Defense Critical Supply Chain Networks and Risk Management with the Inclusion of Labor

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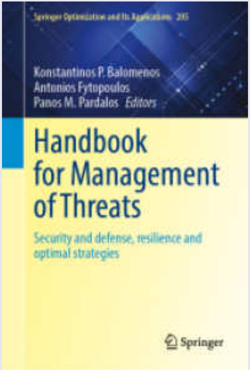
This presentation is based on the paper, "Defense Critical Supply Chain Networks and Risk Management with the Inclusion of Labor: Dynamics and Quantification of Performance and the Ranking of Nodes and Links," A. Nagurney, in press in:



Outline of This Presentation

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- **The Supply Chain Game Theory Model**
- **Nash Equilibrium and Variational Inequality Formulations**
- **The Algorithm**
- **Defense Supply Chain Network Performance / Efficiency**
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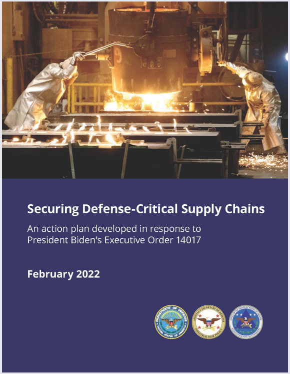
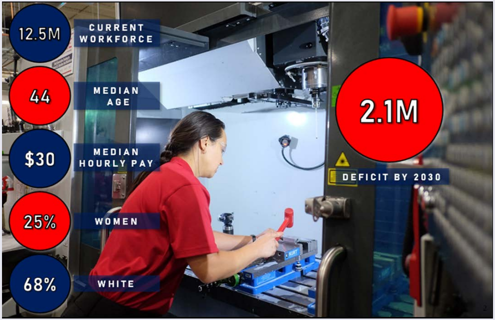
Background and Motivation

On February 24, 2022, the U.S Department of Defense (DoD) issued the long-awaited report:



The report was in response to Executive Order 14017, "America's Supply Chains," signed by President Joseph R. Biden Jr., to identify how to improve supply chain resilience and how to protect against material shortages, which had clearly become exacerbated in the COVID-19 pandemic.

The DoD's report provided an assessment of defense critical supply chains in order to improve the department's capacity to defend the United States.



In the DoD report, manufacturing, as well as the workforce, are considered to be strategic enablers and critical to building overall supply chain resilience.

On the same day of February 24, 2022, as the issuance of the report, Russia launched the full-scale invasion of Ukraine. The geopolitical risk continues to rise globally.

Having a framework for the modeling, analysis, and solution of defense critical supply chains is of major importance.

Of additional relevance is having a methodology to identify which of the nodes and links in such supply chain networks, corresponding, for example, to manufacturing sites and processes, storage facilities, transportation and distribution, are important, since focusing on those can help to preserve the performance of the supply chain networks for critical defense products in the case of disruptions.

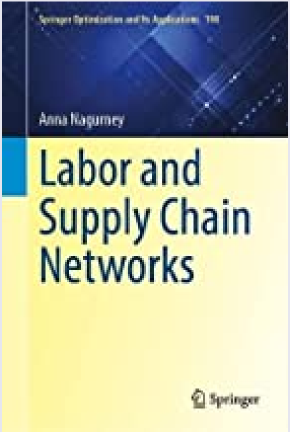
The Supply Chain Network Game Theory Model

• In this paper, a defense critical supply chain network game theory model is constructed in which the defense firms compete noncooperatively in producing, transporting, storing, and distributing their substitutable defense products, which are distinguished by firm or "brand."

• Defense products can be: weaponry, radars, tanks, or even life-saving vests and medical kits. Defense critical products can include high tech elements such as computer chips, which have been in short supply, as well as other raw materials that may be located in places under governance by antagonistic regimes.

• The objective function faced by a defense firm that it wishes to maximize consists of the profit and the weighted total risk associated with its supply chain network. A crucial element of the model is the availability of labor associated with each supply chain network link and a bound on the labor hours available.

