An Integrated Financial and Logistical Game Theory Model for Humanitarian Organizations with Purchasing Costs, Multiple Freight Service Providers, and Budget, Capacity, and Demand Constraints

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ABSTRACT

In this paper, a game theory model for disaster relief is constructed that incorporates both financial and logistical aspects of humanitarian organizations involved in the purchasing and delivery of relief items, post-disaster, using freight services. The model allows for the purchasing of the relief items, both locally and nonlocally, includes a budget constraint for each humanitarian organization, along with imposed lower and upper bound demand constraints at each point of demand by a higher level organization. The governing concept is that of Generalized Nash Equilibrium, since not only does the utility function of a given humanitarian organization depend on its own strategies and the strategies of the other humanitarian organizations, but the constraints do as well. The concept of a variational equilibrium is utilized to derive the variational inequality formulation of the governing equilibrium conditions and the model is analyzed qualitatively. Lagrange analysis of the marginal utilities is conducted to gain insights on the impact of the constraints and an alternative variational inequality

Outline

Introduction

- **Disasters**

  The Emergency Events Database (EM-DAT) defines a disaster as a natural situation or event that overwhelms local capacity and/or necessitates a request for external assistance.

- **Response**

  - Providing primary relief supplies such as water, food, and medicine is crucial.

  - In many cases, it may be very difficult and challenging.

Introduction

- **Humanitarian Logistics**

  Definition: The process of planning, implementing, and controlling the efficient, cost-effective flow and storage of goods and materials and also the related information from the point of origin to the point of demand in order to reduce the suffering of the victims.

Introduction

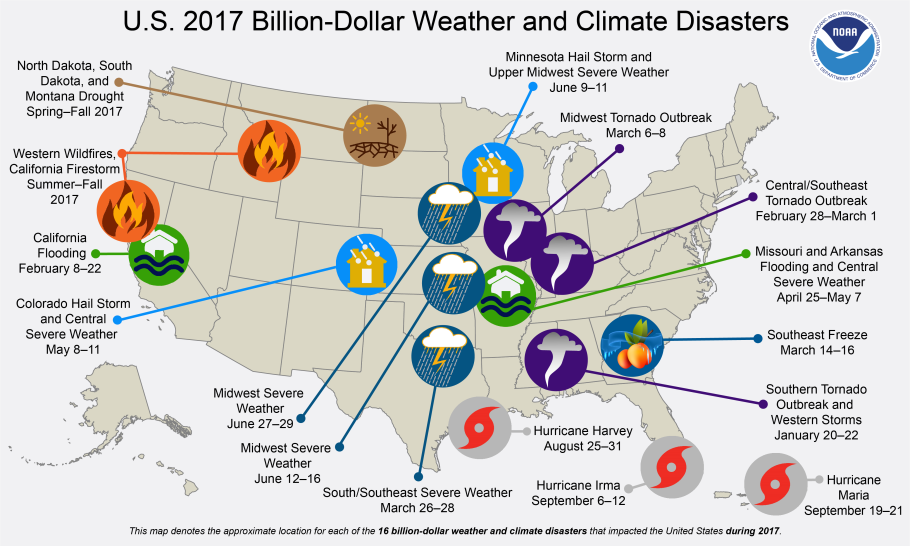
- **Costs**

  - **Purchasing** and **Transportation** costs
  - NGOs are nonprofit and dependent on **donations**.
  - Approximately $300 billion dollars are donated to charities in the United States each year.
  - **Competition** is natural in an environment in which Humanitarian Organizations (HOs) are competing for donor funding.
  - The **visibility** of HOs in terms of disaster relief in the **media** can assist in increasing donations.

- **Coordination**

  - Lack of coordination among agencies may lead to the **duplication** of efforts, **confusion** at the "last mile", and issues of material convergence.
  - Initiatives have emerged such as the United Nations Joint Logistics Center.

Billion Dollar Disasters in the US in 2017



Optimization Models and Disaster Relief

- **Numerous studies focusing on optimization frameworks in the context of disaster relief:**

  Haghani and Oh (1996) - Ozdamar et al. (2004) - Yi and Kumar (2007) - Yi and Ozdamar (2007) - Tzeng et al. (2007) - Balcik, Beamon, and Smilowitz (2008) - Balcik et al. (2010) - Nagurney et al. (2012) - Nagurney and Nagurney (2016).

  - The survey of optimization models in emergency logistics by Caunhye, Nie, and Pokharel (2012).

  - Additional references on models in humanitarian logistics: see Duran et al. (2013) and the survey by Ortuno et al. (2013).

Game Theory Models and Disaster Relief

Although there have been quite a few optimization models developed for disaster relief there are very few game theory models.

Toyasaki and Wakolbinger (2014) constructed **the first models of financial flows that captured the strategic interaction between donors and humanitarian organizations using game theory** and also included earmarked donations.

Muggy and Stamm (2014), in turn, provide **an excellent review of game theory in humanitarian operations** and emphasize that there are many untapped research opportunities for modeling in this area.

Game Theory Models and Disaster Relief

- A. Nagurney, E. Alvarez Flores, and C. Soylu (2016). A Generalized Nash Equilibrium Network Model for Post-Disaster Humanitarian Relief. *Transportation Research E*, **95**, pp 1-18.

- The first Generalized Nash Equilibrium (GNE) model for disaster relief.
- Integrated the financial side and the logistical side.
- The model contains, as a special case, its Nash Equilibrium counterpart.

Game Theory Models and Disaster Relief

- A. Nagurney, P. Daniele, E. Alvarez Flores, and V. Caruso (2018). A Variational Equilibrium Network Framework for Humanitarian Organizations in Disaster Relief: Effective Product Delivery Under Competition for Financial Funds. In: *Dynamics of Disasters: Algorithmic Approaches and Applications*, I.S. Kotsireas, A. Nagurney, and P.M. Pardalos, Editors, Springer International Publishers Switzerland, pp 109-133.

