A Multiperiod, Multicommodity, Capacitated International Agricultural Trade Network Equilibrium Model with Applications to Ukraine in Wartime

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Abstract. The world is facing immense challenges because of increasing strife and the impacts of climate change with accompanying disasters, both sudden-onset as well as slow-onset ones, which have affected the trade of agricultural commodities needed for food security. In this paper, a multiperiod, multicommodity, international, agricultural trade network equilibrium model is constructed with capacity constraints on the production, transportation, and storage of agricultural commodities. The model allows for multiple routes between supply and demand country markets, different modes of transport, and storage in the producing and consuming countries as well as in the intermediate countries. The generality of the underlying functions, coupled with the capacity constraints, allow for the modeling of competition among agricultural commodities for production, transportation, and storage. The capacity constraints also enable the quantification of various disaster-related disruptions to production, transportation, and storage on the volumes of commodity flows as well as on the prices. A series of numerical examples inspired by the effects of Russia's full-scale invasion of Ukraine on agricultural trade is presented, and the results are analyzed to provide insights into food insecurity issues caused by the war.

Keywords: networks • international trade • spatial price equilibrium • agriculture • disruptions • Ukraine

Acknowledgment and Dedication

This presentation is dedicated to farmers in Ukraine and everywhere in the world.

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The War in Ukraine

- **The full-scale invasion of Ukraine by Russia on February 24, 2022 has resulted in immense losses of lives and an increase in human suffering. It has severely impacted the economy of Ukraine with repercussions globally.**

The Impacts on Ukraine's Agricultural Sector

- **Between 20 to 30% of the arable land in Ukraine is estimated to remain idle due to mining and other damages because of the full-scale invasion, resulting in around a 40% decrease in the production of grains in Ukraine.**

The Impacts on Ukraine's Agricultural Sector

- **The blockade of the Ukrainian Black Sea ports, which used to handle around 90% of the grain exports from Ukraine, caused a global shortage of grains.**
- **The war has cost Ukraine around 15% of its grain storage capacity.**

The Impacts on Global Food Security and Economy

- **Reports indicate a rise of around 17% in the population facing food insecurity worldwide due to the full-scale invasion.**
- **The disruptions in the exports of Ukrainian grain can cost the global economy more than 1.6 billion dollars.**
- **Many countries heavily rely on Ukrainian grains, especially vulnerable countries in the Middle East and North Africa (MENA) region.**

The Black Sea Grain Initiative

- **The Black Sea Grain Initiative, facilitated by Turkey and the United Nations, allowed for the limited passage of grain shipments from selected Ukrainian ports on the Black Sea from August 1, 2022.**
- **As of July 17, 2023, Russia has suspended the initiative, imposing a severe food security risk worldwide.**

Literature Review

• The theory of variational inequalities (cf. Nagurney (1999, 2006)) is the methodology used to develop the modeling and algorithmic framework.

• Spatial price equilibrium (SPE) models were first introduced in the groundbreaking contributions of Samuelson (1952) and Takayama and Judge (1964, 1971) and are partial equilibrium models under perfect competition.

• SPE models have been widely used in modeling disaster scenarios and relevant food security issues, the trade of agricultural products, and the impacts of different policy instruments.

