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A VARIATIONAL INEQUALITY TRADE NETWORK MODEL IN PRICES AND QUANTITIES UNDER COMMODITY LOSSES

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Abstract. Multicommodity trade enables the production, consumption, and flow of commodities across the globe from agricultural ones to precious metals. Mathematical formalisms to model, analyze, and solve such problems have advanced and are also relevant to policy and decision-making. In this paper, we construct a variational inequality trade network model in price and quantity variables, which captures possible losses on transportation routes, which can occur because of perishability of commodities, as in the case of agricultural ones, or outright thefts. The equilibrium conditions are stated and the variational inequality formulation derived. Qualitative properties of existence and uniqueness of the equilibrium supply price, commodity shipment, and demand price pattern are provided under reasonable conditions. Illustrative examples help to demonstrate the model. An algorithm that is proposed yields closed form expressions at each iteration and can also be interpreted as a discrete time adjustment process for the evolution of the economic variables. A spectrum of algorithmically solved numerical examples, with full input and output data provided, yields insights into the impacts of commodity losses, increased congestion, as well as enhanced marketing on producers as well as consumers. This new model expands the scope of spatial price equilibrium modeling under commodity losses.

Keywords. Agriculture; Equilibrium problems; Generalized networks; Multicommodity trade networks; Variational inequalities.

1. Introduction

Commodities are products that have not yet been transformed through a production process with important examples being agricultural commodities, such as fresh produce, energy in the form of oil and gas, and metals such as gold and silver. According to Hayes [1] commodities are often classified as being "hard" with such commodities including natural resources that need to be extracted or mined (precious metals, lithium, etc.), whereas "soft" commodities include agricultural products (fruits, vegetables, corn, wheat, etc.) or livestock. Commodities are the fundamental components of the global economy.

Given the importance of commodities to health in the form of agricultural products and as essential raw materials to high technology products, as well as to energy, multicommodity trade has been the subject of mathematical modeling. Of particular interest is the determination of

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Outline of Presentation

- **Introduction and Motivation**
- **Literature Review and Contributions**
- **The Variational Inequality Trade Network Model in Prices and Quantities**
- **The Algorithm**
- **Numerical Examples**
- **Insights and Summary**

Commodity Definition and Classification

- **Commodities are products that have not yet been transformed through a production process with important examples being agricultural commodities, energy in the form of oil and gas, and metals such as gold and silver.**



- **Commodities are often classified as hard which includes natural resources that need to be extracted or mined (e.g., precious metals, lithium), while soft commodities include agricultural products (e.g., fruits, vegetables, corn, wheat) or livestock (Hayes (2024)).**

International Trade Importance and Economic Value

- **Commodities are fundamental to the global economy, essential for health, technology, and energy.**
- **The value of the fresh food commodity market globally, which includes fresh fruits, vegetables, and meats, was estimated at 3,077 billion US dollars in 2021 and the number is expected to increase by 2027 to more than 3,922 billion US dollars.**
- **The global precious metals market is projected to grow from $275.40 billion US dollars in 2021 to $403.08 billion in 2028.**

Challenges to Commodity Trade: Perishability

- **One of the major challenges associated with commodity trade is that of "losses," from that of perishability of fresh produce to outright theft of various commodities.**
- **Approximately 14% of the world's food continues to be lost after harvest and before it reaches retailers.**


Challenges to Commodity Trade: Theft

- **Food and beverage products in the United States in 2023 were top targets for freight theft, with an average loss of $214,000.**
- **Food and drink products were the most commonly stolen items in global supply chains, comprising almost 25% of the reported stolen products in Britain alone**.
- **Thefts in Europe, while food is in transit, was noted as being "commonplace."**.

Challenges to Commodity Trade: Geopolitical and Environmental Risks

- **In an environment of heightened geopolitical risk and negative impacts of climate change, trade is being challenged by a plethora of disasters, both slow-onset and sudden-onset ones.**

Literature Review

• Spatial price equilibrium models, which have served as the foundation for the formulation, analysis, and computation of solutions to commodity trade problems, have significantly advanced since the pioneering work of Samuelson (1952) and Takayama and Judge (1964, 1971).

• Thore (1986) was the first to consider the use of arc multipliers for perishability in the context of spatial price equilibrium problems but in the case of separable functions.