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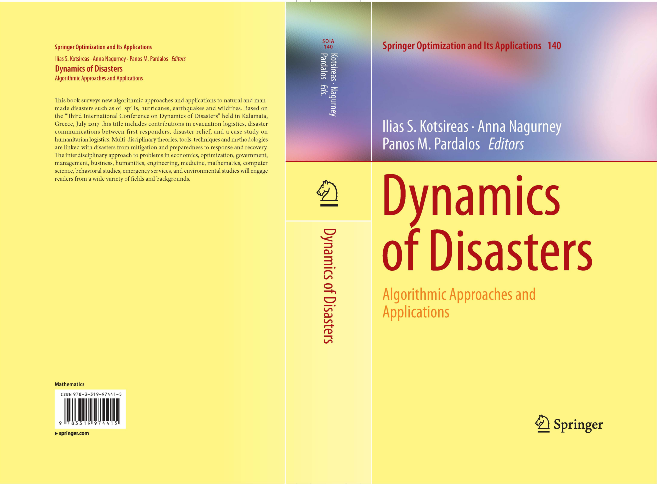

Our Contributions

- A game theory model to capture the competition among humanitarian organizations with the goal of bringing **greater realism**.
- **Purchasing of the products**
  There are **different suppliers** which HOs may be able to purchase products from and these may be **local** or **nonlocal**.
- **Multiple freight service providers**
  There are **different freight service providers** that can ship relief items from the purchase locations of the HOs to the demand points.
- **Capacity constraints**
  The freight service providers' **shipment capacities** are **limited** due to their available facilities and the impacted regions' infrastructures.
- **Budget constraints**
  The budget constraint faced by an HO is the most critical constraint in any humanitarian relief operation.

Our Contributions

- The model allows each NGO to make optimal resource allocation, given a budget constraint, based on local/nonlocal purchase prices and freight service provision costs.
- We do not assume that the relief items are prepositioned, but, rather, that they must be purchased.
- The results in this paper also contribute to the literature on variational inequalities with nonlinear constraints, with a focus on game theory.
- To-date, the only other work that includes Lagrange analysis for a humanitarian logistics model in the context of game theory, is the paper by Nagurney et al. (2018) and therein all the constraints were linear and there were no purchasing costs nor budget constraints.

- m humanitarian organizations, with a typical one denoted by i.
- n demand locations, with a typical location denoted by j.
- t freight service providers (FSPs) with a typical one denoted by l.
- o possible purchase locations, with a typical location denoted by k.

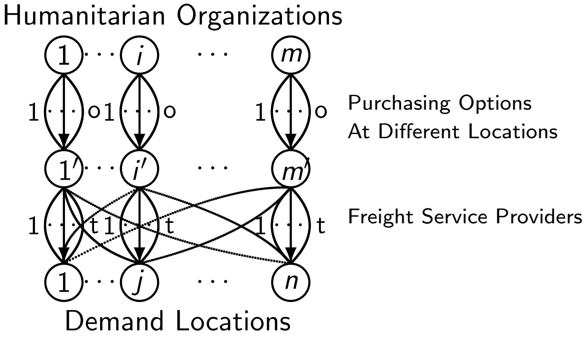

Figure: The Network Structure of the Game Theory Model with Multiple Purchasing Options and Multiple Freight Service Providers.

Purchasing costs

$q_{ijk,l}$: The volume of relief items purchased by HO i at location k and shipped to demand location j by FSP l.
ρ_k: The relief item price at location k

The total financial outlay for purchasing the relief items at the various locations for HO i; $i = 1, \ldots, m$, is

$$\sum_{k=1}^{o} \rho_k \sum_{j=1}^{n} \sum_{l=1}^{t} q_{ijk,l}. \quad (1)$$

Transportation costs

$c_{ijk,l}$: The transportation cost that HO i pays to get its relief items delivered to the demand point j by freight service provider l from purchase location k.

The total outlay associated with the logistical costs, hence, can be expressed for HO i; $i = 1, \ldots, m$, as:

$$\sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{t} c_{ijk,l}(q). \tag{2}$$

Cost functions are assumed to be convex and continuously differentiable.

Monetized benefit functions

The more effectively HOs provide relief at the demand points, the more attention they receive from potential and existing donors.

A benefit function associated with HO i; $i = 1, \dots, m$, is denoted by $B_i(q)$, and with it we associate a nonnegative monetization weight ω_i as follows:

$$\omega_i B_i(q). \tag{3}$$

HOs may benefit not only from their own efforts but also from other HOs' visibility at the demand points.

Altruism functions are assumed to be concave and continuously differentiable.

Utility functions

The utility function $U_i(q)$ for HO i; $i = 1, \dots, m$:

$$U_i(q) = \omega_i B_i(q) - \sum_{k=1}^{o} \rho_k \sum_{j=1}^{n} \sum_{l=1}^{t} q_{ijk,l} - \sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{t} c_{ijk,l}(q). \quad (4)$$

Integrated Financial and Logistical Game Theory Model

Non-negativity constraints

The volume of the relief item of each HO i; $i = 1, \ldots, m$, to any demand point j; $j = 1, \ldots, n$, purchased at location k; $k = 1, \ldots, o$, and transported by FSP l; $l = 1, \ldots, t$, consists of the nonnegativity constraints:

$$q_{ijk,l} \geq 0, \quad \forall j, k, l. \tag{5}$$

Budget constraints

b_i denotes HO i's budget.

$$\sum_{k=1}^{o} \rho_k \sum_{j=1}^{n} \sum_{l=1}^{t} q_{ijk,l} + \sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{t} c_{ijk,l}(q) \leq b_i. \tag{6}$$

Integrated Financial and Logistical Game Theory Model

- While the HOs may be willing to send as much of the relief item as they can, FSPs have limited capacity due to their facilities, vehicle portfolio and availability, and also the disaster regions' infrastructures.

Shipment capacity constraints

$u_{k,l}$: The shipment capacity of FSP l from purchase location k
The capacity constraints faced by HO i; $i = 1, \ldots, m$, are:

$$\sum_{i=1}^{m} \sum_{j=1}^{n} q_{ijk,l} \leq u_{k,l}, \quad k = 1, \ldots, o; l = 1, \ldots, t. \tag{7}$$

Integrated Financial and Logistical Game Theory Model

- At each demand point j; $j = 1, \ldots, n$, the volume of relief items will not be less than $\underline{d}_j$ and, at the same time, it will not exceed $\bar{d}_j$.

Demand lower bound and upper bound constraints

$$\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t} q_{ijk,l} \geq \underline{d}_j, \quad j = 1, \ldots, n, \tag{8}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t} q_{ijk,l} \leq \bar{d}_j \quad j = 1, \ldots, n. \tag{9}$$

Feasible set K_i

We define the feasible set K_i corresponding to HO i as:

$$K_i \equiv \{q_i | (5) \text{ holds}\} \tag{10}$$

and we let $K \equiv \prod_{i=1}^{m} K_i$.

