A Dynamic Oligopoly Model For Pharmaceuticals Supply Chain Networks Under Brand Differentiation and Perishability

Amir H. Masoumi[1]
Doctoral Candidate
Min Yu[2]
Assistant Professor
and
Anna Nagurney[3]
John F. Smith Memorial Professor

1,3 - Department of Finance and Operations Management
Isenberg School of Management, University of Massachusetts, Amherst, MA 01003

2 - Operations & Technology Management Group
Pamplin School of Business Administration, University of Portland, Portland, OR 97203

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Masoumi, A., Yu, M., Nagurney, A. (2012) A supply chain generalized network oligopoly model for pharmaceuticals under brand differentiation and perishability. *Transportation Research E* 48: pp 762-780.

Background: The Pharmaceutical Industry

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But many researchers are skeptical that this is an effective way to fund medical innovation. The drug companies spend only 12 percent of their revenues on innovation. Some of that money goes to innovation, but only 12 percent of it (Washington Post, March 2).

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Although pharmaceutical supply chains have begun to be coupled with sophisticated technologies in order to improve both the quantity and the quality of their associated products, despite all the advances in manufacturing, storage, and distribution methods, pharmaceutical drug companies are far from effectively satisfying market demands on a consistent basis.

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In fact, it has been argued that pharmaceutical drug supply chains are in urgent need of efficient optimization techniques in order to reduce costs and to increase productivity and responsiveness (Shah (2004) and Papageorgiou (2009)).

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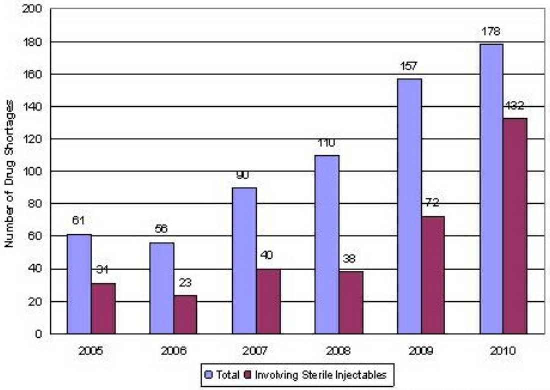
- In 2007, in a warehouse belonging to the Health Department of Chicago, over one million dollars in drugs, vaccines, and other medical supplies were found spoiled, stolen, or unaccounted for.
- In 2009, CVS pharmacies in California, as a result of a settlement of a lawsuit filed against the company, had to offer promotional coupons to customers who had identified expired drugs, including expired baby formula and children's medicines, in more than 42 percent of the stores.

Background: Product Shortages

Ironically, whereas some drugs may be unsold and unused and / or past their expiration dates, the number of drugs that were reported in short supply in the US in the first half of 2011 has risen to 211 – close to an all-time record – with only 58 in short supply in 2004.



(Source: CDER Drug Shortage Program)

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Interestingly, among curative cancer drugs, only the older generic, yet, less expensive, ones, have experienced shortages.

Background: Possible Causes of Shortages

Pharmaceutical companies secure notable returns solely in the early lifetime of a successful drug, before competition takes place. This competition-free time-span, however, has been observed to be shortening, from 5 years to only 1-2 years (Shah (2004)).

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Hence, pharmaceutical companies are forced to make a difficult decision: whether to lose money by continuing to produce a lifesaving product or to switch to a more profitable drug. Unfortunately, the FDA cannot force companies to continue to produce low-profit medicines even if millions of lives rely on them.

Background: Economic and Financial Pressures

Pharmaceutical companies are expected to suffer a significant decrease in their revenues as a result of losing patent protection for ten of the best-selling drugs by the end of 2012 (De la Garza (2011)).

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Lipitor, the most popular brand of cholesterol-lowering drugs and once the top-selling branded drug in the world, lost its patent rights in late 2011. This led to a 50% decrease of net income for Pfizer Inc. in Q4 2011 compared to Q4 2010 (Forbes, February 13).

Background: Safety Issues

- More than 80% of the ingredients of drugs sold in the US are made overseas, mostly in remote facilities located in China and India that are rarely – if not ever – visited by government inspectors.
- The amount of counterfeit drugs in the European pharmaceutical supply chains has considerably increased.

Background: Waste and Environmental Impacts

Another pressure faced by pharmaceutical firms is the environmental impact of their medical waste, which includes the perished excess medicine, and inappropriate disposal on the retailer / consumer end.

Relevant Literature

- Papageorgiou, L., Rotstein, G., Shah, N. (2001) Strategic supply chain optimization for the pharmaceutical industries. *Industrial & Engineering Chemistry Research* 40, 275–286.

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- Niziolek, L. (2008) A simulation-based study of distribution strategies for pharmaceutical supply chains. Ph.D. thesis in Industrial Engineering, Purdue University, Indiana.

- Blackburn, J., Scudder, G. (2009) Supply chain strategies for perishable products: The case of fresh produce. *Production and Operations Management* 18, pp 129-137.

- Nagurney, A., Yu, M. (2012) Sustainable fashion supply chain management under oligopolistic competition and brand differentiation. *International Journal of Production Economics* 135, pp 532-540.

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Some Examples of Oligopolies

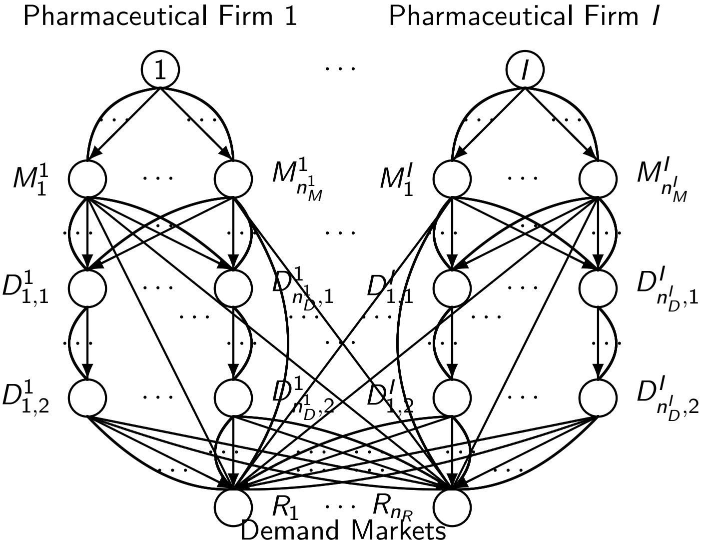
- airlines
- freight carriers
- automobile manufacturers
- oil companies
- beer / beverage companies
- wireless communications
- fast fashion brands
- certain food companies.

A Generalized Network Oligopoly Model for Pharmaceutical Supply Chains

I pharmaceutical firms are considered, with a typical firm denoted by i.

The firms compete non-cooperatively, in an oligopolistic manner, and the consumers can differentiate among the products of the pharmaceutical firms through their individual product brands.

The supply chain network activities include manufacturing, shipment, storage, and, ultimately, the distribution of the brand name drugs to the demand markets.



The Pharmaceutical Supply Chain Network Topology

A Generalized Network Oligopoly Model for Pharmaceutical Supply Chains

Each pharmaceutical firm i; $i = 1, \ldots, I$, utilizes n_M^i manufacturing plants and n_D^i distribution / storage facilities, and the goal is to serve n_R demand markets consisting of pharmacies, retail stores, hospitals, and other medical centers.

L^i denotes the set of directed links corresponding to the sequence of activities associated with firm i. Also, $G = [N, L]$ denotes the graph composed of the set of nodes N, and the set of links L, where L contains all sets of L_is: $L \equiv \cup_{i=1,\ldots,I} L^i$.

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There are direct links connecting manufacturing units with various demand markets in order to capture the possibility of direct mail shipments from manufacturers.

Formulation

Let f_a denote the (initial) flow of product on link a with f'_a denoting the final flow on link a. We have:

$$f'_a = \alpha_a f_a, \qquad \forall a \in L. \tag{1}$$

Associated with waste is a discarding total cost function, $\hat{z}_a$, which is a function of flow on the link, f_a:

$$\hat{z}_a = \hat{z}_a(f_a), \qquad \forall a \in L, \tag{3}$$

and which is assumed to be convex and continuously differentiable.