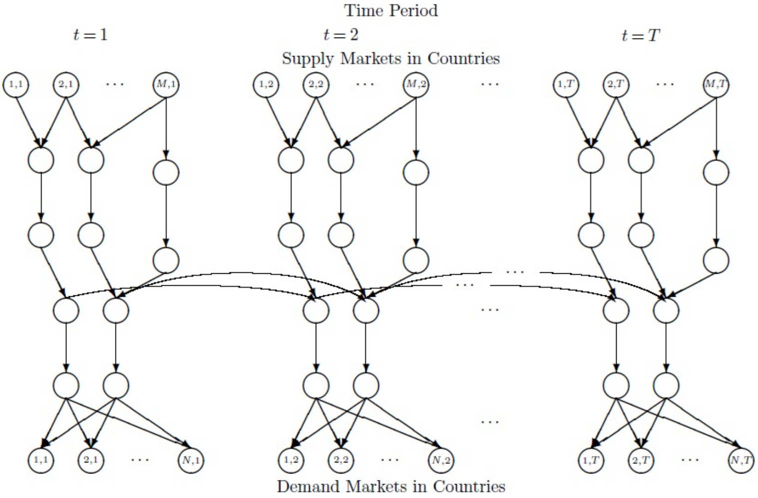
• Nagurney et al. (2023) developed a multicommodity international SPE model with exchange rates in which transportation through multiple intermediate countries was possible; however, the model did not include storage and imposed no capacities.

• Nagurney et al. (2024) constructed a multicommodity international agricultural SPE model with bounds on the production and transportation of commodities, but the model included only one period, and transportation through multiple countries was not considered.

Contributions of This Paper

- The model simultaneously includes bounds on production output, transportation, and storage.
- The model allows for the storage of agricultural commodities in the intermediate countries as the commodities are transported from origin countries to destination countries as well as in the producing origin country and/or the destination consuming country).
- The underlying functions in the model can be nonlinear and asymmetric.
- The generality of the underlying functions, along with the capacity constraints and their associated Lagrange multipliers, allow for the modeling of competition among agricultural commodities for production, transportation, and storage.
- The numerical examples are drawn from an ongoing war with a global impact on food security.

The Multiperiod International Trade Model



Notation for The Multiperiod International Trade Model

Notation	Parameter Definition
$u_{i,t}$	supply capacity at supply node i; $i \in \mathcal{I}$, in time period t; $t \in \mathcal{T}$.
u_a^τ	transportation capacity on transportation link a; $a \in \mathcal{L}^\tau$.
u_a^σ	inventory capacity on inventory link a; $a \in \mathcal{L}^\sigma$.
Notation	**Variable Definition**
Q_p^h	the flow of commodity h; $h \in \mathcal{H}$, on path p; $p \in \mathcal{P}$. We group all the commodity path flows into the vector $Q \in \mathcal{R}_+^{Hn_\mathcal{P}}$, where $n_\mathcal{P}$ is the number of paths in the network.
f_a^h	the flow of commodity h; $h \in \mathcal{H}$, on link a; $a \in \mathcal{L}$. We group all the commodity link flows into the vector $f \in \mathcal{R}_+^{Hn_\mathcal{L}}$, where $n_\mathcal{L}$ is the number of links in the network.
$s_{i,t}^h$	the supply of the commodity h; $h \in \mathcal{H}$, at supply node i; $i \in \mathcal{I}$, in time period t; $t \in \mathcal{T}$. We group all the commodity supplies into the vector $s \in \mathcal{R}_+^{HMT}$.
$d_{j,t'}^h$	the demand for the commodity h; $h \in \mathcal{H}$, at demand node j; $j \in \mathcal{J}$, in time period t'; $t' \in \mathcal{T}$. We group all the commodity demands into the vector $d \in \mathcal{R}_+^{HNT}$.
$\lambda_{i,t}$	the Lagrange multiplier associated with the supply capacity constraint at supply node i; $i \in \mathcal{I}$, in time period t; $t \in \mathcal{T}$. We group all such Lagrange multipliers into the vector $\lambda \in \mathcal{R}_+^{MT}$.
μ_a	the Lagrange multiplier associated with the transportation capacity constraint on transportation link a; $a \in \mathcal{L}^\tau$. We group all such Lagrange multipliers into the vector $\mu \in \mathcal{R}_+^{n_{\mathcal{L}^\tau}}$, where $n_{\mathcal{L}^\tau}$ is the number of transportation links in the trade network.
γ_a	the Lagrange multiplier associated with the inventory capacity constraint on inventory link a; $a \in \mathcal{L}^\sigma$. We group all such Lagrange multipliers into the vector $\gamma \in \mathcal{R}_+^{n_{\mathcal{L}^\sigma}}$, where $n_{\mathcal{L}^\sigma}$ is the number of inventory links in the trade network.
Notation	**Function Definition**
$\pi_{i,t}^h(Q)$	the supply price function for commodity h; $h \in \mathcal{H}$, at supply node i; $i \in \mathcal{I}$, in time period t; $t \in \mathcal{T}$.
$\rho_{j,t'}^h(Q)$	the demand price function for commodity h; $h \in \mathcal{H}$, at demand node j; $j \in \mathcal{J}$, in time period t; $t \in \mathcal{T}$.
$\hat{c}_a^h(Q)$	the unit link cost associated with the commodity h; $h \in \mathcal{H}$, on link a; $a \in \mathcal{L}$.