Feasible set $\mathcal{S}$

We define the feasible set $\mathcal{S}$ of shared constraints as:

$$\mathcal{S} \equiv \{q | (6) \text{ holding for all } i, \text{ and } (8), (9), (10) \text{ hold}\}. \tag{11}$$

- We assume that the sum of the budgets of all the HOs, i.e., $\sum_{i=1}^{m} b_i$ is sufficient to meet the sum of all the minimum demands, that is, $\sum_{j=1}^{n} \underline{d}_j$ so that the set $\mathcal{K} \equiv K \cap \mathcal{S}$ will be nonempty.

Definition 1: Generalized Nash Equilibrium for the Humanitarian Organizations

A relief item flow vector $q^* \in K, q^* \in \mathcal{S}$ *is a Generalized Nash Equilibrium if for each HO* $i; i = 1, \ldots, m,$*:*

$$U_i(q_i^*, \hat{q_i^*}) \geq U_i(q_i, \hat{q_i^*}), \quad \forall q_i \in K_i, \forall q \in \mathcal{S}, \tag{12}$$

where $\hat{q_i^*} \equiv (q_1^*, \ldots, q_{i-1}^*, q_{i+1}^*, \ldots, q_m^*)$.

- Not one of the HOs is willing to deviate from his current relief item flow pattern, given the relief flow item patterns of the other HOs.
- Each HO's utility depends not only on his own strategy but also on that of the others' strategies since their feasible sets are intertwined. The latter condition makes the problem a **Generalized Nash Equilibrium** model.

Integrated Financial and Logistical Game Theory Model

Definition 2: Variational Equilibrium

A relief item flow vector q^* is a Variational Equilibrium of the above Generalized Nash Equilibrium problem if $q^* \in K, q^* \in \mathcal{S}$ is a solution to the following variational inequality:

$$-\sum_{i=1}^{m} \langle \nabla_{q_i} U_i(q^*), q_i - q_i^* \rangle \geq 0, \quad \forall q \in K, \forall q \in \mathcal{S}. \tag{13}$$

Expanding variational inequality (13), we obtain:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left[\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} + \rho_k - \omega_i \frac{\partial B_i(q^*)}{\partial q_{ijk,l}}\right] \times \left[q_{ijk,l} - q^*_{ijk,l}\right] \geq 0, \tag{14}$$

$$\forall q \in K, \forall q \in \mathcal{S}.$$

We now put variational inequality (14) into standard form: determine $X^* \in \mathcal{K}$, such that:

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{15}$$

where $\langle \cdot, \cdot \rangle$ is the inner product in N-dimensional Euclidean space, where $N = mnot$ for our model. We define $X \equiv q$ and $F(X)$ as having components:

$$F_{ijk,l}(X) \equiv \sum_{r=1}^{n} \sum_{p=1}^{o} \sum_{s=1}^{t} \frac{\partial c_{irp,s}(q)}{\partial q_{ijk,l}} + \rho_k - \omega_i \frac{\partial B_i(q)}{\partial q_{ijk,l}}, \quad \forall i, j, k, l, \tag{16}$$

with $\mathcal{K} \equiv K \cap \mathcal{S}$.

Existence

Since the function $F(X)$ that enters our variational inequality problem (15) with components as in (16) is, under the imposed conditions, continuous and, clearly, the feasible set $\mathcal{K}$ is not only convex, but compact because of the demand and budget constraints, we know that a solution X^* exists from the standard theory of variational inequalities (Kinderlehrer and Stampacchia (1980)).

Lagrange Theory and Analysis of the Marginal Utilities

- We investigate the Lagrange theory associated with the variational inequality (14).
- Using the Lagrange multipliers, we analyze the marginal utilities and the role of each constraint in the model.

Optimization problem

$$C(q)=\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left[\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}}+\rho_k-\omega_i\frac{\partial B_i(q^*)}{\partial q_{ijk,l}}\right]\times\left[q_{ijk,l}-q^*_{ijk,l}\right], \quad (17)$$

Variational inequality (14) can be rewritten as the following minimization problem:

$$\min_{\mathcal{K}} C(q)=C(q^*)=0. \quad (18)$$

Lagrange Theory and Analysis of the Marginal Utilities

Constraints and associated Lagrange multipliers

$$e_{k,l} = \sum_{i=1}^{m} \sum_{j=1}^{n} q_{ijk,l} - u_{k,l} \leq 0, \quad \epsilon_{k,l}, \forall k, \forall l,$$

$$f_i = \sum_{k=1}^{o} \rho_k \sum_{j=1}^{n} \sum_{l=1}^{t} q_{ijk,l} + \sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{t} c_{ijk,l}(q) - b_i \leq 0, \quad \gamma_i, \forall i,$$

$$g_{ijk,l} = -q_{ijk,l} \leq 0, \quad \lambda_{ijk,l}, \forall i, \forall j, \forall k, \forall l, \tag{19}$$

$$a_j = \underline{d}_j - \sum_{i=1}^{m} \sum_{k=1}^{o} \sum_{l=1}^{t} q_{ijk,l} \leq 0, \quad \alpha_j, \forall j,$$

$$b_j = \sum_{i=1}^{m} \sum_{k=1}^{o} \sum_{l=1}^{t} q_{ijk,l} - \bar{d}_j \leq 0, \quad \beta_j, \forall j.$$

Lagrange Theory and Analysis of the Marginal Utilities

Lagrange function

$$\mathcal{L}(q,\epsilon,\gamma,\lambda,\alpha,\beta)=\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left[\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}}+\rho_k-\omega_i\frac{\partial B_i(q^*)}{\partial q_{ijk,l}}\right]\times\left[q_{ijk,l}-q^*_{ijk,l}\right]$$

$$+\sum_{k=1}^{o}\sum_{l=1}^{t}e_{k,l}\epsilon_{k,l}+\sum_{i=1}^{m}f_i\gamma_i+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}g_{ijk,l}\lambda_{ijk,l}+\sum_{j=1}^{n}a_j\alpha_j+\sum_{j=1}^{n}b_j\beta_j, \qquad (20)$$

$$\forall q\in R_+^{mnot},\ \forall\alpha\in R_+^{n},\ \forall\beta\in R_+^{n},\ \forall\epsilon\in R_+^{ot},\ \forall\gamma\in R_+^{m},\ \forall\lambda\in R_+^{mnot}.$$

