A Network Economic Model of a Service-Oriented Internet with Choices and Quality Competition

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where a full list of references can be found.

The paper was recently recognized by the Association for Computing Machinery (ACM) Computing Review in its Best of 2013. Specifically, it was recognized in the Computer Applications category and the full list of "notable items published in computing" can be downloaded from the link:

http://www.computingreviews.com/recommend/bestof/2013NotableItems.pdf

In addition, this paper is listed by Springer on the Netnomics homepage under "Popular Content within this Publication."

Outline

- Background and Motivation
- The Network Economic Game Theory Model
- The Projected Dynamical System Model
- The Algorithm
- Numerical Examples
- Summary and Conclusions

Service-Oriented Internet

Background

- Internet as the communication highway.
- Underlying technology is well-understood, but the economics has been less studied.
- Lack of common metrics for economic analysis.
- Internet scope has been expanded.
- Fresh look at future Internet architectures.

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Literature Review

Cournot competition between wireless networks to find an efficient use of all available spectrum bandwidth

- Gibbons, R., Mason, R., & Steinberg, R. (2000). Internet service classes under competition. *IEEE Journal on Selected Areas in Communications*, 18(12), 2490-2498,
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Literature Review

Future Generation Internet

- Nagurney, A., Wolf, T. (2013). A Cournot-Nash-Bertrand game theory model of a service-oriented Internet with price and quality competition among network transport providers. *Computational Management Science* 11(4), 475-502.

- Nagurney, A., Li, D., Saberi, S., Wolf, T. (2013). A dynamic network economic model of a service-oriented Internet with price and quality competition. In *Network Models in Economics and Finance*, V.A. Kalyagin, P.M. Pardalos, and T. M. Rassias, Editors, Springer International Publishing Switzerland, 239-264.

- Saberi, S., Nagurney, A., Wolf, T. (2014). A network economic game theory model of a service-oriented Internet with price and quality competition in both content and network provision. *Service Science*, 6(4), 1-24.

Contribution

Our Contribution

- Oligopolistic Cournot competition among service providers.
- Differentiated services with different quality levels.
- Multiple network providers.
- Modeling the competition in a dynamic way.
- The model is inspired by Zhang et al. (2010) derived a game theoretic formulation with:
  - Two service providers who were Cournot competitors,
  - Two network providers who were Bertrand competitors,
  - Two users.

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The Network of Oligopoly Model

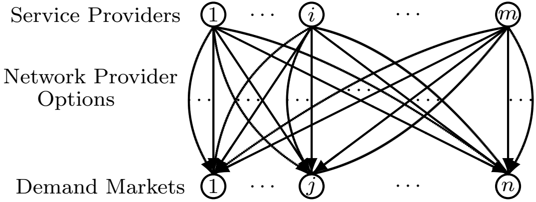

Figure : The structure of the network economic problem

$$s_i = \sum_{j=1}^{n} \sum_{k=1}^{o} Q_{ijk}, \quad i = 1, \ldots, m; \tag{1}$$

$$d_{ij} = \sum_{k=1}^{o} Q_{ijk}, \quad i = 1, \ldots, m; j = 1, \ldots, n, \tag{2}$$

$$Q_{ijk} \geq 0, \quad i = 1, \ldots, m; j = 1, \ldots, n; k = 1, \ldots, o, \tag{3}$$

$$q_i \geq 0, \quad i = 1, \ldots, m. \tag{4}$$

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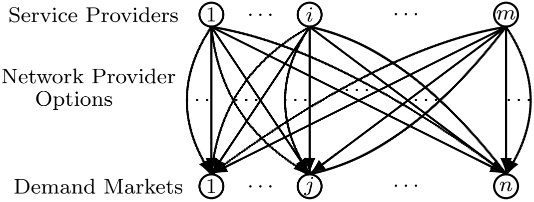

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Service Providers

Each service provider i a *production* cost $\hat{f}_i$:

$$\hat{f}_i = \hat{f}_i(s, q_i), \quad i = 1, \ldots, m. \tag{5}$$

The demand price at a demand market j associated with the service provided by service provider i:

$$\rho_{ij} = \rho_{ij}(d, q, p), \quad i = 1, \ldots, m; j = 1, \ldots, n. \tag{6}$$