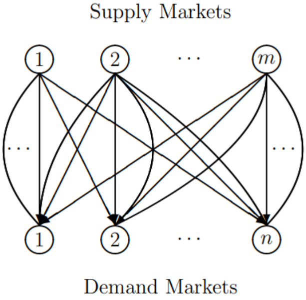
• In the literature, the spatial price equilibrium models in quantity variables have dominated (Floran et al. (1982), Pang (1984), Nagurney (1987), Daniele (2004), Nagurney and Besik (2022), Nagurney et al. (2023), Nagurney et al. (2024), Nagurney et al. (2024), and the references therein). It may be easier, however, to estimate supply and demand functions directly, and, therefore, having a model such as the one introduced here is very relevant.

• Recent research on spatial price equilibrium modeling in the context of disaster scenarios, and associated disruptions, using variational inequality theory, with a focus on international trade, quantity variables, and with no commodity losses, can be found in Passacantando and Raciti (2024) and Nagurney et al. (2024).

Contributions

- **This paper presents a new trade network equilibrium model capturing commodity losses from perishability, theft, accidents, or attacks, like those in the Red Sea, Suez Canal, and Ukraine.**
- **The model considers multiple commodities and multiple routes joining supply and demand markets.**
- **In equilibrium, the model yields the supply prices, demand prices, and the commodity shipments flows.**
- **The underlying functions of the model can be nonlinear and asymmetric.**
- **The model integrates generalized networks and variational inequalities.**
- **Unlike most existing models focused only on quantity variables, this model includes both price and quantity variables.**

The Variational Inequality Trade Network Model in Prices and Quantities



In the trade network model, there are m supply markets and n demand markets, with K commodities. A typical commodity is denoted by k, a typical supply market is denoted by i, and a typical demand market is denoted by j. There are n_{ij} routes joining each pair of supply and demand markets (i, j) with a typical route denoted by r. There is a total of P routes in the trade network.

Parameter and Variables

• Let α_{ijr}^k denote the route multiplier parameter associated with commodity k on route r joining i with j. Each such multiplier lies in the range $(0, 1]$ with 1 representing no loss.

• Let π_i^k denote the supply price for commodity k at supply market i, and all the supply prices are grouped into the vector $\pi \in R_+^{Km}$.

• Let Q_{ijr}^k denote the shipment of the commodity k from supply market i to demand market j on route r. All the commodity shipments are grouped into the vector $Q \in R_+^{KP}$.

• The demand price for commodity k at demand market j is denoted by ρ_j^k, and all the demand prices are grouped into the vector $\rho \in R_+^{Kn}$.

• **In this new trade network model, the variables are the commodity shipments between supply and demand markets as well as the commodity supply market prices and their demand market prices.**

Functions

• $s_i^k(\pi)$ denotes the supply function for commodity k and supply market i.

• $c_{ijr}^k(Q)$ denotes the unit transportation cost function associated with shipping the commodity k from supply market i to demand market j.

• $d_j^k(\rho)$ denotes the demand function for commodity k at demand market j.

• **It is assumed that the supply, demand, and unit transportation cost functions are all continuous.**

Equilibrium Conditions

Definition 1: The International Trade Network Equilibrium Conditions Under Commodity Losses

A multicommodity supply price, shipment, and demand price pattern $(\pi^*, Q^*, \rho^*) \in \mathcal{K}$, *where* $\mathcal{K} \equiv \{(\pi, Q, \rho) | (\pi, Q, \rho) \in R_+^{Km+KP+Kn}\}$ *is a trade network equilibrium under commodity losses if the following conditions hold: for all commodities* k; $k = 1, \ldots, K$; *and for all supply markets* i; $i = 1, \ldots, m$:

$$s_i^k(\pi^*) \begin{cases} = \sum_{j=1}^{n} \sum_{r=1}^{n_{ij}} Q_{ijr}^{k*}, & \text{if } \pi_i^{k*} > 0 \\ \geq \sum_{j=1}^{n} \sum_{r=1}^{n_{ij}} Q_{ijr}^{k*}, & \text{if } \pi_i^{k*} = 0, \end{cases} \tag{1}$$