Lagrange Theory and Analysis of the Marginal Utilities

- Since the feasible set $\mathcal{K}$ is convex and the Slater condition is satisfied, if q^* is a minimal solution to problem (18) the vector $(q^*, \epsilon^*, \gamma^*, \lambda^*, \alpha^*, \beta^*)$ is a saddle point of the Lagrange function.

$$\mathcal{L}(q^*, \epsilon, \gamma, \lambda, \alpha, \beta) \leq \mathcal{L}(q^*, \epsilon^*, \gamma^*, \lambda^*, \alpha^*, \beta^*) \leq \mathcal{L}(q, \epsilon^*, \gamma^*, \lambda^*, \alpha^*, \beta^*) \quad (21)$$

and

$$\begin{aligned} e^*_{k,l}\epsilon^*_{k,l} &= 0, \qquad \forall k, \forall l, \\ f^*_i\gamma^*_i &= 0, \qquad \forall i, \\ g^*_{ijk,l}\lambda^*_{ijk,l} &= 0, \qquad \forall i, \forall j, \forall k, \forall l, \\ a^*_j\alpha^*_j &= 0, \qquad \forall j, \\ b^*_j\beta^*_j &= 0, \qquad \forall j. \end{aligned} \quad (22)$$

Lagrange Theory and Analysis of the Marginal Utilities

$$\frac{\partial \mathcal{L}(q^*, \epsilon^*, \gamma^*, \lambda^*, \alpha^*, \beta^*)}{\partial q_{ijk,l}} = \left[\sum_{r=1}^{n} \sum_{p=1}^{o} \sum_{s=1}^{t} \frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} + \rho_k - \omega_i \frac{\partial B_i(q^*)}{\partial q_{ijk,l}} \right]$$

$$+\epsilon^*_{k,l} + \gamma^*_i \left(\rho_k + \sum_{r=1}^{n} \sum_{p=1}^{o} \sum_{s=1}^{t} \frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} \right) - \lambda^*_{ijk,l} - \alpha^*_j + \beta^*_j = 0, \quad (23)$$

- Conditions (22) and (23) represent an equivalent formulation of variational inequality (14). Indeed, if we multiply (23) by $(q_{ijk,l} - q^*_{ijk,l})$ and sum up with respect to i, j, k, and l we get:

Variational Inequality

Determine $(q^*, \epsilon^*, \gamma^*, \alpha^*, \beta^*) \in R_+^{2mnot+ot+m+2n}$ such that

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}\left[(\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}}+\rho_k)(1+\gamma_i^*)-\omega_i\frac{\partial B_i(q^*)}{\partial q_{ijk,l}}+\epsilon_{k,l}^*-\alpha_j^*+\beta_j^*\right]$$

$$\times\left[q_{ijk,l}-q_{ijk,l}^*\right]+\sum_{k=1}^{o}\sum_{l=1}^{t}\left[u_{k,l}-\sum_{i=1}^{m}\sum_{j=1}^{n}q_{ijk,l}^*\right]\times\left[\epsilon_{k,l}^*-\epsilon_{k,l}^*\right]$$

$$+\sum_{i=1}^{m}\left[b_i-\sum_{k=1}^{o}\rho_k\sum_{j=1}^{n}\sum_{l=1}^{t}q_{ijk,l}^*-\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{t}c_{ijk,l}(q^*)\right]\times[\gamma_i-\gamma_i^*]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t}q_{ijk,l}^*-\underline{d}_j\right]\times\left[\alpha_j-\alpha_j^*\right]+\sum_{j=1}^{n}\left[\bar{d}_j-\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t}q_{ijk,l}^*\right]\times\left[\beta_j-\beta_j^*\right]\geq 0,$$

$$\forall(q,\epsilon,\gamma,\alpha,\beta)\in R_+^{mnot+ot+m+2n}. \qquad (24)$$

The meaning of some of the Lagrange multipliers

- When the constraints are not active:

$$\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t}\frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}}+\rho_k=\omega_i\frac{\partial B_i(q^*)}{\partial q_{ijk,l}}, \tag{25}$$

- The weighted marginal altruism is equal to the sum of the marginal logistical cost and the relief item purchase price (for the respective i, j, k, l).

The meaning of some of the Lagrange multipliers

- The first constraint is active; namely, $\sum_{i=1}^{m}\sum_{j=1}^{n} q^*_{ijk,l} = u_{k,l}$,

$$\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t} \frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} + \rho_k + \epsilon_{k,l} = \omega_i \frac{\partial B_i(q^*)}{\partial q_{ijk,l}}, \qquad (26)$$

- The weighted marginal altruism exceeds the sum of the marginal logistical cost and the relief item purchase price (for the respective i, j, k, l) and this is a good situation.

Lagrange Theory and Analysis of the Marginal Utilities

The meaning of some of the Lagrange multipliers

- If the fourth constraint is active, namely $\sum_{i=1}^{m}\sum_{k=1}^{n}\sum_{l=1}^{t} q^*_{ijk,l} = \underline{d}_j$,

$$\sum_{r=1}^{n}\sum_{p=1}^{o}\sum_{s=1}^{t} \frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} + \rho_k - \alpha_j = \omega_i \frac{\partial B_i(q^*)}{\partial q_{ijk,l}}, \tag{27}$$

which implies a bad situation.

The meaning of some of the Lagrange multipliers

- The fifth constraint is active, namely $\sum_{i=1}^{m} \sum_{k=1}^{n} \sum_{l=1}^{t} q^*_{ijk,l} = \bar{d}_j$,

$$\sum_{r=1}^{n} \sum_{p=1}^{o} \sum_{s=1}^{t} \frac{\partial c_{irp,s}(q^*)}{\partial q_{ijk,l}} + \rho_k + \beta_j = \omega_i \frac{\partial B_i(q^*)}{\partial q_{ijk,l}}, \quad (28)$$

- The weighted marginal altruism exceeds the sum of the marginal logistical cost and the relief item purchase price (for the respective i, j, k, l) and this is again a good situation.