Formulation: Path Flows

x_p represents the (initial) flow of product on path p joining an origin node, i, with a destination node, R_k.

The path flows must be nonnegative:

$$x_p \geq 0, \qquad \forall p \in P_k^i;\ i = 1, \ldots, I; k = 1, \ldots, n_R, \tag{4}$$

where P_k^i is the set of all paths joining the origin node i; $i = 1, \ldots, I$ with destination node R_k.

Formulation: Path Throughput Factors

μ_p denotes the multiplier corresponding to the throughput on path p, defined as:

$$\mu_p \equiv \prod_{a \in p} \alpha_a, \qquad \forall p \in P_k^i;\ i = 1, \ldots, I;\ k = 1, \ldots, n_R. \tag{5}$$

Formulation: Link Flows and Path Flows

We define the multiplier, α_{ap} as follows:

$$\alpha_{ap} \equiv \begin{cases} \delta_{ap} \prod_{a'<a} \alpha_{a'}, & \text{if } \{a' < a\} \neq \emptyset, \\ \delta_{ap}, & \text{if } \{a' < a\} = \emptyset, \end{cases} \tag{6}$$

where $\{a' < a\}$ denotes the set of the links preceding link a in path p, and $\emptyset$ denotes the null set.

Formulation: Link Flows and Path Flows

The relationship between the link flow, f_a, and the path flows, x_p, can be expressed as:

$$f_a = \sum_{i=1}^{I} \sum_{k=1}^{n_R} \sum_{p \in P_k^i} x_p \; \alpha_{ap}, \qquad \forall a \in L, \tag{7}$$

where P_k^i is the set of all paths joining the origin node i; $i = 1, \ldots, I$ with destination node R_k.

Formulation: Demand

Let d_{ik} denote the demand for pharmaceutical firm i's brand drug; $i = 1, \ldots, I$, at demand market R_k; $k = 1, \ldots, n_R$. The consumers differentiate the products by their brands.

The relationship between the path flows and the demands in the supply chain network:

$$\sum_{p \in P_k^i} x_p \mu_p = d_{ik}, \quad i = 1, \ldots, I; k = 1, \ldots, n_R. \tag{8}$$

The demands d_{ik} are grouped into the $n_R \times I$-dimensional vector d.

Formulation: Demand Price Functions

The demand price function associated with firm i's pharmaceutical at demand market k:

$$\rho_{ik} = \rho_{ik}(d), \quad i = 1, \ldots, I; k = 1, \ldots, n_R. \tag{9}$$

The total operational cost on link a is:

$$\hat{c}_a = \hat{c}_a(f), \quad \forall a \in L, \tag{10}$$

where f is the vector of all the link flows. The total cost on each link is assumed to be convex and continuously differentiable.

Formulation: Profit Function

Profit function of firm i:

$$U_i = \sum_{k=1}^{n_R} \rho_{ik}(d) \sum_{p \in P_k^i} \mu_p x_p - \sum_{a \in L^i} \hat{c}_a(f) - \sum_{a \in L^i} \hat{z}_a(f_a). \tag{11}$$

$\hat{U}_i(X) = U_i$ is defined for all firms i; $i = 1, \ldots, I$.
$\hat{U}$: I-dimensional vector of the profits of all the firms:

$$\hat{U} = \hat{U}(X). \tag{12}$$

Supply Chain Generalized Network Cournot-Nash Equilibrium

In the Cournot-Nash oligopolistic market framework, each firm selects its product path flows in a noncooperative manner, seeking to maximize its own profit, until an equilibrium is achieved.

Definition 1: Supply Chain Generalized Network Cournot-Nash Equilibrium

A path flow pattern $X^* \in K = \prod_{i=1}^{I} K_i$ *constitutes a supply chain generalized network Cournot-Nash equilibrium if for each firm* i*;* $i = 1, \ldots, I$*:*

$$\hat{U}_i(X_i^*, \hat{X}_i^*) \geq \hat{U}_i(X_i, \hat{X}_i^*), \quad \forall X_i \in K_i, \tag{13}$$

where $\hat{X}_i^* \equiv (X_1^*, \ldots, X_{i-1}^*, X_{i+1}^*, \ldots, X_I^*)$ *and* $K_i \equiv \{X_i | X_i \in R_+^{n_{P^i}}\}$.

The Variational Inequality Formulation

Theorem 1

Assume that, for each pharmaceutical firm i; $i = 1, \ldots, I$, the profit function $\hat{U}_i(X)$ is concave with respect to the variables in X_i, and is continuously differentiable. Then $X^* \in K$ is a supply chain generalized network Cournot-Nash equilibrium according to Definition 1 if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^{I} \langle \nabla_{X_i} \hat{U}_i(X^*)^T, X_i - X_i^* \rangle \geq 0, \quad \forall X \in K, \tag{14}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in the corresponding Euclidean space and $\nabla_{X_i} \hat{U}_i(X)$ denotes the gradient of $\hat{U}_i(X)$ with respect to X_i.

The Variational Inequality Formulation

Variational Inequality (Path Flows)

Determine $x^* \in K^1$ *such that:*

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\sum_{p\in P_k^i}\left[\frac{(\sum_{q\in\mathcal{P}}\hat{C}_q(x^*))}{\partial x_p}+\frac{(\sum_{q\in\mathcal{P}}\hat{Z}_q(x^*))}{\partial x_p}-\rho_{ik}(x^*)\mu_p-\right.$$

$$\left.\sum_{l=1}^{n_R}\frac{\partial\rho_{il}(x^*)}{\partial d_{ik}}\mu_p\sum_{p\in P_l^i}\mu_p x_p^*\right]\times[x_p-x_p^*]\geq 0,\quad \forall x\in K^1, \qquad (15)$$

where $K^1 \equiv \{x | x \in R_+^{n_P}\}$.

The Variational Inequality Formulation

Variational Inequality (Link Flows)

Determine the vector of equilibrium link flows and the vector of equilibrium demands $(f^*, d^*) \in K^2$, *such that:*

$$\sum_{i=1}^{I} \sum_{a \in L^i} \left[\sum_{b \in L^i} \frac{\partial \hat{c}_b(f^*)}{\partial f_a} + \frac{\partial \hat{z}_a(f_a^*)}{\partial f_a} \right] \times [f_a - f_a^*]$$

$$+ \sum_{i=1}^{I} \sum_{k=1}^{n_R} \left[-\rho_{ik}(d^*) - \sum_{l=1}^{n_R} \frac{\partial \rho_{il}(d^*)}{\partial d_{ik}} d_{il}^* \right] \times [d_{ik} - d_{ik}^*] \geq 0,$$

$$\forall (f, d) \in K^2, \tag{17}$$

where $K^2 \equiv \{(f, d) | x \geq 0, \textit{and } (7), \textit{ and } (8) \textit{ hold}\}$.

The Projected Dynamical System (PDS) Model

Recall the ordinary differential equation:

$$\dot{x} = \Pi_{\mathcal{K}}(x, -F(x)), \qquad x(0) = x_0 \in \mathcal{K}, \tag{23}$$

where, since $\mathcal{K}$ is a convex polyhedron, according to Dupuis and Nagurney (1993), $\Pi_{\mathcal{K}}(X, -F(X))$ is the projection, with respect to $\mathcal{K}$, of the vector $-F(X)$ at X defined as

$$\Pi_{\mathcal{K}}(X, -F(X)) = \lim_{\delta \to 0} \frac{P_{\mathcal{K}}(X - \delta F(X)) - X}{\delta} \tag{24}$$

with $P_{\mathcal{K}}$ denoting the projection map:

$$P(X) = \text{argmin}_{x \in \mathcal{K}} \|Q - x\|, \tag{25}$$

and where $\| \cdot \| = \langle x^T, x \rangle$.

The Projected Dynamical System (PDS) Model

The projected dynamical system, in the context of a classical oligopoly model, can be expressed as:

$$\dot{q} = \Pi_{\mathcal{K}}(q, \frac{\partial U(q)}{\partial q}), \qquad q(0) = q_0 \in \mathcal{K}, \tag{26}$$

where $\frac{\partial U(q)}{\partial q} \equiv (\frac{\partial U_1(q)}{\partial q_1}, \cdots, \frac{\partial U_m(q)}{\partial q_m})$, and $\mathcal{K} = R^m_+$.

