A Relative Total Cost Index for the Evaluation of Transportation Network Robustness in the Presence of Degradable Links and Alternative Travel Behavior

John F. Smith Memorial Professor Anna Nagurney
Doctoral Student Qiang Qiang

Isenberg School of Management
University of Massachusetts
Amherst, Massachusetts 01003

INFORMS Annual Meeting, Washington DC October 12-15, 2008

Acknowledgements

This research was supported in part by

- The National Science Foundation under Grant No.: IIS-0002647 Awarded to John F. Smith Memorial Professor Anna Nagurney.
- John F. Smith Memorial Fund - University of Massachusetts at Amherst.

This support is gratefully acknowledged.

This presentation is based on the paper by the same name in press in the *International Transactions in Operational Research*.

1. Motivation
2. Literature
3. Transportation Network Models
4. Relative Total Cost Indices
5. Numerical Examples
6. Summary and Conclusions

1. Motivation
2. Literature
3. Transportation Network Models
4. Relative Total Cost Indices
5. Numerical Examples
6. Summary and Conclusions

The U.S. Ailing Infrastructure

- The U.S. is experiencing a freight capacity crisis that threatens the strength and productivity of the U.S. economy. According to the American Road & Transportation Builders Association (see Jeanneret (2006)), nearly 75% of US freight is carried in the US on highways and bottlenecks are causing truckers 243 million hours of delay annually with an estimated associated cost of $8 billion.
- It is noted that the US government is facing a $1.6 trillion deficit over the five years in terms of infrastructure repairing and reconstruction according to a recent estimate (Environment News Service (2008)).

The U.S. Ailing Infrastructure

- Over one-quarter of the nation's 590,750 bridges were rated structurally deficient or functionally obsolete. The degradation of transportation networks due to poor maintenance, natural disasters, deterioration over time, as well as unforeseen attacks now lead to estimates of $94 billion in the U.S. in terms of needed repairs for roads alone. Poor road conditions in the U.S. cost motorists $54 billion in repairs and operating costs annually (ASCE Survey (2005)).

The U.S. Ailing Infrastructure

- Due to the constant breakdowns of the U.S. transportation networks and the increasing number of vehicles, American commuters now spend 3.5 billion hours a year stuck in traffic, which translates to a cost of $63.2 billion a year to the economy (ASCE (2005)).
- The number of motor vehicles in the U.S. has risen by 157 million (or 212.16%) since 1960 while the population of licensed drivers grew by 109 million (or 125.28%) (U.S. Department of Transportation (2004)).

Figure: Road Damaged after Flooding, Source: AccuWeather.com

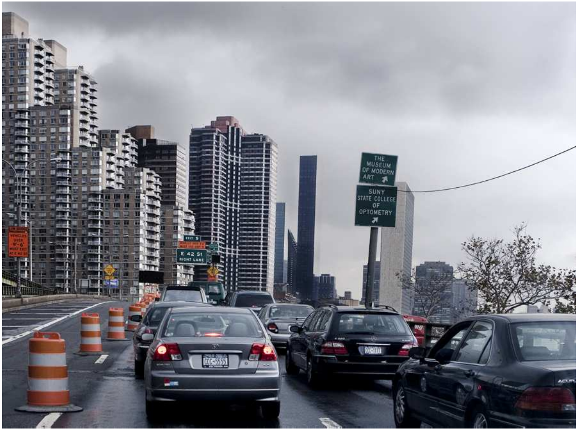


Figure: Congestion Caused by One Lane Closed, Source: Flicker.com

Robustness Measure

- The focus of the robustness of networks (and complex networks) has been on the impact of different network measures when facing the removal of nodes on networks.
- We focus on the degradation of links through reductions in their capacities and the effects on the induced travel costs in the presence of known travel demands and different functional forms for the links.

"Robustness" in Engineering and Computer Science

- IEEE (1990) defined robustness as "the degree to which a system of component can function correctly in the presence of invalid inputs or stressful environmental conditions."

- Gribble (2001) defined system robustness as "the ability of a system to continue to operate correctly across a wide range of operational conditions, and to fail gracefully outside of that range."

- Schillllo et al. (2001) argued that robustness has to be studied "in relation to some definition of the performance measure."

"Robustness" in Transportation

- Sakakibara et al. (2004) proposed a topological index. The authors considered a transportation network to be robust if it is dispersed in terms of the number of links connected to each node.