Lagrange Theory and Analysis of the Marginal Utilities

- From the above analysis of the Lagrange multipliers and marginal utilities at the equilibrium solution, we can conclude that the most convenient situation, in terms of weighted altruism, is the one when $\sum_{i=1}^{m}\sum_{j=1}^{n} q^*_{ijk,l} = u_{k,l}$ and $\sum_{i=1}^{m}\sum_{k=1}^{n}\sum_{l=1}^{t} q^*_{ijk,l} = \overline{d}_j$.

The Algorithm

The algorithm is the Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993). As Dupuis and Nagurney (1993) establish, for convergence of the general iterative scheme, the sequence $\{a_\tau\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$.

Explicit Formulae for the Euler Method Applied to the Game Theory Model

Specifically, at an iteration $\tau + 1$, we have the following closed form expression for the relief item flow that each HO $i = 1, \ldots, m$, purchases at location $k = 1, \ldots, o$, and has then transported to the demand point $j = 1, \ldots, n$, by FSP $l = 1, \ldots, t$:

$$q_{ijk,l}^{\tau+1} = \max\{0, q_{ijk,l}^{\tau} + a_{\tau}(\omega_i \frac{\partial B_i(q^{\tau})}{\partial q_{ijk,l}} - (\sum_{r=1}^{n} \sum_{p=1}^{o} \sum_{s=1}^{t} \frac{\partial c_{irp,s}(q^{\tau})}{\partial q_{ijk,l}} + \rho_k)(1 + \gamma_i^{\tau}) + \alpha_j^{\tau} - \beta_j^{\tau} - \epsilon_{k,l}^{\tau})\}. \tag{30}$$

Explicit Formulae for the Euler Method Applied to the Game Theory Model

The explicit formula for the Lagrange multipliers associated with the budget constraint (6), respectively, for $i = 1, \ldots, m$, is:

$$\gamma_i^{\tau+1} = \max\{0, \gamma_i^{\tau} + a_{\tau}(-b_i + \sum_{k=1}^{o} \rho_k \sum_{j=1}^{n} \sum_{l=1}^{t} q_{ijk,l}^{\tau} + \sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{t} c_{ijk,l}(q^{\tau}))\}. \quad (31)$$

The closed form expression associated with the Lagrange multiplier for each capacity constraint (8) for $k = 1, \ldots, o$; $l = 1, \ldots, t$, is:

$$\epsilon_{k,l}^{\tau+1} = \max\{0, \epsilon_j^{\tau} + a_{\tau}(-u_{k,l} + \sum_{i=1}^{m} \sum_{j=1}^{n} q_{ijk,l}^{\tau})\}. \quad (32)$$

Explicit Formulae for the Euler Method Applied to the Game Theory Model

The Lagrange multiplier for the demand lower bound constraint (9) at demand points $j = 1, \ldots, n$, is computed according to:

$$\alpha_j^{\tau+1} = \max\{0, \alpha_j^{\tau} + a_{\tau}(-\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t} q_{ijk,l}^{\tau} + \underline{d}_j)\}. \tag{33}$$

The Lagrange multiplier for the demand upper bound constraint (10) at demand points $j = 1, \ldots, n$, is computed as follows:

$$\beta_j^{\tau+1} = \max\{0, \beta_j^{\tau} + a_{\tau}(-\bar{d}_j + \sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{t} q_{ijk,l}^{\tau})\}. \tag{34}$$

Numerical Examples: Hurricane Harvey

Hurricane Harvey

- Hurricane Harvey was a Category 4 storm that hit Texas on August 25, 2017.
- Caused $125 billion in damage, affected almost 13 million people, and became the second costliest disaster in U.S history.
- Over 880,000 applications across 41 Texas counties.
  Port Arthur, 13,654 applications were submitted.
  Bay City, with 6500 applications.
  Silsbee, with 3,232 registered applications.
- FEMA, American Red Cross, and The Salvation Army.

Example 1: Baseline Supply Chain Network

- Two humanitarian organizations, the Salvation Army and the American Red Cross, respectively. The Salvation Army is a smaller relief organization as compared to the American Red Cross.
- Three demand points: Port Arthur, Bay City, and Silsbee, respectively. The major devastation occurred in the Port Arthur region.
- The HOs have two options to purchase the relief items from: Purchasing Location 1 (PL 1) and Purchasing Location 2 (PL 2). PL 1: reasonable price, far from the affected area. PL 2: offers a similar product at a higher price, a local market.
- Two Freight Service Providers, denoted by FSP 1 and FSP 2, respectively. In contrast to FSP 2, FSP 1 has less equipment and capability, so it provides service with less capacity as compared to FSP 1.

Example 1: Baseline Supply Chain Network

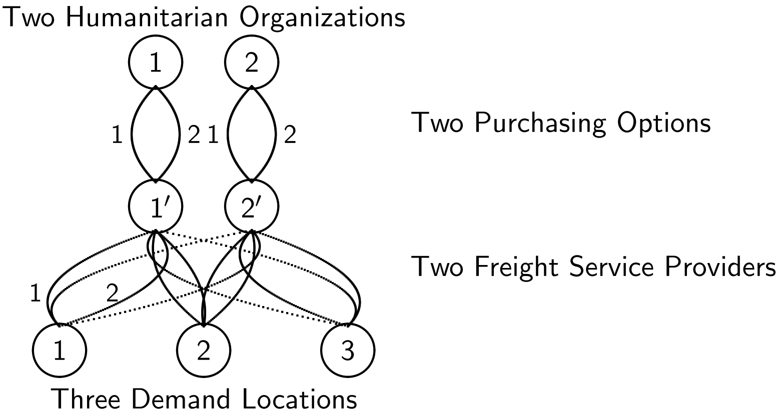

Figure: Example 1: Two Humanitarian Organizations, Three Demand Locations, Two Purchasing Options, Two Freight Service Providers.

