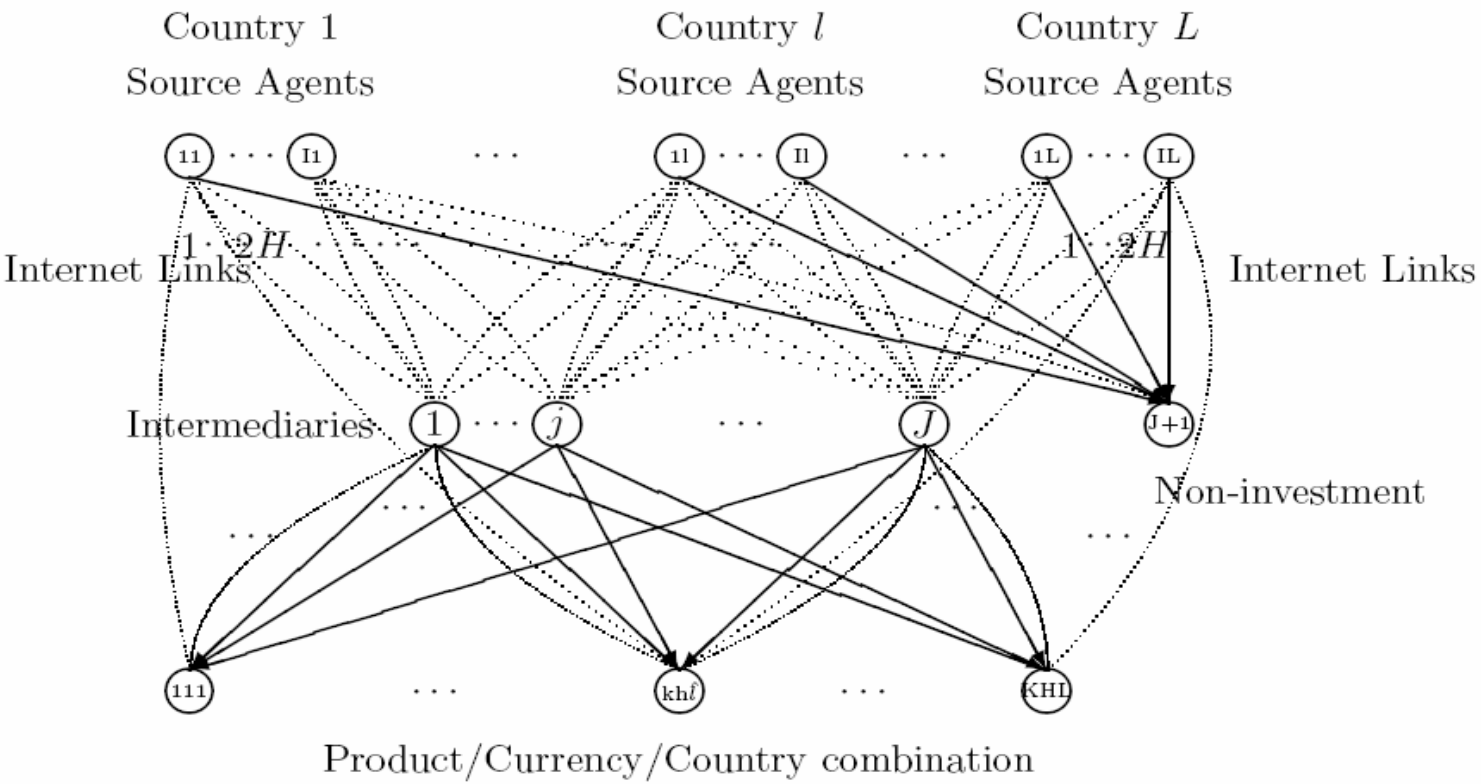
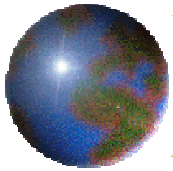


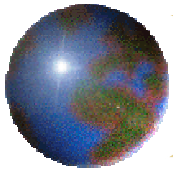
Figure 1: International Financial Network with Internediaries and with Electronic Transactions (Non-investment Allowed)



Characteristics of the Model

The International Financial Network Model with Electronic Transactions:

- Handles as many
  - Countries
  - Currencies
  - Source agents
  - Intermediaries
  - Demand markets
- Considers two modes of transactions:
  - Physical
  - Electronic
- Predicts not only the equilibrium flows but also the equilibrium prices.