The Multiperiod International Trade Model

The commodity path flows must be nonnegative:

$$Q_p^h \geq 0, \quad \forall h \in \mathcal{H}, p \in \mathcal{P}. \tag{1}$$

The cost on a path p for commodity h is given by the following expression:

$$C_p^h = \sum_{a \in \mathcal{L}} \delta_{a,p} c_a^h(Q), \quad \forall h \in \mathcal{H}, p \in \mathcal{P}, \tag{2}$$

where $\delta_{a,p} = 1$, if link a is contained in path p, and is 0, otherwise; i.e., the cost on a path for a commodity is equal to the sum of the costs on the links that make up the path for the commodity.

The Equilibrium Conditions

Definition 1: The Equilibrium Conditions

A path flow and Lagrange multiplier pattern $(Q^*, \lambda^*, \mu^*, \gamma^*) \in \mathcal{K}^1 \equiv \{(Q, \lambda, \mu, \gamma) | Q \in \mathcal{R}_+^{Hn_{\mathcal{P}}}, \lambda \in \mathcal{R}_+^{MT}, \mu \in \mathcal{R}_+^{n_{\mathcal{L}^\tau}}, \gamma \in \mathcal{R}_+^{n_{\mathcal{L}^\sigma}}\}$ *is an equilibrium under capacities if the following conditions hold:*

$$\pi_{i,t}^h(Q^*) + C_p^h(Q^*) + \lambda_{i,t}^* + \sum_{a \in \mathcal{L}^\tau} \delta_{a,p}\mu_a^* + \sum_{a \in \mathcal{L}^\sigma} \delta_{a,p}\gamma_a^* - \rho_{j,t'}^h(Q^*) \geq 0 \ \perp \ Q_p^{h*} \geq 0$$

$$\forall p \in \mathcal{P}_{j,t'}^{i,t}, h \in \mathcal{H} \tag{3}$$

$$u_{i,t} - \sum_{h \in \mathcal{H}} \sum_{p \in \mathcal{P}^{i,t}} Q_p^{h*} \geq 0 \ \perp \ \lambda_{i,t}^* \geq 0 \ \forall i \in \mathcal{I}, t \in \mathcal{T} \tag{4}$$

$$u_a^\tau - \sum_{h \in \mathcal{H}} \sum_{p \in \mathcal{P}} \delta_{a,p} Q_p^{h*} \geq 0 \ \perp \ \mu_a^* \geq 0 \ \forall a \in \mathcal{L}^\tau \tag{5}$$

$$u_a^\sigma - \sum_{h \in \mathcal{H}} \sum_{p \in \mathcal{P}} \delta_{a,p} Q_p^{h*} \geq 0 \ \perp \ \gamma_a^* \geq 0 \ \forall a \in \mathcal{L}^\sigma. \tag{6}$$