• The inclusion of labor into general supply chain networks is a recent contribution, and was motivated by the impacts of the COVID-19 pandemic on workers, their health, loss of productivity, etc., as well as the negative effects of shortages of labor on profits as well as consumers.



The Supply Chain Network Game Theory Model

The Supply Chain Network Game Theory Model

I firms are involved in the production, transportation, storage, and ultimate distribution of the defense products, which are substitutable.

A typical defense firm is denoted by i. Each defense firm i has n^i_M production facilities; can utilize n^i_D distribution centers, and can distribute its defense product to the n_R defense demand markets. L^i represents the links of the supply chain network of defense firm i; $i = 1, \ldots, I$, with n_{L^i} elements.

By $G = [N, L]$ is denoted the graph consisting of the set of nodes N and the set of links L in Figure 1.

The Defense Critical Supply Chain Network Topology

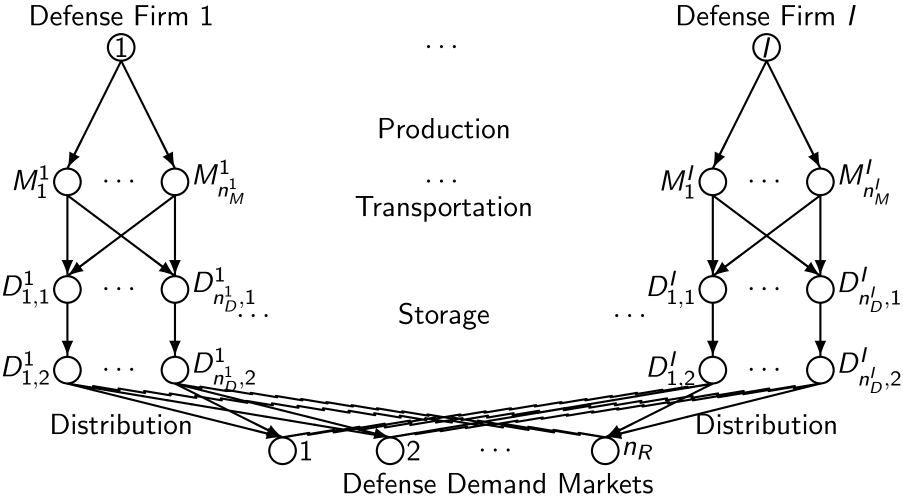


Table: Notation

Notation	Definition
P_k^i	the set of paths in defense firm i's supply chain network ending at defense demand market k; $i = 1, \ldots, I$; $k = 1, \ldots, n_R$.
P^i	the set of n_{P^i} paths of defense firm i; $i = 1, \ldots, I$.
P	the set of n_P paths in the defense supply chain network economy.
x_p; $p \in P_k^i$	the nonnegative flow of the defense product of firm i on path p originating at defense firm node i and ending at defense demand market k; $i = 1, \ldots, I$; $k = 1, \ldots, n_R$. Defense firm i's defense product path flows are grouped into the vector $x^i \in R_+^{n_{P^i}}$. The defense firms' defense product path flows are grouped into the vector $x \in R_+^{n_P}$.
f_a	the nonnegative flow of the defense product on link a, $\forall a \in L$. The defense product link flows are grouped into the vector $f \in R_+^{n_L}$.
l_a	the labor on link a denoted in person hours, $\forall a \in L$.
α_a	positive factor relating input of labor to output of defense product flow on link a, $\forall a \in L$.
$\bar{l}_a$	the upper bound on the availability of labor on link a, $\forall a \in L$.
d_{ik}	the demand for the defense product of defense firm i at defense demand market k; $i = 1, \ldots, I$; $k = 1, \ldots, n_R$. The $\{d_{ik}\}$ elements of defense firm i are grouped into the vector $d^i \in R_+^{n_R}$ and all the defense product demands are grouped into the vector $d \in R_+^{In_R}$.
$\hat{c}_a(f)$	the total operational cost associated with link a, $\forall a \in L$.
$r_a(f)$	the risk function associated with link a, $\forall a \in L$.
β_i	the nonnegative weight applied to the evaluation of the total risk by defense firm i; $i = 1, \ldots, I$. We group all these weights into the vector β.
w_a	the cost (wage) of a unit of labor on link a, $\forall a \in L$.
$\rho_{ik}(d)$	the demand price function for the defense product of defense firm i at defense demand market k; $i = 1, \ldots, I$; $k = 1, \ldots, n_R$.

