Networks in Finance in *Networks* and Beyond: A Half Century Retrospective

Anna Nagurney

John F. Smith Memorial Professor
Director of the Virtual Center for Supernetworks
Isenberg School of Management
University of Massachusetts
Amherst, Massachusetts 01003

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Networks in economics and finance in *Networks* and beyond: A half century retrospective

Anna Nagurney

Department of Operations and Information Management Isenberg, School of Management, University of Massachusetts Amherst, Amherst, Massachusetts

Correspondence
Anna Nagurney, Department of Operations and Information Management Isenberg, School of Management, University of Massachusetts Amherst, Amherst, MA 01003.
Email: nagurney@isenberg.umass.edu

Abstract

This paper presents a panoramic view of research on economic and financial networks in the journal *Networks*, since its inception half a century ago. This paper focuses on both the breadth and depth of the journal articles, and within the context of earlier contributions, as well as more recent related ones in other scientific publications. From network optimization to game theory and a plethora of equilibrium concepts, along with novel dynamical systems frameworks, the journal has led the way in advancing economic and financial network models, algorithms, and applications. Moreover, *Networks* has helped to attract researchers in a variety of disciplines to the science of networks and the formulation and solution of associated problems drawn from the real world.

KEYWORDS

economic networks, financial equilibrium, financial networks, game theory, projected dynamical systems, spatial price equilibrium, variational inequalities

1 | INTRODUCTION

Networks, from transportation and logistical ones, to communications and energy, have provided the foundation and connectivity for the flow of people, and the exchange of goods, information, and services across space and time. Intimately related to such physical networks, in which the identification of nodes, links, and associated flows with physical entities is well-understood, are economic and financial networks. The importance of all such network systems to the functioning of our societies and economies, coupled with the need to understand their interrelationships, has spurred numerous advances in methodologies for their modeling, analysis, and solution, under different behavioral concepts associated with usage and management.

The origins of network theory can be traced back to the 1700s, to the classical paper of Euler [64], the earliest paper on *graph* theory. By a graph in this context is meant, mathematically, a means of abstractly representing a system by its representation in terms of vertices (or nodes) and edges (arcs or, equivalently, links) joining pairs of vertices. Euler sought to determine whether it was possible to walk around Königsberg (later renamed Kaliningrad) by crossing the seven bridges over the River Pregel exactly once. The problem was depicted as a graph with the vertices representing land masses and the edges-bridges.

Interestingly, one of the first network models was for a financial system. Specifically, Quesnay [188], in his *Tableau Economique*, conceptualized the circular flow of financial funds in an economy as a network. His fundamental idea has been utilized in the construction of financial flow of funds accounts, which provide a statistical description of the flows of money and credit in an economy [37]. This work also inspired the first paper on financial networks in *Networks*, by Nagurney and Hughes [144]. The network model, with an accompanying decomposition algorithm, can be applied to calculate reconciled values of outstanding financial instruments, tangible assets, and net worth. The reconciled dataset can then be utilized as a base line for an empirical general equilibrium model and for macromonetary policy analysis.

Cournot [42], in his classical work in economics, was inspired by competition in a spring water product duopoly. His model, which considered two spatially separated markets in which the cost associated with transporting the product was included, implicitly assumed a network. Pigou [184], subsequently, studied a transportation network with two routes and observed that the

Networks and Financial Systems

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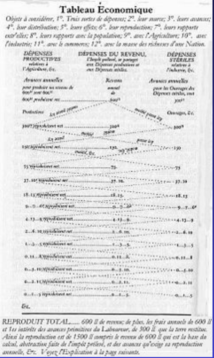