- Scott et al. (2005) examined transportation network robustness by analyzing the increase in the total network cost after removal of certain network components.

1 Motivation

2 Literature

3 Transportation Network Models

4 Relative Total Cost Indices

5 Numerical Examples

6 Summary and Conclusions

Related Literature in Transportation Network Vulnerability and Robustness

- Berdica (2002)
- Murray-Tuite and Mahmassani (2004)
- Sakakibara, Kajitani, and Okada (2004)
- Scott et al. (2006)
- Taylor, Sekhar, and D'Este (2006)
- Murray and Grubesic (2007)
- Jenelius (2007)

Our Research on Network Efficiency, Vulnerability, and Robustness I

- Nagurney, A., Qiang, Q., 2007a. A network efficiency measure for congested networks. *Europhysics Letters 79*, 38005.
- Nagurney, A., Qiang, Q., 2007b. A transportation network efficiency measure that captures flows, behavior, and costs with applications to network component importance identification and vulnerability. *Proceedings of the 18th Annual POMS Conference*.
- Nagurney, A., Qiang, Q., 2007c. Robustness of transportation networks subject to degradable links. *Europhysics Letters 80*, 68001.
- Nagurney, A., Qiang, Q., 2007d. A network efficiency measure with application to critical infrastructure networks. *Journal of Global Optimization 40*, 261-275.

Our Research on Network Efficiency, Vulnerability, and Robustness II

- Nagurney, A., Qiang, Q., 2008a. An efficiency measure for dynamic networks with application to the Internet and vulnerability analysis. *Netnomics*, in press.
- Nagurney, A., Qiang, Q., Nagurney, L. S., 2008b. Environmental impact assessment of transportation networks with degradable links in an era of climate change. Isenberg School of Management, University of Massachusetts at Amherst.
- Qiang, Q., Nagurney, A., 2008c. A unified network performance measure with importance identification and the ranking of network components. *Optimization Letters 2*, 127-142.

Our Research on Network Efficiency, Vulnerability, and Robustness III

- Nagurney, A., Qiang, Q., 2008d. Identification of critical nodes and links in financial networks with intermediation and electronic transactions. In **Computational Methods in Financial Engineering**. Kontoghiorghes, E. J., Rustem, B., Winker, P., Editors, Springer, Berlin, Germany, pp 273-297.

1 Motivation

2 Literature

3 Transportation Network Models

4 Relative Total Cost Indices

5 Numerical Examples

6 Summary and Conclusions

Notation for the Transportation Network Models

- Network $\mathcal{G}$ with the set of links L with n_L elements, the set of paths P with n_P elements, and the set of O/D pairs W with n_W elements.
- The set of (acyclic) paths connecting O/D pair w are denoted by P_w, the links by a, b, etc, the paths by p, q, etc., and the O/D pairs by w_1, w_2, etc.
- The flow on path p is denoted by x_p and the flow on link a by f_a. x is the column vector of all path flows and f is the column vector of all link flows.
- Travel cost on a path p is denoted by C_p and the travel cost on a link a by c_a. C is the column vector of all path costs and c is the column vector of all link costs. Assume that the user link cost functions are continuous.
- Denote the travel demand between O/D pair w by d_w and the travel disutility by λ_w, where d_w is fixed and known for all w.

Conservation of Flow Between Demands and Path Flows

$$d_w = \sum_{p \in P_w} x_p, \quad \forall w \in W, \tag{1}$$

Conservation of Flow Between Link Flows and Path Flows

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L, \tag{2}$$

where $\delta_{ap} = 1$, if path p contains link a, and $\delta_{ap} = 0$, otherwise.

Link Costs and Path Costs

$$C_p = \sum_{a \in L} c_a \delta_{ap}, \quad \forall p \in P, \tag{3}$$

The user cost on link a is denoted by c_a where

$$c_a = c_a(f_a), \quad \forall a \in L. \tag{4}$$

Bureau of Public Roads (BPR) (1964) Link Cost Function

$$c_a(f_a) = t_a^0[1 + k(\frac{f_a}{u_a})^\beta], \quad \forall a \in L, \tag{5}$$

u_a is the “practical” capacity on link a, which also has the interpretation of the level-of-service flow rate; t_a^0 is the free-flow travel time or cost on link a; k and β are the model parameters and both take on positive values. Typically, in applications, $k = .15$ and $\beta = 4$.