Example 1: Baseline Supply Chain Network

The humanitarian organizations' budgets are:

$$b_1 = 3 \times 10^6, \quad b_2 = 6 \times 10^6.$$

The HOs' altruism functions are:

$$B_1(q) = \sum_{k=1}^{2}\sum_{l=1}^{2}(300q_{11k,l} + 200q_{12k,l} + 100q_{13k,l}),$$

$$B_2(q) = \sum_{k=1}^{2}\sum_{l=1}^{2}(400q_{21k,l} + 300q_{22k,l} + 200q_{23k,l}),$$

monetization weights associated with these benefit functions are:

$$\omega_1 = 1, \quad \omega_2 = 1.$$

Example 1: Baseline Supply Chain Network

The lower and upper bounds on demand points are:

$$\underline{d}_1 = 10000, \quad \bar{d}_1 = 20000,$$

$$\underline{d}_2 = 1000, \quad \bar{d}_2 = 10000,$$

$$\underline{d}_3 = 1000, \quad \bar{d}_3 = 10000.$$

Markets at each purchasing location sell the relief items at the following prices:

$$\rho_1 = 50, \quad \rho_2 = 70.$$

Example 1: Baseline Supply Chain Network

The FSPs' capacities are as follows:

$$u_{1,1} = 3000, \quad u_{1,2} = 6000,$$
$$u_{2,1} = 5000, \quad u_{2,2} = 8000.$$

Cost functions:

$$c_{i11,1}(q) = 0.2q_{i11,1}^2 + 2q_{i11,1} + q_{j11,1}, \quad c_{i21,1}(q) = 0.2q_{i21,1}^2 + 5q_{i21,1} + 2.5q_{j21,1},$$
$$c_{i31,1}(q) = 0.2q_{i31,1}^2 + 7q_{i31,1} + 3.5q_{j31,1},$$
$$c_{i12,1}(q) = 0.15q_{i12,1}^2 + 2q_{i12,1} + q_{j12,1}, \quad c_{i22,1}(q) = 0.15q_{i22,1}^2 + 5q_{i22,1} + 2.5q_{j22,1},$$
$$c_{i32,1}(q) = 0.15q_{i32,1}^2 + 7q_{i32,1} + 3.5q_{j32,1},$$
$$c_{i11,2}(q) = 0.15q_{i11,2}^2 + 2q_{i11,2} + q_{j11,2}, \quad c_{i21,2}(q) = 0.15q_{i21,2}^2 + 5q_{i21,2} + 2.5q_{j21,2},$$
$$c_{i31,2}(q) = 0.15q_{i31,2}^2 + 7q_{i31,2} + 3.5q_{j31,2},$$
$$c_{i12,2}(q) = 0.1q_{i12,2}^2 + 2q_{i12,2} + q_{j12,2}, \quad c_{i22,2}(q) = 0.1q_{i22,2}^2 + 5q_{i22,2} + 2.5q_{j22,2},$$
$$c_{i32,2}(q) = 0.1q_{i32,2}^2 + 7q_{i32,2} + 3.5q_{j32,2},$$
$$\forall i = 1, 2; j \neq i.$$

Example 1: Baseline Supply Chain Network

Results:

$$q^*_{ij1,1} = \begin{bmatrix} 874.22 & 362.42 & 107.52 \\ 134.20 & 0.00 & 357.42 \end{bmatrix}, \quad q^*_{ij2,1} = \begin{bmatrix} 1098.80 & 416.54 & 76.70 \\ 1432.14 & 749.74 & 409.87 \end{bmatrix},$$

$$q^*_{ij1,2} = \begin{bmatrix} 1165.47 & 483.20 & 143.28 \\ 1498.80 & 816.10 & 476.53 \end{bmatrix}, \quad q^*_{ij2,2} = \begin{bmatrix} 1648.22 & 624.48 & 115.05 \\ 2148.17 & 1123.9 & 614.48 \end{bmatrix}.$$

The amount of **relief item** received at **each demand point** is:

$$\sum_{i=1}^{2}\sum_{k=1}^{2}\sum_{l=1}^{2} q^*_{i1k,l} = 10000.00, \quad \sum_{i=1}^{2}\sum_{k=1}^{2}\sum_{l=1}^{2} q^*_{i2k,l} = 4576.76, \quad \sum_{i=1}^{2}\sum_{k=1}^{2}\sum_{l=1}^{2} q^*_{i3k,l} = 2300.86.$$

The Lagrange multipliers for lower bound demand constraints:

$$\alpha_1^* = 101.72, \quad \alpha_2^* = \alpha_3^* = 0.$$

The Lagrange multipliers of the upper demand bound constraints:

$$\beta_1^* = \beta_2^* = \beta_3^* = 0.$$

Example 1: Baseline Supply Chain Network

The volumes of relief items carried by each FSP from each purchasing location are:

$$\sum_{i=1}^{2}\sum_{j=1}^{3} q^*_{ijk,l} = \begin{bmatrix} 1835.79 & 4583.70 \\ 4183.79 & 6274.38 \end{bmatrix}.$$

The Lagrange multipliers associated with the shipping capacities:

$$\epsilon^*_{1,1} = \epsilon^*_{1,2} = \epsilon^*_{2,1} = \epsilon^*_{2,2} = 0.$$

The total cost of each organization:

$$\sum_{k=1}^{2} \rho_k \sum_{j=1}^{3}\sum_{l=1}^{2} q^*_{1jk,l} + \sum_{j=1}^{3}\sum_{k=1}^{2}\sum_{l=1}^{2} c_{1jk,l}(q^*) = 1,419,224.00,$$

$$\sum_{k=1}^{2} \rho_k \sum_{j=1}^{3}\sum_{l=1}^{2} q^*_{2jk,l} + \sum_{j=1}^{3}\sum_{k=1}^{2}\sum_{l=1}^{2} c_{2jk,l}(q^*) = 2,208,465.25.$$

The Lagrange multipliers associated with their budget constraints:

$$\gamma^*_1 = \gamma^*_2 = 0.$$

Example 1: Baseline Supply Chain Network

The benefit/altruism that each organization gains from helping the affected people is:

$$B_1(q^*) = 1,857,600.50, \quad B_2(q^*) = 3,264,018.25.$$

Putting all the terms in the respective objective functions together, the utility of each HO, after the disaster relief operation, is:

$$U_1(q^*) = 438,376.50, \quad U_2(q^*) = 1,055,553.00.$$

Analysis and Insights:

- The American Red Cross is more active than The Salvation Army.
- The American Red Cross delivers 9,761.73 relief items, which is more than the 7,115.92 that The Salvation Army has been able to deliver.
- The American Red Cross spends more than The Salvation Army
- The American Red Cross has a higher utility than The Salvation Army.
- FSP 2 achieves a large share of the transportation market by benefiting from its own facilities and larger shipment capacity.
- In the sales market, 10,458.16 relief items are purchased from PL 2 and, despite having a higher price, PL2 is preferred by the HOs due to the lower shipping costs. PL 1 also, because of its lower item price, still has a good share of market. 6,419.48 relief items have been brought to the demand points from PL 1.