Assumptions:

- Source agents can transact either physically or electronically with the intermediaries in different currencies.
- Source agents can transact directly with the demand markets via Internet links.
- Demand in a country can be associated with a particular currency.
- All transaction costs are measured in a base currency (US Dollar).

Some Model Notation

X^{il}_{jhm}	flow on link *h* joining node *il* with node *j in currency h and via m*
y^{j}_{khlm}	flow on link joining node *j* with node *klh via mode m*
X^{il}_{khl}	flow on internet link joining node *il* with node *khl*
ρ^{il}_{1jhm}	price associated with the instrument in currency *h* transacted between source *il* and intermediary *j via mode m*
ρ^{j}_{2khlm}	price associated with intermediary *j* and demand market *k* in currency *h* and country *l and mode m*
ρ^{il}_{1khl}	price associated with the product in currency *h* transacted between source *il* and demand market *khl*
ρ_{3khl}	price of the instrument at demand market *k* in currency *h* and in country *l*
e_h	rate of appreciation of currency *h* against the basic currency

Optimization Problem for Source Agent i in Country l

$$\text{Maximize} \quad U^{il}(x^{il}) = \sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}(\rho_{1jhm}^{il*} + e_h^*)x_{jhm}^{il} + \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}(\rho_{1kh\hat{l}}^{il*} + e_h^*)x_{kh\hat{l}}^{il}$$

$$-\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}c_{jhm}^{il}(x_{jhm}^{il}) - \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}c_{kh\hat{l}}^{il}(x_{kh\hat{l}}^{il}) - r^{il}(x^{il}),$$

subject to $x_{jh}^{il} \geq 0$, $x_{kh\hat{l}m}^{il} \geq 0$ for all $j, h, k, \hat{l}, m$, and to the constraint:

$$\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}x_{jhm}^{il} + \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}x_{kh\hat{l}}^{il} \leq S^{il},$$

Optimization Problem for Intermediary j

$$\text{Maximize} \quad U^j(x_j, y^j) = \sum_{k=1}^{k}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}(\rho^{j*}_{2kh\hat{l}m} + e^*_h)y^j_{kh\hat{l}m} - c_j(x^1) - \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}\hat{c}^{il}_{jhm}(x^{il}_{jhm})$$

$$- \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2} c^j_{kh\hat{l}m}(y^j_{kh\hat{l}m}) - \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}(\rho^{il*}_{1jhm} + e^*_h)x^{il}_{jhm} - r^j(x^1, y)$$

subject to: the nonnegativity constraints: $x^{il}_{jhm} \geq 0$, $y^j_{kh\hat{l}m} \geq 0$, for all $i, l, h, \hat{l}, m$, and

$$\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2} y^j_{kh\hat{l}m} \leq \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2} x^{il}_{jhm}.$$

The Consumers at the Demand Markets

$$\rho^{il*}_{1kh\hat{l}} + e^*_h + \hat{c}^{il}_{kh\hat{l}}(x^{2*}) \begin{cases} = \rho^*_{3kh\hat{l}}, & \text{if} \quad x^{il*}_{kh\hat{l}} > 0 \\ \geq \rho^*_{3kh\hat{l}}, & \text{if} \quad x^{il*}_{kh\hat{l}} = 0 \end{cases}$$

$$\rho^{j*}_{2kh\hat{l}m} + e^*_h + \hat{c}^{j}_{kh\hat{l}m}(y^*) \begin{cases} = \rho^*_{3kh\hat{l}}, & \text{if} \quad y^{j*}_{kh\hat{l}m} > 0 \\ \geq \rho^*_{3kh\hat{l}}, & \text{if} \quad y^{j*}_{kh\hat{l}m} = 0, \end{cases}$$

$$d_{kh\hat{l}}(\rho^*_3) \begin{cases} = \sum_{j=1}^{J} \sum_{m=1}^{2} y^{j*}_{kh\hat{l}m} + \sum_{i=1}^{I} \sum_{l=1}^{L} x^{il*}_{kh\hat{l}}, & \text{if} \quad \rho^*_{3kh\hat{l}} > 0 \\ \leq \sum_{j=1}^{J} \sum_{m=1}^{2} y^{j*}_{kh\hat{l}m} + \sum_{i=1}^{I} \sum_{l=1}^{L} x^{il*}_{kh\hat{l}}, & \text{if} \quad \rho^*_{3kh\hat{l}} = 0. \end{cases}$$

