# Investment with Linear Programming

Xuan Huang

# Investment 101

- Return – capital gain or loss.
- Risk – No free lunch!
- Securities – bond, stock, etc.
- Mutual fund Portfolio – risk diversification; don’t put all your eggs in one basket.

# LP application in Investment

- Portfolio Management
  - Mutual fund portfolio optimization
  - Bond portfolio improvement
- Asset and Liability Management (ALM)

# Portfolio Optimization

- W.F. Sharpe, 1967.
- H. Konno and H. Yamazaki, 1991.
- M.R. Young, 1998.

# Portfolio Optimization

- $x_j$ – decision variables, j= 1,2...n
- $R_j$ – rate of return. $r_{jt}$ , t= 1,2...T;
  $E(R_j) = r_j = \Sigma_t r_{jt} / T$

- Expected return: $E(\Sigma_j R_j x_j) = \Sigma_j r_j x_j$
- Risk of Return: $Var(\Sigma_j R_j x_j)$

# Famous Markowitz's Model (1952)

$$\text{minimize } f(x_1,\ldots x_n) = Var\left(\sum_{j=1}^{n} R_j x_j\right)$$

$$\text{subject to } \sum_{j=1}^{n} r_j x_j \geq \rho M_0$$

$$\sum_{j=1}^{n} x_j = M_0$$

$$x_j \geq 0 \qquad j = 1,\ldots,n$$

# Famous Markowitz's Model (1952)

$$\text{minimize} \quad \boxed{f\left(x_1,\ldots x_n\right) = Var\left(\sum_{j=1}^{n} R_j x_j\right)}$$

$$\text{subject to} \quad \sum_{j=1}^{n} r_j x_j \geq \rho M_0$$

$$\sum_{j=1}^{n} x_j = M_0$$

$$x_j \geq 0 \qquad\qquad j = 1,\ldots,n$$

# Famous Markowitz's Model (1952)

$$\text{minimize } f(x_1,\ldots x_n) = Var\left(\sum_{j=1}^{n} R_j x_j\right)$$

$$\text{subject to } \boxed{\sum_{j=1}^{n} r_j x_j \geq \rho M_0}$$

$$\sum_{j=1}^{n} x_j = M_0$$

$$x_j \geq 0 \qquad\qquad j = 1,\ldots,n$$

# Famous Markowitz's Model (1952)

$$\text{minimize } f(x_1,\ldots x_n) = Var\left(\sum_{j=1}^{n} R_j x_j\right)$$

$$\text{subject to } \sum_{j=1}^{n} r_j x_j \geq \rho M_0$$

$$\boxed{\sum_{j=1}^{n} x_j = M_0}$$

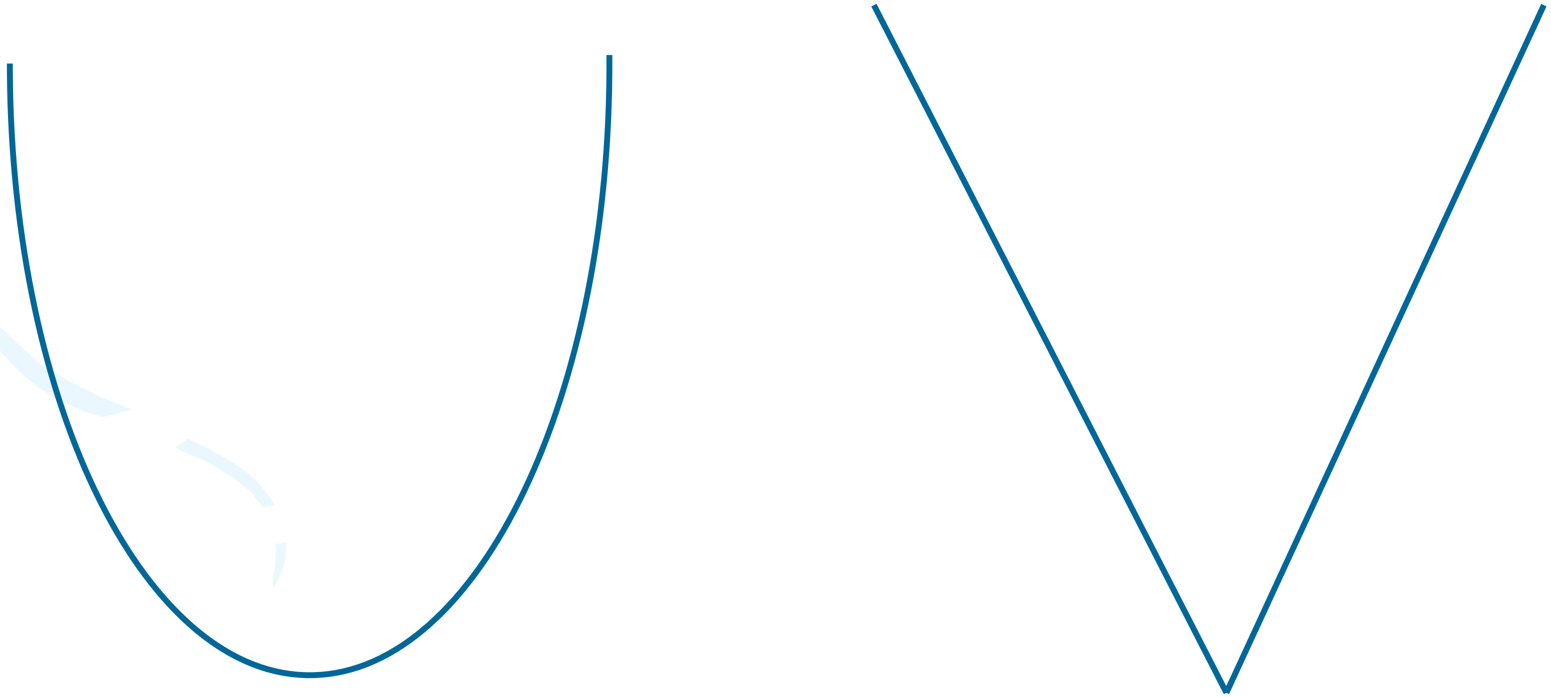
$$x_j \geq 0 \qquad\qquad j = 1,\ldots,n$$

# Sharpe remarked (1971)…

-If the essence of the portfolio analysis problem could be adequately captured in a form for linear programming methods, the prospect for practical application would be greatly enhanced.