Equilibrium Conditions

Definition 1 (continued): The International Trade Network Equilibrium Conditions Under Commodity Losses

and for all commodities k; $k = 1, \ldots, K$; *for all supply and demand market pairs:* (i, j); $i = 1, \ldots, m$; $j = 1, \ldots, n$, *and for all routes* r; $r = 1, \ldots, n_{ij}$:

$$\pi_i^{k*} + c_{ijr}^k(Q^*) \begin{cases} = \alpha_{ijr}^k \rho_j^{k*}, & \text{if } Q_{ijr}^{k*} > 0 \\ \geq \alpha_{ijr}^k \rho_j^{k*}, & \text{if } Q_{ijr}^{k*} = 0, \end{cases} \tag{2}$$

and for all commodities k; $k = 1, \ldots, K$, *and for all demand markets* j; $j = 1, \ldots, n$:

$$d_j^k(\rho^*) \begin{cases} = \sum_{i=1}^m \sum_{r=1}^{n_{ij}} \alpha_{ijr}^k Q_{ijr}^{k*}, & \text{if } \rho_j^{k*} > 0 \\ \leq \sum_{i=1}^m \sum_{r=1}^{n_{ij}} \alpha_{ijr}^k Q_{ijr}^{k*}, & \text{if } \rho_j^{k*} = 0. \end{cases} \tag{3}$$

Variational Inequality Formulation

Theorem 1: Variational Inequality Formulation of the Trade Network Equilibrium Conditions Under Commodity Losses

A multicommodity supply price, shipment, and demand price pattern $(\pi^*, Q^*, \rho^*) \in \mathcal{K}$ is a trade network equilibrium under commodity losses, according to Definition 1, if and only if it satisfies the variational inequality:

$$\sum_{k=1}^{K}\sum_{i=1}^{m}\left[s_i^k(\pi^*) - \sum_{j=1}^{n}\sum_{r=1}^{n_{ij}} Q_{ijr}^{k*}\right] \times (\pi_i^k - \pi_i^{k*})$$

$$+\sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{n_{ij}}\left[\pi_i^{k*} + c_{ijr}^k(Q^*) - \alpha_{ijr}^k \rho_j^{k*}\right] \times (Q_{ijr}^k - Q_{ijr}^{k*})$$

$$+\sum_{k=1}^{K}\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{r=1}^{n_{ij}} \alpha_{ijr}^k Q_{ijr}^{k*} - d_j^k(\rho^*)\right] \times (\rho_j^k - \rho_j^{k*}) \geq 0, \quad \forall (\pi, Q, \rho) \in \mathcal{K}. \quad (4)$$

Considering a Policy Intervention

Remark

Note that in the case of policy interventions, such as that of a unit tariff, τ_{ij}^{k} associated with specific commodities k and supply and demand market pairs (i, j), one can adapt the equilibrium condition (2) as:

$$\pi_i^{k*} + c_{ijr}^{k}(Q^*) + \tau_{ij}^{k} \begin{cases} = \alpha_{ijr}^{k} \rho_j^{k*}, & \textit{if } Q_{ijr}^{k*} > 0 \\ \geq \alpha_{ijr}^{k} \rho_j^{k*}, & \textit{if } Q_{ijr}^{k*} = 0. \end{cases} \quad (5)$$

Variational Inequality Formulation in Standard Form

Standard Form

Variational inequality (4) is now put into standard form (cf. Nagurney (1999)), VI$(F, \mathcal{K})$, where one seeks to determine a vector $X^* \in \mathcal{K} \subset R^{\mathcal{N}}$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{6}$$

with F being a given continuous function from $\mathcal{K}$ to $R^{\mathcal{N}}$, where $\mathcal{K}$ is a given closed, convex set, and $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathcal{N}$-dimensional Euclidean space.

Specifically, define $X \equiv (\pi, Q, \rho)$, and $\mathcal{N} \equiv Km + KP + Kn$. The feasible set $\mathcal{K}$ remains as before. Here $\mathcal{N} = Km + KP + Kn$. $F(X)$ is the vector function mapping $\mathcal{K}$ into $R_+^{Km+KP+Kn}$ and is defined by the vector:

$$F(X) \equiv (S(X), T(X), D(X)), \tag{7}$$

Variational Inequality Formulation in Standard Form

where in equation (7), $S(X)$ has components S_{ki} such that:

$$S_{ki} = s_i^k(\pi) - \sum_{j=1}^{n} \sum_{r=1}^{n_{ij}} Q_{ijr}^k, \quad k = 1, \dots, K; i = 1, \dots, m, \tag{8a}$$

$T(X)$ has components T_{kijr} such that:

$$T_{kijr} = \pi_i^k + c_{ijr}^k(Q) - \alpha_{ijr}^k \rho_j^k$$

$$k = 1, \dots, K; i = 1, \dots, m; j = 1, \dots, n; r = 1, \dots, n_{ij}, \tag{8b}$$

and $D(X)$ has components D_{kj}, such that:

$$D_{kj} = \sum_{i=1}^{m} \sum_{r=1}^{n_{ij}} \alpha_{ijr}^k Q_{ijr}^k - d_j^k(\rho), \quad k = 1, \dots, K; j = 1, \dots, n. \tag{8c}$$

Clearly, variational inequality (4) can be put into standard form (6).