Variational Inequality Formulation

Theorem: The equilibrium conditions governing the international financial network with intermediation are equivalent to the solution of the variational inequality given by: determine $(x^{1*}; x^{2*}; y^*; \gamma^*; \rho^*_3) \in K$, satisfying:

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\frac{\partial r^{il}(x^{il*})}{\partial x^{il}_{jhm}} + \frac{\partial c^{il}_{jhm}(x^{il*}_{jhm})}{\partial x^{il}_{jhm}} + \frac{\partial r^{j}(x^{1*}, y^*)}{\partial x^{il}_{jhm}} + \frac{\partial c_j(x^{1*})}{\partial x^{il}_{jhm}} + \frac{\partial \hat{c}^{il}_{jhm}(x^{il*}_{jhm})}{\partial x^{il}_{jhm}} - \gamma^*_j\right]$$

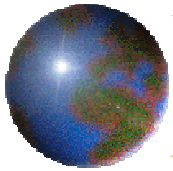
$$\times \left[x^{il}_{jhm} - x^{il*}_{jhm}\right]$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\frac{\partial r^{il}(x^{il*})}{\partial x^{il}_{kh\hat{l}}} + \frac{\partial c^{il}_{kh\hat{l}}(x^{il*}_{kh\hat{l}})}{\partial x^{il}_{kh\hat{l}}} + \hat{c}^{il}_{kh\hat{l}}(x^{2*}) - \rho^*_{3kh\hat{l}}\right] \times \left[x^{il}_{kh\hat{l}} - x^{il*}_{kh\hat{l}}\right]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}\left[\frac{\partial r^{j}(x^{1*}, y^*)}{\partial y^{j}_{kh\hat{l}m}} + \frac{\partial c^{j}_{kh\hat{l}}(y^{j*}_{kh\hat{l}m})}{\partial y^{j}_{kh\hat{l}m}} + \hat{c}^{j}_{kh\hat{l}m}(y^*) - \rho^*_{3kh\hat{l}} + \gamma^*_j\right] \times \left[y^{j}_{kh\hat{l}m} - y^{j*}_{kh\hat{l}m}\right]$$

$$+\sum_{j=1}^{J}\left[\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2} x^{il*}_{jhm} - \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2} y^{j*}_{kh\hat{l}m}\right] \times \left[\gamma_j - \gamma^*_j\right]$$

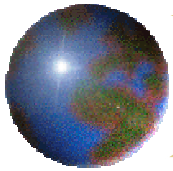
$$+\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\sum_{j=1}^{J}\sum_{m=1}^{2} y^{j*}_{kh\hat{l}m} + \sum_{i=1}^{I}\sum_{l=1}^{L} x^{il*}_{kh\hat{l}} - d_{kh\hat{l}}(\rho^*_3)\right] \times \left[\rho_{3kh\hat{l}} - \rho^*_{3kh\hat{l}}\right] \geq 0,$$



Additional Theoretical Results

We have established:

- Existence of the solution of the VI
- Uniqueness of the solution of the VI
- Convergence of the modified projection method.