The Total Cost on Link a, denoted by $\hat{c}_a$

$$\hat{c}_a = \hat{c}_a(f_a) = c_a(f_a) \times f_a = t_a^0[1 + k(\frac{f_a}{u_a})^\beta] \times f_a, \quad \forall a \in L, \tag{6}$$

The Total Cost on a Network, denoted by TC

$$TC = \sum_{a \in L} \hat{c}_a(f_a), \tag{7}$$

where the link flows f must lie in the feasible set $\mathcal{K}$:
$\mathcal{K} \equiv \{f \in R_+^{n_L} | \exists x \in R_+^{n_P} \text{ satisfying } (1), (2)\}$.

The U-O and S-O models that we utilize are classical (see Beckmann, McGuire, and Winsten (1956) and Dafermos and Sparrow (1969)).

The User-Optimized (U-O) Problem

A path flow pattern $x^* \in \mathcal{K}^1$, where $\mathcal{K}^1 \equiv \{x | x \in R_+^{n_P}$ and $d_w = \sum_{p \in P_w} x_p, \forall w \in W$ holds$\}$, is said to be a transportation network equilibrium, in the case of fixed demands, if the following condition holds for each O/D pair $w \in W$ and every path $p \in P_w$:

$$C_p(x^*) - \lambda_w^* \begin{cases} = 0, & \text{if} \quad x_p^* > 0, \\ \geq 0, & \text{if} \quad x_p^* = 0. \end{cases} \tag{8}$$

Equivalent Optimization Problem

The equilibrium link flow (and path flow pattern) can be obtained via the solution of the following optimization problem:

$$\text{Minimize}_{f \in \mathcal{K}} \quad \sum_{a \in L} \int_0^{f_a} c_a(y) dy. \tag{9}$$

The System-Optimized (S-O) Problem

Assume that there is a central controller of the traffic who routes the traffic in an optimal manner so as to minimize the total cost in the network. That is,

$$\text{Minimize}_{f \in \mathcal{K}} \quad \sum_{a \in L} \hat{c}_a(f_a) \tag{10}$$

The total cost on a path, denoted by $\hat{C}_p$, is the user cost on a path times the flow on a path, that is,

$$\hat{C}_p = C_p x_p, \; \forall p \in P, \tag{11}$$

where the user cost on a path, C_p, is given by (3).
In view of (2), (3), and (4), one may express the cost on a path p as a function of the path flow variables and, hence, an alternative version of the above S-O problem with objective function (10) can be stated in path flow variables only, where one has now the problem:

$$\text{Minimize}_{x \in \mathcal{K}^1} \quad \sum_{p \in P} C_p(x) x_p \tag{12}$$

System-Optimality Conditions

Under the assumption of increasing user link cost functions, the objective function (10) in the S-O problem is convex, and the feasible set $\mathcal{K}^1$ is also convex. Therefore, the optimality conditions, that is, the Kuhn-Tucker conditions are: for each O/D pair $w \in W$ and each path $p \in P_w$, the flow pattern $x \in \mathcal{K}^1$, must satisfy:

$$\hat{C}'_p(x) \begin{cases} = \mu_w, & \text{if} \quad x_p > 0, \\ \geq \mu_w, & \text{if} \quad x_p = 0, \end{cases} \tag{13}$$

where $\hat{C}'_p(x)$ denotes the marginal of the total cost on path p, given by:

$$\hat{C}'_p(x) = \sum_{a \in L} \frac{\partial \hat{c}_a(f_a)}{\partial f_a} \delta_{ap} \tag{14}$$

evaluated in (13) at the solution and μ_w is the Lagrange multiplier associated with constraint (1) for that O/D pair w.