Conservation of Flow Equations

The demand for each defense firm's product at each defense demand market must be satisfied by the defense product flows from the defense firm to the defense demand market, as follows: For each defense firm i: $i = 1, \ldots, I$:

$$\sum_{p \in P_k^i} x_p = d_{ik}, \quad k = 1, \ldots, n_R. \tag{1}$$

The defense product path flows must be nonnegative; where, for each defense firm i; $i = 1, \ldots, I$:

$$x_p \geq 0, \quad \forall p \in P^i. \tag{2}$$

The link product flows of each defense firm i; $i = 1, \ldots, I$, must satisfy the following equations:

$$f_a = \sum_{p \in P^i} x_p \delta_{ap}, \quad \forall a \in L^i, \tag{3}$$

where $\delta_{ap} = 1$, if link a is contained in path p, and 0, otherwise.

Relationship Between Labor and Product Link Flow

As in Nagurney (2021a, b, c), the product output on each link is a linear function of the labor input, where

$$f_a = \alpha_a l_a, \quad \forall a \in L^i, \quad i = 1, \ldots, I. \tag{4}$$

The greater the value of α_a, the more productive the labor on the link. Some economic background on such a construct can be found in Mishra (2007).

We also consider the following constraints on labor, since shortages of skilled labor is a big issue in defense critical supply chains: For each defense firm i; $i = 1, \ldots, I$:

$$l_a \leq \bar{l}_a, \quad \forall a \in L^i. \tag{5}$$

Utility Maximization

The utility function of defense firm i, U^i; $i = 1, \ldots, I$, is the profit, consisting of the difference between its revenue and its total costs, the wages paid out, and the weighted total risk:

$$U^i = \sum_{k=1}^{n_R} \rho_{ik}(d) d_{ik} - \sum_{a \in L^i} \hat{c}_a(f) - \sum_{a \in L^i} w_a l_a - \beta_i \sum_{a \in L^i} r_a(f). \quad (6a)$$

The utility functions are assumed to be concave, with the demand price functions being monotone decreasing and continuously differentiable and the total link cost and risk functions being convex and also continuously differentiable.

Each defense firm i; $i = 1, \ldots, I$ seeks to solve the problem:

$$\text{Maximize} \quad \sum_{k=1}^{n_R} \rho_{ik}(d) d_{ik} - \sum_{a \in L^i} \hat{c}_a(f) - \sum_{a \in L^i} w_a l_a - \beta_i \sum_{a \in L^i} r_a(f), \quad (6b)$$

subject to: (1) – (5).

Utility Maximization

In view of (2) and (3), can redefine the total operational cost link functions as: $\tilde{c}_a(x) \equiv \hat{c}_a(f)$, $\forall a \in L$; the demand price functions as: $\tilde{\rho}_{ik}(x) \equiv \rho_{ik}(d)$, $\forall i$, $\forall k$, and the risk functions $\tilde{r}_a(x) \equiv r_a(f)$, $\forall a \in L$. As noted in Nagurney (2021a,b), it follows from (3) and (4), that $l_a = \frac{\sum_{p \in P} x_p \delta_{ap}}{\alpha_a}$, for all $a \in L$.

Hence, one can redefine the utility functions $\tilde{U}^i(x) \equiv U^i$; $i = 1 \ldots, I$, and group the utilities of all the defense firms into an I-dimensional vector $\tilde{U}$, where

$$\tilde{U} = \tilde{U}(x). \tag{7}$$

Utility Maximization

The optimization problem faced by defense firm i; $i = 1, \ldots, I$, can be expressed as:

$$\text{Maximize } \tilde{U}^i(x) = \sum_{k=1}^{n_R} \tilde{\rho}_{ik}(x) \sum_{p \in P_k^i} x_p - \sum_{a \in L^i} \tilde{c}_a(x) - \sum_{a \in L^i} \frac{w_a}{\alpha_a} \sum_{p \in P^i} x_p \delta_{ap}$$

$$-\beta_i \sum_{a \in L^i} \tilde{r}_a(x), \tag{8}$$

subject to the nonnegativity constraints (1) and the re-expressing of constraints in (5) as:

$$\frac{\sum_{p \in P^i} x_p \delta_{ap}}{\alpha_a} \leq \bar{l}_a, \quad \forall a \in L^i. \tag{9}$$