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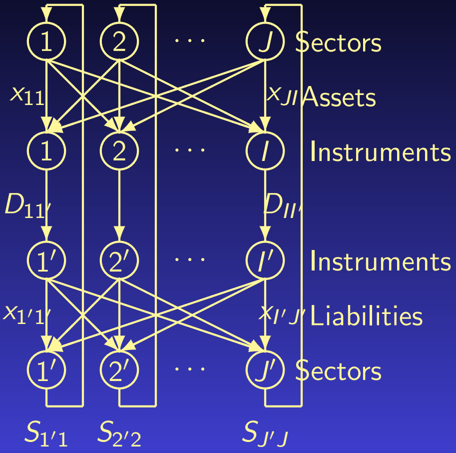
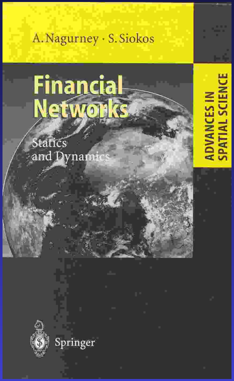
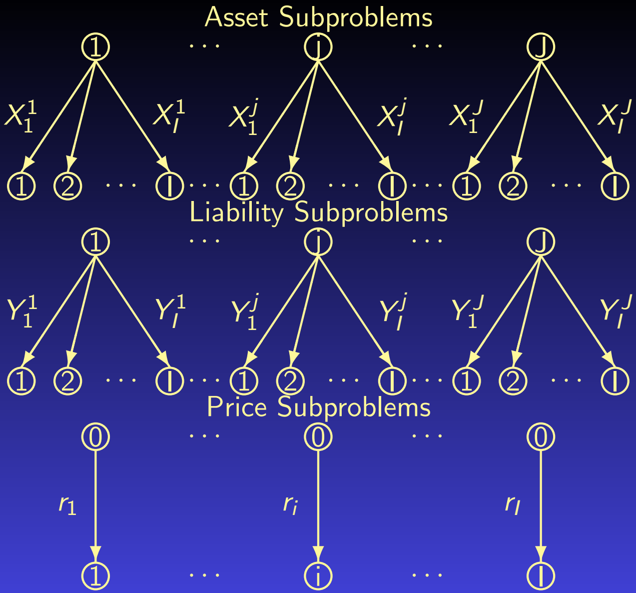


Network Model of Financial Flow of Funds Accounts (Elastic Sector and Instrument Total Volumes)

The financial network model, with decomposition algorithm, can be applied to calculate reconciled values of outstanding financial instruments, tangible assets, and net worth. The reconciled dataset serves as a baseline for an empirical general equilibrium model and for macromonetary policy analysis.

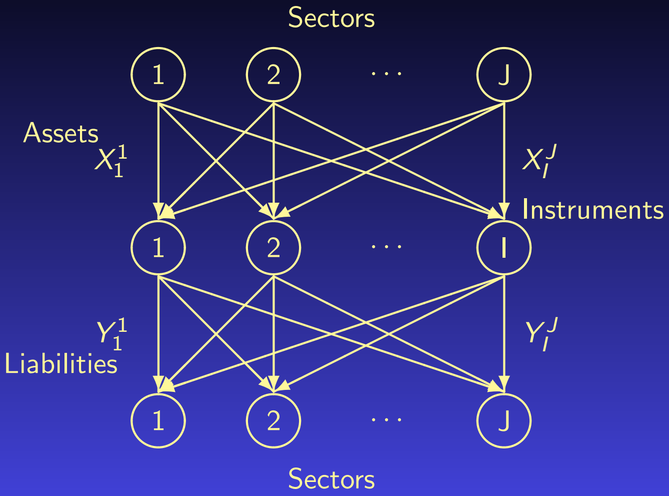


We constructed static, and dynamic, multisector, multi-instrument financial networks models, with a synthesis in our book.



Financial Network Subproblems Induced by the Modified Projection Method (**Nagurney, Dong, and Hughes (1992)**)

The Network Structure at Equilibrium



Nagurney and Siokos (1996)

More Financial Networks in *Networks*

Network Decomposition of General Financial Equilibria with Transaction Costs

Anna Nagurney

Department of Finance and Operations Management, School of Management, University of Massachusetts, Amherst, Massachusetts 01003

June Dong

School of Business, State University of New York at Oswego, Oswego, New York 13126

In this paper, we developed a new general financial equilibrium model with transaction costs which considers multiple sectors of an economy, each of which seeks to determine its optimal composition of instruments held as assets and as liabilities in its portfolio. The governing equilibrium conditions are shown to satisfy a variational inequality problem, which is then studied in terms of existence and other qualitative properties. A decomposition algorithm is proposed which exploits the underlying generalized network structure of the problem and convergence results obtained. Finally, the algorithm is applied to compute the equilibrium asset, liability, and price pattern in several numerical examples. *© 1996 John Wiley & Sons, Inc.*

1. INTRODUCTION

The study of general financial equilibrium problems is concerned principally with the computation of optimal portfolios held by the sectors in an economy as well as the prices of the financial instruments. Typical sectors include the government, households, banks, etc., whereas typical instruments held as assets and/or as liabilities include savings deposits, mutual fund shares, life insurance reserves, and pension fund reserves, among others. Since such problems are typically large-scale, the investigation of effective algorithms for their solution is warranted.

