Tutorial: Game Theory and the COVID-19 Pandemic

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This tutorial is dedicated to all essential workers.

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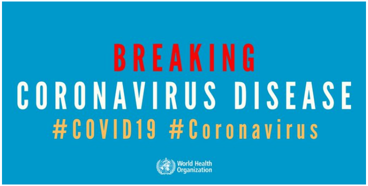
Outline of the Tutorial

- **Some Background and Motivation**
- **Methodology – Variational Inequalities**
- **Commercial Supply Chains and the Inclusion of Labor**
- **Disaster Relief Supply Chains – Competition for Medical Supplies**
- **Cooperation in Disaster Relief**
- **Impacting Policy Through Operations Research**

Some Background and Motivation

The COVID-19 pandemic, declared by the World Health Organization on March 11, 2020, has impacted supply chains, commerce, and trade, employment and work, healthcare, transportation, education, and social activities worldwide.



The pandemic is a disaster not limited in time and location.

Economies and societies have undergone significant transformations in this pandemic, with effects that will linger and will be long studied.

The toll, in terms of lives lost, illnesses, and disruptions, has been immense.

Some Background and Motivation

The Operation Research community has been deeply engaged in the pandemic in research, education, as well as outreach, including speaking with the media, in order to inform the broader community and even to influence policy.

In this tutorial, I hope to provide you with some of the methodological fundamentals, as well as a plethora of applications that have been inspired to address issues and challenges in the pandemic.

The focus here is on game theory because the pandemic has revealed intense competition among various stakeholders and decision-makers, from the local, regional, and national levels to the global arena, as well as opportunities for cooperation.

The tutorial paper, on which this presentation is based, has additional results and many references.

Methodology - Variational Inequalities

Variational Inequalities

Dafermos (1980) identified that the traffic network equilibrium conditions, as formulated by Smith (1979), were a variational inequality (VI) problem. This unveiled the theory for the formulation, analysis, and computation of solutions to numerous equilibrium problems in Operations Research, economics, engineering, and other disciplines.

The paper, available for free download, S. Dafermos (1980), **"Traffic Equilibrium and Variational Inequalities,"** *Transportation Science* **14(1)**, pp 42-54,

was selected by the Editors as one of the 12 most impactful papers in 50 years!

Background

To-date, problems which have been formulated and studied as variational inequality (VI) problems include:

- traffic network equilibrium problems
- spatial price equilibrium problems
- oligopolistic market equilibrium problems
- financial equilibrium problems
- migration equilibrium problems, as well as
- environmental network and ecology problems,
- knowledge network problems,
- electric power generation and distribution networks,
- supply chain network equilibrium problems, and even
- the Internet!

Many of the VI Problems Arise in Network Systems

Variational Inequality Theory

Variational inequality theory provides us with a tool for:

• formulating a variety of equilibrium problems;
• qualitatively analyzing the problems in terms of existence and uniqueness of solutions, stability and sensitivity analysis, and
• providing us with algorithms with accompanying convergence analysis for computational purposes.

It contains, as special cases, such well-known problems in mathematical programming as: systems of nonlinear equations, optimization problems, complementarity problems, and is also related to fixed point problems.

Definition 2.1: Finite-Dimensional Variational Inequality Problem

The finite-dimensional variational inequality problem, $\mathrm{VI}(F,\mathcal{K})$*, is to determine a vector* $X^* \in \mathcal{K} \subset R^N$*, such that*

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{2.1a}$$

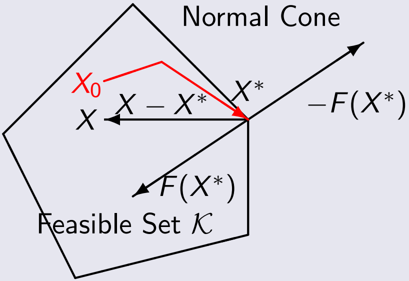
where F *is a given continuous function from* $\mathcal{K}$ *to* R^N*,* $\mathcal{K}$ *is a given closed convex set, and* $\langle \cdot, \cdot \rangle$ *denotes the inner product in* N*-dimensional Euclidean space. In (2.1a),* $F(X) \equiv (F_1(X), F_2(X), \ldots, F_N(X))^T$*, and* $X \equiv (X_1, X_2, \ldots, X_N)^T$*.*

(2.1a) is equivalent to

$$\sum_{i=1}^{N} F_i(X^*) \cdot (X_i - X_i^*) \geq 0, \quad \forall X \in \mathcal{K}. \tag{2.1b}$$

Geometric Interpretation of VI($F, \mathcal{K}$) and a Projected Dynamical System

As shown by Dupuis and Nagurney (1993), there is associated with a VI problem, a *projected dynamical system*, which provides a natural underlying dynamics until an equilibrium state is achieved, under appropriate conditions. In particular, $F(X^*)$ is "orthogonal" to the feasible set $\mathcal{K}$ at the point X^*.



To model the **dynamic behavior of complex network systems**, including supply chains, we utilize *projected dynamical systems* (PDSs) advanced by Dupuis and Nagurney (1993) in the *Annals of Operations Research* and by Nagurney and Zhang (1996) in our book *Projected Dynamical Systems and Variational Inequalities with Applications*.

Such nonclassical dynamical systems are being used for

evolutionary games (Sandholm (2005, 2011)),

ecological predator-prey networks (Nagurney and Nagurney (2011a, b)),

even **neuroscience** (Girard et al. (2008),

dynamic spectrum model for cognitive radio networks (Setoodeh, Haykin, and Moghadam (2012)),

Future Internet Architectures (Saberi, Nagurney, and Wolf (2014); see also Nagurney et al. (2015)).

Variational Inequality Theory

Proposition 2.1: Formulation of a Constrained Optimization Problem as a Variational Inequality

Let X^* be a solution to the optimization problem:

$$\textit{Minimize} \quad f(X) \tag{2.2}$$

subject to:

$$X \in \mathcal{K},$$

where f is continuously differentiable and $\mathcal{K}$ is closed and convex. Then X^* is a solution of the variational inequality problem:

$$\langle \nabla f(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{2.3}$$

where $\nabla f(X)$ is the gradient vector of f with respect to X; that is, $\nabla f(X) \equiv (\frac{\partial f(X)}{\partial X_1}, \ldots, \frac{\partial f(X)}{\partial X_N})^T$.

Variational Inequality Theory

Proposition 2.2: Formulation of an Unconstrained Optimization Problem as a Variational Inequality

If $f(X)$ is a convex function and X^* is a solution to $\mathrm{VI}(\nabla f, \mathcal{K})$, then X^* is a solution to the optimization problem (2.2). In the case that the feasible set $\mathcal{K} = R^N$, then the unconstrained optimization problem is also a variational inequality problem.

Variational Inequality Theory

The variational inequality problem can be reformulated as an optimization problem under certain symmetry conditions. Several definitions are now recalled, followed by a theorem presenting the above relationship.

Definition 2.2: Positive Semi-Definiteness and Definiteness

An $N \times N$ matrix $M(X)$, whose elements $m_{ij}(X); i, j = 1, ..., N$, are functions defined on the set $\mathcal{T} \subset R^N$, is said to be positive-semidefinite on $\mathcal{T}$ if

$$v^T M(X) v \geq 0, \quad \forall v \in R^N, X \in \mathcal{T}. \tag{2.4}$$

It is said to be positive-definite on $\mathcal{T}$ if

$$v^T M(X) v > 0, \quad \forall v \neq 0, v \in R^N, X \in \mathcal{T}. \tag{2.5}$$

Finally, it is said to be strongly positive-definite on $\mathcal{T}$ if

$$v^T M(X) v \geq \alpha \|v\|^2, \textit{ for some } \alpha > 0, \quad \forall v \in R^N, X \in \mathcal{T}. \tag{2.6}$$

Variational Inequality Theory

Theorem 2.1: Reformulation of a Variational Inequality Problem as an Optimization Problem Under Symmetry Assumption

Assume that $F(X)$ is continuously differentiable on $\mathcal{K}$ and that the Jacobian matrix

$$\nabla F(X) = \begin{bmatrix} \frac{\partial F_1}{\partial X_1} & \cdots & \frac{\partial F_1}{\partial X_N} \\ \vdots & \cdots & \vdots \\ \frac{\partial F_N}{\partial X_1} & \cdots & \frac{\partial F_N}{\partial X_N} \end{bmatrix} \tag{2.7}$$

is symmetric and positive-semidefinite. Then there is a real-valued convex function $f : \mathcal{K} \longmapsto R^1$ satisfying

$$\nabla f(X) = F(X) \tag{2.8}$$

with X^* the solution of $\mathrm{VI}(F, \mathcal{K})$ also being the solution of the mathematical programming problem:

Variational Inequality Theory

$$\textit{Minimize} \quad f(X)$$

subject to:

$$X \in \mathcal{K},$$

where $f(X) = \int F(X)^T dx$, *and* $\int$ *is a line integral.*

Hence, the variational inequality is a more general problem formulation than an optimization problem formulation, since it can also handle a function $F(X)$ with an asymmetric Jacobian.

This enriches the breadth of applications that can be rigorously handled in different disciplines. Next, certain qualitative properties associated with variational inequality problems are presented.

Qualitative Properties

Existence of a solution to a variational inequality problem follows from continuity of the function $F(X)$ that enters the variational inequality, provided that the feasible set $\mathcal{K}$ is compact as stated in Theorem 2.2.

Theorem 2.2: Existence of a Solution

If $\mathcal{K}$ is a compact convex set and $F(X)$ is continuous on $\mathcal{K}$, then the variational inequality problem admits at least one solution X^*.

Theorem 2.3: Existence of a Solution Using a Coercivity Condition

Suppose that $F(X)$ satisfies the coercivity condition

$$\frac{\langle F(X) - F(X_0), X - X_0 \rangle}{\|X - X_0\|} \to \infty \tag{2.9}$$

as $\|X\| \to \infty$ for $X \in \mathcal{K}$ and for some $X_0 \in \mathcal{K}$. Then $\mathrm{VI}(F, \mathcal{K})$ has a solution.

Important Definitions

Definition 2.3: Monotonicity

$F(X)$ is monotone on $\mathcal{K}$ if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{2.10}$$

Definition 2.4: Strict Monotonicity

$F(X)$ is strictly monotone on $\mathcal{K}$ if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle > 0, \quad \forall X^1, X^2 \in \mathcal{K}, \; X^1 \neq X^2. \tag{2.11}$$

Important Definitions

Definition 2.5: Strong Monotonicity

$F(X)$ is strongly monotone on $\mathcal{K}$ if

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq \alpha \|X^1 - X^2\|^2, \quad \forall X^1, X^2 \in \mathcal{K}, \qquad (2.12)$$

where $\alpha > 0$.

Definition 2.6: Lipschitz Continuity

$F(X)$ is Lipschitz continuous on $\mathcal{K}$ if there exists an $L > 0$, such that

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \leq L \|X^1 - X^2\|^2, \quad \forall X^1, X^2 \in \mathcal{K}, \qquad (2.13)$$

where L is known as the Lipschitz constant.

Qualitative Properties

Theorem 2.4: Uniqueness of a Solution Under Strict Monotonicity

Suppose that $F(X)$ is strictly monotone on $\mathcal{K}$. Then the solution to the $\mathrm{VI}(F, \mathcal{K})$ problem is unique, if one exists.

Theorem 2.5: Existence and Uniqueness Under Strong Monotonicity

Suppose that $F(X)$ is strongly monotone on $\mathcal{K}$. Then there exists precisely one solution X^* to $\mathrm{VI}(F, \mathcal{K})$.

Note that, according to Theorem 2.5, strong monotonicity of the function F guarantees both existence and uniqueness of a solution, in the case of an unbounded feasible set $\mathcal{K}$. If the feasible set $\mathcal{K}$ is compact, that is, closed and bounded, the continuity of F guarantees the existence of a solution. The strict monotonicity of F is then sufficient to guarantee the uniqueness of a solution, provided that it exists.

Relationships Between VIs and Game Theory

Nash (1950, 1951) developed noncooperative game theory, involving multiple players, each of whom acts in her own interest.

Consider a game with m players, each player i having, without loss of generality, a strategy vector $X_i = \{X_{i1}, ..., X_{in}\}$ selected from a closed, convex set $K_i \subset R^n$. Each player i seeks to maximize her utility function, U_i: $\mathcal{K} \rightarrow R$, where $\mathcal{K}$=$K_1 \times K_2 \times \cdots \times K_m \subset R^{mn}$. The utility of player i, U_i, depends not only on her own strategy vector, X_i, but also on the strategy vectors of the other players, $(X_1, \ldots, X_{i-1}, X_{i+1}, \ldots, X_m)$. An equilibrium is achieved if no one can increase her utility by unilaterally altering the value of her strategy vector.

Nash Equilibrium

Definition 2.7: Nash Equilibrium

A Nash equilibrium is a strategy vector

$$X^* = (X_1^*, \dots, X_m^*) \in \mathcal{K}, \tag{2.14}$$

where

$$U_i(X_i^*, \hat{X}_i^*) \geq U_i(X_i, \hat{X}_i^*), \quad \forall X_i \in K_i, \forall i, \tag{2.15}$$

and $\hat{X}_i^* = (X_1^*, \dots, X_{i-1}^*, X_{i+1}^*, \dots, X_m^*)$.

The Relationships between VIs and Game Theory

It has been shown by Hartman and Stampacchia (1966) and Gabay and Moulin (1980) that, given continuously differentiable and concave utility functions, U_i, $\forall i$, the Nash equilibrium problem can be formulated as a variational inequality problem defined on $\mathcal{K}$.

Theorem 2.6: Variational Inequality Formulation of Nash Equilibrium

Under the assumption that each utility function U_i is continuously differentiable and concave, X^* is a Nash equilibrium if and only if $X^* \in \mathcal{K}$ is a solution of the variational inequality

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad X \in \mathcal{K}, \tag{2.16}$$

where $F(X) \equiv (-\nabla_{X_1} U_1(X), \ldots, -\nabla_{X_m} U_m(X))^T$, and $\nabla_{X_i} U_i(X) = (\frac{\partial U_i(X)}{\partial X_{i1}}, \ldots, \frac{\partial U_i(X)}{\partial X_{in}})$.