The total provision/transportation cost for provider i's services for demand market j:

$$\hat{c}_{ij} = \sum_{k=1}^{o} \hat{c}_{ijk}(Q_{ijk}), \quad i = 1, \ldots, m; j = 1, \ldots, n. \tag{7}$$

The profit or utility U_i of service provider i:

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Nash Equilibrium

Definition 1

A Network Economic Cournot-Nash Equilibrium with Service Differentiation, Network provision Choices, and Quality Levels

A service transport volume and quality level pattern $(Q^*, q^*) \in K$ *is said to constitute a Cournot-Nash equilibrium if for each service provider* i*,*

$$U_i(Q_i^*, q_i^*, \hat{Q}_i^*, \hat{q}_i^*) \geq U_i(Q_i, q_i, \hat{Q}_i^*, \hat{q}_i^*), \quad \forall (Q_i, q_i) \in K^i, \tag{9}$$

where

$$\hat{Q}_i^* \equiv (Q_1^*, \ldots, Q_{i-1}^*, Q_{i+1}^*, \ldots, Q_m^*); \text{ and } \hat{q}_i^* \equiv (q_1^*, \ldots, q_{i-1}^*, q_{i+1}^*, \ldots, q_m^*). \tag{10}$$

Variational Inequality Formulations

Theorem 1

$$-\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\frac{\partial U_i(Q^*,q^*)}{\partial Q_{ijk}}\times(Q_{ijk}-Q^*_{ijk})-\sum_{i=1}^{m}\frac{\partial U_i(Q^*,q^*)}{\partial q_i}\times(q_i-q^*_i)\geq 0, \quad (11)$$

or, equivalently,

$$\sum_{i=1}^{m}\frac{\partial \hat{f}_i(s^*,q^*_i)}{\partial s_i}\times(s_i-s^*_i)-\sum_{i=1}^{m}\sum_{j=1}^{n}\rho_{ij}(d^*,q^*,p)\times(d_{ij}-d^*_{ij})$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\left[\frac{\partial \hat{c}_{ijk}(Q^*_{ijk})}{\partial Q_{ijk}}-\sum_{l=1}^{n}\frac{\partial \rho_{il}(d^*,q^*,p)}{\partial d_{ij}}\times d^*_{il}\right]\times(Q_{ijk}-Q^*_{ijk})$$

$$+\sum_{i=1}^{m}\left[\frac{\partial \hat{f}_i(s^*,q^*_i)}{\partial q_i}-\sum_{l=1}^{n}\frac{\partial \rho_{il}(d^*,q^*,p)}{\partial q_i}\times d^*_{il}\right]\times(q_i-q^*_i)\geq 0, \ \forall (s,d,Q,q)\in\mathcal{K}^1, \quad (12)$$

where $\mathcal{K}^1\equiv\{(s,d,Q,q)\,|\,Q\geq 0,\ q\geq 0, \text{and (1) and (2) hold}\}$.

Standard Form of VI

Determine $X^* \in \mathcal{K} \subset R^N$, such that

Standard VI

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{13}$$

We define the $(mno + m)$-dimensional vector $X \equiv (Q, q)$ and the $(mn + m)$-dimensional row vector $F(X) = (F^1(X), F^2(X))$ with the (i, j, k)-th component, F^1_{ijk}, of $F^1(X)$ given by

$$F^1_{ijk}(X) \equiv -\frac{\partial U_i(Q, q)}{\partial Q_{ijk}}, \tag{14}$$

the i-th component, F^2_i, of $F^2(X)$ given by

$$F^2_i(X) \equiv -\frac{\partial U_i(Q, q)}{\partial q_i}, \tag{15}$$

and with the feasible set $\mathcal{K} \equiv K$.

The Projected Dynamical System Model

For a current service volume and quality level pattern at time t, $X(t) = (Q(t), q(t))$,

$$-F^1_{ijk}(X(t)) = \frac{\partial U_i(Q(t), q(t))}{\partial Q_{ijk}}, \qquad -F^2_i(X(t)) = \frac{\partial U_i(Q(t), q(t))}{\partial q_i}. \qquad (16-17)$$

Rate of Q_{ijk} change

$$\dot{Q}_{ijk} = \begin{cases} \frac{\partial U_i(Q,q)}{\partial Q_{ijk}}, & \text{if} \quad Q_{ijk} > 0 \\ \max\{0, \frac{\partial U_i(Q,q)}{\partial Q_{ijk}}\}, & \text{if} \quad Q_{ijk} = 0, \end{cases} \qquad (18)$$

where $\dot{Q}_{ijk}$ denotes the rate of change of Q_{ijk}.