The exploitation of network structure is a well-established approach toward developing efficient algorithms for the solution of problems with such a characteristic. Examples of competitive equilibrium problems for which network-based formulations and computational schemes have been proposed and applied include such well-known problems as the traffic network equilibrium problem, spatial price equilibrium problems, market disequilibrium problems, and problems of human migration (cf. [6] and the references therein).

In this paper, we focus on the discipline of finance where networks in an equilibrium framework have been introduced only recently and their full potential remains as yet unexplored. In particular, we use as the foundation the recent work of Nagurney [7] and Nagurney and Dong [8] (see, also, Nagurney et al. [9]). The former paper introduced a general financial equilibrium model, but without transaction costs, whereas the latter paper considered a financial equilibrium model with specific, quadratic utility functions, but included transaction costs. In that paper, however, no convergence results were given for the proposed algorithm.

Here, we extend the general financial equilibrium modeling framework to include transaction costs and provide supporting convergence results for the network decomposition algorithm. As is well known in both economics and finance, the introduction of transaction costs creates market imperfections and considerably complexifies existing models (see, e.g., [1, 2, 4] and the references

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Dynamic Multi-Sector, Multi-Instrument Financial Networks with Futures: Modeling and Computation

Anna Nagurney,[1] Stavros Siokos[2]

[1] Department of Finance and Operations Management, School of Management, University of Massachusetts, Amherst, Massachusetts 01003

[2] Department of Mechanical and Industrial Engineering, University of Massachusetts, Amherst, Massachusetts 01003

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Abstract: In this paper, we develop a dynamic model of financial behavior in the case of multiple sectors and multiple instruments in the presence of financial futures, which demonstrates the evolution of the underlying networks through time. The dynamic model is formulated as a projected dynamical system whose set of stationary points coincides with the set of solutions to a variational inequality problem. We identify the network structure of the individual sectors' portfolio optimization problems out of equilibrium and then prove that the equilibrium solution can be reformulated as the solution to a network optimization problem, in which the network represents a merger of the individual networks. We subsequently provide a discrete time algorithm for the solution of the continuous time financial model, which exploits the network structure, and provide convergence results. The model and algorithm are then illustrated through numerical examples. © 1999 John Wiley & Sons, Inc. Networks 33: 93–108, 1999

1. INTRODUCTION

Networks have been recognized as a conceptual tool in modeling economic activity as early as Pigou [33] and Beckmann et al. [4], who studied transportation networks, and Enke [12] and Samuelson [34], who focused on spatial production networks. Network theory provides one not only with a powerful technique in identifying the characteristic structure underlying complex systems with accompanying qualitative analysis, but can also suggest efficient computational techniques. Moreover, network theory provides one with a common language to enhance communication across disciplines. For background on network economic systems, see Nagurney [24].

In this paper, we focus on a class of economic problems arising in the field of finance in which sectors in the economy seek to determine their optimal portfolio composition of hedged (''futures'') and unhedged financial instruments held as assets and/or liabilities. A futures contract is a firm commitment to deliver or receive a specific quantity of a financial instrument at a predetermined price. Since the last several decades have been witness to a large increase in the volatility of interest rates, investors have increasingly sought to hedge returns against changes in the prices of financial instruments.

For this problem, we develop a dynamic model of financial behavior and utilize for the first time in this application the methodologies of projected dynamical systems and network theory in order to investigate both

Correspondence to: A. Nagurney
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The Markowitz Model

A theoretical breakthrough was by Markowitz in 1952, who is credited with the *birth of modern portfolio theory*. Markowitz determined that one of the principal objectives of investors, besides the maximization of the returns of their portfolios, is to diversify away as much risk as possible.

He claimed that investors select assets in such a way that the risk of their portfolio matches their risk preferences.

His **work also suggested that the tradeoff between risk and return is different for each investor**, but the preferences of all people lie upon a fictitious curve which is usually called the "frontier of efficient portfolios."