The conditions for existence and uniqueness of a Nash equilibrium can be readily obtained from the previous Theorems.

Generalized Nash Equilibrium

We now turn to a discussion of Generalized Nash Equilibrium (GNE) in which the constraints underlying the players' strategies also depend on the strategies of their rivals.

Definition 2.10: Generalized Nash Equilibrium

A strategy vector $X^* \in K \equiv \prod_{i=1}^{m} K_i, X^* \in \mathcal{S}$, *constitutes a Generalized Nash Equilibrium if for each player* i; $i = 1, ..., m$:

$$U_i(X_i^*, \hat{X}_i^*) \geq U_i(X_i, \hat{X}_i^*), \quad \forall X_i \in K_i, \forall X \in \mathcal{S}, \tag{2.17}$$

where

$$\hat{X}_i^* \equiv (X_1^*, \ldots, X_{i-1}^*, X_{i+1}^*, \ldots, X_m^*),$$

K_i *is the feasible set of individual player* i *and* $\mathcal{S}$ *is the feasible set consisting of the shared constraints.*

Generalized Nash Equilibrium

Definition 2.11: Variational Equilibrium

A strategy vector X^* is said to be a variational equilibrium of the above Generalized Nash Equilibrium game if $X^* \in \mathcal{K}$, where $\mathcal{K} \equiv K \cap \mathcal{S}$, is a solution of the variational inequality:

$$-\sum_{i=1}^{m} \langle \nabla_{X_i} \hat{U}_i(X^*), X_i - X_i^* \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{2.18}$$

Bensoussan (1974) formulated the GNE problem as a quasivariational inequality. GNE problems are challenging to solve as quasivariational inequality problems since the state-of-the-art in terms of algorithms is not as advanced as that for variational inequality problems. Kulkarni and Shanbhag (2012) provide sufficient conditions to establish the theory of a *Variational Equilibrium* as a refinement of the GNE, which is highly relevant to applications in the COVID-19 pandemic.

An Algorithm

There are many algorithms for the computation of solutions to variational inequality problems, including those based on the general iterative schemes of Dafermos (1983) and Dupuis and Nagurney (1993).

In this tutorial, we use the modified projection method of Korpelevich (1977), which requires only Lipschitz continuity and monotonicity of $F(X)$ for convergence, provided a solution exists.

We especially are interested in algorithms that resolve the variational inequality problem into subproblems that can be solved easily and exactly in closed form.

Geometric Interpretation of a Projection

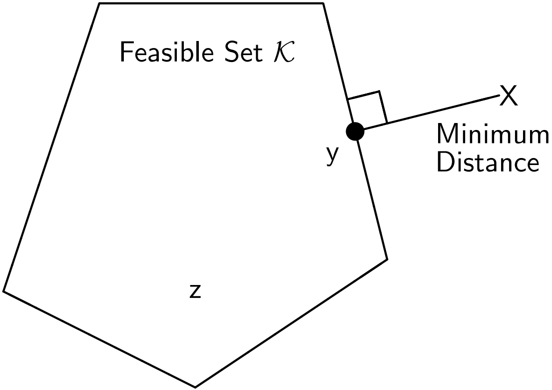


Figure: The projection y of X on the set $\mathcal{K}$

An Algorithm

The modified projection method, with τ denoting an iteration counter, is presented below.

Step 0: Initialization

Set $X^0 \in \mathcal{K}$. Let $\tau = 1$ and let β be a scalar such that $0 < \beta \leq \frac{1}{L}$, where L is the Lipschitz continuity constant (cf. (2.13)).

Step 1: Computation

Compute $\bar{X}^\tau$ by solving the variational inequality subproblem:

$$\langle \bar{X}^\tau + \beta F(X^{\tau-1}) - X^{\tau-1}, X - \bar{X}^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{2.19}$$

Step 2: Adaptation

Compute X^τ by solving the variational inequality subproblem:

$$\langle X^\tau + \beta F(\bar{X}^\tau) - X^{\tau-1}, X - X^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{2.20}$$

Step 3: Convergence Verification

If $\max |X_l^\tau - X_l^{\tau-1}| \leq \epsilon$, for all l, with $\epsilon > 0$, a prespecified tolerance, then stop; else, set $\tau := \tau + 1$, and go to Step 1.

Convergence

Theorem 2.11: Convergence of the Modified Projection Method

If $F(X)$ is monotone and Lipschitz continuous (and a solution exists), the modified projection algorithm converges to a solution of variational inequality (2.1a).

Commercial Supply Chains and the Inclusion of Labor

Some Motivation - Food Supply Chains

Food is essential to our health and well-being. During the COVID-19 pandemic, the associated supply chains suffered major disruptions.

Food Supply Chain Disruptions Due to COVID-19

The COVID-19 pandemic impacted food supply chains in a dramatic and sustained manner.

- Infections at three of the nation's largest meat processors were significant in 2020. At Tysons Foods, the largest meat processor in the US, **the number of Tyson employees with the coronavirus exploded from less than 1,600 in April 2020 to more than 7,000 by May 25, 2020.**
- **Millions of farm animals had to be culled** because of the shutdown of several big meat processing plants. Enhanced cleaning, redesign, and emphasis on social distancing was slowing down the processing, causing additional delays.
- **Shortages of many types of meats, even organic chicken, were experienced, with price increases**.

Food Supply Chain Disruptions Due to COVID-19

- **Fresh produce (oranges, potatoes, strawberries, etc.) on some farms, had to be discarded** because of lack of timely processing capabilities at food processing plants.
- **Labor needed to pick ripened produce was less available due to migrant labor restrictions, illnesses, etc.**
- With the closures of schools, restaurants, businesses, etc., during part of the pandemic outlets for perishable food changed dramatically. **Distribution channels were being reinvisioned and redesigned.**
- **Food insecurity was rising nationally.**

Food Supply Chain Disruptions Due to COVID-19

USA TODAY

AMERICA'S FOOD CHAIN

As coronavirus pandemic spikes orange juice sales, a Florida citrus grower gets squeezed

Janine Zeitlin, USA TODAY Network - Florida
Updated 8:07 p.m. EDT May 14, 2020

CNN

An Idaho farm is giving away 2 million potatoes because coronavirus has hurt demand

By Alisha Ebrahimji, CNN
Updated 1:33 PM ET, Thu April 16, 2020

Lacking seasonal workers, Italy elevates its long-shunned migrants

The Christian Science Monitor

THEDENVERCHANNEL.COM

Farms encountering guest worker shortage amid new coronavirus restrictions

REUTERS

Piglets aborted, chickens gassed as pandemic slams meat sector

The Washington Post
Democracy Dies in Darkness

The meat industry is trying to get back to normal. But workers are still getting sick — and shortages may get worse.

There are now more than 11,000 coronavirus cases tied to Tyson Foods, Smithfield Foods and JBS

Germany Struggles To Fill Its Farm Labor Shortage After Closing Its Borders

May 20, 2020 · 10:58 AM ET
ROB SCHMITZ

npr

Game Theory Supply Chain Network Model with Labor

This part of the tutorial is based on the paper, **"Supply Chain Game Theory Network Modeling Under Labor Constraints: Applications to the Covid-19 Pandemic,"** A. Nagurney, *European Journal of Operational Research*, **293(3)**, (2021), pp. 880-891, in which a game theory model for supply chains with labor was constructed, under three different sets of constraints, building on our previous work.

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Production, Manufacturing, Transportation and Logistics

Supply chain game theory network modeling under labor constraints: Applications to the Covid-19 pandemic

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ABSTRACT

The Covid-19 pandemic has brought attention to supply chain networks due to disruptions for many reasons, including that of labor shortages as a consequences of illnesses, death, risk mitigation, as well as travel restrictions. Many sectors of the economy from food to healthcare have been competing for workers, as a consequence. In this paper, we construct a supply chain game theory network framework that captures labor constraints under three different scenarios. The appropriate equilibrium constructs are defined, along with their variational inequality formulations. Computed solutions to numerical examples inspired by shortages of migrant labor to harvest fresh produce; specifically, blueberries, in the United States, reveal the impacts of a spectrum of disruptions to labor on the product flows and the profits of the firms in the supply chain network economy. This research adds to the literature in both economics and operations research.

1. Introduction

The Covid-19 pandemic, officially declared by the World Health Organization on March 11, 2020, has demonstrated dramatically the importance of the health of workers to the global economy (Organization, 2020). Product supply chains as varied as those for Personal Protective Equipment (PPEs) (Burki, 2020) and other medical supplies (Ranney, Griffieth, & Jha, 2020), toilet paper and cleaning supplies (Gao, 2020), meat (Corkery & Yaffe-Bellany, 2020) and fresh produce (Knight, 2020; Laborde, Martin, Swinnen, & Vos, 2020), and even blood (Nagurney, 2020a), have been disrupted for reasons including that of the reduction of labor availability because of illnesses and death of workers, along with workers' fear of contracting the disease. Some workers have not been able to travel for seasonal employment because of travel restrictions to mitigate the spread of the coronavirus that causes the Covid-19 disease, resulting in both economic and personal losses (IHS-Markit, 2020). According to Elflein (2020), as of August 14, 2020, Covid-19 had spread to six continents, with over 750,000 succumbing to the coronavirus that causes the Covid-19 disease. Just over 2 months after, as of October 27, 2020, there were 1,160,421 documented deaths globally due to Covid-19 (JHU-CSSE2020, 2020). The negative impact of this global healthcare and economic crisis has affected different tiers of supply chains and associated network economic activities of production, transportation, storage, and distribution.

Labor has emerged as an essential factor in the functioning of supply chains in the pandemic. Without farmers, food cannot be produced. Without workers to harvest, food may end up rotting in the fields or be discarded (see Associated Press, 2020); without freight service provision food cannot be delivered to food processors; without food processors food cannot be processed, resulting in additional waste (see Polansek & Huffstutter, 2020). Ultimately, freight service provision is also essential for deliveries to retail outlets for purchase by consumers or even for direct transport to consumers using, for example, electronic commerce. Freight service workers have also been impacted by Covid-19 (Parker, 2020). Employees at distribution centers have been felled by Covid-19 (Daniels, 2020), as well, and Covid-19 has further exacerbated worker shortages at a time of greater demand because of electronic commerce (see Hardwick, 2020). Shortages of labor in manufacturing facilities were an issue even earlier on in February 2020 in China, since that country was impacted first by the coronavirus, which was to have originated in Wuhan, with implications for global supply chains (see Bloomberg, 2020). Labor shortages arose due to illnesses, need for social distancing in facilities, as well as adherence to quarantines. Each link of a product supply chain requires labor, and, therefore, reduction in labor capacity can propagate through paths of a supply chain network, affecting product flows as well as prices (see, e.g., OECD, 2020). Furthermore, unhealthy workers cannot be fully productive.

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Game Theory Supply Chain Network Model with Labor

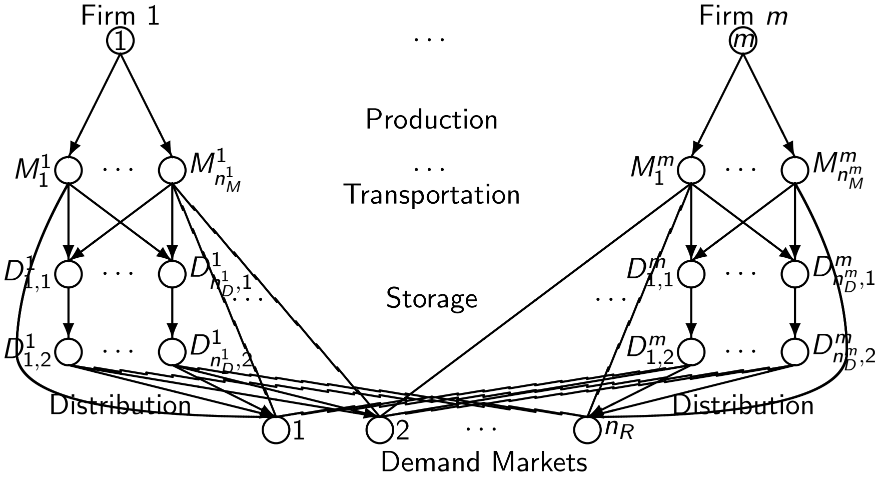


Figure: The Supply Chain Network Topology of the Game Theory Model with Labor

Game Theory Supply Chain Network Model Notation

Table: Game Theory Supply Chain Network Model Notation

Notation	Definition
L^i	The set of links in firm i's supply chain network, with L being all the links.
$G = [\mathcal{N}, L]$	the graph of the supply chain network consisting of all nodes $\mathcal{N}$ and all links L.
P^i_k	set of paths in firm i's supply chain network terminating in demand market k; $\forall i, k$.
P^i	set of all n_{P^i} paths of firm i; $i = 1, \ldots, m$.
P	set of all n_P paths in the supply chain network economy.
x_p; $p \in P^i_k$	nonnegative flow on path p originating at firm node i and terminating at k; $\forall i, k$. Group firm i's path flows into vector $x^i \in R_+^{n_{P^i}}$. Then group all firms' path flows into vector $x \in R_+^{n_P}$.
f_a	nonnegative flow of the product on link a, $\forall a \in L$. Group all link flows into vector $f \in R_+^{n_L}$.
l_a	labor on link a (usually denoted in person hours).
α_a	positive factor relating input of labor to output of product flow on link a, $\forall a \in L$.
$\bar{l}_a$	bound on the availability of labor on link a, $\forall a \in L$
d_{ik}	demand for the product of firm i at demand market k; $\forall i, k$. Group $\{d_{ik}\}$ elements for firm i into vector $d^i \in R_+^{n_R}$ and all demands into vector $d \in R_+^{m \times n_R}$.
$\hat{c}_a(f)$	total operational cost associated with link a, $\forall a \in L$.
π_a	cost of a unit of labor on link a, $\forall a$.
$\rho_{ik}(d)$	demand price function for the product of firm i at demand market k; $\forall i, k$.