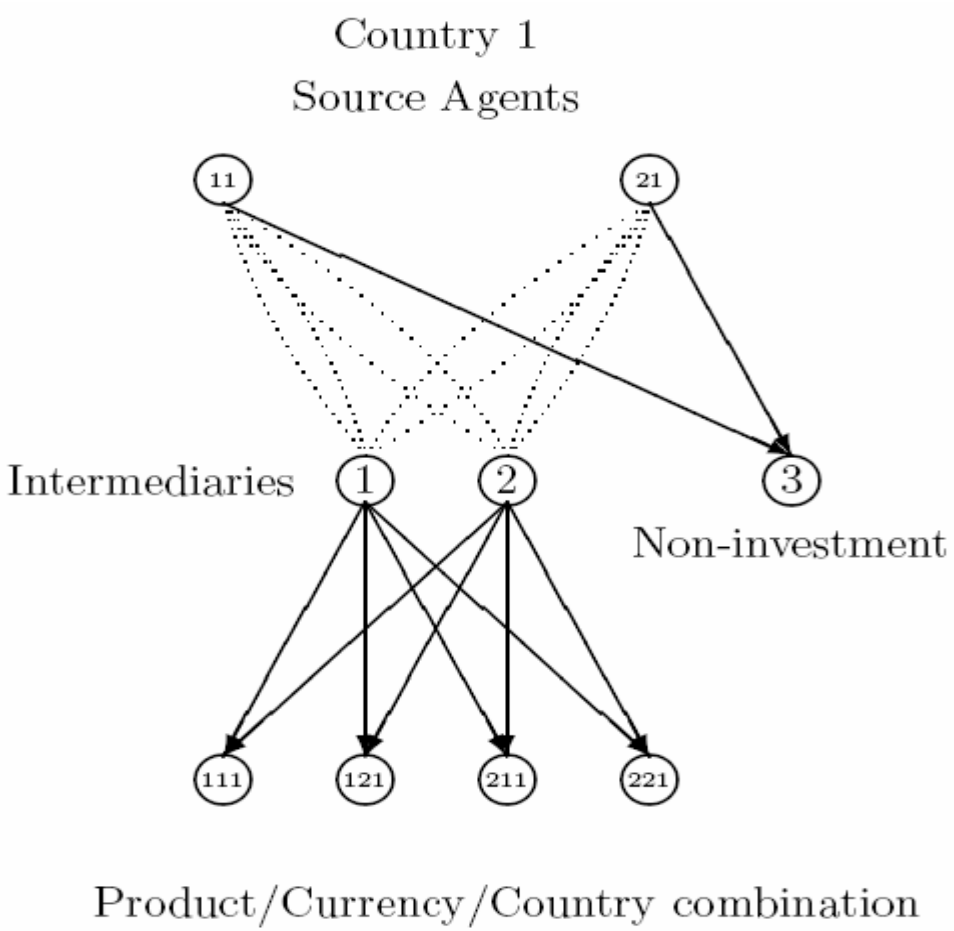


Algorithm

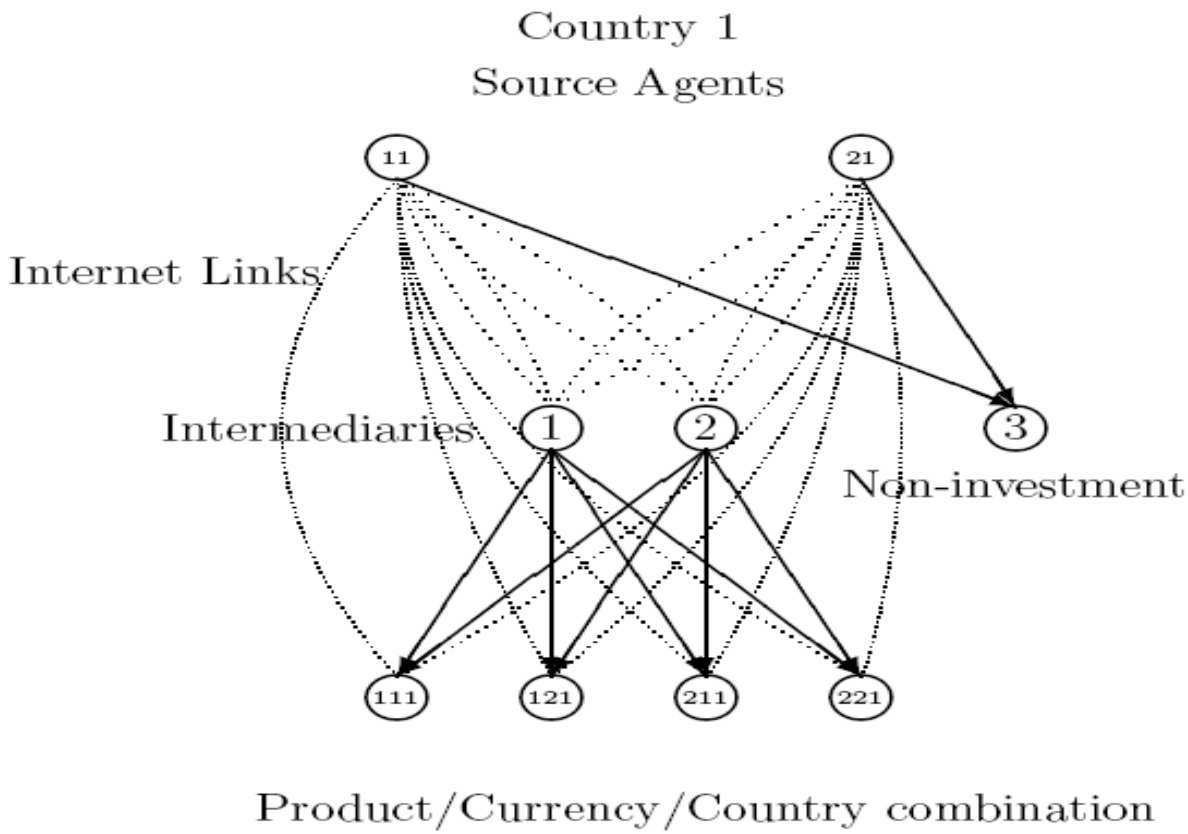
- Modified projection method

The notable feature of this algorithm is that it resolves the VI subproblems into network optimization problems with special structure that can be solved exactly in closed form.

Network Structure of Example 1 (without electronic transactions)



Network Structure of Example 2 (with electronic transactions)



Input Data for Examples 1 and 2

The transaction cost functions of the source agents	$c_{jhm}^{il}(x_{jhm}^{il}) = 0.5(x_{jhm}^{il})^2 + 3.5x_{jhm}^{il}$
The handling costs of the intermediaries	$c_j(x^1) = 0.5\sum_{i=1}^{2}\sum_{h=1}^{2}(x_{jh1}^{i1})^2$
The transaction costs of the intermediaries	$\hat{c}_{jhm}^{il}(x_{jh}^{il}) = 1.5(x_{jhm}^{il})^2 + 3x_{jhm}^{il}$
The transaction costs between the intermediaries and the consumers	$\hat{c}_{kh\hat{l}\hat{m}}^{j}(y) = y_{kh\hat{l}\hat{m}}^{j} + 5$
The variance-covariance matrices were equal to the identity matrices	Q^{il} and Q^{j}

Additional transaction costs for Example 2:

The transaction cost functions of the source agents for dealing with demand markets:	$c^{il}_{kh\hat{l}}(x^{il}_{kh\hat{l}}) = 0.5(x^{il}_{kh\hat{l}})^2 + x^{il}_{kh\hat{l}}$
Transaction cost functions from the demand markets:	$\hat{c}_{kh\hat{l}}{}^{il}(x^2) = .1x^{il}_{kh\hat{l}} + 1$

The demand functions at the demand markets:

$d_{111}(\rho_3) = -2\rho_{3111} - 1.5\rho_{3121} + 1000$	$d_{121}(\rho_3) = -2\rho_{3121} - 1.5\rho_{3111} + 1000$
$d_{211}(\rho_3) = -2\rho_{3211} - 1.5\rho_{3221} + 1000$	$d_{221}(\rho_3) = -2\rho_{3221} - 1.5\rho_{3211} + 1000$

Solutions to Examples 1, 2

	Example 1	Example 2
$x^{1*}=x^{il}_{jhm}, \forall i, l, j, h, m$	5.000	0.372
x^{2*}	--	4.637
y^{1*}_{1111}	5.000	0.372
y^{1*}_{1211}	5.000	0.372
$y^{1*}_{2111}=y^{1*}_{2211}$	5.000	0.372
y^{2*}_{1111}	5.000	0.372
y^{2*}_{1211}	5.000	0.372
$y^{2*}_{2111}= y^{2*}_{2211}$	5.000	0.372
$\gamma^*_1= \gamma^*_2$	262.8566	276.738
ρ^*_{3111}	282.856	282.857
ρ^*_{3121}	282.856	282.857
$\rho^*_{3211}= \rho^*_{3221}$	282.856	282.857
ρ^{1*}	214.8566	270.385