Example 2: New Purchasing Location is Added

- The HOs have a new location option for purchasing the relief items.
- The new purchasing location, denoted by PL 3, is a local one. It charges a lower price than the existing local purchasing location, PL 2, but its price is still higher than PL 1's price.
- The relief items are sold at the new purchasing location at the price:

$$\rho_3 = 60.$$

Example 2: New Purchasing Location is Added

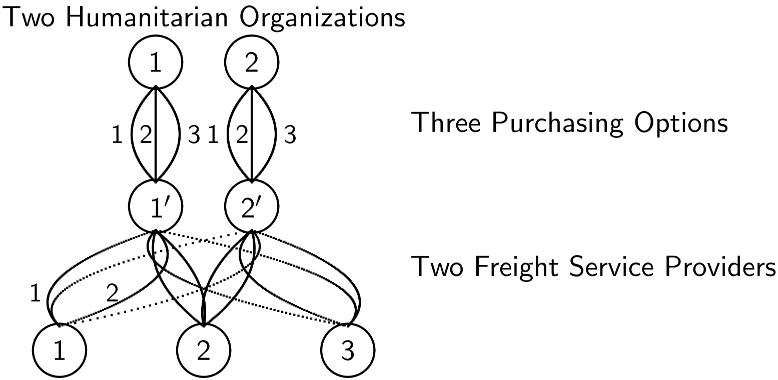

Figure: Example 2: Two Humanitarian Organizations, Three Demand Locations, Three Purchasing Options, Two Freight Service Providers.

Example 2: New Purchasing Location is Added

The FSPs' capacities:

$$u_{1,1} = 3000, \quad u_{1,2} = 6000,$$
$$u_{2,1} = 4000, \quad u_{2,2} = 7000,$$
$$u_{3,1} = 4000, \quad u_{3,2} = 7000.$$

The HOs' altruism functions:

$$B_1(q) = \sum_{k=1}^{3}\sum_{l=1}^{2}(300q_{11k,l} + 200q_{12k,l} + 100q_{13k,l}),$$

$$B_2(q) = \sum_{k=1}^{3}\sum_{l=1}^{2}(400q_{21k,l} + 300q_{22k,l} + 200q_{23k,l}).$$

The transportation cost functions:

$$c_{i13,1}(q) = 0.15q_{i13,1}^2 + 2q_{i13,1} + q_{j13,1}, \quad c_{i23,1}(q) = 0.15q_{i23,1}^2 + 5q_{i23,1} + 2.5q_{j23,1},$$
$$c_{i33,1}(q) = 0.15q_{i33,1}^2 + 7q_{i33,1} + 3.5q_{j33,1},$$
$$c_{i13,2}(q) = 0.1q_{i13,2}^2 + 2q_{i13,2} + q_{j13,2}, \quad c_{i23,2}(q) = 0.1q_{i23,2}^2 + 5q_{i23,2} + 2.5q_{j23,2},$$
$$c_{i33,2}(q) = 0.1q_{i33,2}^2 + 7q_{i33,2} + 3.5q_{j33,2}.$$
$$\forall i = 1, 2; j \neq i.$$

Results:

$$q^*_{ij1,1} = \begin{bmatrix} 620.14 & 362.42 & 107.48 \\ 0.00 & 0.00 & 357.42 \end{bmatrix}, \quad q^*_{ij2,1} = \begin{bmatrix} 760.20 & 416.54 & 76.69 \\ 1092.91 & 749.74 & 409.87 \end{bmatrix},$$

$$q^*_{ij1,2} = \begin{bmatrix} 826.80 & 483.20 & 143.28 \\ 1159.56 & 816.40 & 476.53 \end{bmatrix}, \quad q_{ij2,2} = \begin{bmatrix} 1139.12 & 624.48 & 114.94 \\ 1639.00 & 1123.96 & 614.48 \end{bmatrix},$$

$$q^*_{ij3,1} = \begin{bmatrix} 793.51 & 449.87 & 109.97 \\ 1126.623 & 783.07 & 443.20 \end{bmatrix},$$

$$q^*_{ij3,2} = \begin{bmatrix} 1189.10 & 674.47 & 164.88 \\ 1688.99 & 1173.96 & 664.47 \end{bmatrix}.$$

The amount of **relief items** received at **each demand point** is:

$$\sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i1k,l} = 12,035.55, \quad \sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i2k,l} = 7,658.10, \quad \sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i3k,l} = 3,683.22.$$

The Lagrange multipliers for the lower bound demand constraints:

$$\alpha^*_1 = \alpha^*_2 = \alpha^*_3 = 0.$$

Example 2: New Purchasing Location is Added

The Lagrange multipliers of the upper demand bound constraints:

$$\beta_1^* = \beta_2^* = \beta_3^* = 0.$$

The amount of relief items carried by each FSP from each purchasing location is:

$$\sum_{i=1}^{2} \sum_{j=1}^{3} q_{ijk,l}^* = \begin{bmatrix} 1447.47 & 3905.76 \\ 3505.94 & 5255.98 \\ 3705.85 & 5555.87 \end{bmatrix}.$$

Not one of them has reached the capacity and, hence, we have that:

$$\epsilon_{1,1}^* = \epsilon_{1,2}^* = \epsilon_{2,1}^* = \epsilon_{2,2}^* = \epsilon_{3,1}^* = \epsilon_{3,2}^* = 0.$$

Example 2: New Purchasing Location is Added

The total cost of each organization in this operation:

$$\sum_{k=1}^{3} \rho_k \sum_{j=1}^{3} \sum_{l=1}^{2} q^*_{1jk,l} + \sum_{j=1}^{3} \sum_{k=1}^{3} \sum_{l=1}^{2} c_{1jk,l}(q^*) = 1,454,783.75,$$

$$\sum_{k=1}^{3} \rho_k \sum_{j=1}^{3} \sum_{l=1}^{2} q^*_{2jk,l} + \sum_{j=1}^{3} \sum_{k=1}^{3} \sum_{l=1}^{2} c_{2jk,l}(q^*) = 2,821,781.50.$$