For each firm i; $i = 1, \ldots, m$, we must have that:

$$\sum_{p \in P_k^i} x_p = d_{ik}, \quad k = 1, \ldots, n_R. \tag{3.1}$$

The path flows must be nonnegative; that is, for each firm i; $i = 1, \ldots, m$:

$$x_p \geq 0, \quad \forall p \in P^i. \tag{3.2}$$

The link flows of each firm i; $i = 1, \ldots, m$, are related to the path flows as:

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L^i, \tag{3.3}$$

where $\delta_{ap} = 1$, if link a is contained in path p, and 0, otherwise.
We now discuss how labor is related to product flow. We assume a linear production function:

$$f_a = \alpha_a l_a, \quad \forall a \in L^i, \quad i = 1, \ldots, m. \tag{3.4}$$

Game Theory Supply Chain Network Model with Labor

The utility function of firm i, U^i; $i = 1, \ldots, m$, is the profit, given by the difference between its revenue and its total costs:

$$U^i = \sum_{k=1}^{n_R} \rho_{ik}(d) d_{ik} - \sum_{a \in L^i} \hat{c}_a(f) - \sum_{a \in L^i} \pi_a l_a. \quad (3.5a)$$

The functions U_i; $i = 1, \ldots, m$, are assumed to be concave, with the demand price functions being monotone decreasing and continuously differentiable and the total link cost functions being convex and also continuously differentiable.

The Optimization Problem of Each Firm

The optimization problem of each firm i; $i = 1, \ldots, m$, is:

$$\text{Maximize} \quad \sum_{k=1}^{n_R} \rho_{ik}(d) d_{ik} - \sum_{a \in L^i} \hat{c}_a(f) - \sum_{a \in L^i} \pi_a l_a, \quad (3.5b)$$

subject to: (3.1), (3.2), (3.3), and (3.4).

Game Theory Supply Chain Network Model with Labor

A Bound on Labor on Each Supply Chain Network Link

The additional constraints on the fundamental model are:

$$l_a \leq \bar{l}_a, \quad \forall a \in L. \tag{3.6}$$

Game Theory Supply Chain Network Model with Labor

Recall that x^i denotes the vector of strategies, which are the path flows, for each firm i; $i = 1, \ldots, m$. We can redefine the utility/profit functions $\tilde{U}^i(x) \equiv U^i$; $i = 1 \ldots, m$ and group the profits of all the firms into an m-dimensional vector $\tilde{U}$, such that

$$\tilde{U} = \tilde{U}(x). \tag{3.7}$$

Objective function (3.5b), in lieu of the above, can now be expressed as:

$$\text{Maximize} \quad \tilde{U}^i(x) = \sum_{k=1}^{n_R} \tilde{\rho}_{ik}(x) \sum_{p \in P_k^i} x_p - \sum_{a \in L^i} \tilde{c}_a(x) - \sum_{a \in L^i} \frac{\pi_a}{\alpha_a} \sum_{p \in P} x_p \delta_{ap}. \tag{3.8}$$

Supply Chain Nash Equilibrium Conditions

Each firm competes noncooperatively until the following equilibrium is achieved.

Definition 3.1: Supply Chain Network Nash Equilibrium

A path flow pattern $x^* \in K$ *is a supply chain network Nash Equilibrium if for each firm* i; $i = 1, \ldots, m$:

$$\tilde{U}^i(x^{i*}, \hat{x}^{i*}) \geq \tilde{U}^i(x^i, \hat{x}^{i*}), \quad \forall x^i \in K_i, \tag{3.9}$$

where $\hat{x}^{i*} \equiv (x^{1*}, \ldots, x^{i-1*}, x^{i+1*}, \ldots, x^{m*})$, *the feasible set* K_i *for firm* i:
$K_i \equiv \{x^i | x^i \in R_+^{n_{P^i}}, \frac{\sum_{p \in P^i} x_p \delta_{ap}}{\alpha_a} \leq \bar{l}_a, \forall a \in L^i\}$, *for* $i = 1, \ldots, m$, *and*
$K \equiv \prod_{i=1}^m K_i$..

Variational Inequality Formulations

Applying the classical theory of Nash equilibria and variational inequalities, under our imposed assumptions on the underlying functions, it follows that (cf. Gabay and Moulin (1980) and Nagurney (1999)) the solution to the above Nash Equilibrium problem (see Nash (1950, 1951)) coincides with the solution of the variational inequality problem: determine $x^* \in K$, such that

$$-\sum_{i=1}^{m} \langle \nabla_{x^i} \tilde{U}^i(x^*), x^i - x^{i*} \rangle \geq 0, \quad \forall x \in K, \tag{3.10}$$

where $\langle \cdot, \cdot \rangle$ represents the inner product in the corresponding Euclidean space, which here is of dimension n_P, and $\nabla_{x^i} \tilde{U}^i(x)$ is the gradient of $\tilde{U}^i(x)$ with respect to x^i.

We introduce Lagrange multipliers λ_a associated with constraint (3.6), $\forall a \in L$ and group the Lagrange multipliers for each firm i's network L^i into the vector λ^i. Group all such vectors for firms into vector $\lambda \in R_+^{n_L}$. Define feasible sets: $K_i^1 \equiv \{(x^i, \lambda^i) | (x^i, \lambda^i) \in R_+^{n_{P^i} + n_{L^i}}\}$; $i = 1, \ldots, m$, and $K^1 \equiv \prod_{i=1}^{m} K_i^1$.

Variational Inequality Formulations

Theorem 3.1: Alternative VI of Nash Equilibrium

The supply chain network Nash Equilibrium satisfying the Definition 3.1 is equivalent to the solution of the variational inequality: determine vectors of path flows and Lagrange multipliers, $(x^*, \lambda^*) \in K^1$*, where:*

$$\sum_{i=1}^{m}\sum_{k=1}^{n_R}\sum_{p\in P_k^i}\left[\frac{\partial \tilde{C}_p(x^*)}{\partial x_p}+\sum_{a\in L^i}\frac{\lambda_a^*}{\alpha_a}\delta_{ap}+\sum_{a\in L^i}\frac{\pi_a}{\alpha_a}\delta_{ap}-\tilde{\rho}_{ik}(x^*)-\sum_{l=1}^{n_R}\frac{\partial \tilde{\rho}_{il}(x^*)}{\partial x_p}\sum_{q\in P_l^i}x_q^*\right]\times[x_p-x_p^*]$$

$$+\sum_{a\in L}\left[\bar{l}_a-\frac{\sum_{p\in P}x_p^*\delta_{ap}}{\alpha_a}\right]\times[\lambda_a-\lambda_a^*]\geq 0, \quad \forall (x,\lambda)\in K^1; \tag{3.11}$$

where for each path p; $p \in P_k^i$; $i = 1, \ldots, m$; $k = 1, \ldots, n_R$:

$$\frac{\partial \tilde{C}_p(x)}{\partial x_p}\equiv\sum_{a\in L^i}\sum_{b\in L^i}\frac{\partial \hat{c}_b(f)}{\partial f_a}\delta_{ap}, \tag{3.12a}$$

$$\frac{\partial \tilde{\rho}_{ik}(x)}{\partial x_p}\equiv\frac{\partial \rho_{il}(d)}{\partial d_{ik}}. \tag{3.12b}$$

Application of the Modified Projection Method

Realization of the Modified Projection Method Computation Step (2.19) for VI (3.11)

Specifically, at iteration τ, we compute each of the path flows $\bar{x}_p^\tau$, $\forall P_k^i$, $\forall i$, $\forall k$, according to:

$$\bar{x}_p^\tau = \max\{0, x_p^{\tau-1} - \beta(\frac{\partial \tilde{C}_p(x^{\tau-1})}{\partial x_p} + \sum_{a \in L^i} \frac{\lambda_a^{\tau-1}}{\alpha_a}\delta_{ap} + \sum_{a \in L^i} \frac{\pi_a}{\alpha_a}\delta_{ap}$$

$$-\tilde{\rho}_{ik}(x^{\tau-1}) - \sum_{l=1}^{n_R} \frac{\partial \tilde{\rho}_{il}(x^{\tau-1})}{\partial x_p} \sum_{q \in P_l^i} x_q^{\tau-1})\} \quad (3.13)$$

and each of the Lagrange multipliers $\bar{\lambda}_a^\tau$, $\forall a \in L$, according to:

$$\bar{\lambda}_a^\tau = \max\{0, \lambda_a^{\tau-1} - \beta(\bar{l}_a - \frac{\sum_{p \in P} x_p^{\tau-1}\delta_{ap}}{\alpha_a})\}. \quad (3.14)$$

Application of the Modified Projection Method

Realization of the Modified Projection Method Computation Step (2.20) for VI (3.11)

At iteration τ, we compute each of the path flows x_p^τ, $\forall P_k^i$, $\forall i$, $\forall k$, according to:

$$x_p^\tau = \max\{0, x_p^{\tau-1} - \beta(\frac{\partial \tilde{C}_p(\bar{x}^\tau)}{\partial x_p} + \sum_{a \in L^i} \frac{\bar{\lambda}_a^\tau}{\alpha_a}\delta_{ap} + \sum_{a \in L^i} \frac{\pi_a}{\alpha_a}\delta_{ap} - \tilde{\rho}_{ik}(\bar{x}^\tau)$$

$$- \sum_{l=1}^{n_R} \frac{\partial \tilde{\rho}_{il}(\bar{x}^\tau)}{\partial x_p} \sum_{q \in P_l^i} \bar{x}_q^\tau)\} \tag{3.15}$$

and each of the Lagrange multipliers λ_a^τ, $\forall a \in L$, according to:

$$\lambda_a^\tau = \max\{0, \lambda_a^{\tau-1} - \beta(\bar{l}_a - \frac{\sum_{p \in P} \bar{x}_p^\tau \delta_{ap}}{\alpha_a})\}. \tag{3.16}$$

Numerical Experiments

Our numerical examples are based on disruptions in migrant labor in the blueberry supply chain in the Northeast of the US in the summer of 2020.

The numerical examples investigate:

- Modifications in demand price functions;
- Disruptions in labor on a supply chain network link,

with additional numerical examples presented in the paper version of this tutorial.

Numerical Examples

Examples 3.1, 3.2, and 3.3 have the supply chain network topology given below. There are two competing food firms (blueberry farms), each with two production locations, and with a single distribution center. There are two demand markets.

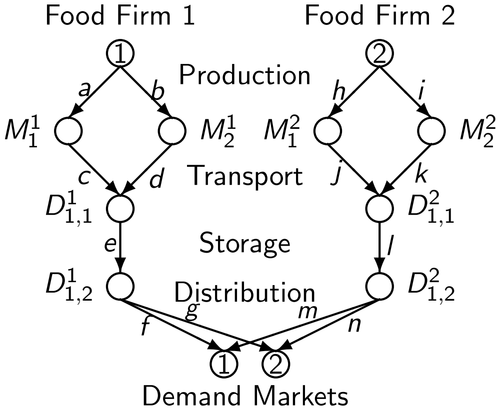


Figure: The Supply Chain Network Topology for the Numerical Examples

Example 3.1 - Baseline Example

The total operational cost functions for Food Firm 1 on its supply chain network L^1 are:

$$\hat{c}_a(f) = .0006f_a^2, \quad \hat{c}_b(f) = .0007f_b^2, \quad \hat{c}_c(f) = .001f_c^2, \quad \hat{c}_d(f) = .001f_d^2,$$

$$\hat{c}_e(f) = .002f_e^2, \quad \hat{c}_f(f) = .005f_f^2, \quad \hat{c}_g(f) = .005f_g^2.$$

Also, the total operational costs associated with Food Firm 2's supply chain network L^2 are:

$$\hat{c}_h(f) = .00075f_h^2, \quad \hat{c}_i(f) = .0008f_i^2, \quad \hat{c}_j(f) = .0005f_j^2, \quad \hat{c}_k(f) = .0005f_k^2,$$

$$\hat{c}_l(f) = .0015f_l^2, \quad \hat{c}_m(f) = .01f_m^2, \quad \hat{c}_n(f) = .01f_n^2.$$

The costs for labor (wages) for Food Firm 1 are:

$$\pi_a = 10, \quad \pi_b = 10, \quad \pi_c = 15, \quad \pi_d = 15, \quad \pi_e = 20, \quad \pi_f = 17, \quad \pi_g = 18,$$

and for Food Firm 2:

$$\pi_h = 11, \quad \pi_i = 22, \quad \pi_j = 15, \quad \pi_k = 15, \quad \pi_l = 18, \quad \pi_m = 18, \quad \pi_n = 18.$$

Example 3.1 - Baseline Example

The link labor productivity factors for the first firm are:

$$\alpha_a = 24, \quad \alpha_b = 25, \quad \alpha_c = 100, \quad \alpha_d = 100, \quad \alpha_e = 50, \quad \alpha_f = 100, \quad \alpha_g = 100,$$

and for the second firm:

$$\alpha_h = 23, \quad \alpha_i = 24, \quad \alpha_j = 100, \quad \alpha_k = 100, \quad \alpha_l = 70, \quad \alpha_m = 100, \quad \alpha_n = 100.$$

The bounds on labor for the first firm are:

$$\bar{l}_a = 10, \quad \bar{l}_b = 200, \quad \bar{l}_c = 300, \quad \bar{l}_d = 300, \quad \bar{l}_e = 100, \quad \bar{l}_f = 120, \quad \bar{l}_g = 120,$$

and for the second firm:

$$\bar{l}_h = 800, \quad \bar{l}_i = 90, \quad \bar{l}_j = 200, \quad \bar{l}_k = 200, \quad \bar{l}_l = 300, \quad \bar{l}_m = 100, \quad \bar{l}_n = 100.$$

Observe that the labor availability on link a is low. This is done in order to capture a disruption to labor in the pandemic.