Nash Equilibrium

The feasible set K_i for defense firm i is defined as:
$K_i \equiv \{x^i | x^i \in R_+^{n_{P^i}}, \frac{\sum_{p \in P^i} x_p \delta_{ap}}{\alpha_a} \leq \bar{l}_a, \forall a \in L^i\}$, for $i = 1, \ldots, I$, with $K \equiv \prod_{i=1}^{I} K_i$. Clearly, K is a convex set.

Definition 1: Defense Supply Chain Network Nash Equilibrium

A defense product path flow pattern $x^* \in K$ is a Defense Supply Chain Network Nash Equilibrium if for each defense firm i; $i = 1, \ldots, I$:

$$\tilde{U}^i(x^{i*}, \hat{x}^{i*}) \geq \tilde{U}^i(x^i, \hat{x}^{i*}), \quad \forall x^i \in K_i, \tag{10}$$

where $\hat{x}^{i*} \equiv (x^{1*}, \ldots, x^{i-1*}, x^{i+1*}, \ldots, x^{I*})$.

Conditions (10) state that a Defense Supply Chain Nash Equilibrium is achieved if no defense firm can improve upon its utility unilaterally.

Variational Inequality Formulations

It follows from the classical theory of Nash equilibria and variational inequalities that, under the imposed assumptions (cf. Gabay and Moulin (1980) and Nagurney (1999)), the solution to the above Defense Supply Chain Nash Equilibrium problem (see Nash (1950, 1951)) coincides with the solution of the variational inequality problem:

Determine $x^* \in K$, such that

$$-\sum_{i=1}^{I}\langle \nabla_{x^i} \tilde{U}^i(x^*), x^i - x^{i*}\rangle \geq 0, \quad \forall x \in K, \tag{11}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in the corresponding Euclidean space (here, of dimension n_P), and $\nabla_{x^i}\tilde{U}^i(x)$ is the gradient of $\tilde{U}^i(x)$ with respect to x^i.

Variational Inequality Formulations

We associate Lagrange multipliers λ_a with the constraint (9) for each link $a \in L$ and group the Lagrange multipliers for each defense firm i's supply chain network L^i into the vector λ^i. All such vectors for the defense firms are then grouped into the vector $\lambda \in R_+^{n_L}$.

Also, we introduce the feasible sets:
$K_i^1 \equiv \{(x^i, \lambda^i) | (x^i, \lambda^i) \in R_+^{n_{P^i}+n_{L^i}}\}$; $i = 1, \ldots, I$, and
$K^1 \equiv \prod_{i=1}^I K_i^1$.

We define:

$$\frac{\partial \tilde{C}_p(x)}{\partial x_p} \equiv \sum_{a \in L^i} \sum_{b \in L^i} \frac{\partial \hat{c}_b(f)}{\partial f_a} \delta_{ap}, \quad \forall p \in P^i, \tag{12}$$

$$\frac{\partial \tilde{R}_p(x)}{\partial x_p} \equiv \sum_{a \in L^i} \sum_{b \in L^i} \frac{\partial r_b(f)}{\partial f_a} \delta_{ap}, \quad \forall p \in P^i. \tag{13}$$

Variational Inequality Formulations

Theorem 1: Alternative Variational Inequality Formulation

The Defense Supply Chain Network Nash Equilibrium satisfying Definition 1 is equivalent to the solution of the variational inequality: Determine the vector of equilibrium defense product path flows and the vector of optimal Lagrange multipliers, $(x^*, \lambda^*) \in K^1$, *such that:*