1 Motivation

2 Literature

3 Transportation Network Models

4 Relative Total Cost Indices

5 Numerical Examples

6 Summary and Conclusions

Relative Total Cost Index for Assessing the Robustness of a Transportation Network Under the U-O Flow Pattern

The relative total cost index under the U-O flow pattern for a transportation network G with the vector of demands d, the vector of user link cost functions c, and the vector of link capacities u is defined as the relative total cost increase under a given uniform capacity retention ratio γ ($\gamma \in (0,1]$) so that the new capacities (cf. 5) are given by γu. Let c denote the vector of BPR user link cost functions and let d denote the vector of O/D pair travel demands.

$$\mathcal{I}^{\gamma}_{U-O} = \mathcal{I}_{U-O}(G, c, d, \gamma, u) = \frac{TC^{\gamma}_{U-O} - TC_{U-O}}{TC_{U-O}} \times 100\% \quad (15)$$

where TC_{U-O} and TC^{γ}_{U-O} are the total network costs (7) evaluated under the user-optimizing flow pattern with the original capacities and the remaining capacities (i.e., γu), respectively.

Relative Total Cost Index for Assessing the Robustness of a Transportation Network Under the S-O Flow Pattern

The relative total cost index under the S-O flow pattern for a transportation network G with the vector of demands d, the vector of user link cost functions c, and the vector of link capacities u is defined as the relative total cost increase under a given uniform capacity retention ratio γ ($\gamma \in (0, 1]$) so that the new capacities (cf. 5) are given by γu. Let c denote the vector of BPR user link cost functions and let d denote the vector of O/D pair travel demands.

$$\mathcal{I}^{\gamma}_{S-O} = \mathcal{I}_{S-O}(G, c, d, \gamma, u) = \frac{TC^{\gamma}_{S-O} - TC_{S-O}}{TC_{S-O}} \times 100\% \qquad (16)$$

where TC_{S-O} and TC^{γ}_{S-O} are the total network costs (7) evaluated under the system-optimizing flow pattern with the original capacities and the remaining capacities (i.e., γu), respectively.

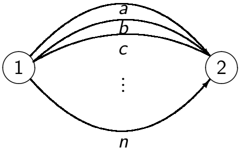
Remark

From the above definition(s), a transportation network, under a given capacity retention/deterioration ratio γ (and either the S-O or the U-O travel behavior) is considered to be robust if the index $\mathcal{I}^{\gamma}$ is low. This means that the relative total cost does not change much and, hence, the transportation network may be viewed as being more robust than if the relative total cost is small.

Theoretical Results I

Theorem

Consider a network consisting of two nodes 1 and 2 as in the figure below, which are connected by n parallel links. If the free-flow term, t_a^0, $\forall a \in L$, is the same for all links $a \in L$ in the BPR link cost function, the S-O flow pattern coincides with the U-O flow pattern and, therefore, $\mathcal{I}_{U-O} = \mathcal{I}_{S-O}$.



Theorem

The upper bound for $\mathcal{I}^{\gamma}_{S-O}$ for a transportation network with BPR link cost functions is $\frac{1-\gamma^{\beta}}{\gamma^{\beta}} \times 100\%$.

Theoretical Results II

Theorem

Consider a network consisting of two nodes 1 and 2 as in the previous figure, which are connected by n parallel links. Assume that the associated BPR link cost functions have $\beta = 1$. Furthermore, assume that there are positive flows on all the links at both the original and the partially degraded capacity levels given, respectively, by u and γu. Then the relative total cost index under the U-O flow pattern is given by the explicit formula:

$$\mathcal{I}^{\gamma}_{U-O} = \left(\frac{\gamma U + k d_w}{\gamma U + k \gamma d_w} - 1\right) \times 100\%, \tag{17}$$

where d_w is the given demand for O/D pair $w = (1, 2)$ and $U \equiv u_a + u_b + \cdots + u_n$. Moreover, the network robustness $\mathcal{I}^{\gamma}_{U-O}$ is bounded from above by $\frac{1-\gamma}{\gamma} \times 100\%$.

Relationship with Price of Anarchy

- The price of anarchy (PoA) (cf. Roughgarden (2005)), denoted by ρ, is defined as:

$$\rho = \frac{TC_{U-O}}{TC_{S-O}}. \tag{18}$$

- Observe that ρ captures the relationship between total costs *across* distinct behavioral principles whereas the proposed indices are focused on the degradation of network performance *within* the U-O or the S-O behavior.
- We have the following relationship between the ratio of the two indices and the PoA:

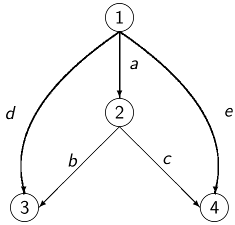
$$\frac{I^{\gamma}_{S-O}}{I^{\gamma}_{U-O}} = \frac{[TC^{\gamma}_{S-O} - TC_{S-O}]}{[TC^{\gamma}_{U-O} - TC_{U-O}]} \times \rho. \tag{19}$$