Network Structure of Example 3 (without electronic transactions)

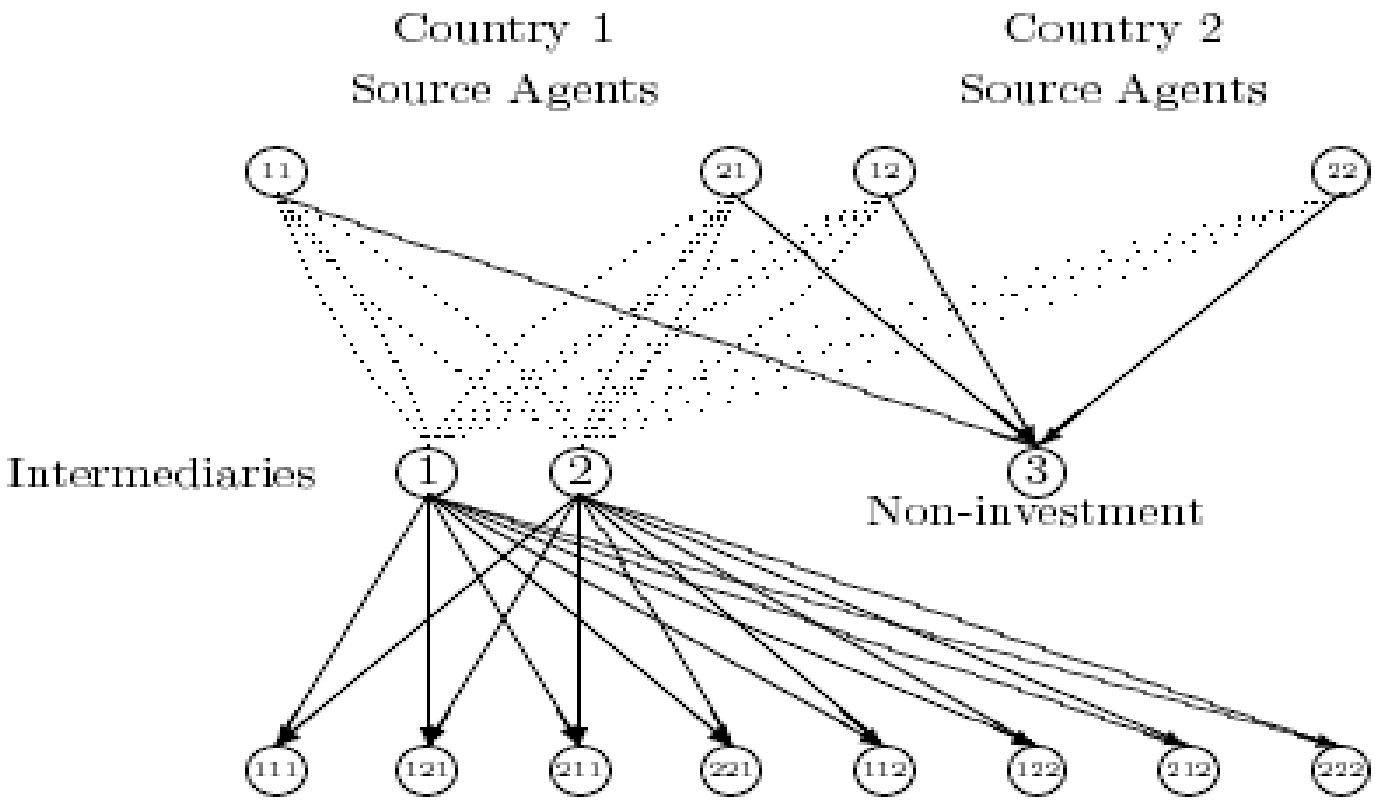


Figure 4: International Financial Network for Example 3

Network Structure of Example 4 (with electronic transactions)