The Projected Dynamical System (PDS) Model

The adjustment process in (26) can be re-expressed as:

PDS Model

$$\dot{q}_i = \begin{cases} \frac{\partial \hat{U}_i(q)}{\partial q}, & \text{if} \quad q_i > 0 \\ \max\{0, \frac{\partial \hat{U}_i(q)}{\partial q}\}, & \text{if} \quad q_i = 0. \end{cases} \tag{27}$$

The Projected Dynamical System (PDS) Model

Now we present a dynamic adjustment process for the evolution of the pharmaceutical firms' strategies.

Substituting the utility function (27) with its equivalent functions, our PDS model can now be rewritten in terms of path flows. For each path p, we have:

PDS Model

$$\dot{x}_p = \begin{cases} \frac{\partial \hat{U}_i}{\partial x_p}, & \text{if} \quad x_p > 0 \\ \max\{0, \frac{\partial \hat{U}_i}{\partial x_p}\}, & \text{if} \quad x_p = 0. \end{cases} \tag{28}$$

Where:

$$\frac{\partial \hat{U}_i}{\partial x_p} = \rho_{ik}(d)\mu_p + \sum_{l=1}^{n_R} \frac{\partial \rho_{il}(d)}{\partial d_{ik}} \mu_p d_{il} - \sum_{a \in L^i} \sum_{b \in L^i} \frac{\partial \hat{c}_b(f)}{\partial f_a} \alpha_{ap} - \sum_{a \in L^i} \frac{\partial \hat{z}_a(f_a)}{\partial f_a} \alpha_{ap}.$$

Theorem 2: Equilibrium Condition

From Dupuis and Nagurney (1993), the equilibrium points of the PDS coincide with the solution of its corresponding VI$(F, \mathcal{K})$, provided that $\mathcal{K}$ is a convex polyhedron.

Thus, X^* which is a Cournot-Nash equilibrium solution to VI (15) is also a stationary point of the adjustment process defined in (28).

Corollaries

Corollary 1: Homogeneous Drug

Let d_k and ρ_k denote the demand for the homogeneous drug and its demand price at demand market R_k, respectively. One can derive:

$$\sum_{i=1}^{I} \sum_{p \in P_k^i} x_p \mu_p = d_k, \quad k = 1, \ldots, n_R. \tag{30}$$

Then, the profit function (11) can be rewritten as:

$$U_i = \sum_{k=1}^{n_R} \rho_k(d) \sum_{p \in P_k^i} \mu_p x_p - \sum_{a \in L^i} \hat{c}_a(f) - \sum_{a \in L^i} \hat{z}_a(f_a). \tag{31}$$

Corollaries

Corollary 1 (cont'd): Homogeneous Drug

The corresponding variational inequality (15) in terms of path flows can be rewritten as: determine $x^* \in K^1$ such that:

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\sum_{p\in P_k^i}\left[\frac{\partial(\sum_{q\in\mathcal{P}}\hat{C}_q(x^*))}{\partial x_p}+\frac{\partial(\sum_{q\in\mathcal{P}}\hat{Z}_q(x^*))}{\partial x_p}-\rho_k(x^*)\mu_p\right.$$

$$\left.-\sum_{l=1}^{n_R}\frac{\partial\rho_l(x^*)}{\partial d_k}\mu_p\sum_{p\in P_l^i}\mu_p x_p^*\right]\times[x_p-x_p^*]\geq 0,\qquad \forall x\in K^1. \quad (32)$$

Corollaries

Corollary 2: Fixed Demand

Assume that the demand d_{ik} for firm i's pharmaceutical is fixed. Then, the demand price of this product at demand market R_k will then also be fixed. One can derive:

$$U_i = \sum_{k=1}^{n_R} \bar{\rho}_{ik} d_{ik} - \sum_{a \in L^i} \hat{c}_a(f) - \sum_{a \in L^i} \hat{z}_a(f_a), \tag{33}$$

Corollaries

Corollary 2 (cont'd): Fixed Demand

Therefore, the corresponding variational inequality (15) in terms of path flows simplifies, in this case, to: determine $x^* \in K^3$ *such that:*

$$\sum_{i=1}^{I} \sum_{k=1}^{n_R} \sum_{p \in P_k^i} \left[\frac{\partial (\sum_{q \in \mathcal{P}} \hat{C}_q(x^*))}{\partial x_p} + \frac{\partial (\sum_{q \in \mathcal{P}} \hat{Z}_q(x^*))}{\partial x_p} \right] \times [x_p - x_p^*] \geq 0,$$

$$\forall x \in K^3, \tag{34}$$

where

$$K^3 \equiv \{x | x \geq 0, \mathit{and}\,(8)\,\mathit{is\,satisfied\ with\ the}\,d_{ik}\mathit{s\ known\ and\ fixed}, \forall i, k.\}$$

Corollaries

Corollary 3: Homogeneous Drug and Fixed Demand

Assume that the firms produce a homogeneous drug for which the demand d_k at market R_k is fixed, as well as the demand price $\bar{\rho}_k$. One has:

$$U_i = \sum_{k=1}^{n_R} \bar{\rho}_k \sum_{p \in P_k^i} \mu_p x_p - \sum_{a \in L^i} \hat{c}_a(f) - \sum_{a \in L^i} \hat{z}_a(f_a). \qquad (37)$$

The corresponding variational inequality is: determine $x^* \in K^5$ such that:

$$\sum_{i=1}^{I} \sum_{k=1}^{n_R} \sum_{p \in P_k^i} \left[\frac{\partial(\sum_{q \in \mathcal{P}} \hat{C}_q(x^*))}{\partial x_p} + \frac{\partial(\sum_{q \in \mathcal{P}} \hat{Z}_q(x^*))}{\partial x_p} \right] \times [x_p - x_p^*] \geq 0, \; \forall x \in K^5, \qquad (38)$$

where $K^5 \equiv \{x | x \geq 0,$ and (36) is satisfied with the d_ks known and fixed, $\forall k.\}$.

Solution Algorithm

Iteration τ of the Euler method (see also Nagurney and Zhang (1996)) is given by:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau} F(X^{\tau})), \tag{42}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem (15).
The sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \to 0$, as $\tau \to \infty$.

Solution Algorithm

Explicit Formulae for the Euler Method Applied to the Supply Chain Generalized Network Oligopoly Variational Inequality (15)

For all the path flows $p \in P_k^i, \forall i, k$, one has:

$$x_p^{\tau+1} = \max\{0, x_p^\tau + a_\tau(\rho_{ik}(x^\tau)\mu_p + \sum_{l=1}^{n_R} \frac{\partial \rho_{il}(x^\tau)}{\partial d_{ik}} \mu_p \sum_{p \in P_l^i} \mu_p x_p^\tau$$

$$- \frac{\partial(\sum_{q \in \mathcal{P}} \hat{C}_q(x^\tau))}{\partial x_p} - \frac{\partial(\sum_{q \in \mathcal{P}} \hat{Z}_q(x^\tau))}{\partial x_p})\}. \quad (43)$$

Computational Details

The Euler method for the solution of variational inequality (15) was implemented in Matlab on a Microsoft Windows 7 System with a Dell PC at the University of Massachusetts Amherst.

We set the sequence $a_\tau = .1(1, \frac{1}{2}, \frac{1}{2}, \cdots)$, and the convergence tolerance was 10^{-6}. In other words, the absolute value of the difference between each path flow in two consecutive iterations was less than or equal to this tolerance.

We initialized the algorithm by setting the path flows equal to 10.