Example 3.1 - Baseline Example

The demand price functions for Food Firm 1 are:

$$\rho_{11}(d) = -.0001d_{11} - .00005d_{21} + 6, \quad \rho_{12}(d) = -.0002d_{12} - .0001d_{22} + 8.$$

The demand price functions for Food Firm 2 are:

$$\rho_{21}(d) = -.0003d_{21} + 7, \quad \rho_{22}(d) = -.0002d_{22} + 7.$$

The paths are: $p_1 = (a, c, e, f)$, $p_2 = (b, d, e, f)$, $p_3 = (a, c, e, g)$, path $p_4 = (b, d, e, g)$, $p_5 = (h, j, l, m)$, $p_6 = (i, k, l, m)$, $p_7 = (h, j, l, n)$, and $p_8 = (i, k, l, n)$.

Example 3.1 - Baseline Example

The modified projection method converges to the path flow equilibrium pattern reported in Table 2; see also the equilibrium link labor values reported in Table 3. All the Lagrange multipliers are equal to 0.00 except for $\lambda_a^* = 4.925$ with the labor equilibrium value on link a equal to its upper bound of 10.00.

The product prices at equilibrium are:

$$\rho_{11} = 5.97, \quad \rho_{12} = 7.91, \quad \rho_{21} = 6.94, \quad \rho_{22} = 6.96,$$

with equilibrium demands of:

$$d_{11}^* = 172.07, \quad d_{12}^* = 359.15, \quad \rho_{21} = 195.94, \quad \rho_{22} = 197.86.$$

The profit of Food Firm 1 is: 1,671.80 and the profit of Food Firm 2 is: 1,145.06.

Example 3.2 – Modification of Demand Price Functions

Example 3.2 has the same data as Example 3.1 except that we modify the demand price functions for the second firm to include a cross term, so that:

$$\rho_{21}(d) = -.0003d_{21} - .0001d_{11} + 6, \quad \rho_{22}(d) = -.0002d_{22} - .0001d_{12} + 7.$$

The computed equilibrium path flows are reported in Table 2, with the computed equilibrium link labor values given in Table 3.

The Lagrange multipliers are all equal to 0.00 except for $\lambda_a^* = 4.93$.

Example 3.2 – Modification of Demand Price Functions

The product prices at equilibrium are now:

$$\rho_{11} = 5.97, \quad \rho_{12} = 7.91, \quad \rho_{21} = 6.92, \quad \rho_{22} = 6.92,$$

with the equilibrium demands:

$$d_{11}^* = 172.07, \quad d_{12}^* = 359.16, \quad d_{21}^* = 195.48, \quad d_{22}^* = 196.48.$$

The profit for Food Firm 1 is: 1,671.86 and the profit for Food Firm 2 is: 1,134.61. The profit for Food Firm 1 rises ever so slightly, whereas that for Food Firm 2 decreases.

Example 3.3 – Disruptions in Storage Facilities

Example 3.3 has the same data as Example 3.2 except that we now consider a sizable disruption in terms of the spread of COVID-19 at the distribution centers of both food firms with the bounds on labor corresponding to the associated respective links being reduced to:

$$\bar{l}_e = 5, \quad \bar{l}_l = 5.$$

The computed equilibrium path flows for this example are reported in Table 2 with Table 3 having the computed equilibrium link labor values for this example, as well.

All computed equilibrium Lagrange multipliers are now equal to 0 except for those associated with the distribution center links, since the equilibrium labor values attain the imposed upper bounds onn links e and l, with the respective equilibrium Lagrange multiplier values being:

$$\lambda_e^* = 157.2138, \quad \lambda_l^* = 43.6537.$$

Example 3.3 – Disruptions of Labor in Storage Facilities

The product prices at equilibrium are now:

$$\rho_{11} = 5.99, \quad \rho_{12} = 7.94, \quad \rho_{21} = 6.94, \quad \rho_{22} = 6.94,$$

with the equilibrium demands:

$$d_{11}^* = 30.03, \quad d_{12}^* = 219.96, \quad d_{21}^* = 174.61, \quad d_{22}^* = 175.39.$$

The profit for Food Firm 1 is now dramatically reduced to 1,218.74 and the profit for Food Firm 2 also declines, but by a much smaller amount, to 1,126.73.

Equilibrium Path Flows

Table: Equilibrium Product Path Flows for Examples 3.1 Through 3.3

Equilibrium Product Path Flows	Ex. 3.1	Ex. 3.2	Ex. 3.3
$x^*_{p_1}$	73.23	73.22	15.65
$x^*_{p_2}$	98.85	98.85	14.38
$x^*_{p_3}$	166.77	166.78	110.60
$x^*_{p_4}$	192.38	192.38	109.35
$x^*_{p_5}$	142.85	142.62	131.97
$x^*_{p_6}$	53.08	52.86	42.63
$x^*_{p_7}$	143.81	143.12	132.36
$x^*_{p_8}$	54.04	53.36	43.02

Equilibrium Link Labor Values

Table: Equilibrium Link Labor Values for Examples 3.1 Through 3.3

Equilibrium Link Labor Values	Ex. 3.1	Ex. 3.2	Ex. 3.3
l_a^*	10.00	10.00	5.26
l_b^*	11.65	11.65	4.95
l_c^*	2.40	2.40	1.26
l_d^*	2.91	2.91	1.24
l_e^*	10.62	10.62	5.00
l_f^*	1.72	1.72	0.30
l_g^*	3.59	3.59	2.20
l_h^*	12.46	12.42	11.49
l_i^*	4.46	4.43	3.57
l_j^*	2.87	2.86	2.64
l_k^*	1.07	1.06	0.86
l_l^*	5.63	5.60	5.00
l_m^*	1.96	1.95	1.75
l_n^*	1.98	1.96	1.75

Farmers should do everything possible to secure the health of workers at their production/harvesting and other facilities, so that the blueberries can be harvested in a timely manner and so that profits do not suffer. Keeping workers healthy, through appropriate measures, impacts the bottom line!

Some Additional Research

In a recent paper of ours, **"Wage-Dependent Labor and Supply Chain Networks,"** we retained the link productivity factors, so that:

$$f_a = \alpha_a l_a, \quad \forall a \in L,$$

but had labor being wage-dependent, that is:

$$l_a = \gamma_a \pi_a, \forall a \in L.$$

We introduced a supply chain network game theory model without wage bounds on links and one with wage bounds on links:

$$\pi_a \leq \bar{\pi}_a, \quad \forall a \in L.$$

Some Additional Research

The numerical results therein clearly reveal the importance of a holistic approach to supply chain network modeling since decisions made by a specific firm can have unexpected impacts on other competing firms in the supply chain network economy.

Our results strongly suggest that having wages and labor equilibrate without any wage ceilings can be beneficial for an individual firm and also for firms engaged in competition.

Disaster Relief Supply Chains – Competition for Medical Supplies

Some Motivation

The COVID-19 pandemic is a healthcare disaster in which we have seen intense competition for medical supplies.

CORONAVIRUS

Coronavirus USA: Federal fix sought for 'Wild West' COVID-19 PPE competition

By Chuck Goudie and Barb Markoff, Christine Tressel and Ross Weidner

Thursday, April 2, 2020

Global contest for medical equipment amidst the COVID19 pandemic

DHRITI KAMDAR

While the coronavirus pandemic continues to stay strong, the demand and supply for crucial medical equipment is highly unlikely to disappear.

Competition among state, local governments creates bidding war for medical equipment

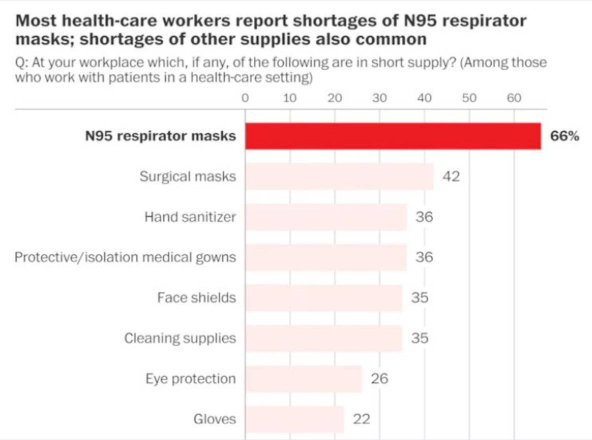
Gov. Cuomo has called for the creation of a 'nationwide buying consortium.'

By ABC News

April 3, 2020, 12:33 PM • 8 min read

Where Are the PPEs?

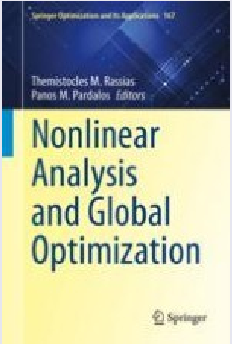
Results of a survey of healthcare workers reported in *The Washington Post*, May 20, 2020.



Some Motivation

- China has historically produced half of the world's face masks, but with the coronavirus originating in Wuhan, China, the country dedicated the majority of the supply for their own citizens.
- Countries, such as Germany, even banned the export of PPEs.
- The intense competition for PPEs led to a dramatic increase in the price.
- The price of N95 masks grew from \$0.38 to \$5.75 each (a 1,413% increase) (Diaz, Sands, and Alesci (2020) and Berklan (2020)).
- Isolation protective gowns experienced a price increase from \$0.25 to \$5.00 (a 1900% increase).
- The price of reusable face shields going from \$0.50 to \$4.00 (a 700% increase).

This part of the tutorial is based on the paper, **"Competition for Medical Supplies Under Stochastic Demand in the Covid-19 Pandemic: A Generalized Nash Equilibrium Framework,"** A. Nagurney, M. Salarpour, J. Dong, and P. Dutta, In: *Nonlinear Analysis and Global Optimization*, T.M. Rassias and P.M. Pardalos, Editors, Springer Nature Switzerland AG, 2021, pp 331-356.



Background

- The **competitive game theory network model** for medical supplies is inspired by the **COVID-19 pandemic**.
- It features salient characteristics of the realities of this pandemic in terms of **competition among organizations/institutions** for supplies under **limited capacities globally** as well as **uncertain demands**.
- The model includes general transportation costs.
- Since organizations, notably, healthcare ones, compete with one another for the limited supplies, given the prices and their associated logistical costs as well as the expected loss due to possible shortages or surpluses, the model is a **Generalized Nash Equilibrium (GNE)** model.
- In the case of GNE models not only do the objective functions of the players in the game depend on the strategies of the other players but the feasible sets do as well.

Relevance to a Plethora of Medical Supplies

The Generalized Nash Equilibrium Network Model for Medical Supplies under Stochastic Demand

The network consists of m supply locations for the medical supplies, with a typical supply point denoted by i, and n locations that are demand points, with a typical demand point denoted by j.

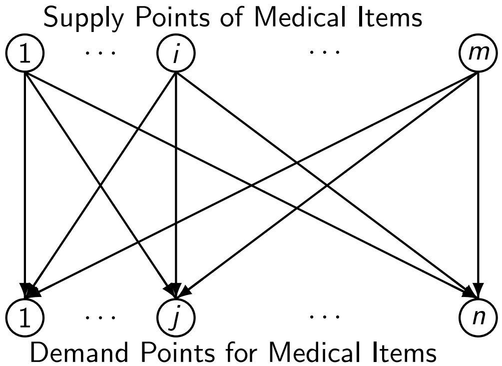


Figure: The Network Structure of the Competitive Game Theory Model for Medical Supplies

The Generalized Nash Equilibrium Network Model for Medical Supplies under Stochastic Demand

Table 1: Notation for the Medical Supply Generalized Nash Equilibrium Network Model

Notation	Definition
q_{ij}	the amount of the medical item purchased from supply location i by j. We first group all the i elements $\{q_{ij}\}$ into the vector q_j and then we group such vectors for all j into the vector $q \in R^{mn}_+$.
v_j	the projected demand at demand point j; $j = 1, \ldots, n$.
d_j	the actual (uncertain) demand for the medical item at demand location j; $j = 1, \ldots, n$.
Δ^-_j	the amount of shortage of the medical item at demand point j; $j = 1, \ldots, n$.
Δ^+_j	the amount of surplus of the medical item at demand point j; $j = 1, \ldots, n$.
λ^-_j	the unit penalty associated with a shortage of the the medical item at demand point j; $j = 1, \ldots, n$.
λ^+_j	the unit penalty associated with a surplus of the medical item at demand point j; $j = 1, \ldots, n$.
ρ_i	the price of the medical item at supply location i; $i = 1, \ldots, m$.
$c_{ij}(q)$	the generalized cost of transportation associated with transporting the the medical item from supply location i to demand location j, which includes the financial cost, any tariffs/taxes, time, and risk. We group all the generalized costs into the vector $c(q) \in R^{mn}$.
S_i	the nonnegative amount of the medical item available for purchase at supply location i; $i = 1, \ldots, m$.
μ_i	the nonnegative Lagrange multiplier associated with the supply constraint at supply location i. We group the Lagrange multipliers into the vector $\mu \in R^m_+$.

The Generalized Nash Equilibrium Network Model for Medical Supplies under Stochastic Demand

Stochastic Demand

Since d_j denotes the actual (uncertain) demand at destination point j, we have:

$$P_j(D_j) = P_j(d_j \leq D_j) = \int_0^{D_j} \mathcal{F}_j(t)dt, \qquad j = 1, \ldots, n, \tag{4.1}$$

where P_j and $\mathcal{F}_j$ denote the probability distribution function, and the probability density function of demand at point j, respectively. v_j is the "projected demand" for the medical item at demand point j; $j = 1, \ldots, n$.

The Generalized Nash Equilibrium Network Model for Medical Supplies under Stochastic Demand

Note that v_j is the "projected demand" for the medical item at demand point $j; j = 1, \ldots, n$.