Variational Inequality Formulation

The Variational Inequality Formulation in Path Flows and Lagrange Multipliers

A path flow and Lagrange multiplier pattern $(Q^*, \lambda^*, \mu^*, \gamma^*) \in \mathcal{K}^1$ *is an equilibrium under capacities according to Definition 1 if and only if it satisfies the variational inequality:*

$$\sum_{h\in\mathcal{H}}\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{t\in\mathcal{T}}\sum_{t'\in\mathcal{T}}\sum_{p\in\mathcal{P}^{i,t}_{j,t'}}\left[\pi^h_{i,t}(Q^*)+C^h_p(Q^*)+\lambda^*_{i,t}+\sum_{a\in\mathcal{L}^\tau}\delta_{a,p}\mu^*_a+\sum_{a\in\mathcal{L}^\sigma}\delta_{a,p}\gamma^*_a-\rho^h_{j,t'}(Q^*)\right]$$

$$\times\left[Q^h_p-Q^{h*}_p\right]+\sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}\left[u_{i,t}-\sum_{h\in\mathcal{H}}\sum_{p\in\mathcal{P}^{it}}Q^{h*}_p\right]\times\left[\lambda_{i,t}-\lambda^*_{i,t}\right]$$

$$+\sum_{a\in\mathcal{L}^\tau}\left[u^\tau_a-\sum_{h\in\mathcal{H}}\sum_{p\in\mathcal{P}}\delta_{a,p}Q^{h*}_p\right]\times\left[\mu_a-\mu^*_a\right]$$

$$+\sum_{a\in\mathcal{L}^\sigma}\left[u^\sigma_a-\sum_{h\in\mathcal{H}}\sum_{p\in\mathcal{P}}\delta_{a,p}Q^{h*}_p\right]\times\left[\gamma_a-\gamma^*_a\right]\geq 0,\quad\forall(Q,\lambda,\mu,\gamma)\in\mathcal{K}^1. \tag{7}$$

Numerical Examples

• A series of numerical examples inspired by the disruptions to the international trade of agricultural commodities caused by Russia's full-scale invasion of Ukraine are presented. **All data is contained in our paper**.

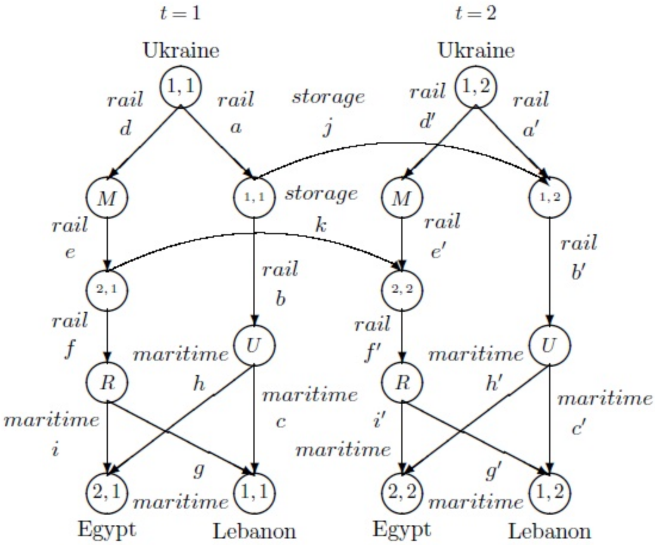
• The examples focus on the export of agricultural commodities of wheat and corn from Ukraine to Lebanon and Egypt.

• All examples consist of two periods corresponding to the projected yearly commodity shipments in metric tons.

• Lebanon and Egypt are two representative MENA countries significantly affected by the war in terms of the food security of their populations.

• The algorithmic framework used to solve the examples is that of the Modified Projection Method of Korpelevich (1977).

Network Topology



Path Descriptions

To Lebanon, $t = 1$

• $p_1 = (a, b, c)$: Rail transport from farms to a storage facility in Ukraine and to a Black Sea port in Ukraine, then maritime transport to Lebanon.

• $p_2 = (d, e, f, g)$: Rail transport from farms to Moldova, to a storage facility in Moldova, and to a Black Sea port in Romania, then maritime transport to Lebanon.