Numerical Cases

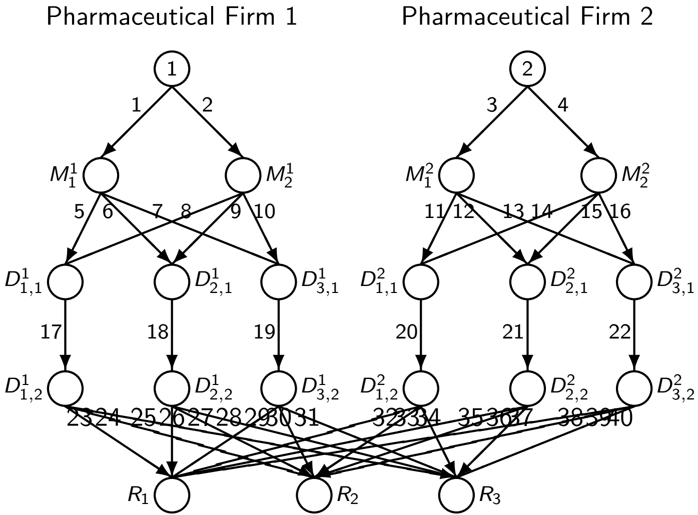
Case I

This case is assumed occur in the third quarter of 2011 prior to the expiration of the patent for Lipitor.

Firm 1 represents a multinational pharmaceutical giant, hypothetically, Pfizer, Inc., which still possesses the patent for Lipitor, the most popular brand of cholesterol-lowering drug.

Firm 2, on the other hand, which might represent, for example, Merck & Co., Inc., been producing Zocor, another cholesterol regulating brand, whose patent expired in 2006.

The Pharmaceutical Supply Chain Network Topology for Case I



Case I (cont'd)

The demand price functions were as follows:

$$\rho_{11}(d) = -1.1d_{11} - 0.9d_{21} + 275; \; \rho_{21}(d) = -1.2d_{21} - 0.7d_{11} + 210;$$

$$\rho_{12}(d) = -0.9d_{12} - 0.8d_{22} + 255; \; \rho_{22}(d) = -1.0d_{22} - 0.5d_{12} + 200;$$

$$\rho_{13}(d) = -1.4d_{13} - 1.0d_{23} + 265; \; \rho_{23}(d) = -1.5d_{23} - 0.4d_{13} + 186.$$

The Euler method for the solution of variational inequality was implemented in Matlab. The results can be seen in the following tables.

Link Multipliers, Total Cost Functions and Link Flow Solution for Case I

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	f_a^*
1	.95	$5f_1^2 + 8f_1$	$.5f_1^2$	13.73
2	.97	$7f_2^2 + 3f_2$	$.4f_2^2$	10.77
3	.96	$6.5f_3^2 + 4f_3$	$.3f_3^2$	8.42
4	.98	$5f_4^2 + 7f_4$	$.35f_4^2$	10.55
5	1.00	$.7f_5^2 + f_5$	$.5f_5^2$	5.21
6	.99	$.9f_6^2 + 2f_6$	$.5f_6^2$	3.36
7	1.00	$.5f_7^2 + f_7$	$.5f_7^2$	4.47
8	.99	$f_8^2 + 2f_8$	$.6f_8^2$	3.02
9	1.00	$.7f_9^2 + 3f_9$	$.6f_9^2$	3.92
10	1.00	$.6f_{10}^2 + 1.5f_{10}$	$.6f_{10}^2$	3.50
11	.99	$.8f_{11}^2 + 2f_{11}$	$.4f_{11}^2$	3.10
12	.99	$.8f_{12}^2 + 5f_{12}$	$.4f_{12}^2$	2.36
13	.98	$.9f_{13}^2 + 4f_{13}$	$.4f_{13}^2$	2.63
14	1.00	$.8f_{14}^2 + 2f_{14}$	$.5f_{14}^2$	3.79
15	.99	$.9f_{15}^2 + 3f_{15}$	$.5f_{15}^2$	3.12
16	1.00	$1.1f_{16}^2 + 3f_{16}$	$.6f_{16}^2$	3.43
17	.98	$2f_{17}^2 + 3f_{17}$	$.45f_{17}^2$	8.20
18	.99	$2.5f_{18}^2 + f_{18}$	$.55f_{18}^2$	7.25
19	.98	$2.4f_{19}^2 + 1.5f_{19}$	$.5f_{19}^2$	7.97
20	.98	$1.8f_{20}^2 + 3f_{20}$	$.3f_{20}^2$	6.85

Link Multipliers, Total Cost Functions and Solution for Case I (cont'd)

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	f_a^*
21	.98	$2.1f_{21}^2 + 3f_{21}$	$.35f_{21}^2$	5.42
22	.99	$1.9f_{22}^2 + 2.5f_{22}$	$.5f_{22}^2$	6.00
23	1.00	$.5f_{23}^2 + 2f_{23}$	$.6f_{23}^2$	3.56
24	1.00	$.7f_{24}^2 + f_{24}$	$.6f_{24}^2$	1.66
25	.99	$.5f_{25}^2 + .8f_{25}$	$.6f_{25}^2$	2.82
26	.99	$.6f_{26}^2 + f_{26}$	$.45f_{26}^2$	3.34
27	.99	$.7f_{27}^2 + .8f_{27}$	$.4f_{27}^2$	1.24
28	.98	$.4f_{28}^2 + .8f_{28}$	$.45f_{28}^2$	2.59
29	1.00	$.3f_{29}^2 + 3f_{29}$	$.55f_{29}^2$	3.45
30	1.00	$.75f_{30}^2 + f_{30}$	$.55f_{30}^2$	1.28
31	1.00	$.65f_{31}^2 + f_{31}$	$.55f_{31}^2$	3.09
32	.99	$.5f_{32}^2 + 2f_{32}$	$.3f_{32}^2$	2.54
33	.99	$.4f_{33}^2 + 3f_{33}$	$.3f_{33}^2$	3.43
34	1.00	$.5f_{34}^2 + 3.5f_{34}$	$.4f_{34}^2$	0.75
35	.98	$.4f_{35}^2 + 2f_{35}$	$.55f_{35}^2$	1.72
36	.98	$.3f_{36}^2 + 2.5f_{36}$	$.55f_{36}^2$	2.64
37	.99	$.55f_{37}^2 + 2f_{37}$	$.55f_{37}^2$	0.95
38	1.00	$.35f_{38}^2 + 2f_{38}$	$.4f_{38}^2$	3.47
39	1.00	$.4f_{39}^2 + 5f_{39}$	$.4f_{39}^2$	2.47
40	.98	$.55f_{40}^2 + 2f_{40}$	$.6f_{40}^2$	0.00

Case I: Result Analysis

The computed equilibrium demands for each of the two brands were:

$$d_{11}^* = 10.32, \; d_{21}^* = 7.66,$$

$$d_{12}^* = 4.17, \; d_{22}^* = 8.46,$$

$$d_{13}^* = 8.41, \; d_{23}^* = 1.69.$$

The incurred equilibrium prices associated with the branded drugs at each demand market were as follows:

$$\rho_{11}(d^*) = 256.75, \; \rho_{21}(d^*) = 193.58,$$

$$\rho_{12}(d^*) = 244.48, \; \rho_{22}(d^*) = 189.46,$$

$$\rho_{13}(d^*) = 251.52, \; \rho_{23}(d^*) = 180.09.$$

Case I: Result Analysis

Firm 1, which produces the top-selling product, captures the majority of the market share at demand markets 1 and 3, despite the higher price. In fact, it has almost entirely seized demand market 3 forcing several links connecting Firm 2 to demand market 3 to have insignificant flows including link 40 with a flow equal to zero.

Firm 2 dominates demand market 2, due to the consumers' willingness to lean towards this product there, perhaps as a consequence of the lower price, or the perception of quality, etc.

The profits of the two firms are:

$$U_1(X^*) = 2,936.52 \text{ and } \quad U_2(X^*) = 1,675.89.$$

Case II

Firm 1 has just lost the exclusive patent right of its highly popular cholesterol regulator. A manufacturer of generic drugs, say, Ranbaxy Laboratories, here denoted by Firm 3, has recently introduced a generic substitute for Lipitor by reproducing its active ingredients.