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\sum_{p\in P_k^i}\left[\frac{\partial \tilde{C}_p(x^*)}{\partial x_p} + \beta_i \frac{\partial \tilde{R}_p(x^*)}{\partial x_p} + \sum_{a\in L^i}\frac{\lambda_a^*}{\alpha_a}\delta_{ap} + \sum_{a\in L^i}\frac{w_a}{\alpha_a}\delta_{ap} - \tilde{\rho}_{ik}(x^*)\right.$$

$$\left. - \sum_{l=1}^{n_R}\frac{\partial \tilde{\rho}_{il}(x^*)}{\partial x_p}\sum_{q\in P_l^i} x_q^*\right] \times [x_p - x_p^*]$$

$$+ \sum_{a\in L}\left[\bar{l}_a - \frac{\sum_{p\in P} x_p^*\delta_{ap}}{\alpha_a}\right]\times[\lambda_a - \lambda_a^*] \geq 0, \quad \forall (x,\lambda)\in K^1. \qquad (14)$$

The Algorithm

The Euler Method

This algorithm is due to Dupuis and Nagurney (1993). It can be applied to solve a variational inequality problem, in standard form, VI$(F, \mathcal{K})$, where one seeks to determine a vector $X^* \in \mathcal{K} \subset R^{\mathcal{N}}$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{15}$$

where F is a given continuous function from $\mathcal{K}$ to $R^{\mathcal{N}}$, $\mathcal{K}$ is a given closed, convex set, and $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathcal{N}$-dimensional Euclidean space.

The Euler Method

Initialize with $X^0 \in \mathcal{K}$ and set $\tau = 0$. Compute:

$$X^{\tau+1} = P_{\mathcal{K}}(X^\tau - a_\tau F(X^\tau)), \tag{16}$$

where: $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to \infty$, as $\tau \to \infty$ and $P_{\mathcal{K}}$ is the projection operator.

The Euler Method

Explicit Formulae for the Defense Product Path Flows at an Iteration

At iteration $\tau + 1$, one computes the following for each path p; $p \in P^i_k, \forall i, k$:

$$x_p^{\tau+1} = \max\{0, x_a^\tau - a_\tau(\frac{\partial \tilde{C}_p(x^\tau)}{\partial x_p} + \beta_i \frac{\partial \tilde{R}_p(x^\tau)}{\partial x_p} + \sum_{a \in L^i} \frac{\lambda_a^\tau}{\alpha_a}\delta_{ap} + \sum_{a \in L^i} \frac{w_a}{\alpha_a}\delta_{ap}$$

$$-\tilde{\rho}_{ik}(x^\tau) - \sum_{l=1}^{n_R} \frac{\partial \tilde{\rho}_{il}(x^\tau)}{\partial x_p} \sum_{q \in P^i_l} x_q^\tau)\}; \quad (17)$$

Explicit Formulae for the Lagrange Multipliers at an Iteration

At iteration $\tau + 1$, one computes the following for each Lagrange multiplier $a \in L$:

$$l_a^{\tau+1} = \max\{0, l_a^\tau - a_\tau(\bar{l}_a - \frac{\sum_{p \in P} x_p^\tau \delta_{ap}}{\alpha_a})\}. \quad (18)$$

Defense Supply Chain Network Efficiency / Performance

Defense Supply Chain Network Efficiency / Performance

It is important to recognize that, in matters of defense, a government, in preparing for conflicts and/or in times of war, may need to acquire defense supplies from a country other than its own.

Our defense supply chain network model allows for this and, we see that this is happening now as the war by Russia against Ukraine rages.

Hence, we believe that an adaptation of the constructs for supply chain network performance / efficiency of Nagurney and Qiang (2009) and of Nagurney and Li (2016) can also be applied for the new model in this paper, with note that the new model, unlike the previous ones in the above citations, includes labor; plus, we also have explicit weighted risk functions, since risk is of high relevance in the defense sector.