The Algorithm

The Euler Method

For the solution of additional numerical examples, the Euler Method is utilized, which is induced by the general iterative scheme of Dupuis and Nagurney (1993). Specifically, the statement at iteration $\tau + 1$ of this algorithm is (see also Nagurney and Zhang (1996)):

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau} F(X^{\tau})), \tag{11}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem (7).

For convergence, the sequence a_{τ} must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \to 0$, as $\tau \to \infty$. (Dupuis and Nagurney (1993), Kinderlehrer and Stampacchia (1980))

The Algorithm

Explicit Formulae for the Euler Method Applied to the Trade Network Model

The closed form expressions for the commodity supply prices at iteration $\tau + 1$, for $k = 1, \ldots K$; $i = 1, \ldots, m$ are:

$$\pi_i^{k,\tau+1} = \max\{0, a_\tau(\sum_{j=1}^{n} \sum_{r=1}^{n_{ij}} Q_{ijr}^{k,\tau} - s_i^k(\pi^\tau)) + \pi_i^{k,\tau}\}. \tag{12}$$

The closed form expressions for the commodity flows at iteration $\tau + 1$: for $k = 1, \ldots, K$; $i = 1, \ldots, m$; $j = 1, \ldots, n$ are:

$$Q_{ijr}^{k,\tau+1} = \max\{0, a_\tau(\alpha_{ijr}^k \rho_j^{k,\tau} - c_{ijr}^k(Q^\tau) - \pi_i^{k,\tau}) + Q_{ijr}^{k,\tau}\}. \tag{13}$$

The closed form expressions for the commodity demand prices at iteration $\tau + 1$, for $k = 1, \ldots, K$; $j = 1, \ldots, n$ are:

$$\rho_j^{k,\tau+1} = \max\{0, a_\tau(d_j^k(\rho^\tau) - \sum_{i=1}^{m} \sum_{r=1}^{P} \alpha_{ijr}^k Q_{ijr}^{k,\tau}) + \rho_j^{k,\tau}\}. \tag{14}$$

Convergence

Theorem 2: Convergence

If the $F(X)$ in the trade network model is strongly monotone and F is Lipschitz continuous, that is,

$$\|F(X^1) - F(X^2)\| \leq L\|X^1 - X^2\|, \quad \forall X^1, X^2 \in \mathcal{K}, \tag{15}$$

where L is a positive number known as the Lipschitz constant, then there is a unique equilibrium $(\pi^*, Q^*, \rho^*) \in \mathcal{K}$ and any sequence generated by the algorithm as in (11), where $\{a_\tau\}$ satisfies $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$ converges to (π^*, Q^*, ρ^*).

In the following numerical examples that are solved via the Euler Method implemented in MATLAB on a Microsoft Windows 11 system at the University of Massachusetts Amherst, the $\{a_\tau\}$ series was set to: $\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \dots\}$. The algorithm was deemed to have converged if the absolute value of each variable at two successive iterations differed by no more than 10^{-5}.

Example Set 1: 2 Supply Markets and 2 Demand Markets

The first set of algorithmically solved numerical examples consists of problems with two supply markets and two demand markets. There is a single commodity and a single route joining each pair of supply and demand markets. Consider the commodity to be an agricultural one.

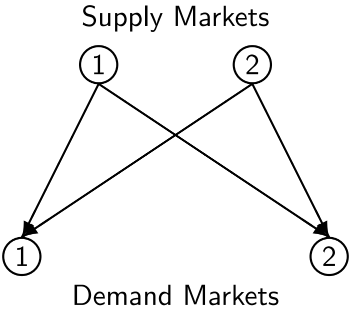


Figure: The Trade Network Topology for Numerical Examples 1 Through 3

Numerical Example Scenarios

Example 1: There Are No Losses.

Example 1 serves as the baseline and, it is assumed that there are no losses; hence, $\alpha_{111} = \alpha_{121} = \alpha_{211} = \alpha_{221} = 1$.

Example 2: Losses From the First Supply Market to the First Demand Market.

Example 2 has the same data as Example 1 except that it is assumed that there are losses associated with transport from the first supply market to the first demand market with $\alpha_{111} = 0.8$.

Example 3: The Most Disrupted Scenario.