Shortage and Surplus

The amounts of shortage and surplus at demand point j are calculated, respectively, according to:

$$\Delta_j^- \equiv \max\{0, d_j - v_j\}, \qquad j = 1, \ldots, n, \tag{4.2a}$$

$$\Delta_j^+ \equiv \max\{0, v_j - d_j\}, \qquad j = 1, \ldots, n. \tag{4.2b}$$

The expected values of shortage and surplus at each demand point are, hence:

$$E(\Delta_j^-) = \int_{v_j}^{\infty} (t - v_j)\mathcal{F}_j(t)dt, \qquad j = 1, \ldots, n, \tag{4.3a}$$

$$E(\Delta_j^+) = \int_0^{v_j} (v_j - t)\mathcal{F}_j(t)dt, \qquad j = 1, \ldots, n. \tag{4.3b}$$

The Generalized Nash Equilibrium Network Model for Medical Supplies under Stochastic Demand

Expected Penalties

The expected penalty incurred by demand point j due to the shortage and surplus of the medical item is equal to:

$$E(\lambda_j^- \Delta_j^- \ + \ \lambda_j^+ \Delta_j^+) = \lambda_j^- E(\Delta_j^-) + \lambda_j^+ E(\Delta_j^+), \qquad j = 1, \ldots, n. \quad (4.4)$$

Projected Demand

The projected demand at demand point j, v_j, is equal to the sum of flows of the medical item to j, that is:

$$v_j \equiv \sum_{i=1}^{m} q_{ij}, \qquad j = 1, \ldots, n. \quad (4.5)$$

The Generalized Nash Equilibrium Network Model for Medical Supplies under Stochastic Demand

Objective Function

The objective function of each demand point j is, hence, given by:

$$\text{Minimize} \quad \sum_{i=1}^{m} \rho_i q_{ij} + \sum_{i=1}^{m} c_{ij}(q) + \lambda_j^- E(\Delta_j^-) + \lambda_j^+ E(\Delta_j^+) \tag{4.6}$$

Constraints

$$\sum_{j=1}^{n} q_{ij} \leq S_i, \quad i = 1, \ldots, m, \tag{4.7}$$

$$q_{ij} \geq 0, \quad i = 1, \ldots, m. \tag{4.8}$$

The Generalized Nash Equilibrium Network Model for Medical Supplies under Stochastic Demand

- We assume that the total generalized transportation cost functions are continuously differentiable and convex.
- In our model, the transportation costs can, in general, depend upon the vector of medical item flows since there is competition for freight service provision in the pandemic.
- In the paper, we present some preliminaries that allow us to express the partial derivatives of the expected total shortage and discarding costs of the medical items at the demand points only in terms of the medical item flow variables.
- We prove that the third term in the Objective Function (4.6) is also convex.

The Generalized Nash Equilibrium Network Model for Medical Supplies under Stochastic Demand

Feasible Set

We define the feasible sets $K_j \equiv \{q_j \geq 0\}$; $j = 1, \ldots, n$. We define $K \equiv \prod_{i=1}^{I} K_i$. We also define the feasible set $\mathcal{S} \equiv \{q | q \,\text{satisfying}\, (4.7))\}$, which consists of the shared constraints.

Definition 4.1: Generalized Nash Equilibrium for Medical Items

A vector of medical items $q^* \in K \cap \mathcal{S}$ is a Generalized Nash Equilibrium if for each demand point j; $j = 1, \ldots, n$:

$$DU_j(q_j^*, \hat{q}_j^*) \leq DU_j(q_j, \hat{q}_j^*), \quad \forall q_j \in K_j \cap \mathcal{S}, \tag{4.9}$$

where $\hat{q}_j^* \equiv (q_1^*, \ldots, q_{j-1}^*, q_{j+1}^*, \ldots, q_n^*)$.

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- According to (4.9), an equilibrium is established if no demand point has any incentive to unilaterally change its vector of medical item purchases/shipments.
- In our model not only does the objective function of a demand point depend not only on the vector of strategies of its own strategies and on those of the other demand points, but the feasible set does as well.
- This model is not a Nash (1950, 1951) model, but, rather, it is a Generalized Nash Equilibrium model.
- We define the feasible set $\mathcal{K} \equiv K \cap \mathcal{S}$.
- Our model captures the reality of the intense competitive landscape in the COVID-19 pandemic.

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Definition 4.2: Variational Equilibrium

A vector of medical items $q^* \in \mathcal{K}$ is a Variational Equilibrium of the above Generalized Nash Equilibrium problem if it is a solution to the following variational inequality:

$$\sum_{j=1}^{n} \sum_{i=1}^{m} \frac{\partial DU_j(q^*)}{q_{ij}} \times (q_{ij} - q^*_{ij}) \geq 0, \quad \forall q \in \mathcal{K}, \tag{4.10}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in mn-dimensional Euclidean space.

In expanded form, the variational inequality in (10) is: determine $q^* \in \mathcal{K}$ such that

$$\sum_{j=1}^{n} \sum_{i=1}^{m} \left[\rho_i + \sum_{l=1}^{m} \frac{\partial c_{lj}(q^*)}{\partial q_{ij}} + \lambda_j^+ P_j(\sum_{l=1}^{m} q^*_{lj}) - \lambda_j^- (1 - P_j(\sum_{l=1}^{m} q^*_{lj})) \right] \times \left[q_{ij} - q^*_{ij} \right] \geq 0, \tag{4.11}$$

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Standard Form

We know that finite-dimensional variational inequality problem, VI$(F, \mathcal{K})$, is to determine a vector $X^* \in \mathcal{K} \subset R^N$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{4.12}$$

where F is a given continuous function from $\mathcal{K}$ to R^N, and $\mathcal{K}$ is a given closed, convex set.

We let $X \equiv q$ and $F(X)$ be the vector with elements: $\{\frac{\partial DU_j(q^*)}{q_{ij}}\}$, $\forall j, i$ with $\mathcal{K}$ as originally defined and $N = mn$. Then, clearly, variational inequality (4.11) can be put into standard form (4.12).

The Generalized Nash Equilibrium Network Model for Medical Supplies under Stochastic Demand

We associate a nonnegative Lagrange multiplier μ_i with constraint (4.7), for each supply location $i = 1, \ldots, m$. We group all the Lagrange multipliers into the vector $\mu \in R^m_+$. We define the feasible set $\mathcal{K}^2 \equiv \{(q, \mu) | q \geq 0, \mu \geq 0\}$.

Then, using arguments as in Nagurney, Salarpour, and Daniele (2019), an alternative variational inequality for (4.11) is: determine $(q^*, \mu^*) \in \mathcal{K}^2$ such that

$$\sum_{j=1}^{n} \sum_{i=1}^{m} \left[\rho_i + \sum_{l=1}^{m} \frac{\partial c_{lj}(q^*)}{\partial q_{ij}} + \lambda_j^+ P_j(\sum_{l=1}^{m} q^*_{lj}) - \lambda_j^- (1 - P_j(\sum_{l=1}^{m} q^*_{lj}) + \mu^*_i \right] \times [q_{ij} - q^*_{ij}]$$

$$+ \sum_{i=1}^{m} \left[S_i - \sum_{j=1}^{n} q^*_{ij} \right] \times [\mu_i - \mu^*_i] \geq 0, \quad \forall (q, \mu) \in \mathcal{K}^2. \tag{4.13}$$

The Generalized Nash Equilibrium Network Model for Medical Supplies under Stochastic Demand

Variational inequality (4.13) can also be put into standard form (4.12) if we define $X \equiv (q, \mu)$ and $F(X) \equiv (F^1(X), F^2(X))$ where $F^1(X)$ has as its (i, j)-th component:

$$\rho_i + \sum_{l=1}^m \frac{\partial c_{lj}(q)}{\partial q_{ij}} + \lambda_j^+ P_j(\sum_{l=1}^m q_{lj}) - \lambda_j^-(1 - P_j(\sum_{l=1}^m q_{lj}) + \mu_i; \; i = 1, \ldots, m;$$

$j = 1, \ldots, n$, and the i-th component of $F^2(X)$ is $S_i - \sum_{j=1}^n q_{ij}$, for $i = 1, \ldots, m$. Furthermore, $\mathcal{K} \equiv \mathcal{K}^2$ and $N = mn + m$.

Algorithm - Realization of the Modified Projection Method

Explicit Formula for the Medical Item Flows for Step 1

Determine $\bar{q}_{ij}^{\tau}$ for each i, j at Step 1 iteration τ according to:

$$\bar{q}_{ij}^{\tau} = \max\{0, q_{ij}^{\tau-1} + \beta(-\rho_i - \sum_{l=1}^{m} \frac{\partial c_{lj}(q^{\tau-1})}{\partial q_{ij}} - \lambda_j^{+} P_j(\sum_{l=1}^{m} q_{lj}^{\tau-1}) +$$

$$\lambda_j^{-}(1 - P_j(\sum_{l=1}^{m} q_{lj}^{\tau-1})) - \mu_i^{\tau-1})\}. \tag{4.14}$$

Explicit Formula for the Lagrange Multiplier for Step 1

Determine $\bar{\mu}_i^{\tau}$ for each i at Step 1 iteration τ according to:

$$\bar{\mu}_i^{\tau} = \max\{0, \mu_i^{\tau-1} + \beta(-S_i + \sum_{j=1}^{n} q_{ij}^{\tau-1})\}. \tag{4.15}$$

Step 2 of the modified projection method can be easily derived (see the second part of this tutorial).

Numerical Examples

The numerical examples (with additional ones reported in Nagurney et al. (2021)) are focused on the procurement of N95 masks but in the scenario of increasing demand among smaller healthcare organizations in the form of medical practices. The q_{ij}s are in units since these medical practices are small relative to hospitals, etc.

Example 4.1: One Supply Point and Two Demand Points

The supply chain network topology for this example is given in the Figure.

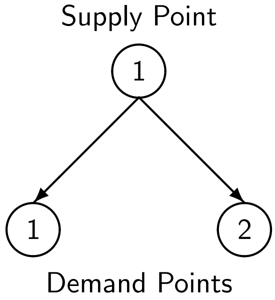


Figure: Supply Chain Network Topology for Example 4.1

Numerical Example 4.1: One Supply Point and Two Demand Points

We assume a uniform probability distribution in the range $[100, 1000]$ at the first demand point. The probability distribution at the second demand point has the same lower and upper bounds as in the first demand point. The additional data are, for the first demand point:

$$\rho_1 = 2, \quad S_1 = 1000, \quad c_{11}(q) = .005q_{11}^2 + .01q_{11}, \quad \lambda_1^- = 1000, \quad \lambda_1^+ = 10,$$

and for the second demand point:

$$c_{12}(q) = .01q_{12}^2 + .02, \quad \lambda_2^- = 1000, \quad \lambda_2^+ = 10.$$

The modified projection method converges to the following equilibrium solution:

$$q_{11}^* = 502.20, \quad q_{12}^* = 497.80, \quad \mu_1^* = 541.61.$$

The available supply of 1000 N95 masks is exhausted between the two demand points, and the associated Lagrange multiplier μ_1^* is positive. The equilibrium conditions hold with excellent accuracy.

Example 4.2: Two Supply Points and Two Demand Points

In Example 4.2, we consider the impacts of the addition of a second supply point to Example 4.1. The supply chain network topology is now as in the Figure.

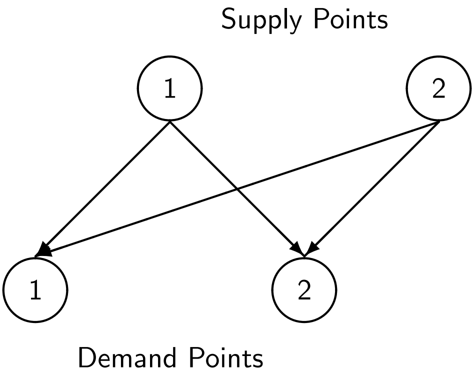


Figure: Supply Chain Network Topology for Example 4.2

Example 4.2: Two Supply Points and Two Demand Points

The data are as in Example 4.1 with the following additions:

$$S_2 = 500, \quad \rho_2 = 3, \quad c_{21}(q) = .015q_{21}^2 + .03, \quad c_{22}(q) = .02q_{22}^2 + .04q_{22}.$$

The modified projection method converges to the equilibrium solution:

$$q_{11}^* = 526.31,\ q_{12}^* = 473.69,\ q_{21}^* = 225.57,\ q_{22}^* = 274.43,$$

$$\mu_1^* = 261.17,\ \mu_2^* = 258.65.$$

With the addition of a new supply point for medical supplies, both demand points gain significantly in terms of the volume of N95 masks that each procures. Furthermore, the supplies of the medical item at each supply point are fully sold out. Hence, both equilibrium Lagrange multipliers are positive.

Example 4.3: Two Supply Points and Three Demand Points

Example 4.3 is constructed from Example 4.2 with Demand Point 3 added, as depicted in the Figure.

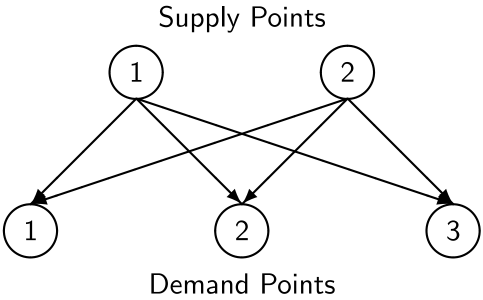


Figure: Supply Chain Network Topology for Example 4.3

Example 4.3: Two Supply Points and Three Demand Points

Example 4.3 has the same data as Example 4.2 but with the addition of data for Demand Point 3 as follows:

$c_{13}(q) = .01q_{13}^2 + .02q_{13}$, $c_{23}(q) = .015q_{23}^2 + .03q_{23}$, $\lambda_3^- = 1000$, $\lambda_3^+ = 10$.

The probability distribution for the N95 masks associated with Demand Point 3 is uniform with a lower bound of 200 and an upper bound of 1000.

The modified projection method converges to the equilibrium solution:

$$q_{11}^* = 360.11, \quad q_{12}^* = 318.83, \quad q_{13}^* = 321.06,$$

$$q_{21}^* = 122.29,\ q_{22}^* = 161.10,\ q_{23}^* = 216.62,\ \mu_1^* = 565.25,\ \mu_2^* = 564.16.$$

Note that, with increasing competition for the N95 masks with another demand point, both Demand Points 1 and 2 experience decreases in procurement of supplies. The two supply points again fully sell out of their N95 masks and the associated equilibrium Lagrange multipliers are both positive.

Example 4.4: Two Supply Points and Four Demand Points

In Example 4.4, yet another demand point is added to the supply chain network topology of Example 4.3. Smaller medical practices are increasingly concerned about being able to secure the much needed PPEs to protect the health of their employees and the viability of their practices.