Rate of q_i change

$$\dot{q}_i = \begin{cases} \frac{\partial U_i(Q,q)}{\partial q_i}, & \text{if} \quad q_i > 0 \\ \max\{0, \frac{\partial U_i(Q,q)}{\partial q_i}\}, & \text{if} \quad q_i = 0, \end{cases} \qquad (19)$$

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The Ordinary Differential Equation (ODE)

Pertinent ODE for the adjustment processes of the service transport volumes and quality levels

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)). \tag{20}$$

Vector $-F(X)$ at X defined as

$$\Pi_{\mathcal{K}}(X, -F(X)) = \lim_{\delta \to 0} \frac{P_{\mathcal{K}}(X - \delta F(X)) - X}{\delta}, \tag{21}$$

with $P_{\mathcal{K}}$ denoting the projection map:

$$P(X) = \operatorname{argmin}_{z \in \mathcal{K}} \|X - z\|, \tag{22}$$

and where $\| \cdot \| = \langle x, x \rangle$.

Stability Under Monotonicity

Theorem 2

X^* solves the variational inequality problem if and only if it is a stationary point of the ODE, that is,

$$\dot{X} = 0 = \Pi_{\mathcal{K}}(X^*, -F(X^*)). \tag{23}$$

Assumption 1

Suppose that in our network economic model there exists a sufficiently large M, such that for any (i, j, k),

$$\frac{\partial U_i(Q, q)}{\partial Q_{ijk}} < 0, \tag{24}$$

for all service transport volume patterns Q with $Q_{ijk} \geq M$ and that there exists a sufficiently large $\bar{M}$, such that for any i,

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for all quality level patterns q with $q_i \geq \bar{M}$.

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Existence and Uniqueness

Proposition 1: Existence

Any network economic problem that satisfies Assumption 1 possesses at least one equilibrium service transport volume and quality level pattern.

Proposition 2: Uniqueness

Suppose that F is strictly monotone at any equilibrium point of the variational inequality problem defined in (12). Then it has at most one equilibrium point.

Theorem 3

(i). If $-\nabla U(Q, q)$ is monotone, then every network economic Cournot-Nash equilibrium, provided its existence, is a global monotone attractor for the utility gradient process.
(ii). If $-\nabla U(Q, q)$ is strictly monotone, then there exists at most one network economic Cournot-Nash equilibrium. Furthermore, provided existence, the unique spatial Cournot-Nash equilibrium is a strictly global monotone attractor for the utility gradient process.
(iii). If $-\nabla U(Q, q)$ is strongly monotone, then there exists a unique network economic Cournot-Nash equilibrium, which is globally exponentially stable for the utility gradient process.

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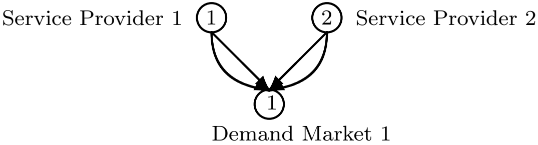


Figure : Example 1

The production cost functions are:

$$\hat{f}_1(s, q_1) = s_1^2 + s_1 + s_2 + 2q_1^2 + 39, \quad \hat{f}_2(s, q_2) = 2s_2^2 + 2s_1 + s_2 + q_2^2 + 37,$$

the total transportation cost functions are:

$$\hat{c}_{111} = 0.5Q_{111}^2 + 0.4Q_{111}, \quad \hat{c}_{112} = 0.7Q_{112}^2 + 0.5Q_{112},$$

$$\hat{c}_{211} = 0.6Q_{211}^2 + 0.4Q_{211}, \quad \hat{c}_{212} = 0.4Q_{212}^2 + 0.2Q_{212},$$

and the demand price functions are:

$$\rho_{11}(d, q) = 100 - d_{11} - 0.4d_{21} + 0.3q_1 + 0.05q_2 - p,$$

$$\rho_{21}(d, q) = 100 - 0.6d_{11} - 1.5d_{21} + 0.1q_1 + 0.5q_2 - p,$$

where $p = 30$.