The paper by Markowitz (1952), however, did not provide any specific techniques for determining this set of efficient portfolios although it contained a small illustration of how this set can be determined geometrically. In the original model by Markowitz short sales were excluded, and, thus,

$$X_i \geq 0, \quad i = 1, \dots, n,$$

and the amount of capital available was limited up to a budget.

Hence, the summation over all relative amounts invested in all securities had to be equal to one, that is,

$$\sum_{i=1}^{n} X_i = 1.$$

According to Markowitz (1952), the efficient frontier had to be identified and then every investor had to select a portfolio through a mean-variance analysis that fitted his preferences. This notion was then extended and presented as a mathematical optimization model by **Markowitz (1959)**, where every investor had to determine his optimal portfolio holdings through the solution of a quadratic programming model similar to:

$$\text{Maximize} \quad \alpha R - (1-\alpha)\, V$$

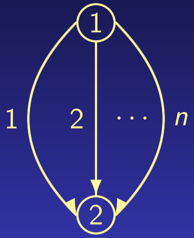
subject to:

$$\sum_{i=1}^{n} X_i = 1$$

$$X_i \geq 0, \quad i = 1, \ldots, n,$$

where α denotes an indicator of how risk-averse a specific investor is, R is the expected value of the return and V is the variance.

Network Structure of the Classical Markowitz Model



Network Structure of the Classical Markowitz Model

Some Additional Background

• **Thore (1969)** proposed networks for the study of systems of linked portfolios, with his work recognizing the contributions of **Charnes and Cooper (1961)**, who had showed that systems of linked accounts could be represented as a network. In such a financial network, the nodes correspond to balance sheets and the links to the credit and debit entries.

• **Thore (1980)**, in his book, which appears to be the first book on financial networks, further investigated network models of linked portfolios, financial intermediation, and decomposition theory.

• **Mulvey (1987)** not only identified that the **Markowitz (1952, 1959)** mean-variance minimization problem was, in fact, a network optimization problem with a nonlinear objective function, but he also presented a collection of nonlinear financial network models that were based on previous cash flow and portfolio models in which the original authors had not identified and, consequently, had not exploited the underlying network structure.

The New Millennium

With the advent of the new millennium, it was appropriate and reflective, to have the 30th anniversary paper by the journals first Editor-in-Chief Frisch (2001) on the early days of *Networks* published in the journal.

On page 6 of the article, he wrote: *When we started Networks, we did not anticipate the explosion of the field of networks.*

This statement resonates and is as true today as it was at the beginnings of the journal.

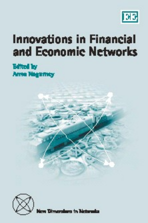
The New Millennium

Nagurney (2003) reported in *Networks* on recent developments in Network Economics based on papers presented at the 2002 Computing in Economics and Finance Conference, which took place in Aix en Provence, France. As emphasized in the article, the role of networks in economics and finance was gaining prominence for multiple reasons.

Geunes and Pardalos (2003), in *Networks*, presented an elegant annotated bibliography on network optimization in supply chain management and financial engineering, emphasizing their real-world relevance, including in the context of large-scale problems. They noted that, in addition to "microlevel" or "single-investor" problems, a substantial body of literature addresses equilibrium in financial networks with multiple sectors and financial intermediaries.

The New Millennium

With the growth of the Internet and innovations in supply chains and in financial networks, along with growing interactions among network systems (as supernetworks), there was a new momentum.



The volume contains work by **Boginski, Butenko, and Pardalos (2003)** for the massive stock market graph, focusing on the US. They establish, for the first time in the field of finance, the power law model, a construct from the network science literature, introduced by physicists. Several papers are on stochastic network approaches for financial optimization problems.

The New Millennium

At the cusp of the new millennium, "network science" was becoming a term that was receiving growing attention with a (2006) report by the National Research Council (NRC) noting that this "new" research field was focusing on an interdisciplinary perspective for complex network systems.