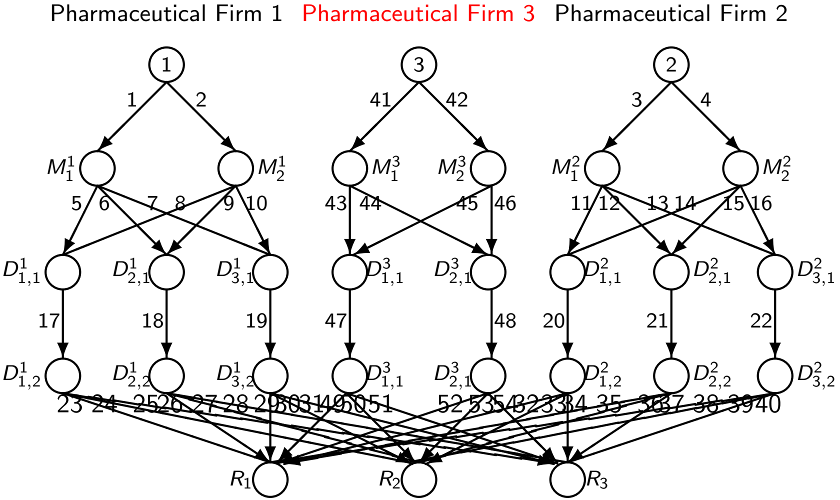
The demand price functions for the products of Firm 1 and 2 will stay the same as in Case I. The demand price functions corresponding to the product of Firm 3 are as follows:

$$\rho_{31}(d) = -0.9d_{31} - 0.6d_{11} - 0.8d_{21} + 150;$$

$$\rho_{32}(d) = -0.8d_{32} - 0.5d_{12} - 0.6d_{22} + 130;$$

$$\rho_{33}(d) = -0.9d_{33} - 0.7d_{13} - 0.5d_{23} + 133.$$

The Pharmaceutical Supply Chain Network Topology for Cases II and III



Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	f_a^*
1	.95	$5f_1^2 + 8f_1$	$.5f_1^2$	13.73
2	.97	$7f_2^2 + 3f_2$	$.4f_2^2$	10.77
3	.96	$6.5f_3^2 + 4f_3$	$.3f_3^2$	8.42
4	.98	$5f_4^2 + 7f_4$	$.35f_4^2$	10.55
5	1.00	$.7f_5^2 + f_5$	$.5f_5^2$	5.21
6	.99	$.9f_6^2 + 2f_6$	$.5f_6^2$	3.36
7	1.00	$.5f_7^2 + f_7$	$.5f_7^2$	4.47
8	.99	$f_8^2 + 2f_8$	$.6f_8^2$	3.02
9	1.00	$.7f_9^2 + 3f_9$	$.6f_9^2$	3.92
10	1.00	$.6f_{10}^2 + 1.5f_{10}$	$.6f_{10}^2$	3.50
11	.99	$.8f_{11}^2 + 2f_{11}$	$.4f_{11}^2$	3.10
12	.99	$.8f_{12}^2 + 5f_{12}$	$.4f_{12}^2$	2.36
13	.98	$.9f_{13}^2 + 4f_{13}$	$.4f_{13}^2$	2.63
14	1.00	$.8f_{14}^2 + 2f_{14}$	$.5f_{14}^2$	3.79
15	.99	$.9f_{15}^2 + 3f_{15}$	$.5f_{15}^2$	3.12
16	1.00	$1.1f_{16}^2 + 3f_{16}$	$.6f_{16}^2$	3.43
17	.98	$2f_{17}^2 + 3f_{17}$	$.45f_{17}^2$	8.20
18	.99	$2.5f_{18}^2 + f_{18}$	$.55f_{18}^2$	7.25
19	.98	$2.4f_{19}^2 + 1.5f_{19}$	$.5f_{19}^2$	7.97
20	.98	$1.8f_{20}^2 + 3f_{20}$	$.3f_{20}^2$	6.85
21	.98	$2.1f_{21}^2 + 3f_{21}$	$.35f_{21}^2$	5.42
22	.99	$1.9f_{22}^2 + 2.5f_{22}$	$.5f_{22}^2$	6.00
23	1.00	$.5f_{23}^2 + 2f_{23}$	$.6f_{23}^2$	3.56
24	1.00	$.7f_{24}^2 + f_{24}$	$.6f_{24}^2$	1.66
25	.99	$.5f_{25}^2 + .8f_{25}$	$.6f_{25}^2$	2.82
26	.99	$.6f_{26}^2 + f_{26}$	$.45f_{26}^2$	3.34
27	.99	$.7f_{27}^2 + .8f_{27}$	$.4f_{27}^2$	1.24

Link Multipliers,
Total Cost Functions
and Link Flow Solution
for Case II

Link Multipliers, Total Cost Functions and Solution for Case II (cont'd)

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	f_a^*
28	.98	$.4f_{28}^2 + .8f_{28}$	$.45f_{28}^2$	2.59
29	1.00	$.3f_{29}^2 + 3f_{29}$	$.55f_{29}^2$	3.45
30	1.00	$.75f_{30}^2 + f_{30}$	$.55f_{30}^2$	1.28
31	1.00	$.65f_{31}^2 + f_{31}$	$.55f_{31}^2$	3.09
32	.99	$.5f_{32}^2 + 2f_{32}$	$.3f_{32}^2$	2.54
33	.99	$.4f_{33}^2 + 3f_{33}$	$.3f_{33}^2$	3.43
34	1.00	$.5f_{34}^2 + 3.5f_{34}$	$.4f_{34}^2$	0.75
35	.98	$.4f_{35}^2 + 2f_{35}$	$.55f_{35}^2$	1.72
36	.98	$.3f_{36}^2 + 2.5f_{36}$	$.55f_{36}^2$	2.64
37	.99	$.55f_{37}^2 + 2f_{37}$	$.55f_{37}^2$	0.95
38	1.00	$.35f_{38}^2 + 2f_{38}$	$.4f_{38}^2$	3.47
39	1.00	$.4f_{39}^2 + 5f_{39}$	$.4f_{39}^2$	2.47
40	.98	$.55f_{40}^2 + 2f_{40}$	$.6f_{40}^2$	0.00
41	.97	$3f_{41}^2 + 12f_{41}$	$.3f_{41}^2$	6.17
42	.96	$2.7f_{42}^2 + 10f_{42}$	$.4f_{42}^2$	6.23
43	.98	$1.1f_{43}^2 + 6f_{43}$	$.45f_{43}^2$	3.23
44	.98	$.9f_{44}^2 + 5f_{44}$	$.45f_{44}^2$	2.75
45	.97	$1.3f_{45}^2 + 6f_{45}$	$.5f_{45}^2$	3.60
46	.99	$1.5f_{46}^2 + 7f_{46}$	$.55f_{46}^2$	2.38
47	.98	$1.5f_{47}^2 + 4f_{47}$	$.4f_{47}^2$	6.66
48	.98	$2.1f_{48}^2 + 6f_{48}$	$.45f_{48}^2$	5.05
49	.99	$.6f_{49}^2 + 3f_{49}$	$.55f_{49}^2$	3.79
50	1.00	$.7f_{50}^2 + 2f_{50}$	$.7f_{50}^2$	1.94
51	.98	$.6f_{51}^2 + 7f_{51}$	$.45f_{51}^2$	0.79
52	.99	$.9f_{52}^2 + 9f_{52}$	$.5f_{52}^2$	1.43
53	1.00	$.55f_{53}^2 + 6f_{53}$	$.55f_{53}^2$	1.23
54	.98	$.8f_{54}^2 + 4f_{54}$	$.5f_{54}^2$	2.28

Case II: Result Analysis

The equilibrium product flows of Firms 1 and 2 on links 1 through 40 are identical to the corresponding values in Case I.

When the new product produced by Firm 3 is just introduced, the manufacturers of the two existing products will not experience an immediate impact on their respective demands of branded drugs.

The equilibrium computed demands for the products of Firms 1 and 2 at the demand markets will remain as in Case I, and the equilibrium amounts of demand for the new product of Firm 3 at each demand market is equal to:

$$d_{31}^* = 5.17, \quad d_{32}^* = 3.18, \quad \text{and } d_{33}^* = 3.01.$$

Case II: Result Analysis

The equilibrium prices associated with the branded drugs 1 and 2 at the demand markets will not change, whereas the incurred equilibrium prices of generic drug 3 are as follows:

$$\rho_{31}(d^*) = 133.02, \quad \rho_{32}(d^*) = 120.30, \quad \text{and } \rho_{33}(d^*) = 123.55,$$

which is significantly lower than the respective prices of its competitors in all the demand markets.