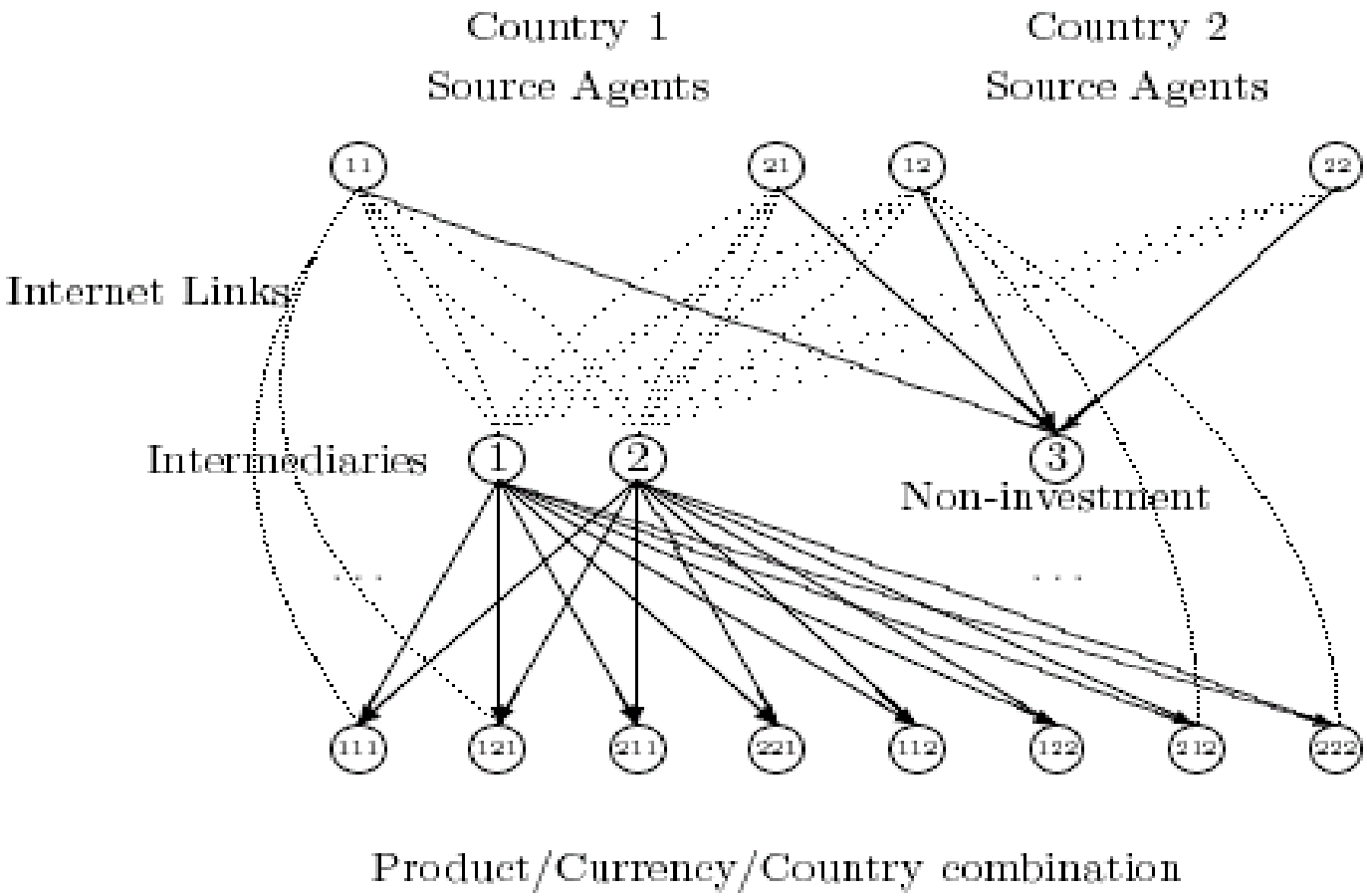


Figure 5: International Financial Network for Example 4

Input Data for Examples 3 and 4

The transaction cost functions of the source agents	$c_{jhm}^{il}(x_{jhm}^{il}) = 0.5(x_{jhm}^{il})^2 + 3.5x_{jhm}^{il}$
The handling costs of the intermediaries	$c_j(x^1) = 0.5\sum_{i=1}^{2}\sum_{h=1}^{2}(x_{jh1}^{i1})^2$
The transaction costs of the intermediaries	$\hat{c}_{jhm}^{il}(x_{jh}^{il}) = 1.5(x_{jhm}^{il})^2 + 3x_{jhm}^{il}$
The transaction costs between the intermediaries and the consumers	$\hat{c}_{kh\hat{l}\hat{m}}^{j}(y) = y_{kh\hat{l}\hat{m}}^{j} + 5$
The variance-covariance matrices were equal to the identity matrices	Q^{il} and Q^{j}