To Egypt, $t = 1$

• $p_3 = (a, b, h)$: Rail transport from farms to a storage facility in Ukraine and to a Black Sea port in Ukraine, then maritime transport to Egypt.

• $p_4 = (d, e, f, i)$: Rail transport from farms to Moldova, to a storage facility in Moldova, and to a Black Sea port in Romania, then maritime transport to Egypt.

To Lebanon and Egypt, $t = 2$

Paths p_5, p_6, p_7, and p_8 are the same as the first four paths, respectively, but all in the second period and with links denoted with a'.

Paths with Storage Links Joining the Two Periods

- $p_9 = (a, j, b', c')$: Rail transport from farms to a storage facility in Ukraine in the first period, storage to the second period, rail transport from the storage facility to a Black Sea port in Ukraine, and maritime transport to Lebanon.
- $p_{10} = (d, e, k, f', g')$: Rail transport from farms to Moldova and to a storage facility in Moldova in the first period, storage to the second period, rail transport from the storage facility to a Black Sea port in Romania, and maritime transport to Lebanon.
- $p_{11} = (a, j, b', h')$: Rail transport from farms to a storage facility in Ukraine in the first period, storage to the second period, rail transport from the storage facility to a Black Sea port in Ukraine, and maritime transport to Egypt.
- $p_{12} = (d, e, k, f', i)$: Rail transport from farms to Moldova and to a storage facility in Moldova in the first period, storage to the second period, rail transport from the storage facility to a Black Sea port in Romania, and maritime transport to Egypt.

Numerical Example Scenarios

Example 1: Single Commodity of Wheat, Prior to Full-Scale Invasion

Example 2: Two Commodities of Wheat and Corn, Prior to Full-Scale Invasion

Similar capacities to those in Example 1 are imposed on two commodities.

Example 3: Two Commodities, First Period Prior to the Full-Scale Invasion, Second Period After

The second period corresponds to when prices were significantly affected, maritime transportation from Ukrainian Black Sea ports was blockaded, and production was severely disrupted due to damages to arable land.

Example 4: Two Commodities: First Period is Before the Black Sea Gran Initiative, Second Period is After It

The transportation capacity on Ukrainian Black Sea ports are restored to those before the invasion. The damages to storage facilities in Ukraine have resulted in a decrease in the storage capacity in Ukraine.

Results for Commodity 1 (Wheat)

	Example 1	Example 2	Example 3	Example 4
$Q_{p_1}^{1*}$	571,868	532,483	512,066	0
$Q_{p_2}^{1*}$	0	0	0	0
$Q_{p_3}^{1*}$	1,917,419	1,799,439	1,730,628	0
$Q_{p_4}^{1*}$	0	0	0	0
$Q_{p_5}^{1*}$	571,870	532,484	0	185,660
$Q_{p_6}^{1*}$	0	0	0	0
$Q_{p_7}^{1*}$	1,917,410	1,799,431	0	518,442
$Q_{p_8}^{1*}$	0	0	0	0
$Q_{p_9}^{1*}$	0	0	0	27,213
$Q_{p_{10}}^{1*}$	0	0	91,669	118,989
$Q_{p_{11}}^{1*}$	0	0	0	359,996
$Q_{p_{12}}^{1*}$	0	0	303,271	180,273
$s_{1,1}^{1*}$	2,489,287	2,331,922	2,637,635	686,470
$s_{1,2}^{1*}$	2,489,280	2,331,916	0	704,102
$d_{1,1}^{1*}$	571,868	532,483	512,066	0
$d_{1,2}^{1*}$	571,870	532,484	91,669	331,861
$d_{2,1}^{1*}$	1,917,419	1,799,439	1,730,628	0
$d_{2,2}^{1*}$	1,917,410	1,799,431	303,271	1,058,711
$\pi_{1,1}^{1}$	\$262.45	\$262.56	\$264.16	\$103.75
$\pi_{1,2}^{1}$	\$262.45	\$262.56	\$100	\$103.82
$\rho_{1,1}^{1}$	\$344.27	\$346.04	\$346.96	\$530
$\rho_{1,2}^{1}$	\$344.27	\$346.04	\$525.87	\$515.07
$\rho_{2,1}^{1}$	\$341.24	\$343.01	\$344.04	\$530
$\rho_{2,2}^{1}$	\$341.24	\$343.01	\$525.45	\$514.12