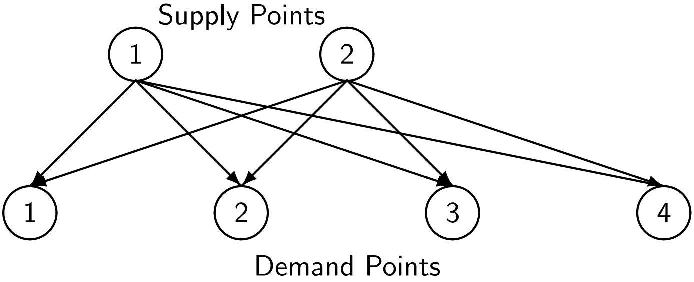


Figure: Supply Chain Network Topology for Example 4.4

Example 4.4: Two Supply Points and Four Demand Points

The data for this example are the same as those for Example 4.3, and the probability distribution structure for the demand at Demand Point 4 is the same, with the following additional data for the new Demand Point 4:

$$c_{14}(q) = .015q_{14}^2 + .03q_{14},\ c_{24}(q) = .025q_{24}^2 + .05q_{24},\ \lambda_4^- = 1000,\ \lambda_4^+ = 10.$$

The modified projection method converges to:

$$q_{11}^* = 260.73,\ q_{12}^* = 229.36,\ q_{13}^* = 251.22,\ q_{14}^* = 258.69,\ q_{21}^* = 79.57,$$

$$q_{22}^* = 109.17,\ q_{23}^* = 160.46,\ q_{24}^* = 150.81,\ \mu_1^* = 725.71,\ \mu_2^* = 724.91.$$

The suppliers of the N95 sell out their supplies. However, the demand points lose in terms of supply procurement for their organizations with the increased demand and competition from yet another demand point.

Some Comments and Insights

The obtained numerical results are consistent with what is being observed in practice and the results also provide managerial insights.

• The numerical results confirm that more supply points with sufficient supplies are needed to guarantee that organizations are not deprived of critical supplies due to competition.

• As a result of this competition and limited local availability, in particular, in the case of supplies such as masks, ventilators, and even coronavirus test kits, we are seeing multiple countries now setting up local production sites with even some companies switching from their usual product manufacturing to the production of much needed medical supplies, including PPEs.

• This model can be applied to study the network economics of a spectrum of medical items, both in the near term, and in the longer term, as in the case of vaccines as well as COVID-19 therapeutics.

Cooperation in Disaster Relief

Cooperation in Disaster Relief

In the previous parts of this tutorial, the focus of the modeling parts was on noncooperative game theory and supply chain network models inspired by the COVID-19 pandemic.

We now turn to cooperation among organizations in a disaster setting. The COVID-19 pandemic is a healthcare disaster and is exacerbating the challenges associated with disaster management of other disasters, including those fueled by climate change.

Cooperation in Disaster Relief

Opportunities for cooperation among organizations engaged in disaster response may exist in their supply chains from procurement to storage and even in the case of transportation and distribution.

Plus, cooperation among organizations may reduce materiel convergence and release resources, including personnel, for more important life-saving tasks.

There is also great promise in the COVID-19 pandemic of enhanced partnerships and these even may be between private companies, including pharmaceutical ones.

Lessons learned from disaster management are, hence, potentially of great benefit to pandemic preparedness, response, and even recovery.

The Goal Here

This part of the tutorial is based on the paper, **"Quantifying Supply Chain Network Synergy for Humanitarian Organizations,"** A. Nagurney and Q. Qiang, *IBM Journal of Research and Development*, **64(1/2)**, 2020, pp. 12:1-12:16.

The Multiproduct Supply Chain Network Models

The Case without Horizontal Cooperation Multiproduct Supply Chain Network Model

We first formulate the multiproduct decision-making optimization problems faced by m organizations without horizontal cooperation. This model is Case 0. Each organization is represented as a network of its supply chain activities, as depicted in the next Figure.

Each organization i; $i = 1, \ldots, m$, has available n^i_M procurement facilities, n^i_S storage facilities, and serves n^i_D disaster areas.

Let $G_i = [N_i, L_i]$ denote the graph consisting of nodes $[N_i]$ and directed links $[L_i]$ representing the supply chain activities associated with each organization i; $i = 1, \ldots, m$.

Let L^0 denote the links: $L_1 \cup L_2 \cup \cdots L_m$ as in the Figure. Each organization is involved in the procurement, transportation, storage, and distribution of J products, with a typical product denoted by j.

Case Without Cooperation

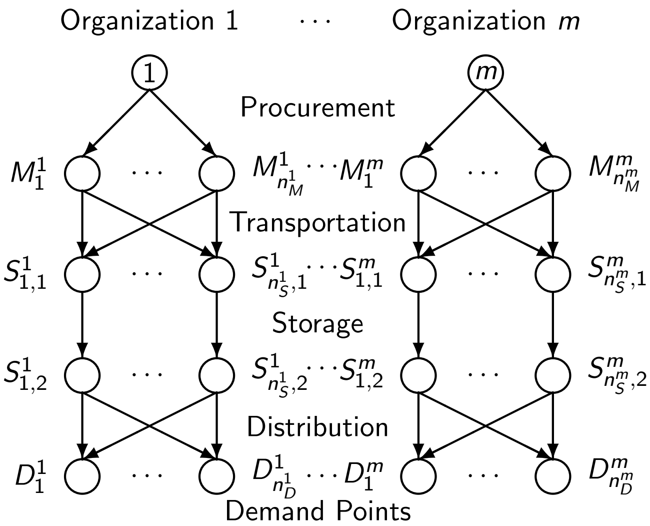


Figure: Supply Chains of Organizations 1 through m Prior to Cooperation

The notation and discussion below build upon those for the previous model.

Demands for the products are assumed to be random and are associated with each product, and each demand point.

Let d_{ik}^{j} denote the random variable representing the actual demand for product j and let v_{ik}^{j} denote the projected random demand for product j; $j = 1, \ldots, J$, at demand point D_k^i for $i = 1, \ldots, m$; $k = 1, \ldots, n_D^i$.

In addition, the probability density function of the actual demand for product j is $\mathcal{F}_{ik}^{j}(t)$ at disaster area D_k^i; $i = 1, \ldots, m$; $k = 1, \ldots, n_D^i$.

Hence, we can define the cumulative probability distribution function of d_{ik}^j as $\mathcal{P}_{ik}^j(v_{ik}^j) = \mathcal{P}_{ik}^j(d_{ik}^j \leq v_{ik}^j) = \int_0^{v_{ik}^j} \mathcal{F}_{ik}^j(t)d(t)$.

Following Masoumi, Yu, and Nagurney (2017) and Dong, Zhang, and Nagurney (2004), we also define the supply shortage and surplus for product j; $j = 1, \ldots, J$, at disaster area D_k^i; $i = 1, \ldots, m$; $k = 1, \ldots, n_D^i$, as

$$\Delta_{ik}^{j-} \equiv \Delta_{ik}^{j-}(v_{ik}^j) \equiv \max\{0, d_{ik}^j - v_{ik}^j\} \qquad (5.1a)$$

$$\Delta_{ik}^{j+} \equiv \Delta_{ik}^{j+}(v_{ik}^j) \equiv \max\{0, v_{ik}^j - d_{ik}^j\}. \qquad (5.1b)$$

The Multiproduct Supply Chain Network Models

The expected value of the shortage Δ_{ik}^{j-}, denoted by $E(\Delta_{ik}^{j-})$, and of the surplus Δ_{ik}^{j+}, denoted by $E(\Delta_{ik}^{j+})$, for $j = 1, \ldots, J$; D_k^i; $i = 1, \ldots, m$; $k = 1, \ldots, n_D^i$, are

$$E(\Delta_{ik}^{j-}) = \int_{v_{ik}^j}^{\infty} (t - v_{ik}^j)\mathcal{F}_{ik}^j(t)d(t), \quad E(\Delta_{ik}^{j+}) = \int_0^{v_{ik}^j} (v_{ik}^j - t)\mathcal{F}_{ik}^j(t)d(t). \tag{5.2}$$

The penalty associated with the shortage and the surplus of the demand for product j; $j = 1, \ldots, J$, at the disaster area D_k^i is denoted by λ_{ik}^{j-} and λ_{ik}^{j+}, respectively, where $i = 1, \ldots, m$; $k = 1, \ldots, n_D^i$.

Case Without Cooperation

A path consists of a sequence of links originating at a node i; $i = 1, \ldots, m$, corresponding to supply chain activities of: procurement, transportation, storage, and distribution of the products to the disaster area nodes.

Let x_p^j denote the nonnegative flow of product j on path p. Let $P^0_{D^i_k}$ denote the set of all paths joining an origin node i with (destination) disaster area node D^i_k.

The conservation of flow equations are: for each organization $i; i = 1, \ldots, m$, each product $j; j = 1, \ldots, J$, and each disaster area $D^i_k; k = 1, \ldots, n^i_D$:

$$\sum_{p \in P^0_{D^i_k}} x_p^j = v^j_{ik}, \quad i = 1, ..., m; \quad j = 1, \ldots, J; \quad k = 1, \ldots, n^i_D. \tag{5.3}$$

Case Without Cooperation

Links are denoted by a, b, etc. Let f_a^j denote the flow of product j on link a.

We also have the following conservation of flow equations:

$$f_a^j = \sum_{p \in P^0} x_p^j \delta_{ap}, \quad j = 1 \ldots, J; \quad \forall a \in L^0, \tag{5.4}$$

where $\delta_{ap} = 1$ if link a is contained in path p and $\delta_{ap} = 0$, otherwise.

P^0 denotes the set of *all* paths in the Figure, that is, $P^0 = \cup_{i=1,\ldots,I; k=1,\ldots,n_D^i} P^0_{D_k^i}$. The path flows must be nonnegative, that is,

$$x_p^j \geq 0, \quad j = 1, \ldots, J; \quad \forall p \in P^0. \tag{5.5}$$

The path flows are grouped into the vector x.

Case Without Cooperation

There is a total cost associated with each product j; $j = 1, \ldots, J$, and each link of the network of each organization i; $i = 1, \ldots, m$.

The total cost on a link a associated with product j is denoted by $\hat{c}_a^j$. The total costs can be influenced by uncertainty factors.

The total cost on link a, $\hat{c}_a^j$, takes the form:

$$\hat{c}_a^j = \hat{c}_a^j(f_a^1, \ldots, f_a^J, \omega_a^j), \quad j = 1, \ldots, J; \quad \forall a \in L^i, \forall i, \tag{5.6}$$

where ω_a^j is a random variable associated with various disaster events, which have an impact on the total cost of link a, $\forall a$, and product j; $j = 1, \ldots, J$. It is assumed that the distribution of the ω_a^js is known.

Case Without Cooperation

The Optimization Problem of Each Organization

Each organization i; $i = 1, \ldots, m$ seeks to determine the link flows and the projected random demands that solve the following optimization problem:

$$\text{Minimize} \quad \Bigg[E(\sum_{j=1}^{J} \sum_{a \in L_i} \hat{c}_a^j(f_a^1, \ldots, f_a^J, \omega_a^j)) + \xi_i(V(\sum_{j=1}^{J} \sum_{a \in L_i} \hat{c}_a^j(f_a^1, \ldots, f_a^J, \omega_a^j)))$$

$$+ \sum_{j=1}^{J} \sum_{k=1}^{n_D^i} (\lambda_{ik}^{j-} E(\Delta_{ik}^{j-}) + \lambda_{ik}^{j+} E(\Delta_{ik}^{j+})) \Bigg] \tag{5.7}$$

subject to: (5.3) – (5.5) and the following capacity constraints:

$$\sum_{j=1}^{J} \gamma_j f_a^j \leq u_a, \quad \forall a \in L_i, \tag{5.8}$$

where γ_j in (5.8) is the volume taken up by product j and u_a is the capacity of a.

Case Without Cooperation

The total operational costs and the variances in (5.7) are assumed to be convex. We know that $\sum_{k=1}^{n_D^i}(\lambda_{ik}^{j-}E(\Delta_{ik}^{j-})+\lambda_{ik}^{j+}E(\Delta_{ik}^{j+}))$ is also convex (see, also, Nagurney, Masoumi, and Yu (2012)). We know then that the objective function (5.7) is convex for each i; $i=1,\ldots,m$. Also, the individual terms in (5.7) are continuously differentiable.

Under the above imposed assumptions, the optimization problem is a convex optimization problem and, clearly, the feasible set underlying the problem represented by the constraints (5.3) – (5.5) and (5.8) is non-empty, so it follows from the standard theory of nonlinear programming that an optimal solution exists.

The objective function (5.7) is referred to as the total generalized cost TGC_i^0 for $i = 1, \ldots, m$. We associate the Lagrange multiplier η_a with constraint (5.8) for each $a \in L^0$ with $\eta_a \geq 0, \forall a \in L^0$ and we denote the associated optimal Lagrange multiplier by $\eta_a^*, \forall a \in L^0$. We group the link flows into the vector f, the projected demands into the vector v, and the Lagrange multipliers into the vector η.

Let $\mathcal{K}^0$ denote the set where
$\mathcal{K}^0 \equiv \{(f, v, \eta) | \exists x \, \text{such that} \, (5.3) - (5.5) \, \text{and} \, \eta \geq 0 \, \text{hold}\}$.

Since we are considering Case 0, we denote the solution of variational inequality (VI) (5.9) below as $(f^{0*}, v^{0*}, \eta^{0*})$ and we refer to the corresponding vectors of variables with superscripts of 0. We now state a theorem, due to Nagurney and Qiang (2020).