# Konno and Yamazaki's treatment

$$\min f(x)=x^2 \xrightarrow{\textbf{Monotonic transform}} \min g(x)=|x|$$



# Konno and Yamazaki's treatment

minimize $\boxed{\sum_{t=1}^{T}\left|\sum_{j=1}^{n} a_{jt} x_j\right| / T}$

subject to ......

# Konno and Yamazaki's treatment

minimize $\sum_{t=1}^{T} \left| \sum_{j=1}^{n} a_{jt} x_j \right| / T$

subject to ......

minimize $|x|$

subject to ......

# Konno and Yamazaki's treatment

minimize $\sum_{t=1}^{T}\left|\sum_{j=1}^{n} a_{jt} x_j\right| / T$

subject to ......

minimize $\boxed{|x|}$ $\longrightarrow$ $\boxed{y}$

subject to ......

$-y \le x \le y$

......

# Konno and Yamazaki's treatment

$$\text{minimize } \sum_{t=1}^{T} \left| \sum_{j=1}^{n} a_{jt} x_j \right| / T$$

subject to ......

minimize $y$

subject to $-y \le x \le y$

......

minimize $\boxed{|x|}$

subject to ......

# Konno and Yamazaki's treatment

$$\text{minimize} \quad \sum_{t=1}^{T} y_t / T$$

$$\text{subject to} \quad y_t \geq \sum_{j=1}^{n} a_{jt} x_j \qquad t=1,2,\ldots T$$

$$y_t \geq -\sum_{j=1}^{n} a_{jt} x_j \qquad t=1,2,\ldots T$$

$$\sum_{j=1}^{n} r_j x_j \geq \rho M_0$$

$$\sum_{j=1}^{n} x_j = M_0$$

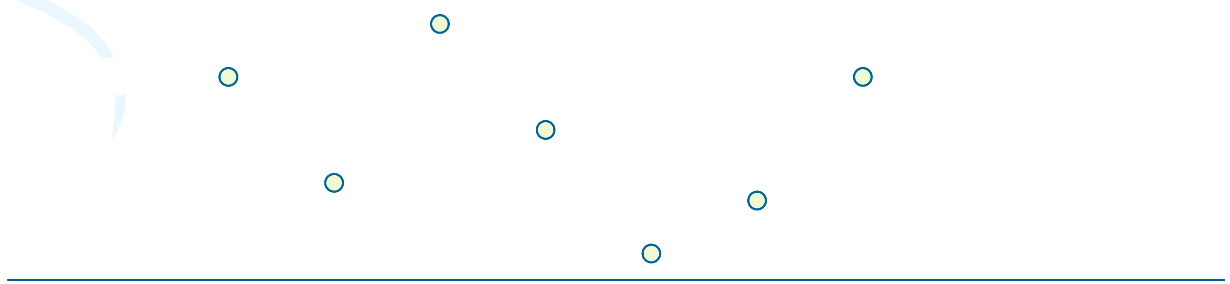
$$x_j \geq 0 \qquad j = 1, \ldots, n$$

$$y_t \geq 0 \qquad t = 1, \ldots, T$$

# Young's treatment

- At t, expected return of the portfolio is $\Sigma_j r_{jt} x_j$

- Maximize ( $\min_t \; \Sigma_j r_{jt} x_j$ )



# Young's treatment

$$\text{maximize} \quad \min_{t} \sum_{j=1}^{n} r_{jt} x_j$$

subject to ......

# Young's treatment

maximize $\boxed{\min_{t} \sum_{j=1}^{n} r_{jt} x_j}$

subject to ......

$\boxed{M_p}$

# Young's treatment

maximize $\boxed{\min_{t} \sum_{j=1}^{n} r_{jt} x_j}$

subject to ......

$M_p$

$$\boxed{\sum_{j=1}^{n} r_{jt} x_j \geq M_p \quad \forall t}$$

# Young's treatment

$$\text{maximize} \quad \boxed{\min_{t} \sum_{j=1}^{n} r_{jt} x_j}$$

subject to ……

$$\text{maximize} \quad \boxed{M_p}$$

$$\text{subject to} \quad \sum_{j=1}^{n} r_{jt} x_j \geq M_p \quad \forall t$$

……

# Sharpe's Improvement of the model

- Modify the objective function to

  $$Z=(1-\lambda)\text{Return}-\lambda\text{Risk}$$

- Adding more constraints with industry consideration.

# Reference


- H. Konno and H. Yamazaki, 1991.
  - "Mean-Absolute Deviation Portfolio Optimization Model and its Application to Tokyo Stock Market".
- W.F. Sharpe, 1967.
  - "A linear Programming Algorithm for Mutual Fund Portfolio Selection".
- M.R. Young, 1998.
  - "A Minimax Portfolio Selection Rule with Linear Programming Solution".

# Thank you!