The Lagrange multipliers associated with the budget constraints:

$$\gamma_1^* = \gamma_2^* = 0.$$

The benefit/altruism of each organization:

$$B_1(q^*) = 2,272,585.50, \quad B_2(q^*) = 4,670,005.50.$$

The utility of each HO:

$$U_1(q^*) = 817,796.75, \quad U_2(q^*) = 1,848,224.00.$$

Analysis and Insights:

- Both organizations take advantage of this opportunity and buy more relief items.
- The American Red Cross experiences an increase of almost 5,000, as compared to Example 1. The Salvation Army provides 9,057.09 relief item kits.
- Both organizations pay more, but this higher cost has led to a significant increase in their utility.
- The new PL was able to take a large market share due to its lower price than the other local PL and the lower transportation rates than the nonlocal PL. Both of the previous purchasing locations experience a decrease in sales with the arrival of the new PL.
- The increase in the purchasing power of HOs has also boosted the transportation market with the major increase being in the shipments of the relief items from newly added PL to the affected region.

Example 3: Additional Disruptions in Transportation

- We now consider additional disruptions in transportation.
- All the logistical costs are as in Example 2 except that the nonlinear component is multiplied by a factor of 10.
- The new computed equilibrium relief item flows $i = 1, 2; j = 1, 3$ are:

$$q^*_{ij1,1} = \begin{bmatrix} 575.14 & 249.96 & 0.00 \\ 0.00 & 0.00 & 266.55 \end{bmatrix}, \quad q^*_{ij2,1} = \begin{bmatrix} 700.21 & 266.54 & 0.00 \\ 1032.92 & 599.75 & 199.94 \end{bmatrix},$$

$$q^*_{ij1,2} = \begin{bmatrix} 766.86 & 333.21 & 0.00 \\ 1099.57 & 666.41 & 266.55 \end{bmatrix}, \quad q_{ij2,2} = \begin{bmatrix} 1049.17 & 399.73 & 0.00 \\ 1549.01 & 899.45 & 299.75 \end{bmatrix},$$

$$q^*_{ij3,1} = \begin{bmatrix} 0.00 & 299.88 & 0.00 \\ 1066.25 & 633.08 & 233.27 \end{bmatrix},$$

$$q^*_{ij3,2} = \begin{bmatrix} 0.00 & 499.73 & 0.00 \\ 1599.00 & 949.45 & 349.74 \end{bmatrix}.$$

Example 3: Additional Disruptions in Transportation

The amount of **relief items** received at **each demand point** is:

$$\sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i1k,l} = 11,270.81, \quad \sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i2k,l} = 5,747.18, \quad \sum_{i=1}^{2}\sum_{k=1}^{3}\sum_{l=1}^{2} q^*_{i3k,l} = 1,549.20.$$

The Lagrange multipliers for the lower bound demand constraints:

$$\alpha^*_1 = \alpha^*_2 = \alpha^*_3 = 0.$$

The Lagrange multipliers of the upper demand bound constraints:

$$\beta^*_1 = \beta^*_2 = \beta^*_3 = 0.$$

The amount of relief items carried by each FSP from each purchasing location is:

$$\sum_{i=1}^{2}\sum_{j=1}^{3} q^*_{ijk,l} = \begin{bmatrix} 1025.06 & 3132.59 \\ 2799.36 & 4197.11 \\ 2966.01 & 4447.06 \end{bmatrix}.$$

Not one of them has reached the capacity and, hence, we have that:

$$\epsilon^*_{1,1} = \epsilon^*_{1,2} = \epsilon^*_{2,1} = \epsilon^*_{2,2} = \epsilon^*_{3,1} = \epsilon^*_{3,2} = 0.$$

Example 3: Additional Disruptions in Transportation

The total cost of each organization in this operation:

$$\sum_{k=1}^{3} \rho_k \sum_{j=1}^{3} \sum_{l=1}^{2} q^*_{1jk,l} + \sum_{j=1}^{3} \sum_{k=1}^{3} \sum_{l=1}^{2} c_{1jk,l}(q^*) = 1,459,130.25,$$

$$\sum_{k=1}^{3} \rho_k \sum_{j=1}^{3} \sum_{l=1}^{2} q^*_{2jk,l} + \sum_{j=1}^{3} \sum_{k=1}^{3} \sum_{l=1}^{2} c_{2jk,l}(q^*) = 2,657,446.75.$$

The Lagrange multipliers associated with the budget constraints:

$$\gamma_1^* = \gamma_2^* = 0.$$

The benefit/altruism of each organization:

$$B_1(q^*) = 1,877,028.75, \quad B_2(q^*) = 3,972,979.50.$$

The utility of each HO:

$$U_1(q^*) = 417,898.50, \quad U_2(q^*) = 1,315,532.75.$$

Analysis and Insights:

- The additional disruption leads to a sharp decrease in the number of items sent by each organization.
- The American Red Cross and the Salvation Army send, respectively, 2,675 and 2,133 fewer relief item kits, than in Example 2.
- The costs of the operations are almost the same as in Example 2 for both organizations, but they experience reduced incurred altruism, as compared to Example 2, and also lower utilities.
- All three PLs face a drop in their sales. Similarly, FSPs transport an average of 2400 fewer items from PLs to the demand regions as compared to the situation of not having the additional disruption in transportation.

Conclusions

- In this paper, we develop an integrated financial and logistical game theory model for humanitarian organizations.
- The model includes both relief item purchasing costs and freight service shipping costs, with the former being possible both locally and nonlocally, if feasible, and with the latter including competition, under capacity constraints, among the humanitarian organizations.
- The governing equilibrium conditions, given common/shared constraints associated with the demands for relief items at the demand points, plus the freight capacity constraints, yield a Generalized Nash equilibrium, which can be challenging to solve. Nevertheless, through the concept of a variational equilibrium, we construct a variational inequality formulation.

Conclusions

- The model is qualitatively analyzed and a Lagrange analysis provided, which yields insights on the impacts of the constraints.
- The proposed algorithm yields closed form expressions, at each iteration, which enables ease of computer implementation.
- The numerical examples, inspired by Hurricane Harvey, one of the most expensive natural disasters to ever hit the United States, illustrate the modeling and computational framework.
- The framework adds to the literature on game theory and disaster relief as well as to the literature on variational inequalities with nonlinear constraints.

Thank you!

For more information: https://supernet.isenberg.umass.edu/