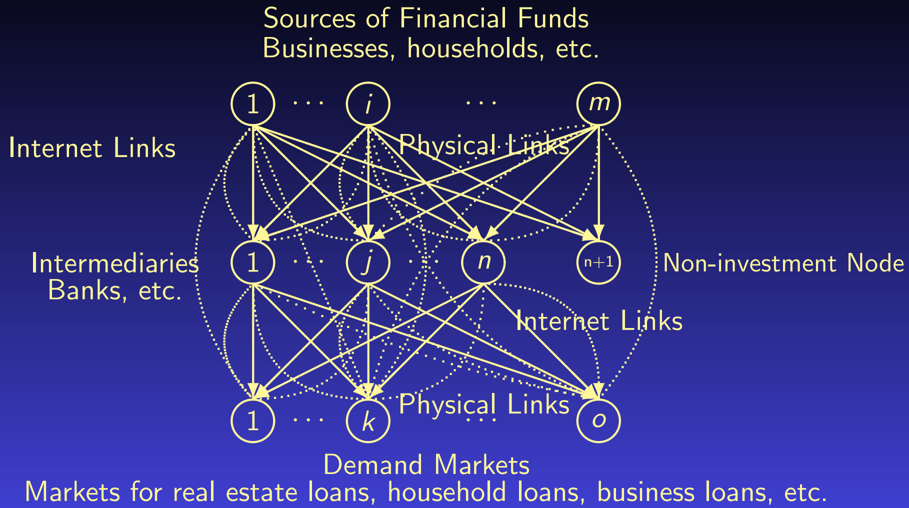
As emphasized in the overview by **Alderson (2008)**, operations research and its fundamental and wide ranging contributions to networks were essentially ignored except for an introductory chapter in the anthology by Newman, Barabási, and Watts (2006) that cited **Ahuja, Magnanti, and Orlin (1993)** and **Nagurney (1993)** as "exemplars."

The Financial Collapse

In 2008 and 2009, the world reeled from the effects of the financial credit crisis, with major banks and lending institutions closing, including Lehman Brothers; others merging, and the financial services landscape forever altered.

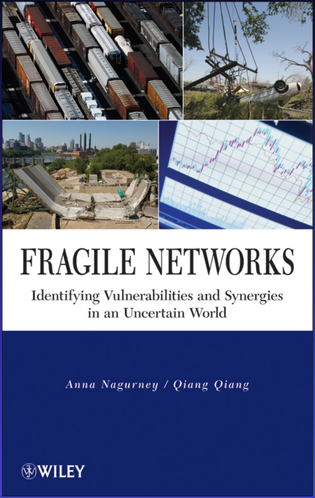
Given the importance of financial networks to the global economy, researchers, including physicists, were attracted to their study.

The Financial Network Model with Intermediation



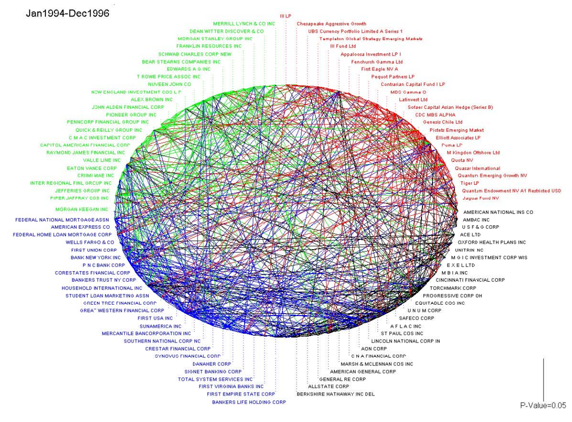
The Structure of the Financial Network with Intermediation (**Liu and Nagurney (2007)**)

Fragile Networks



The book was published in 2009.

Empirical Evidence: Jan. 1994 - Dec. 1996 - Connectivity, Vulnerability



Granger Causality Results: Green Broker, Red Hedge Fund, Black Insurer, Blue Bank Source: **Billio, Getmansky, Lo, and Pelizzon (2011)**

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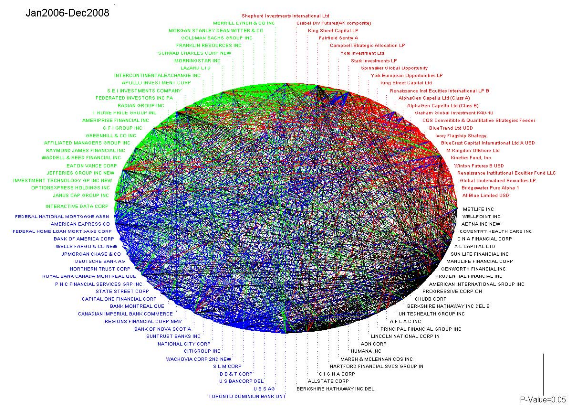