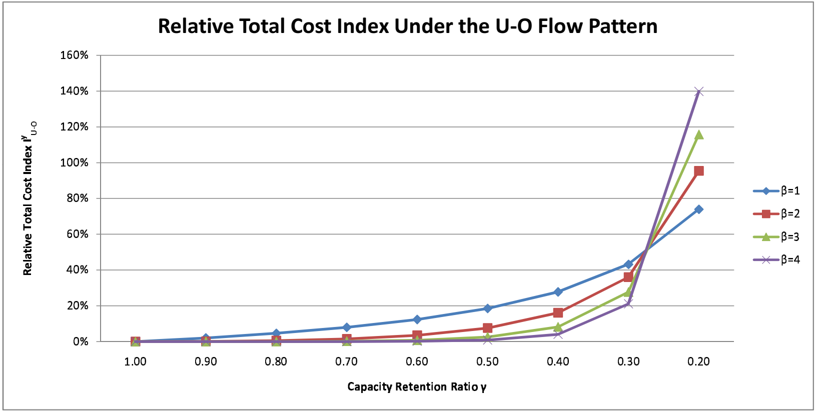
1. Motivation
2. Literature
3. Transportation Network Models
4. Relative Total Cost Indices
5. Numerical Examples
6. Summary and Conclusions

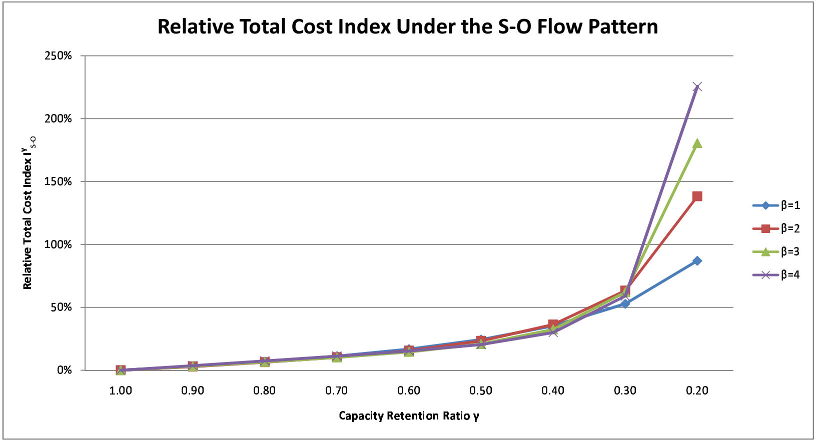
Example 1

Consider the following simple transportation network as shown in the figure on the right. There are two O/D pairs, namely, w_1=(1,3) and w_2=(1,4). The demands for the two O/D pairs are: $d_{w_1} = 10$ and d_{w_2}=20. The paths connecting O/D pair w_1 are: $p_1 = (a, b)$ and $p_2 = d$. The paths connecting O/D pair w_2 are: $p_3 = (a, c)$ and $p_4 = e$. The capacities for links a, b, c, d, and e are: 100, 50, 60, 10, and 20, respectively. Let t_0 and k be identical for all the links and equal to 10 and 1, respectively. The BPR link cost functions for the links are, hence, given by:

$c_a(f_a) = 10[1 + (\frac{f_a}{100})^\beta]$, $c_b(f_b) = 10[1 + (\frac{f_b}{50})^\beta]$,
$c_c(f_c) = 10[1 + (\frac{f_c}{60})^\beta]$,
$c_d(f_d) = 10[1 + (\frac{f_d}{10})^\beta]$, $c_e(f_e) = 10[1 + (\frac{f_e}{20})^\beta]$.