Example 1: Solution

The Jacobian matrix of $-\nabla U(Q,q)$, denoted by $J(Q_{11}, Q_{21}, q_1, q_2)$, is

$$J = \begin{pmatrix} 5 & 4 & 0.4 & 0.4 & -0.3 & -0.05 \\ 4 & 5.4 & 0.4 & 0.4 & -0.3 & -0.05 \\ 0.6 & 0.6 & 8.2 & 7 & -0.1 & -0.5 \\ 0.6 & 0.6 & 7 & 7.8 & -0.1 & -0.5 \\ -0.3 & -0.3 & 0 & 0 & 4 & 0 \\ 0 & 0 & -0.5 & -0.5 & 0 & 2 \end{pmatrix}.$$

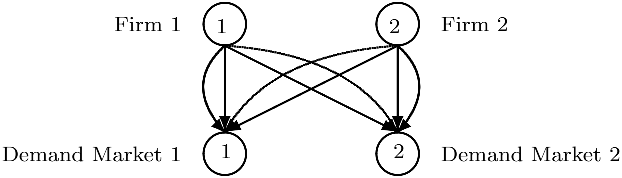
This Jacobian matrix is positive-definite. The equilibrium solution is:

$$Q^*_{111} = 8.40, \qquad Q^*_{112} = 5.93, \qquad Q^*_{211} = 3.18, \qquad Q^*_{212} = 5.01,$$

$$q^*_1 = 1.08, \qquad q^*_2 = 2.05,$$

and it is globally exponentially stable.

Example 2



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$$\hat{c}_{211} = 0.6Q_{211}^2 + 0.4Q_{211}, \quad \hat{c}_{212} = 0.4Q_{212}^2 + 0.2Q_{212},$$
$$\hat{c}_{121} = 0.3Q_{121}^2 + 0.1Q_{121}, \quad \hat{c}_{122} = 0.5Q_{122}^2 + 0.3Q_{122},$$
$$\hat{c}_{221} = 0.4Q_{221}^2 + 0.3Q_{221}, \quad \hat{c}_{222} = 0.4Q_{222}^2 + 0.2Q_{222}.$$

Example 2

The demand price functions are:

$$\rho_{11}(d,q) = 100 - d_{11} - 0.4d_{21} + 0.3q_1 + 0.05q_2 - p,$$

$$\rho_{12}(d,q) = 100 - 2d_{12} - d_{22} + 0.4q_1 + 0.2q_2 - p,$$

$$\rho_{21}(d,q) = 100 - 0.6d_{11} - 1.5d_{21} + 0.1q_1 + 0.5q_2 - p,$$

$$\rho_{22}(d,q) = 100 - 0.7d_{12} - 1.7d_{22} + 0.01q_1 + 0.6q_2 - p,$$

where $p = 30$.
The utility function of firm 1 is:

$$U_1(Q,q) = \rho_{11}d_{11} + \rho_{12}d_{12} - \hat{f}_1 - (\hat{c}_{111} + \hat{c}_{121} + \hat{c}_{112} + \hat{c}_{122}),$$

with the utility function of firm 2 being:

$$U_2(Q,q) = \rho_{21}d_{21} + \rho_{22}d_{22} - \hat{f}_2 - (\hat{c}_{211} + \hat{c}_{221} + \hat{c}_{212} + \hat{c}_{222}).$$