Thus, the profit that Firm 3 derived from manufacturing and delivering the new generic substitute to these 3 markets is:

$$U_3(X^*) = 637.38,$$

while the profits of Firms 1 and 2 remain unchanged.

Case III

The generic product of Firm 3 has now been well-established, and has affected the behavior of the consumers through the demand price functions of the relatively more recognized products of Firms 1 and 2. The demand price functions associated are now given by:

$$\text{Firm 1: } \rho_{11}(d) = -1.1d_{11} - 0.9d_{21} - 1.0d_{31} + 192;$$

$$\rho_{21}(d) = -1.2d_{21} - 0.7d_{11} - 0.8d_{31} + 176;$$

$$\rho_{31} = -0.9d_{31} - 0.6d_{11} - 0.8d_{21} + 170;$$

$$\text{Firm 2: } \rho_{12}(d) = -0.9d_{12} - 0.8d_{22} - 0.7d_{32} + 166;$$

$$\rho_{22}(d) = -1.0d_{22} - 0.5d_{12} - 0.8d_{32} + 146;$$

$$\rho_{32}(d) = -0.8d_{32} - 0.5d_{12} - 0.6d_{22} + 153;$$

$$\text{Firm 3: } \rho_{13}(d) = -1.4d_{13} - 1.0d_{23} - 0.5d_{33} + 173;$$

$$\rho_{23}(d) = -1.5d_{23} - 0.4d_{13} - 0.7d_{33} + 164;$$

$$\rho_{33}(d) = -0.9d_{33} - 0.7d_{13} - 0.5d_{23} + 157.$$

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	f_a^*
1	.95	$5f_1^2 + 8f_1$	$.5f_1^2$	8.42
2	.97	$7f_2^2 + 3f_2$	$.4f_2^2$	6.72
3	.96	$6.5f_3^2 + 4f_3$	$.3f_3^2$	6.42
4	.98	$5f_4^2 + 7f_4$	$.35f_4^2$	8.01
5	1.00	$.7f_5^2 + f_5$	$.5f_5^2$	3.20
6	.99	$.9f_6^2 + 2f_6$	$.5f_6^2$	2.07
7	1.00	$.5f_7^2 + f_7$	$.5f_7^2$	2.73
8	.99	$f_8^2 + 2f_8$	$.6f_8^2$	1.85
9	1.00	$.7f_9^2 + 3f_9$	$.6f_9^2$	2.44
10	1.00	$.6f_{10}^2 + 1.5f_{10}$	$.6f_{10}^2$	2.23
11	.99	$.8f_{11}^2 + 2f_{11}$	$.4f_{11}^2$	2.42
12	.99	$.8f_{12}^2 + 5f_{12}$	$.4f_{12}^2$	1.75
13	.98	$.9f_{13}^2 + 4f_{13}$	$.4f_{13}^2$	2.00
14	1.00	$.8f_{14}^2 + 2f_{14}$	$.5f_{14}^2$	2.84
15	.99	$.9f_{15}^2 + 3f_{15}$	$.5f_{15}^2$	2.40
16	1.00	$1.1f_{16}^2 + 3f_{16}$	$.6f_{16}^2$	2.60
17	.98	$2f_{17}^2 + 3f_{17}$	$.45f_{17}^2$	5.02
18	.99	$2.5f_{18}^2 + f_{18}$	$.55f_{18}^2$	4.49
19	.98	$2.4f_{19}^2 + 1.5f_{19}$	$.5f_{19}^2$	4.96
20	.98	$1.8f_{20}^2 + 3f_{20}$	$.3f_{20}^2$	5.23
21	.98	$2.1f_{21}^2 + 3f_{21}$	$.35f_{21}^2$	4.11
22	.99	$1.9f_{22}^2 + 2.5f_{22}$	$.5f_{22}^2$	4.56
23	1.00	$.5f_{23}^2 + 2f_{23}$	$.6f_{23}^2$	2.44
24	1.00	$.7f_{24}^2 + f_{24}$	$.6f_{24}^2$	1.47
25	.99	$.5f_{25}^2 + .8f_{25}$	$.6f_{25}^2$	1.02
26	.99	$.6f_{26}^2 + f_{26}$	$.45f_{26}^2$	2.48
27	.99	$.7f_{27}^2 + .8f_{27}$	$.4f_{27}^2$	1.31

Link Multipliers,
Total Cost Functions
and Link Flow Solution
for Case III

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	f_a^*
28	.98	$.4f_{28}^2 + .8f_{28}$	$.45f_{28}^2$	0.66
29	1.00	$.3f_{29}^2 + 3f_{29}$	$.55f_{29}^2$	2.29
30	1.00	$.75f_{30}^2 + f_{30}$	$.55f_{30}^2$	1.29
31	1.00	$.65f_{31}^2 + f_{31}$	$.55f_{31}^2$	1.28
32	.99	$.5f_{32}^2 + 2f_{32}$	$.3f_{32}^2$	2.74
33	.99	$.4f_{33}^2 + 3f_{33}$	$.3f_{33}^2$	0.00
34	1.00	$.5f_{34}^2 + 3.5f_{34}$	$.4f_{34}^2$	2.39
35	.98	$.4f_{35}^2 + 2f_{35}$	$.55f_{35}^2$	1.82
36	.98	$.3f_{36}^2 + 2.5f_{36}$	$.55f_{36}^2$	0.00
37	.99	$.55f_{37}^2 + 2f_{37}$	$.55f_{37}^2$	2.21
38	1.00	$.35f_{38}^2 + 2f_{38}$	$.4f_{38}^2$	3.46
39	1.00	$.4f_{39}^2 + 5f_{39}$	$.4f_{39}^2$	0.00
40	.98	$.55f_{40}^2 + 2f_{40}$	$.6f_{40}^2$	1.05
41	.97	$3f_{41}^2 + 12f_{41}$	$.3f_{41}^2$	8.08
42	.96	$2.7f_{42}^2 + 10f_{42}$	$.4f_{42}^2$	8.13
43	.98	$1.1f_{43}^2 + 6f_{43}$	$.45f_{43}^2$	4.21
44	.98	$.9f_{44}^2 + 5f_{44}$	$.45f_{44}^2$	3.63
45	.97	$1.3f_{45}^2 + 6f_{45}$	$.5f_{45}^2$	4.62
46	.99	$1.5f_{46}^2 + 7f_{46}$	$.55f_{46}^2$	3.19
47	.98	$1.5f_{47}^2 + 4f_{47}$	$.4f_{47}^2$	8.60
48	.98	$2.1f_{48}^2 + 6f_{48}$	$.45f_{48}^2$	6.72
49	.99	$.6f_{49}^2 + 3f_{49}$	$.55f_{49}^2$	3.63
50	1.00	$.7f_{50}^2 + 2f_{50}$	$.7f_{50}^2$	3.39
51	.98	$.6f_{51}^2 + 7f_{51}$	$.45f_{51}^2$	1.41
52	.99	$.9f_{52}^2 + 9f_{52}$	$.5f_{52}^2$	1.12
53	1.00	$.55f_{53}^2 + 6f_{53}$	$.55f_{53}^2$	2.86
54	.98	$.8f_{54}^2 + 4f_{54}$	$.5f_{54}^2$	2.60

Link Multipliers, Total Cost Functions and Solution for Case III (cont'd)

Case III: Results

The computed equilibrium demands and sales prices for the products of Firms 1, 2, and 3 are as follows:

$$d^*_{11} = 7.18, \quad d^*_{21} = 7.96, \quad d^*_{31} = 4.70,$$
$$d^*_{12} = 4.06, \quad d^*_{22} = 0.00, \quad d^*_{32} = 6.25,$$
$$d^*_{13} = 2.93, \quad d^*_{23} = 5.60, \quad \text{and } d^*_{33} = 3.93.$$

$$\rho_{11}(d^*) = 172.24, \quad \rho_{21}(d^*) = 157.66, \quad \rho_{31}(d^*) = 155.09,$$
$$\rho_{12}(d^*) = 157.97, \quad \rho_{22}(d^*) = 138.97, \quad \rho_{32}(d^*) = 145.97,$$
$$\rho_{13}(d^*) = 161.33, \quad \rho_{23}(d^*) = 151.67, \quad \text{and } \rho_{33}(d^*) = 148.61.$$

The computed amounts of firms' profits:

$$U_1(X^*) = 1,199.87, \quad U_2(X^*) = 1,062.73, \quad \text{and } U_3(X^*) = 980.83.$$

Case III: Result Analysis

As a result of the consumers' growing inclination towards the generic substitute of the previously popular Lipitor, Firm 2 has lost its entire share of market 2 to its competitors, resulting in zero flows on several links. Similarly, Firm 1 now has declining sales of its brand in demand markets 1 and 3.