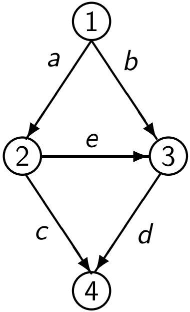



Relative Total Cost Index Under the U-O Flow Pattern
160%
140%
120%
100%
80%
60%
40%
20%
0%
Relative Total Cost Index I^γ_U-O
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
Capacity Retention Ratio γ
β=1
β=2
β=3
β=4

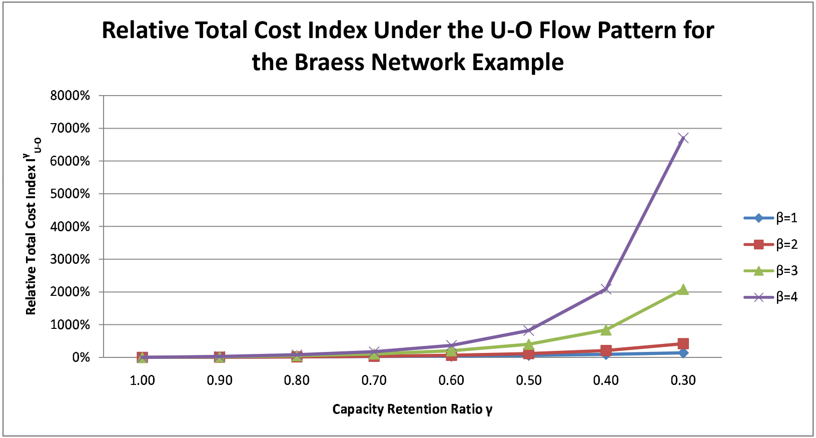


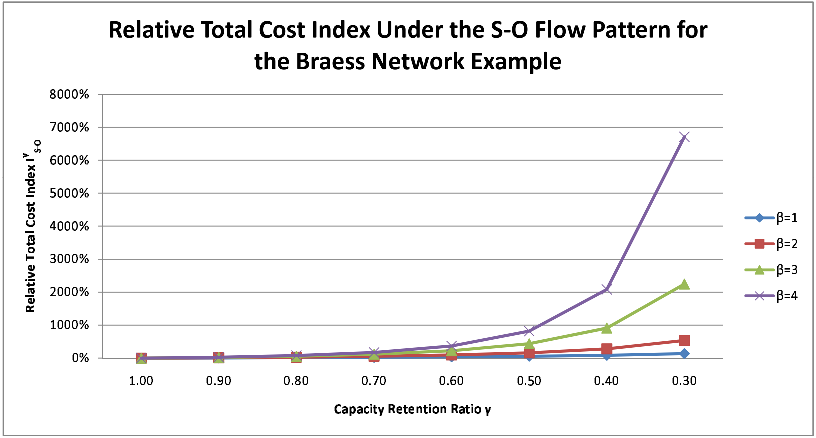
This transportation network under the U-O solution concept is more robust than under the S-O solution concept.

The Braess (1968) Inspired Example

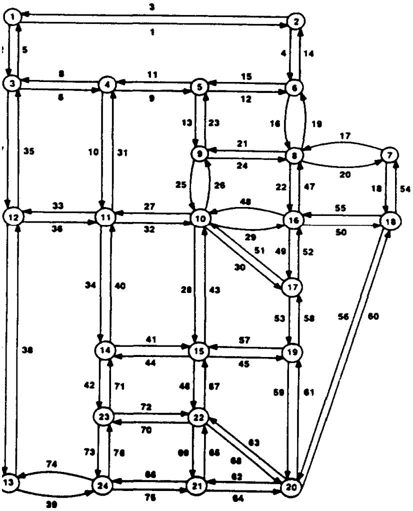


Assume that $k = 1$. Let $t_a^0 = t_d^0 = 1$; $t_b^0 = t_c^0 = 50$, and $t_e^0 = 10$. Furthermore, let $u_a = u_d = 20$; $u_b = u_c = 50$, and $u_e = 100$. The user link cost functions are, thus, given by: $c_a(f_a) = 1 + (\frac{f_a}{20})^\beta$, $c_b(f_b) = 50[1 + (\frac{f_b}{50})^\beta]$, $c_c(f_c) = 50[1 + (\frac{f_b}{50})^\beta]$, $c_d(f_d) = 1 + (\frac{f_d}{20})^\beta$, $c_e(f_e) = 10[1 + (\frac{f_e}{100})^\beta]$. Recall $w_1 = (1, 4)$. The demand is given by $d_{w_1} = 110$.