Input Data for Examples 3 and 4

Additional transaction costs for Example 4:

The transaction cost functions of the source agents for dealing with demand markets:	$c^{il}_{kh\hat{l}}(x^{il}_{kh\hat{l}}) = 0.5(x^{il}_{kh\hat{l}})^2 + x^{il}_{kh\hat{l}}$
Transaction cost functions from perspective of demand markets:	$\hat{c}_{kh\hat{l}}{}^{il}(x^2) = .1x^{il}_{kh\hat{l}} + 1$

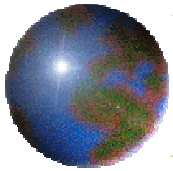
The demand functions at the demand markets:

$d_{111}(\rho_3) = -2\rho_{3111} - 1.5\rho_{3121} + 1000$	$d_{121}(\rho_3) = -2\rho_{3121} - 1.5\rho_{3111} + 1000$
$d_{211}(\rho_3) = -2\rho_{3211} - 1.5\rho_{3221} + 1000$	$d_{221}(\rho_3) = -2\rho_{3221} - 1.5\rho_{3211} + 1000$
$d_{112}(\rho_3) = -2\rho_{3112} - 1.5\rho_{3122} + 1000$	$d_{122}(\rho_3) = -2\rho_{3122} - 1.5\rho_{3112} + 1000$
$d_{212}(\rho_3) = -2\rho_{3212} - 1.5\rho_{3222} + 1000$	$d_{222}(\rho_3) = -2\rho_{3222} - 1.5\rho_{3212} + 1000$

Solutions to Examples 3, 4

	Example 3	Example 4
$x^{1*} = x^{il}_{jhm}, \forall i, l, j, h, m$	5.0000	0.0000
$x^{2*} = x^{il}_{khl}, \forall i, l, k, h, l$	--	2.5000
$y^{1*}_{1111} = y^{1*}_{1211} = y^{1*}_{2111} = y^{1*}_{2211}$	5.0000	0.0000
y^{1*}_{1121}	5.0000	0.0000
y^{1*}_{1221}	5.0000	0.0000
y^{1*}_{2121}	5.0000	0.0000
y^{1*}_{2221}	5.0000	0.0000
$y^{2*}_{1111} = y^{2*}_{1211} = y^{2*}_{2111} = y^{2*}_{2211}$	5.0000	0.0000
y^{2*}_{1121}	5.0000	0.0000
y^{2*}_{1221}	5.0000	0.0000
y^{2*}_{2121}	5.0000	0.0000
y^{2*}_{2221}	5.0000	0.0000

	Example 3	Example 4
$\gamma^*_1 = \gamma^*_2$	262.8486	278.0899
$\rho^*_{3111} = \rho^*_{3121} = \rho^*_{3211} = \rho^*_{3221}$	282.8591	282.8568
ρ^*_{3112}	282.8591	282.8568
ρ^*_{3122}	282.8591	282.8568
ρ^*_{3212}	282.8591	282.8568
ρ^*_{3222}	282.8591	282.8568
$\rho 1^*$	194.8486	194.8486



Summary and Conclusions

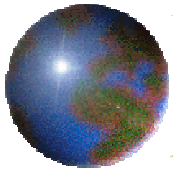
- We developed a framework for the ***modeling, analysis, and computation*** of solutions to international financial problems with intermediaries in the presence of electronic transactions. The framework makes use of the ***supernetwork*** concept.

- The international financial network framework allows for the handling of as many countries, source agents,intermediaries and as many currencies as needed.

Future Research

- We generalized the recent work of Nagurney and Cruz (2002) and Nagurney and Ke (2001, 2003).

- Multicriteria decision-makers
  - Net revenue maximization
  - Risk minimization
- Dynamic adjustment process
  - Evolution of financial flows
  - Evolution of price



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