Case Without Cooperation

Theorem 5.1: VI Formulation of Case 0: No Cooperation

The vector $(f^{0*}, v^{0*}, \eta^{0*}) \in \mathcal{K}^0$ is an optimal solution to (5.7), for all organizations i; $i = 1, \ldots, m$, subject to their constraints (5.3)–(5.5) and (5.8), if and only if it satisfies the variational inequality problem:

$$\sum_{i=1}^{m}\sum_{j=1}^{J}\sum_{a\in L_i}\Big[\frac{\partial E(\sum_{l=1}^{J}\sum_{a\in L_i}\hat{c}_a^l(f_a^{1*},\ldots,f_a^{J*},\omega_a^l))}{\partial f_a^j}$$

$$+\xi_i\frac{\partial V(\sum_{l=1}^{J}\sum_{a\in L_i}\hat{c}_a^l(f_a^{1*},\ldots,f_a^{J*},\omega_a^l))}{\partial f_a^j}+\gamma_j\eta_a^*\Big]\times[f_a^j-f_a^{j*}]$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{J}\sum_{k=1}^{n_D^i}\left[\lambda_{ik}^{j+}\mathcal{P}_{ik}^j(v_{ik}^{j*})-\lambda_{ik}^{j-}(1-\mathcal{P}_{ik}^j(v_{ik}^{j*})\right]\times[v_{ik}^j-v_{ik}^{j*}]$$

$$+\sum_{a\in L^0}[u_a-\sum_{j=1}^{J}\gamma_j f_a^{j*}]\times[\eta_a-\eta_a^*]\geq 0,\quad \forall (f^0,v^0,\eta^0)\in\mathcal{K}^0. \tag{5.9}$$

Case with Cooperation

We now formulate the case with horizontal cooperation of the multiproduct supply chain network model, referred to as Case 1.

The next Figure represents the supply chain network topology for Case 1.

There is a *supersource* node 0, which represents the "teaming/merging" in terms of cooperation of the organizations in terms of their supply chain networks with additional links connecting node 0 to nodes 1 through m.

Case with Cooperation

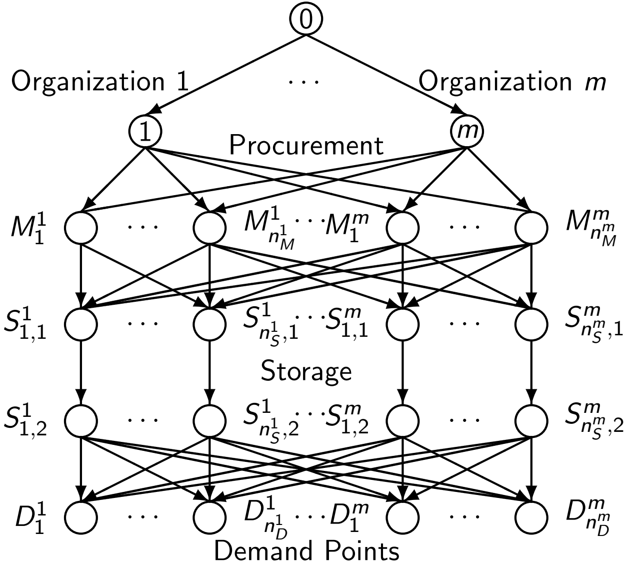


Figure: Supply Chain Network after Cooperation

Case with Cooperation

The optimization problem in Case 1 is also concerned with cost and risk minimization.

We refer to the network in the latest Figure, underlying this integration, as $G^1 = [N^1, L^1]$ where $N^1 \equiv N^0 \cup$ node 0 and $L^1 \equiv L^0 \cup$ the additional links as in the Figure and we associate total cost functions as in (5.6) with the new links, for each product j.

If the total cost functions on the cooperation links connecting node 0 to node 1 through node m are set equal to zero, this means that the cooperation is *costless* in terms of the integrated supply chain network of the organizations.

Case with Cooperation

A path p now originates at node 0 and ends in one of the bottom disaster nodes. Let x_p^j, under the cooperation network configuration given in the Figure, denote the flow of product j on path p joining (origin) node 0 with a disaster area node.

Then, the following conservation of flow equations must hold for each i, j, k:

$$\sum_{p \in P^1_{D^i_k}} x_p^j = v_{ik}^j, \tag{5.10}$$

where $P^1_{D^i_k}$ denotes the set of paths connecting node 0 with disaster area node D^i_k in the Figure. Because of cooperation, the disaster areas can obtain each product j from any procurement facility, and any storage facility. The set of paths $P^1 \equiv \cup_{i=1,m;k=1,\ldots,n^i_D} P^1_{D^i_k}$.

Case with Cooperation

As previously, let f_a^j denote the flow of product j on link a.

We must also have the following conservation of flow equations satisfied:

$$f_a^j = \sum_{p \in P^1} x_p^j \delta_{ap}, \quad j = 1, \ldots, J; \quad \forall a \in L^1. \tag{5.11}$$

In addition, the path flows must be nonnegative for each product j:

$$x_p^j \geq 0, \quad j = 1, \ldots, J; \quad \forall p \in P^1. \tag{5.12}$$

The supply chain network activities have nonnegative capacities, denoted as u_a, $\forall a \in L^1$, with γ_j representing the volume factor for product j. The following constraints must, hence, hold:

$$\sum_{j=1}^{J} \gamma_j f_a^j \leq u_a, \quad \forall a \in L^1, \tag{5.13}$$

where ξ is the associated risk aversion factor of the teamed organizations under cooperation.

Case with Cooperation

The Optimization Problem for Cooperation

$$\text{Minimize} \quad E(\sum_{j=1}^{J}\sum_{a\in L^1}\hat{c}_a^j(f_a^1,\ldots,f_a^J,\omega_a^j)) + \xi\left[V(\sum_{j=1}^{J}\sum_{a\in L^1}\hat{c}_a^j(f_a^1,\ldots,f_a^J,\omega_a^j))\right]$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{J}\sum_{k=1}^{n_D^i}(\lambda_{ik}^{j-}E(\Delta_{ik}^{j-})+\lambda_{ik}^{j+}E(\Delta_{ik}^{j+})) \tag{5.14}$$

subject: (5.10) – (5.13).

The solution to the optimization problem (5.14) subject to the constraints can also be obtained as a solution to a VI problem, similar to (5.9), where now links $a \in L^1$. The vectors f, v, and η keep their prior definitions, but are re-dimensioned accordingly and superscripted with 1. Instead of the feasible set $\mathcal{K}^0$ we now have

$\mathcal{K}^1 \equiv \{(f, v, \eta) | \exists x \text{ such that } (5.10) - (5.12) \text{ hold and } \eta \geq 0\}$.

Case with Cooperation

We denote the solution to the VI problem (5.15) governing Case 1 by $(f^{1*}, v^{1^*}, \eta^{1*})$ and the vectors of corresponding variables as (f^1, v^1, η^1).

Theorem 5.2: VI Formulation of Case 1: Cooperation

The vector $(f^{1*}, v^{1*}, \eta^{1*}) \in \mathcal{K}^1$ is an optimal solution to (5.14), subject to constraints (5.10)–(5.13), if and only if it satisfies the variational inequality problem:

$$\sum_{j=1}^{J} \sum_{a \in L^1} \frac{\partial E(\sum_{l=1}^{J} \sum_{a \in L^1} \hat{c}_a^l(f_a^1, \ldots, f_a^J, \omega_a^l))}{\partial f_a^j} + \xi \frac{\partial V(\sum_{l=1}^{J} \sum_{a \in L^1} \hat{c}_a^l(f_a^{1*}, \ldots, f_a^{J*}, \omega_a^l))}{\partial f_a^j}$$

$$\times [f_a^j - f_a^{j*}] + \sum_{i=1}^{m} \sum_{j=1}^{J} \sum_{k=1}^{n_D^i} \left[\lambda_{ik}^{j+} \mathcal{P}_{ik}^j(v_{ik}^{j*}) - \lambda_{ik}^{j-}(1 - \mathcal{P}_{ik}^j(v_{ik}^{j*}) \right] \times [v_{ik}^j - v_{ik}^{j*}]$$

$$+ \sum_{a \in L^1} [u_a - \sum_{j=1}^{J} \gamma_j f_a^{j*}] \times [\eta_a - \eta_a^*] \geq 0, \quad \forall (f^1, v^1, \eta^1) \in \mathcal{K}^1. \tag{5.15}$$

Total Generalized Cost Definitions

Definition 5.1: Total Generalized Costs at the Optimal Solutions to the Supply Chain Network Problems without and with Cooperation

Let TGC^{0*} *denote the total generalized cost:*
$\sum_{i=1}^{m} TGC_i^0 = E(\sum_{j=1}^{J} \sum_{a \in L^0} \hat{c}_a^j(f_a^1, \dots, f_a^J, \omega_a^j)) +$
$\sum_{i=1}^{m} \xi_i \left[V(\sum_{j=1}^{J} \sum_{a \in L_i} \hat{c}_a^j(f_a^1, \dots, f_a^J, \omega_a^j)) \right] + \sum_{i=1}^{m} \sum_{j=1}^{J} \sum_{k=1}^{n_D^i} (\lambda_{ik}^{j-} E(\Delta_{ik}^{j-}) +$
$\lambda_{ik}^{j+} E(\Delta_{ik}^{j+}))$*, evaluated at the optimal solution* $(f^{0*}, v^{0*}, \eta^{0*})$ *to (5.9).*

Also, let $TGC^{1*} =$
$E(\sum_{j=1}^{J} \sum_{a \in L^1} \hat{c}_a^j(f_a^1, \dots, f_a^J, \omega_a^j)) + \xi \left[V(\sum_{j=1}^{J} \sum_{a \in L^1} \hat{c}_a^j(f_a^1, \dots, f_a^J, \omega_a^j)) \right]$
$+ \sum_{i=1}^{m} \sum_{j=1}^{J} \sum_{k=1}^{n_D^i} (\lambda_{ik}^{j-} E(\Delta_{ik}^{j-}) + \lambda_{ik}^{j+} E(\Delta_{ik}^{j+}))$*, denote the total generalized cost evaluated at the solution* $(f^{1*}, v^{1*}, \eta^{1*})$ *to (5.15).*

Synergy Quantification

We denote the synergy by $\mathcal{S}^{TGC}$. It is the percentage difference between the total generalized cost without *vs*. with the horizontal cooperation (evaluated at the respective optimal solutions):

$$\mathcal{S}^{TGC} \equiv [\frac{TGC^{0*} - TGC^{1*}}{TGC^{0*}}] \times 100\%. \tag{5.16}$$

Observe from (5.16) that the lower the total generalized cost TGC^{1*}, the higher the synergy associated with the supply chain network cooperation and, therefore, the greater the total cost savings resulting from the cooperation.

Synergy Quantification

The total generalized costs include not only the monetary costs, but also the risks and uncertainties involved in the supply chain as well as the associated penalties of shortages and surpluses.

In specific disaster relief operations, including in the pandemic, one may evaluate the integration of supply chain networks with only a subset of the links connecting the original supply chain networks.

A Theoretical Result

We now recall an interesting theorem, due also to Nagurney and Qiang (2020), which reveals that, under certain assumptions related to the total operational costs associated with the supply chain integration and risk factors, the associated synergy can never be negative.

Theorem 5.3

If the total generalized cost functions associated with the cooperation links from node 0 *to nodes* 1 *through* m *for each product are identically equal to zero, and if the risk aversion factors* ξ_i*;* $i = 1, \ldots, m$*, are all equal and set to* ξ*, then the associated synergy,* $\mathcal{S}^{TGC}$*, can never be negative.*

VI in Path Flows

Because of the conservation of flow equations (5.10) and (5.11), and constraints (5.12) and (5.13), we can construct a variational inequality formulation equivalent to the one in (5.15), but in path flows, rather than in links flows (the same holds for a path flow version of VI (5.9)). The alternative variational inequality enables a nice application of the modified projection method.

We group the path flows into the vector $x \in R^{n_{P^1}}$, where n_{P^1} is the number of paths in P^1. We let n_{L^1} denote the number of links in L^1.

We define the feasible set $\mathcal{K}^2 \equiv \{(x, \eta) | x \geq 0, \eta \geq 0\}$.

VI in Path Flows

VI in Path Flows

A vector of path flows and Lagrange multipliers $(x^*, \eta^*) \in \mathcal{K}^2$ is an optimal solution to problem (5.14) subject to (5.10) – (5.13) if and only if it satisfies the variational inequality:

$$\sum_{j=1}^{J} \sum_{p \in P^1} \left[\frac{\partial TGC^1(x^*)}{\partial x_p^j} + \gamma_j \sum_{a \in L^1} \eta_a^* \delta_{ap} \right] \times \left[x_p^j - x_p^{j*} \right]$$

$$+ \sum_{a \in L^1} \left[u_a - \sum_{j=1}^{J} \gamma_j \sum_{p \in P^1} x_p^{j*} \delta_{ap} \right] \times [\eta_a - \eta_a^*] \geq 0, \quad \forall (x, \eta) \in \mathcal{K}^2. \quad (5.17)$$

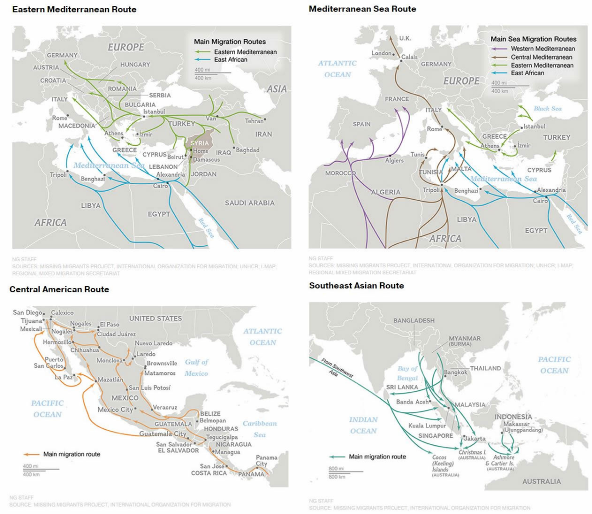
Numerical Examples

The numerical examples are inspired, in part, by ongoing refugee/migrant crises as in Central America and Mexico, which are ongoing and have been exacerbated in the COVID-19 pandemic.

Slow-onset, ongoing disasters are providing huge challenges for various organizations, including humanitarian ones, and governments, to provide the necessary food, water, medicines, etc., to the needy in a variety of shelters.