As expected, the introduction of the generic substitute has also caused remarkable drops in the prices of the existing brands. Interestingly, the decrease in the price of Lipitor in demand markets 2 and 3 exceeds 35%.

Note that simultaneous declines in the amounts of demand and sales price has caused a severe reduction in the profits of Firms 1 and 2. This decline for Firm 1 is observed to be as high as 60%.

Validation of Results: Observations

As noted by Johnson (2011), the market share of a branded drug may decrease by as much as 40%-80% after the introduction of its generic rival. Thus, the model captures the observed decrease in the US market share.

Validation of Results: Observations

As noted by Johnson (2011), the market share of a branded drug may decrease by as much as 40%-80% after the introduction of its generic rival. Thus, the model captures the observed decrease in the US market share.

The reduction in demand and price due to the patent expiration has been observed in the market sales. The US sales of Lipitor have dropped over 75% (Forbes (2012) and Firecepharma (2012)).

Paths Definition and Optimal Path Flow Pattern for Case III - Firm 1

	Path Definition	Path Flow
O/D Pair $(1, R_1)$	$p_1 = (1, 5, 17, 23)$	$x^*_{p_1} = 1.87$
	$p_2 = (1, 6, 18, 26)$	$x^*_{p_2} = 1.46$
	$p_3 = (1, 7, 19, 29)$	$x^*_{p_3} = 1.57$
	$p_4 = (2, 8, 17, 23)$	$x^*_{p_4} = 0.73$
	$p_5 = (2, 9, 18, 26)$	$x^*_{p_5} = 1.17$
	$p_6 = (2, 10, 19, 29)$	$x^*_{p_6} = 0.87$
O/D Pair $(1, R_2)$	$p_7 = (1, 5, 17, 24)$	$x^*_{p_7} = 0.89$
	$p_8 = (1, 6, 18, 27)$	$x^*_{p_8} = 0.57$
	$p_9 = (1, 7, 19, 30)$	$x^*_{p_9} = 0.66$
	$p_{10} = (2, 8, 17, 24)$	$x^*_{p_{10}} = 0.68$
	$p_{11} = (2, 9, 18, 27)$	$x^*_{p_{11}} = 0.82$
	$p_{12} = (2, 10, 19, 30)$	$x^*_{p_{12}} = 0.71$
O/D Pair $(1, R_3)$	$p_{13} = (1, 5, 17, 25)$	$x^*_{p_{13}} = 0.60$
	$p_{14} = (1, 6, 18, 28)$	$x^*_{p_{14}} = 0.16$
	$p_{15} = (1, 7, 19, 31)$	$x^*_{p_{15}} = 0.64$
	$p_{16} = (2, 8, 17, 25)$	$x^*_{p_{16}} = 0.49$
	$p_{17} = (2, 9, 18, 28)$	$x^*_{p_{17}} = 0.53$
	$p_{18} = (2, 10, 19, 31)$	$x^*_{p_{18}} = 0.72$

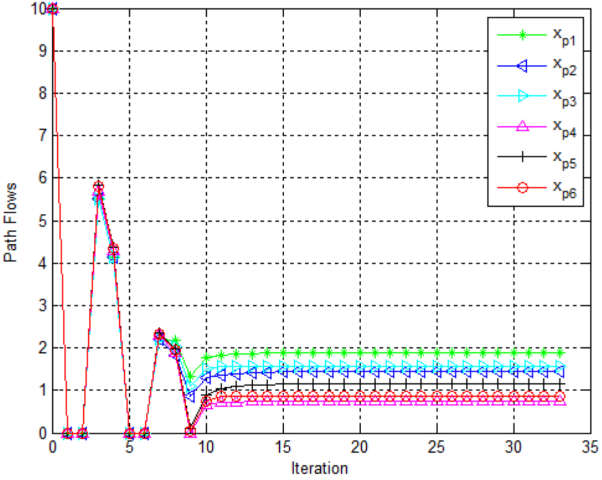
Paths Definition and Optimal Path Flow Pattern for Case III - Firm 2

	Path Definition	Path Flow
O/D Pair $(2, R_1)$	$p_{19} = (3, 11, 20, 32)$	$x^*_{p_{19}} = 1.26$
	$p_{20} = (3, 12, 21, 35)$	$x^*_{p_{20}} = 0.77$
	$p_{21} = (3, 13, 22, 38)$	$x^*_{p_{21}} = 1.51$
	$p_{22} = (4, 14, 20, 32)$	$x^*_{p_{22}} = 1.63$
	$p_{23} = (4, 15, 21, 35)$	$x^*_{p_{23}} = 1.16$
	$p_{24} = (4, 16, 22, 38)$	$x^*_{p_{24}} = 2.12$
O/D Pair $(2, R_2)$	$p_{25} = (3, 11, 20, 33)$	$x^*_{p_{25}} = 0.00$
	$p_{26} = (3, 12, 21, 36)$	$x^*_{p_{26}} = 0.00$
	$p_{27} = (3, 13, 22, 39)$	$x^*_{p_{27}} = 0.00$
	$p_{28} = (4, 14, 20, 33)$	$x^*_{p_{28}} = 0.00$
	$p_{29} = (4, 15, 21, 36)$	$x^*_{p_{29}} = 0.00$
	$p_{30} = (4, 16, 22, 39)$	$x^*_{p_{30}} = 0.00$
O/D Pair $(2, R_3)$	$p_{31} = (3, 11, 20, 34)$	$x^*_{p_{31}} = 1.26$
	$p_{32} = (3, 12, 21, 37)$	$x^*_{p_{32}} = 1.05$
	$p_{33} = (3, 13, 22, 40)$	$x^*_{p_{33}} = 0.57$
	$p_{34} = (4, 14, 20, 34)$	$x^*_{p_{34}} = 1.26$
	$p_{35} = (4, 15, 21, 37)$	$x^*_{p_{35}} = 1.29$
	$p_{36} = (4, 16, 22, 40)$	$x^*_{p_{36}} = 0.54$

Paths Definition and Optimal Path Flow Pattern for Case III - Firm 3

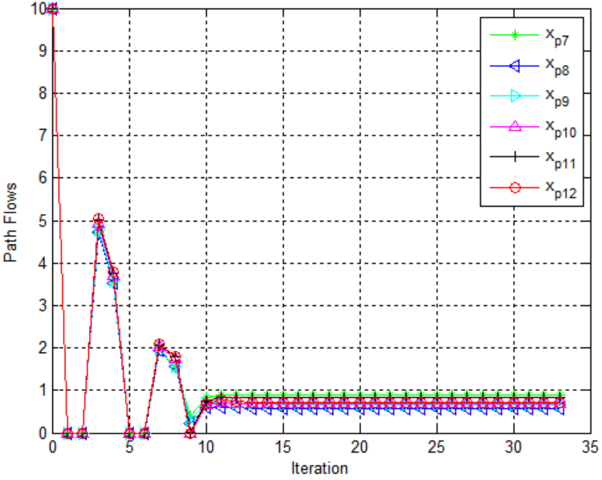
	Path Definition	Path Flow
O/D Pair $(3, R_1)$	$p_{37} = (41, 43, 47, 49)$	$x^*_{p_{37}} = 1.87$
	$p_{38} = (41, 44, 48, 52)$	$x^*_{p_{38}} = 1.78$
	$p_{39} = (42, 45, 47, 49)$	$x^*_{p_{39}} = 0.70$
	$p_{40} = (42, 46, 48, 52)$	$x^*_{p_{40}} = 0.68$
O/D Pair $(3, R_2)$	$p_{41} = (41, 43, 47, 50)$	$x^*_{p_{41}} = 1.61$
	$p_{42} = (41, 44, 48, 53)$	$x^*_{p_{42}} = 1.46$
	$p_{43} = (42, 45, 47, 50)$	$x^*_{p_{43}} = 2.07$
	$p_{44} = (42, 46, 48, 53)$	$x^*_{p_{44}} = 1.90$
O/D Pair $(3, R_3)$	$p_{45} = (41, 43, 47, 51)$	$x^*_{p_{45}} = 0.84$
	$p_{46} = (41, 44, 48, 54)$	$x^*_{p_{46}} = 0.53$
	$p_{47} = (42, 45, 47, 51)$	$x^*_{p_{47}} = 1.46$
	$p_{48} = (42, 46, 48, 54)$	$x^*_{p_{48}} = 1.33$