Example 3 considers the most disrupted scenario with all four route multipliers equal to 0.8.

Results for Example Set 1

	Ex. 1 (No Losses)	Ex. 2 ($\alpha_{111} = 0.8$)	Ex. 3 (The Most Disrupted)
π_1^*	354.3692	323.4289	285.9640
π_2^*	346.8834	347.0309	279.5817
ρ_1^*	371.0841	377.5154	378.0861
ρ_2^*	366.0271	359.8981	370.8422
Q_{111}^*	12.7150	0.0000	12.5050
Q_{121}^*	0.8290	13.2346	0.3549
Q_{211}^*	16.2007	22.4845	14.8872
Q_{221}^*	13.1437	6.8671	11.0921
$c_{111}(Q^*)$	16.7150	4.0000	16.5050
$c_{121}(Q^*)$	11.6579	36.4693	10.7098
$c_{211}(Q^*)$	24.2007	30.4845	22.8872
$c_{221}(Q^*)$	19.1437	12.8671	17.0921

Discussion of Results

• **Example 2:** **Shipment on the route with losses drops to 0, with supply price decreasing at the first market and slightly increasing at the second. Demand prices increase at the first market and decrease at the second.**

• **Example 3:** **With all routes incurring losses, supply volumes and prices are at their lowest, while demand prices at both markets are at their highest.**

These examples highlight the importance of minimizing commodity losses on routes to prevent negative impacts on both producers and consumers.

Example Set 2: 3 Supply Markets and 2 Demand Markets

The second set of algorithmically solved numerical examples consists of three supply markets and two demand markets.

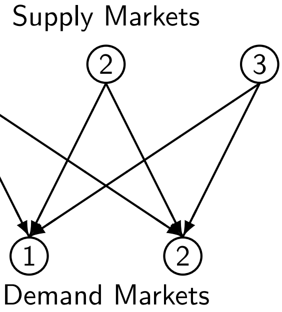


Figure: The Trade Network Topology for Numerical Examples 4 Through 8

Numerical Example Scenarios

Example 4: Supply Market 3 Added.

The data for Example 4 are identical to the data in Example 3 but with supply market 3 added.

Example 5: With Impacts of Heightened Congestion on the Routes From Supply Market 3 to the Demand Markets Using BPR Function.

Example 5 has the same data as that in Example 4 except now *with impacts of heightened congestion on the routes from supply market 3 to the demand markets.* **In particular, the Bureau of Public Road (BPR) type cost functions are used (cf. National Research Council (2010)), which are useful in capturing congestion on transportation networks. Hence, a term to the fourth power is added, as follows:**

$$c_{311}(Q_{311}) = .2Q_{311}^4 + 2Q_{311} + 5, \quad c_{321}(Q_{321}) = .2Q_{321}^4 + 2Q_{321} + 6.$$

Numerical Example Scenarios

Example 6: Additional Term in the Supply Function at Supply Market 1.

Example 6 has the same data as Example 5, but it considers the impact of increased competition on the supply side, such that:

$$s_1(\pi) = .01\pi_1 + .008\pi_2 + 10.$$

Example 7: Increase in Demand at Both Demand Markets.

Example 7 has the same data as in Example 6, but now consider that some marketing has taken place and the demand in both demand markets has increased.

Example 8: Full Circle.

Example 8 has identical data to that in Example 7 except that all the route multipliers are equal to 1.

Results for Example Set 2

	Ex. 4	Ex. 5	Ex. 6	Ex. 7	Ex. 8
	3 Supply Market	**Heightened Congestion**	**Increased Competition**	**Increased Demand**	**Full Circle**
π_1^*	278.9547	280.9207	278.7560	355.8278	440.2250
π_2^*	272.9058	274.6300	273.9659	350.0780	433.5483
π_3^*	342.9806	0.0000	0.0000	0.0000	0.0000
ρ_1^*	368.7333	371.7355	370.7449	468.1761	459.6409
ρ_2^*	363.0880	364.5031	363.7920	461.3979	455.1326
Q_{111}^*	12.0319	12.4674	13.8396	14.7126	15.4156
Q_{121}^*	0.7579	0.3409	1.1387	1.6452	2.4537
Q_{211}^*	14.0808	14.7582	14.6299	16.4627	18.0924
Q_{221}^*	11.5647	10.9721	11.0673	13.0399	15.5838
Q_{311}^*	10.3764	6.4852	6.4807	6.8850	6.8523
Q_{321}^*	7.0538	6.4475	6.4442	6.8552	6.8310
$c_{111}(Q^*)$	16.0319	16.4674	17.8396	18.7126	19.4156
$c_{121}(Q^*)$	11.5157	10.6817	12.2775	13.2904	14.9074
$c_{211}(Q^*)$	22.0808	22.7582	22.6299	24.4627	26.0924
$c_{221}(Q^*)$	17.5647	16.9721	17.0673	19.0399	21.5838
$c_{311}(Q^*)$	25.7527	371.7355	370.7449	468.1761	459.6409
$c_{321}(Q^*)$	20.1075	364.5031	363.7920	461.3979	455.1326