Relative Total Cost Index Under the U-O Flow Pattern for the Braess Network Example
8000%
7000%
6000%
5000%
4000%
3000%
2000%
1000%
0%
Relative Total Cost Index I$^{\gamma}_{U-O}$
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
Capacity Retention Ratio γ
β=1
β=2
β=3
β=4


Relative Total Cost Index Under the S-O Flow Pattern for the Braess Network Example
8000%
7000%
6000%
5000%
4000%
3000%
2000%
1000%
0%
Relative Total Cost Index $I^{\gamma}_{S\text{-}O}$
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
Capacity Retention Ratio γ
β=1
β=2
β=3
β=4

The Sioux-Falls Network



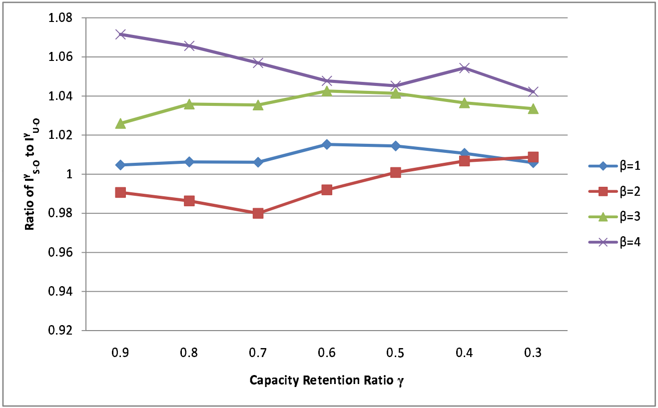
There are 528 O/D pairs, 24 nodes, and 76 links in the Sioux-Falls network.

Algorithms

- The projection method (cf. Dafermos (1980) and Nagurney (1999)) with the embedded Dafermos and Sparrow (1969) equilibration algorithm (see also, e.g., Nagurney (1984)) and the column generation algorithm (cf. Leventhal, Nemhauser, and Trotter (1973)) were utilized to compute the equilibrium solutions.

- Based on the equilibrium solutions, the network efficiency was determined and the importance values and the importance rankings of the links were computed.

The Ratio of I^{γ}_{S-O} to I^{γ}_{U-O}



From the above figure, we can see that the Sioux-Falls network is always more robust under the U-O behavior except when β is equal to 2 and the capacity retention ratio is between 0.5 and 0.9.

1 Motivation

2 Literature

3 Transportation Network Models

4 Relative Total Cost Indices

5 Numerical Examples

6 Summary and Conclusions

- We proposed two relative total cost indices to analyze transportation network robustness associated with different user behaviors.
- We examined the impacts of the degradation of the links in terms of capacity reductions on the relative total cost in the network at the original capacities and at the degraded capacities by assuming that the user link cost functions for the transportation network are of BPR form.
- We established new theoretical results and also provided numerical examples.
- For future research, we plan on assessing the robustness of transportation networks with asymmetric BPR-type link cost functions and with multiple modes of transportation. We would also like to study the robustness of transportation networks in the case of stochastic costs (and demands).


The Virtual Center for Supernetworks
The Virtual Center for Supernetworks
Supernetworks for Optimal Decision-Making and Improving the Global Quality of Life
Home About Background Activities Publications Media Links What's New
Funding Transportation Infrastructure Conference La Defense - Paris, France
The Virtual Center for Supernetworks at the Isenberg School of
the directorship of Anna Nagurney, the John F. Smith Memorial Professor, is an
center, and includes the Supernetworks Laboratory for Computation and Visualization
Mission: The mission of the Virtual Center for Supernetworks is to foster
application of supernetworks and to serve as a resource to academia, industry,
on networks ranging from transportation, supply chains, telecommunication, and
networks to economic, environmental, financial, knowledge and social networks.
The Applications of Supernetworks Include: multimodal
critical infrastructure, energy and the environment, the Internet and electronic
supply chain management, international financial networks, web-based advertising,
networks and decision-making, integrated social and economic networks, network
network metrics.
Announcements and Notes from the Center Director Professor Anna Nagurney
Updated: October 4, 2008
Fall 2008 Operations Research / Management Science Seminar Series
The Braess Paradox
Translation
Information
Photos
FULBRIGHT
Professor Nagurney's Fulbright Senior Specialist Award at the University of Catania March 2008
Humanitarian Logistics: Networks for Africa
Photos of Center Activities

Thank You!

For more information,
see http://supernet.som.umass.edu