Example 2: Solution

The Jacobian of $-\nabla U(Q,q)$ is

$$J(Q_{111},Q_{112},Q_{121},Q_{122},Q_{211},Q_{212},Q_{221},Q_{222},q_1,q_2)$$

$$= \begin{pmatrix} 5 & 4 & 2 & 2 & 0.4 & 0.4 & 0 & 0 & -0.3 & -0.05 \\ 4 & 5.4 & 2 & 2 & 0.4 & 0.4 & 0 & 0 & -0.3 & -0.05 \\ 2 & 2 & 6.6 & 6 & 0 & 0 & 1 & 1 & -0.4 & -0.2 \\ 2 & 2 & 6 & 7 & 0 & 0 & 1 & 1 & -0.4 & -0.2 \\ 0.6 & 0.6 & 0 & 0 & 8.2 & 7 & 4 & 4 & -0.1 & -0.5 \\ 0.6 & 0.6 & 0 & 0 & 7 & 7.8 & 4 & 4 & -0.1 & -0.5 \\ 0 & 0 & 0.7 & 0.7 & 4 & 4 & 8.2 & 7.4 & -0.01 & -0.6 \\ 0 & 0 & 0.7 & 0.7 & 4 & 4 & 7.4 & 8.2 & -0.01 & -0.6 \\ -0.3 & -0.3 & -0.4 & -0.4 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & -0.5 & -0.5 & -0.6 & -0.6 & 0 & 2 \end{pmatrix}.$$

Clearly, this Jacobian matrix is also positive-definite. the equilibrium solution (stationary point) is:

$$Q^*_{111} = 6.97, \quad Q^*_{112} = 4.91, \quad Q^*_{121} = 2.40, \quad Q^*_{122} = 3.85,$$

$$Q^*_{211} = 3.58, \quad Q^*_{212} = 1.95, \quad Q^*_{221} = 2.77, \quad Q^*_{222} = 2.89,$$

$$q^*_1 = 1.52, \qquad q^*_2 = 3.08,$$

and it is globally exponentially stable.

Iteration τ of the Euler method is given by:

Euler Algorithm

$$X^{\tau+1} = p_{\mathcal{K}}(X^{\tau} - a_{\tau}F(X^{\tau})). \tag{26}$$

For convergence of the general iterative scheme, which induces the Euler method, the sequence $\{a_\tau\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$.

Explicit Formulae for the Euler Method

For all the service volume $i = 1, \ldots, m; j = 1, \ldots, n; k = 1, \ldots, o$:

Closed form for Q_{ijk}

$$Q_{ijk}^{\tau+1} = \max\{0, Q_{ijk}^{\tau} + a_\tau(\rho_{ij}(d^\tau, q^\tau, p) + \sum_{l=1}^{n} \frac{\partial \rho_{il}(d^\tau, q^\tau, p)}{\partial d_{ij}} d_{il}^{\tau} - \frac{\partial \hat{f}_i(s^\tau, q_i^\tau)}{\partial s_i} - \frac{\partial \hat{c}_{ijk}(Q_{ijk}^{\tau})}{\partial Q_{ijk}})\}, \quad (27)$$

and for all the quality levels $i = 1, \ldots, m$:

Closed form for q_i

$$q_i^{\tau+1} = \max\{0, q_i^{\tau} + a_\tau(\sum_{l=1}^{n} \frac{\partial \rho_{il}(d^\tau, q^\tau, p)}{\partial q_i} d_{il}^{\tau} - \frac{\partial \hat{f}_i(s^\tau, q_i^\tau)}{\partial q_i})\}. \quad (28)$$

Numerical Examples

We implemented the Euler method using Matlab.

The convergence criterion was $\epsilon = 10^{-6}$; that is, the Euler method was considered to have converged if, at a given iteration, the absolute value of the difference of each service volume and each quality level differed from its respective value at the preceding iteration by no more than ϵ.

The sequence $\{a_\tau\}$ was: $.1(1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \ldots)$. We initialized the algorithm by setting each service volume $Q_{ijk} = 2.5$, $\forall i, j, k$, and by setting the quality level of each firm $q_i = 0.00$, $\forall i$.

Example 1 Revisited

The Euler method required 72 iterations for convergence.

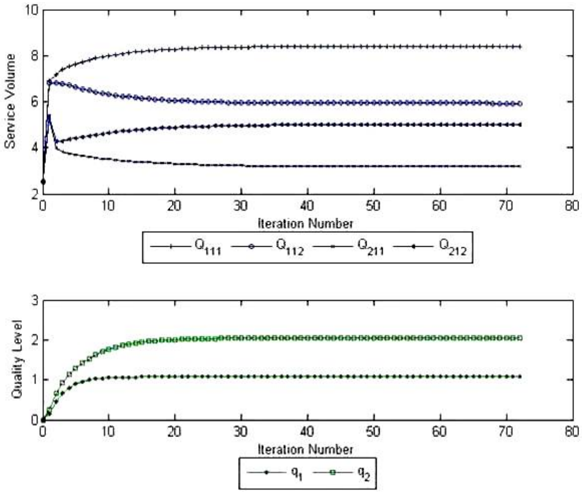

Figure : Service volumes and quality levels for Example 1

Example 2 Revisited

The Euler method required 84 iterations for convergence.