The numerical examples are stylized but reflect real-world features. Furthermore, as in the case of the refugee/migrant crisis emanating from Central America, numerous organizations are involved in providing assistance and, hence, it is valuable to be able to assess possible synergies since the demand is so great. Using carefully calibrated historical data and information, the models can be used to assist the organizations on how to cooperate in terms of the delivery of relief products in a cost-effective manner.

Migration Routes



Source: National Geographic via IOM UN Migration Blog - 2015 data

Motivation and Some Background

Vivid depictions of people fleeing their origin locations permeate the news, whether attempting to escape the great strife and suffering in Syria; the violence in parts of Central America, the economic collapse of Venezuela, and even flooding in parts of Asia as well as droughts in parts of Africa. And we have seen the news about refugees from Afghanistan.

Motivation and Some Background

At times, refugees will travel in extremely dangerous conditions to escape the dire circumstances at their origin nodes.

In 2015, the UN Refugee Agency reported a maritime refugee crisis with, in the first half of that year, 137,000 refugees crossing the Mediterranean Sea to Europe, via very risky transport modes, and with many more unsuccessfully attempting such a passage. 800 died in the largest refugee shipwreck on record that April.

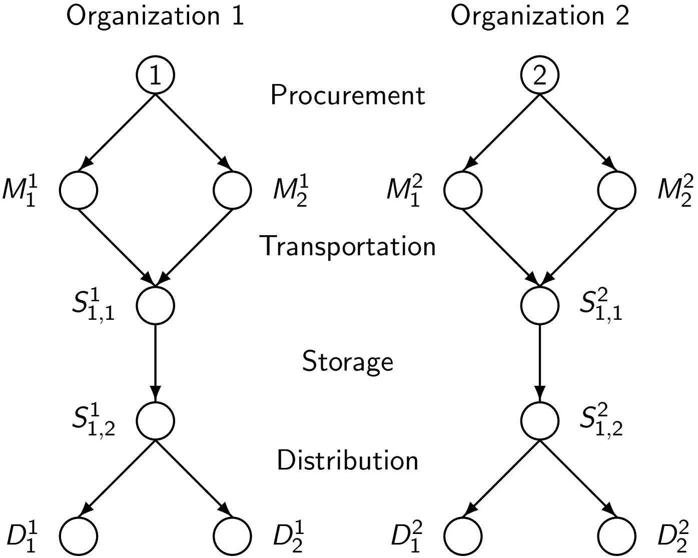


Figure: Pre-Cooperation Supply Chain Network Topology for the Numerical Examples

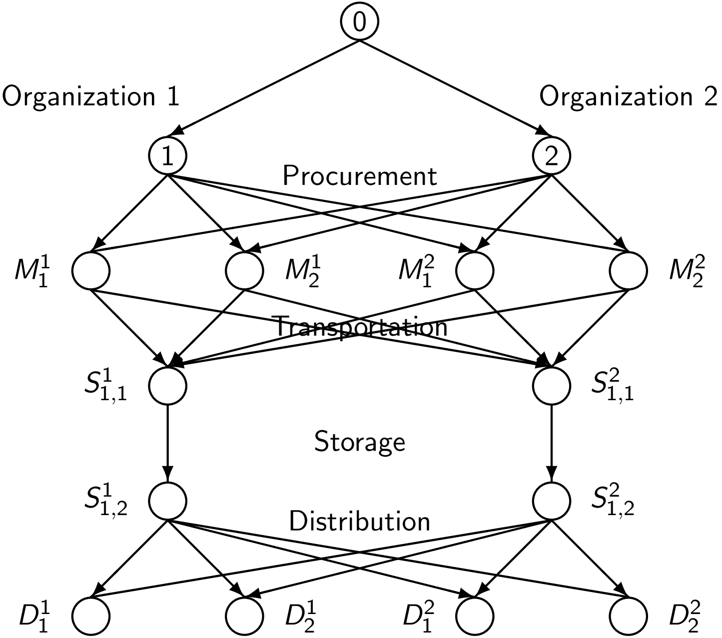


Figure: Cooperation Supply Chain Network Topology for the Examples

Numerical Examples

In the paper version of this tutorial a series of numerical examples, with complete input and output data are reported.

• We find that the relief item flows to the demand points are all greater than the lower value of the interval of the respective probability distribution and the volume is higher under cooperation than before. Hence, victims benefit from the cooperation of organizations.

• Also, we find that the generalized total synergy that can be achieved is substantial, as high as 99% in several of the examples.

This work quantifies the benefits to both organizations and victims of cooperation among organizations involved in disaster relief.

Impacting Policy Through Operations Research

Writing OpEds

THE CONVERSATION
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How disaster relief efforts could be improved with game theory

March 9, 2017 11.21pm EST

Buddhist monks and family members of victims of the Fukushima tsunami and earthquake face the sea to pray on March 11, 2016 while mourning the victims of the March 11, 2011 disaster. REUTERS/Kyodo

Email

The number of disasters has doubled globally since the 1980s, with the

Author

Anna Nagurney
John F. Smith Memorial Professor of Operations Management, University of Massachusetts Amherst

Disclosure statement

Anna Nagurney does not work for, consult, own shares in or receive funding from any company or organization that would benefit from this article, and has disclosed no relevant affiliations beyond the academic appointment above.

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Chicago Tribune

SPORTS BREAKING HOY MOST POPULAR OPINION SUBURBS ENTERTAINMENT ADVERTISING

Response to natural disasters like Harvey could be helped with game theory

By Anna Nagurney
Associated Press

AUGUST 29, 2017 | BOSTON, MASSACHUSETTS

(The Conversation is an independent and nonprofit source of news, analysis and commentary from academic experts.)

Anna Nagurney, University of Massachusetts Amherst

(THE CONVERSATION) The devastation by Hurricane Harvey continues, with the National Weather Service calling the event unprecedented, thus making the response even more complicated.

Nearly half a million people are expected to seek federal aid in the aftermath of the Category 4 hurricane, which already has dumped more than 30 inches on the Houston

SALON VIDEO NEWS POLITICS ENTERTAINMENT LIFE

THURSDAY, AUG 31, 2017 10:38 AM EST

Time for some game theory: How responses to natural disasters like Harvey could be improved

The damage and losses from natural disasters are estimated to cost us $100 billion a year since 2000

ANNA NAGURNEY, THE CONVERSATION

Homeland Security News Wire

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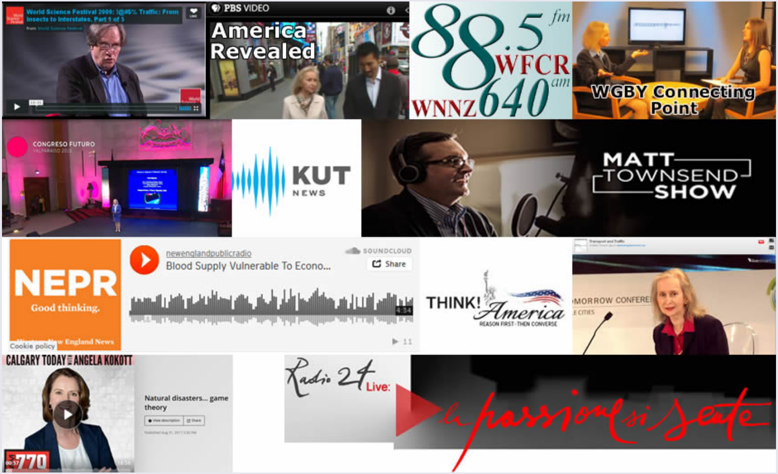
Disaster response

Coverage by the Media



Writing OpEds in the Pandemic

On March 11, 2020 the WHO declared the pandemic. On March 12 my article on blood supply chains in *The Conversation* appeared and, on March 24 my article in INFORMS *Analytics Coronavirus Chronicles*.

THE CONVERSATION

Academic rigor, journalistic flair

How coronavirus is upsetting the blood supply chain

March 12, 2020 8.05am EDT

Email
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Print

The coronavirus, which causes the disease COVID-19, has created enormous anxiety, uncertainty, and disruption to our lives. Much has already been written about potential shortages of medicines and face masks, but little has been said about something only you and I can provide – lifesaving blood.

Our nation's blood supply is essential to our health care security. Blood transfusions are integral parts of major surgeries. Blood is used in the treatment of diseases, particularly sickle cell anemia and some cancers. Blood is needed for victims who have injuries caused by accidents or natural disasters. Every day, the U.S. needs 36,000 units of red blood cells, 7,000 units of platelets, and 10,000 units of plasma.

I am a professor and director of the Virtual Center for Supernetworks at the University of Massachusetts Amherst. Because of the escalating coronavirus health care crisis, I am deeply concerned the U.S. blood supply chain is under stress. The timing could hardly be worse; the COVID-19 outbreak coincides with our seasonal flus and colds.

Analytics

DRIVING BETTER BUSINESS DECISIONS

informs

March 24, 2020 in Coronavirus Chronicles

The COVID-19 Pandemic and the Stressed Blood Supply Chain

By Anna Nagurney

SHARE: PRINT ARTICLE: https://doi.org/10.1287/LYTX.2020.02.18

Blood is essential to our nation's healthcare security. It is a life-saving product that cannot be manufactured and comes solely from volunteer donors. No substitute for blood has yet been invented. Blood transfusions are integral parts of major surgeries. Blood is a must for saving victims of accidents and natural disasters. Blood is also used in the treatment of certain diseases, including certain cancers. In the United States, 36,000 units of red blood cells are needed daily as are 7,000 units of platelets and 10,000 units of plasma. A typical donation of one pint, which can be divided into red blood cells, plasma and platelets, can save up to three lives. Adults have 8-12 pints of blood.

Even in the best of times, the complex blood supply chain in the United States is under stress. Although 38% of the U.S. population is eligible to donate blood, less than 10% actually does so in a year. Furthermore, issues of seasonality come into play with flu and colds cutting donations; the same for weather-related events and holidays. To further complicate matters, blood is perishable; platelets last five days and red blood cells have a shelf life of 42 days.

The blood banking industry, entrusted with maintaining a sufficient supply of blood, is facing a battle of the century with the COVID-19 pandemic. The timing could not be worse with this year's heavy flu and cold season, and the blood banking industry having recently undergone a massive transformation due to both economics and changes in medical procedures [1]. For example, there is increased competition among blood service organizations for donors [2]. The American Red Cross has closed some testing facilities and even eliminated mobile collection units in parts of the country. There have also been mergers and acquisitions of blood service organizations [3]. On the other hand, hospitals are now requiring less blood for certain procedures as compared to a few years ago because of changes in medical practices. This has resulted in requests for lower prices for blood from blood banks, who still have to cover costs, and some of the new costs include higher testing costs due to diseases such as Zika. And now, because of the COVID-19 pandemic, a major source of blood donations – schools – is removed.

Writing OpEds in the Pandemic

On August 4, 2020, I published an article in *The Conversation*,

"The Raging Competition for Medical Supplies is not a Game, but Game Theory Can Help."


On September 18, 2020, I published another article in *The Conversation*,

"Keeping Coronavirus Vaccines at Subzero Temperatures During Distribution Will Be Hard, but Likely Key to Ending Pandemic."

Writing OpEds in the Pandemic

On January 8, 2021, my article,

"Vaccine Delays Reveal Unexpected Weak Link in Supply Chains: A Shortage of Workers," appeared in *The Conversation*.

On April 5, 2021, I published the article,

"Today's Global Economy Runs on Standardized Containers, as the Ever Given Fiasco Illustrates," also in *The Conversation*.

Some of the Media Coverage of Our Work During the Pandemic

Impacting Policy Through Operations Research

On April 22, 2020, a letter from California Attorney General Xavier Becerra to the Admiral Brett Giroir, the Assistant Secretary of the US Department of Health & Human Services, and signed by US Attorney Generals of 21 other states, requested updates, because of the pandemic blood shortages, to blood donation policies that discriminate.

My article on blood supply chains in *The Conversation*, **which was reprinted in LiveScience, was the first reference and was cited on the first page.**

Impacting Policy Through Operations Research

State of California
Office of the Attorney General
XAVIER BECERRA
ATTORNEY GENERAL

April 22, 2020

Via Electronic Mail

The Honorable Admiral Brett Giroir, MD
Assistant Secretary for Health
U.S. Department of Health & Human Services
Mary E. Switzer Building
330 C Street SW, Room L600
Washington, DC 20024
Attn: ACBTSA-PAHPAIA Sec. 209
ACBTSA@hhs.gov

RE: "Solicitation for Public Comments on Section 209 of the Pandemic and All-Hazards Preparedness and Advancing Innovation Act," 85 Fed. Reg. 16,372 (March 23, 2020)

Dear Assistant Secretary Giroir:

The undersigned State Attorneys General from California, Colorado, Connecticut, Delaware, the District of Columbia, Hawaii, Illinois, Iowa, Maine, Massachusetts, Michigan, Minnesota, Nevada, New Jersey, New Mexico, New York, Oregon, Pennsylvania, Vermont, and Virginia submit this letter in response to the federal government's "Solicitation for Public Comments on Section 209 of the Pandemic and All-Hazards Preparedness and Advancing Innovation Act," (85 Fed. Reg. 16,372). We support the Office of the Assistant Secretary for Health in the U.S. Department of Health and Human Services' (HHS) efforts and work in maintaining an adequate national blood supply during the COVID-19 pandemic.

An adequate blood supply is critical to the nation's healthcare. Blood transfusions and blood products are needed for major surgeries, to treat diseases such as sickle cell anemia and some cancers, and to treat victims who have injuries caused by accidents or natural disasters.[1] Every day, the United States needs approximately 36,000 units of red blood cells, nearly 7,000

[1] Anna Nagurney, How Coronavirus is Upsetting the Blood Supply Chain, Live Science (Mar. 13, 2020), https://www.livescience.com/coronavirus-blood-supply-chain.html/.

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Impacting Policy Through Operations Research

Hon. Brett Giroir
April 22, 2020
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New Jersey Attorney General

HECTOR BALDERAS
New Mexico Attorney General

LETITIA JAMES
New York Attorney General

Xavier Becerra, previously California's Attorney General, is now President Joe Biden's Health and Human Services Secretary!

Thank You!

For more information: https://supernet.isenberg.umass.edu/