Path Flow Trajectories



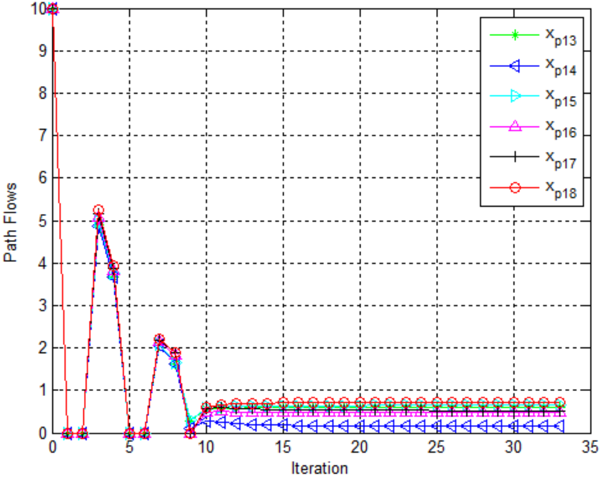
The Trajectories of Product Flows on Paths $p_1 - p_6$ for Case III

Path Flow Trajectories



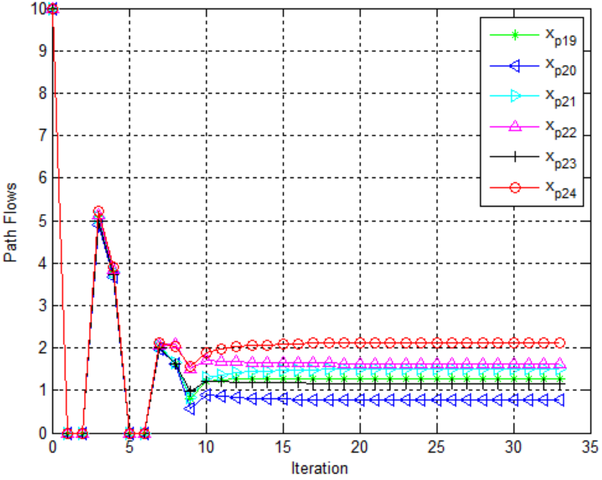
The Trajectories of Product Flows on Paths $p_7 - p_{12}$ for Case III

Path Flow Trajectories



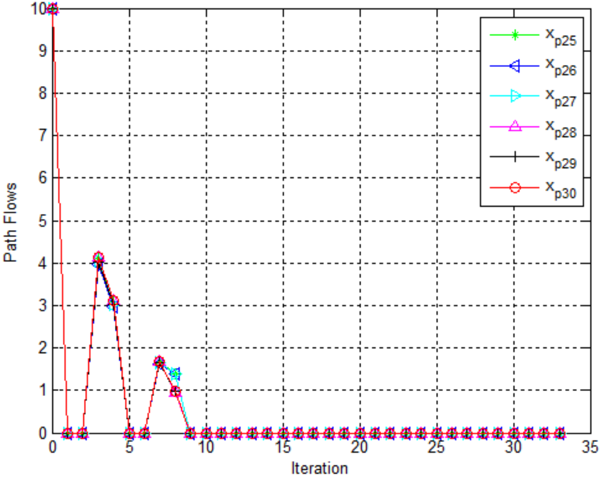
The Trajectories of Product Flows on Paths $p_{13} - p_{18}$ for Case III

Path Flow Trajectories



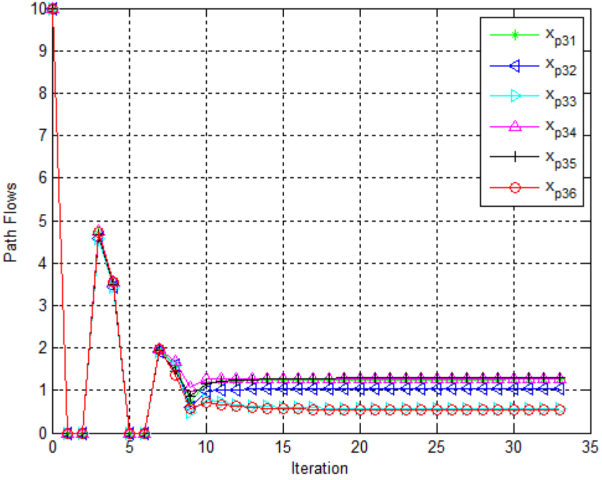
The Trajectories of Product Flows on Paths $p_{19} - p_{24}$ for Case III

Path Flow Trajectories



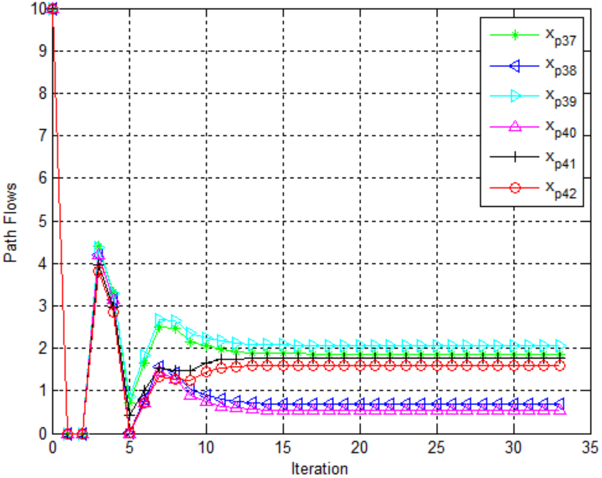
The Trajectories of Product Flows on Paths $p_{25} - p_{30}$ for Case III

Path Flow Trajectories



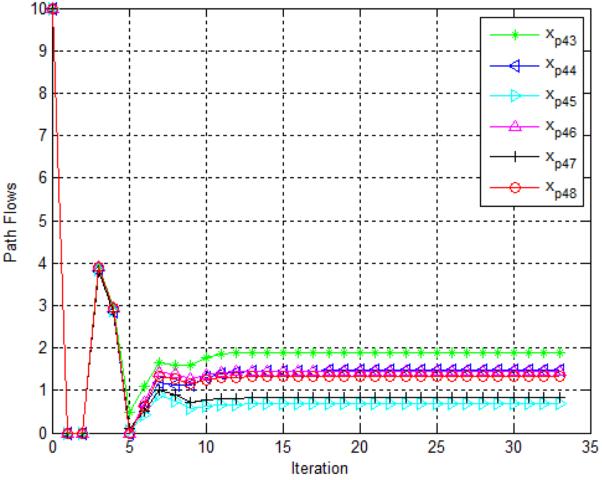
The Trajectories of Product Flows on Paths $p_{31} - p_{36}$ for Case III

Path Flow Trajectories



The Trajectories of Product Flows on Paths $p_{37} - p_{42}$ for Case III

Path Flow Trajectories



The Trajectories of Product Flows on Paths $p_{43} - p_{48}$ for Case III

Comparison of Convergence in Numerical Cases

With the convergence tolerance being set at 10^{-6}, the oligopoly problem in:

- Case I (two firms): converged in 36 iterations.
- Case II (three firms): converged in 36 iterations.
- Case III (three firms): converged in 33 iterations.

Summary and Conclusions

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- allows analyzing the impact of substitution of a popular branded drug by a generic one on market demands, as a result of the patent rights expiration of the brand.

Thank You!

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