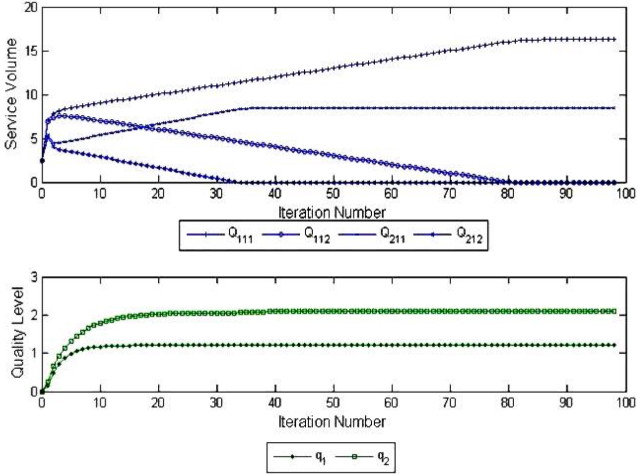

Figure : Service volumes and quality levels for Example 2

Example 3

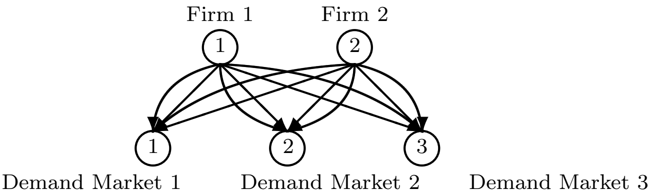

Figure : Example 3

The total transportation cost functions are:

$$\hat{c}_{111} = 0.5Q_{111}^2 + 0.4Q_{111}, \quad \hat{c}_{112} = 0.7Q_{112}^2 + 0.5Q_{112}$$

$$\hat{c}_{211} = 0.6Q_{211}^2 + 0.4Q_{211}, \quad \hat{c}_{212} = 0.4Q_{212}^2 + 0.2Q_{212},$$

$$\hat{c}_{121} = 0.3Q_{121}^2 + 0.1Q_{121}, \quad \hat{c}_{122} = 0.5Q_{122}^2 + 0.3Q_{122},$$

$$\hat{c}_{221} = 0.4Q_{221}^2 + 0.3Q_{221}, \quad \hat{c}_{222} = 0.4Q_{222}^2 + 0.2Q_{222},$$

$$\hat{c}_{131} = Q_{131}^2 + 0.5Q_{131}, \quad \hat{c}_{132} = Q_{132}^2 + 0.6Q_{132},$$

$$\hat{c}_{231} = 0.8Q_{231}^2 + 0.5Q_{231}, \quad \hat{c}_{232} = Q_{232}^2 + 0.7Q_{232}.$$

Example 3

The production cost functions are:

$$\hat{f}_1(s, q_1) = s_1^2 + s_1 + s_2 + 2q_1^2 + 39, \quad \hat{f}_2(s, q_2) = 2s_2^2 + 2s_1 + s_2 + q_2^2 + 37.$$

The demand price functions are:

$$\rho_{11}(d, q) = 100 - d_{11} - 0.4d_{21} + 0.3q_1 + 0.05q_2 - p,$$

$$\rho_{12}(d, q) = 100 - 2d_{12} - d_{22} + 0.4q_1 + 0.2q_2 - p,$$

$$\rho_{13}(d, q) = 100 - 1.7d_{13} - 0.7d_{23} + 0.5q_1 + 0.1q_2 - p,$$

$$\rho_{21}(d, q) = 100 - 0.6d_{11} - 1.5d_{21} + 0.1q_1 + 0.5q_2 - p,$$

$$\rho_{22}(d, q) = 100 - 0.7d_{12} - 1.7d_{22} + 0.01q_1 + 0.6q_2 - p,$$

$$\rho_{23}(d, q) = 100 - 0.9d_{13} - 2d_{23} + 0.2q_1 + 0.7q_2 - p.$$

The utility function expressions of firm 1 is:

$$U_1(Q, q) = \rho_{11}d_{11} + \rho_{12}d_{12} + \rho_{13}d_{13} - \hat{f}_1 - (\hat{c}_{111} + \hat{c}_{121} + \hat{c}_{112} + \hat{c}_{122} + \hat{c}_{131} + \hat{c}_{132}),$$

with the utility function of firm 2 being:

$$U_2(Q, q) = \rho_{21}d_{21} + \rho_{22}d_{22} + \rho_{23}d_{23} - \hat{f}_2 - (\hat{c}_{211} + \hat{c}_{221} + \hat{c}_{212} + \hat{c}_{222} + \hat{c}_{231} + \hat{c}_{232}).$$

Example 3: Solution

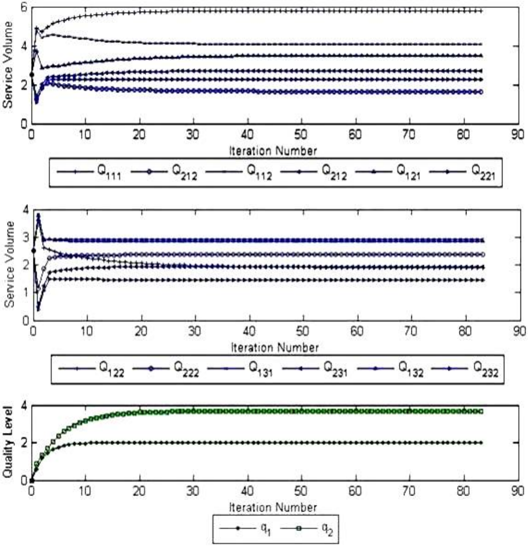

Figure : Service volumes and quality levels for Example 3

Sensitivity Analysis for Example 3

How the changes in the network transmission price p influence the equilibrium solutions and the profit?

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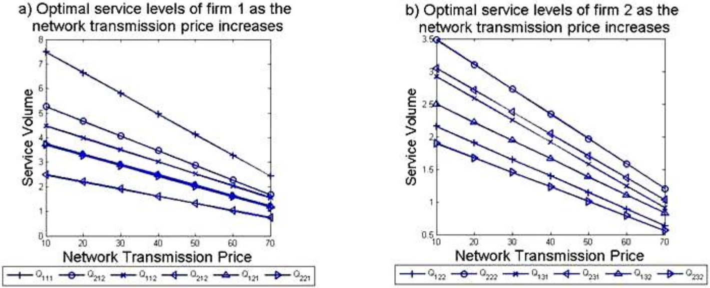

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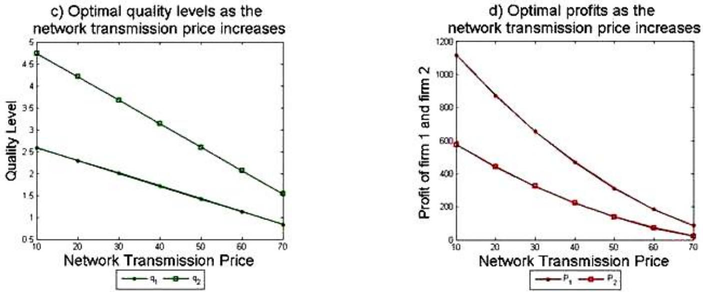

Figure : Sensitivity Analysis for Example 3

Sensitivity Analysis Summary

- Service volumes, quality levels and the profits are negatively related to the network transmission price.
- As the network transmission price becomes higher, consumers would purchase less from the network providers as well as the service providers, which leads to the decreasing in service volumes.
- As the service volumes decrease, there would be less incentive for the firms to improve their quality levels, so the quality levels would also decrease.

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Summary and Conclusions

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- Developed a new dynamic network economic game theory model of a service-oriented Internet.
- Proposed a continuous-time adjustment process.
- Projected dynamical systems model guarantees that the service volumes and quality levels remain nonnegative.
- Described an algorithm, which yields closed form expressions for the service volumes and quality levels at each iteration.
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Thank you!

For more information, see:
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