Discussion of Results

- **Example 5:** **Greater congestion sensitivity causes shipments from supply market 3 to decrease, with its supply price dropping to 0. Demand prices rise at both markets, and transportation costs from supply market 3 increase tenfold.**

- **Example 6:** **Increased competition on the supply side leads to lower demand prices at both markets, while the supply price at market 3 remains 0.**

- **Example 7: Increased consumer demand leads to higher shipments and increased supply and demand prices, except for the supply price at market 3, which remains 0.**

- **Example 8:** **All shipments are higher than in Example 7, except from supply market 3. Supply prices are also higher, except at market 3 (which remains 0), while demand prices are lower. Both producers and consumers benefit from no commodity losses in the network.**

Here, a sensitivity analysis on the intercept of the supply function for supply market 3 is conducted, which is: $s_3(\pi_3) = .01\pi_3 + 14$. Specifically, the data as in Example 8 are retained but the fixed term is changed from 2 through 14.

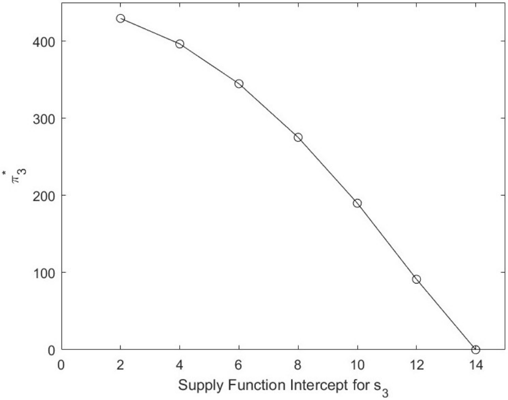


Figure: Sensitivity Analysis for the Supply Function s_3 intercept based on Example 8

Sensitivity Analysis

Note that the equilibrium supply price at supply market 1 ranges from 443.74 for the intercept at 2 to 440.22 for the intercept of 14. Also, the equilibrium supply price at supply market 2 ranges from 437.03 to 433.55 in that same range of supply price intercept values. The high transportation costs of shipping the commodity from supply market 3 to the demand markets come to dominate as the intercept increases and, in order for transactions to take place in the form of commodity shipments, the equilibrium supply price at supply market 3 decreases until it reaches its lower bound of 0.0000.

The numerical results reinforce the importance of reducing commodity losses in transportation as well as the negative impacts of congestion.

Insights

• The proposed variational inequality trade network model in prices and quantities captures the economic impacts of commodity losses due to perishability, theft, accidents, or attacks.

• By integrating generalized networks and considering multiple commodities and routes, the model provides a comprehensive framework to evaluate and address the challenges in global trade.

• The trade network model, unlike previous spatial price equilibrium models with losses, is in price and quantity variables.

• Through numerical examples, the model demonstrates how commodity losses can significantly affect supply prices and demand prices, leading to economic challenges for both producers and consumers.

• This model offers valuable insights for policy and decision-making in the context of global commodity trade.

Summary

• This paper expands the theoretical framework for trade networks under commodity losses using a variational inequality model with generalized networks.
• Commodity losses include perishability in agricultural products and theft of high-value minerals.
• The paper provides equilibrium conditions for supply prices, shipment quantities, and demand prices, along with conditions for existence and uniqueness.
• The proposed algorithm provides a time-discretization of what may be interpreted as a continuous time evolution of the underlying economic variables with conditions for convergence.
• Numerical examples reveal the various scenarios from changes in the route multipliers associated with the losses to an increase in the number of supply markets, greater competition, marketing on the demand side, as well as greater sensitivity to congestion on transportation routes.
• The inclusion of the presence of commodity losses from supply markets to demand markets, as in transportation, quantitatively reveals the negative impacts on producers as well as on consumers in terms of prices and volumes of commodity shipments.

Thank You Very Much!