Results for Commodity 2 (Corn)

	Example 2	Example 3	Example 4
$Q^{2*}_{p_1}$	162,194	157,295	0
$Q^{2*}_{p_2}$	0	0	0
$Q^{2*}_{p_3}$	733,834	710,866	0
$Q^{2*}_{p_4}$	0	0	0
$Q^{2*}_{p_5}$	162,194	0	72,929
$Q^{2*}_{p_6}$	0	0	0
$Q^{2*}_{p_7}$	733,836	0	223,055
$Q^{2*}_{p_8}$	0	0	0
$Q^{2*}_{p_9}$	0	0	6,938
$Q^{2*}_{p_{10}}$	0	17,862	40,491
$Q^{2*}_{p_{11}}$	0	0	105,854
$Q^{2*}_{p_{12}}$	0	87,217	160,248
$s^{2*}_{1,1}$	896,028	973,241	313,530
$s^{2*}_{1,2}$	896,030	0	295,983
$d^{2*}_{1,1}$	162,194	157,295	0
$d^{2*}_{1,2}$	162,194	17,862	120,358
$d^{2*}_{2,1}$	733,834	710,866	0
$d^{2*}_{2,2}$	733,836	87,217	489,156
$\pi^2_{1,1}$	\$253.62	\$255.01	\$94.51
$\pi^2_{1,2}$	\$253.62	\$90	\$94.37
$\rho^2_{1,1}$	\$345.41	\$345.84	\$520
$\rho^2_{1,2}$	\$345.41	\$518.39	\$509.17
$\rho^2_{2,1}$	\$341.65	\$342.23	\$520
$\rho^2_{2,2}$	\$341.65	\$517.82	\$507.77

Insights and Summary

• A zero path flow means that no trade is happening on that specific path. Blockades and damages to transportation infrastructure or high transportation and storage costs due to risks result in a path not being used.

• In Example 1 and 2, no storage is observed since the demand in both demand nodes can be met more cheaply from the supply of the same period.

• In Example 3, due to the blockade of the Ukrainian Black Sea ports in the second period, commodities are stored in Moldova to be exported from Romanian Black Sea ports in the second period. Production and storage in the first period and transport in the second period are preferred since the war has affected transportation costs.

• In Example 4, with the initiative in place in the second period and the transport of commodities via Ukrainian Black Sea ports facilitated, all the wheat and corn harvests in the first period are stored inside Ukraine and in Moldova to be carried to the second period.

Insights and Summary

• The examples shed light on the importance of efficient maritime transportation of grains from the Black Sea ports of Ukraine, and the consequences of war in terms of the production and storage of agricultural commodities.

• The results reveal the impacts in terms of the significantly decreasing earnings of Ukrainian farmers and the increasing consumer prices.

• The examples indicate that, as a result of the ongoing war, Lebanon and Egypt are now essentially competing over the severely limited production, transportation, and storage capacity of Ukraine.

• The results show the priority of wheat over corn in both country demand markets since wheat is an essential part of most staple foods in MENA countries.

• One of the examples highlights the importance of keeping maritime routes from Ukraine for export of agricultural products operational.

Future Research

Future research can include:

- Adding exchange rates.
- Adding uncertainties associated with supplies, capacities, and costs.
- Including nonagricultural commodities to assess the impacts on transportation capacities.
- Incorporating quality of the agricultural commodities over time and associated demands.

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