Granger Causality Results: Green Broker, Red Hedge Fund, Black Insurer, Blue Bank Source: **Billio, Getmansky, Lo, and Pelizzon (2011)**

The Last Decade - From 2010 - Special Issue

In 2013, I edited a special issue of the journal *Computational Management Science* **devoted to financial networks.**

Topics included in the special issue, among others, were:

• advances in the empirical market graph for the US (**Shirokikh, Pastukhov, Boginski, Butenko**);
• a study of the stock market as a complex network (**Bautin, Kalyagin, Koldanov, Koldanov, Pardalos**);
• the use of financial networks for the study of contagion (**Halaj and Kok, and Solorzano-Margain, Martinez-Jaramillo, Lopez-Gallo**);
• dynamic network formation game theory modeling of borrowers and sellers (**Fique and Page**),
• a multitiered financial network model (**Ke, Qiang, Hu**), and
•a framework to address impacts of corporate financial networks on supply chain networks (**Liu**).

And in *Networks*, **Boyles and Waller (2010)** utilized ideas from portfolio optimization to study a minimum cost flow problem in which the arc costs are uncertain, and the decision-maker wishes to minimize both the expected flow cost and the variance of this cost. In addition to providing algorithms, they also quantified the value of information.

Also writing in *Networks*, **Matsypura and Timkovsky (2012)** developed a heuristic network flow algorithm for an extension of the problem of margining option portfolios in practice and demonstrated a high efficiency of the proposed algorithm in a computational study.

The Last Decade - From 2010

Vulnerability issues in networks with financial underpinnings continued to be explored by researchers in *Networks* in this decade in a spectrum of critical node detection and related problems; see: **Veremyev, Prokopyev, Pasiliao (2015)**, **Aringhieri, Grosso, Hosteins, Scatamacchia (2016)**, **Gillen, Veremyev, Prokopyev, Pasiliao, (2018)**, **Walteros, Veremyev, Pardalos, Pasiliao (2019)**.

As mentioned earlier, physicists, as well as those in finance and economics were contributing to the study of financial networks.

The paper, "The Price of Complexity in Financial Networks," **Battiston, Caldarelli, May, Roukny, Stiglitz, *PNAS* (2016)**.

Conclusions

• The article in *Networks* upon which this talk is based provides a panoramic view and discussion of the contributions of many researchers to the study of economic and financial networks in *Networks* since its establishment a half a century ago.

• The research has been placed in the context of the earliest relevant publications, some dating back centuries, in order to provide a proper historical scientific perspective, along with the accentuation of highlights of more recent research in other outlets.

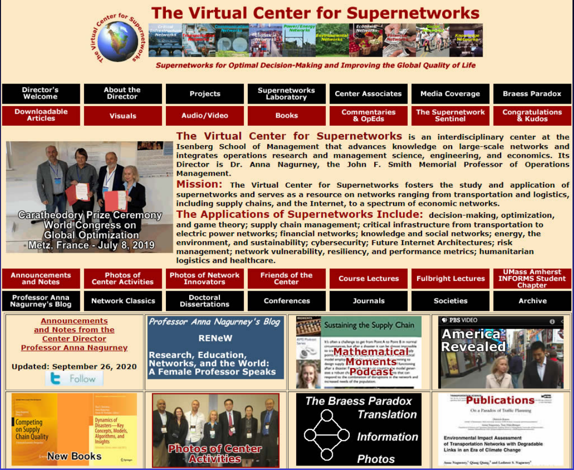
• **The goal was to demonstrate the broad reach of the power of networks and the impact in abstracting complex phenomena in the real world.**

Conclusions

• New synergies are being made possible, as well as insights, through the recognition of research on economic and financial networks in *Networks* by not only operations researchers and management scientists, but also by the finance community, by economists (including regional scientists), and by physicists, computer scientists, and, of course, engineers.

• **It is the expectation that the next half century will see further dramatic interest in economic and financial networks, and the science of networks, with the journal continuing to be a prominent outlet for highly original, creative research on these as well as many other related topics.**

Thank You!



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