Efficiency/Performance of a Defense Supply Chain Network

The efficiency/performance of a defense supply chain network, denoted by *efficiency*, $\mathcal{E}$, is defined as:

$$\mathcal{E} = \mathcal{E}(G, \hat{c}, \rho, w, r, \beta, \alpha, \bar{l}) \equiv \sum_{i=1}^{I} \sum_{k=1}^{n_R} \frac{\frac{d^*_{ik}}{\rho_{ik}(d^*)}}{In_R}, \qquad (19)$$

with the demands, d^*, and the incurred defense demand market prices in (22), evaluated at the solution to (12).

Observe that, given a defense supply chain network economy, and the various parameters and functions, the corresponding multi-firm supply chain network is considered as performing better if, on the average, it can handle higher demands at lower prices.

Importance Identification of a Network Component

Following then Nagurney and Qiang (2009) for results therein for supply chains and Nagurney and Li (2016), one can then define the importance of a component g (node, link, or a combination of nodes and links), $I(g)$, which represents the efficiency drop when g is removed from the defense supply chain network, as:

$$I(g) = \frac{\Delta \mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}(G, \hat{c}, \rho, w, r, \beta, \alpha, \bar{l}) - \mathcal{E}(G - g, \hat{c}, \rho, w, r, \beta, \alpha, \bar{l})}{\mathcal{E}(G, \hat{c}, \rho, w, r, \beta, \alpha, \bar{l})}. \tag{20}$$

One can rank the importance of nodes or links, using (20).

This formalism can be quite valuable for those engaged in decision-making and policy-making in the military and defense. Those defense supply chain network components that are of higher importance should be paid greater attention to since a disruption to those components will have a bigger overall impact.

Resilience Measure Associated with Labor Disruptions

We can adapt the measure proposed in Nagurney and Ermagun (2022). As therein, let $\bar{l}\gamma$ denote the reduction of labor availability with $\gamma \in (0, 1]$ so if $\gamma = .8$ this means that the labor availability associated with the labor constraints is now 80% of the original labor availability as in $\mathcal{E}$.

Resilience Measure Capturing Labor Availability

One can define the resilience measure with respect to labor availability, $\mathcal{R}^{\bar{l}\gamma}$, as

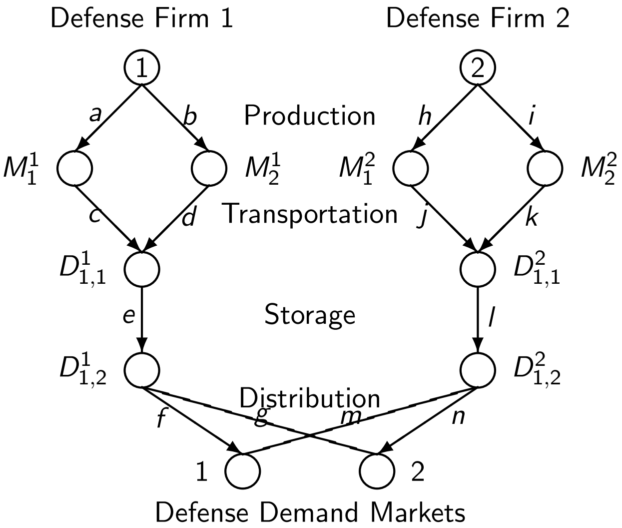
$$\mathcal{R}^{\bar{l}\gamma} \equiv \mathcal{R}^{\bar{l}\gamma}(G, \hat{c}, \rho, w, r, \beta, \alpha, \bar{l}) = \frac{\mathcal{E}^{\bar{l}\gamma}}{\mathcal{E}} \times 100\%, \tag{21}$$

with $\mathcal{E}$ as in (19).

The expression (21) quantifies the resilience of the defense supply chain network subject to reduction of labor availability. The closer the value is to 100%, the greater the resilience.

Numerical Examples

Supply Chain Network Topology



Example 1 - Baseline

The first example, which serves as the baseline, has the following data. Note that, in this example, we assume that the firms are not concerned about risk, so that all the risk functions are identically equal to 0.00**. The defense product is a defensive one, such as helmets or protective vests.**

The total operational cost functions associated with Defense Firm 1's supply chain network links L^1 are:

$$\hat{c}_a(f) = .006f_a^2,\ \hat{c}_b(f) = .007f_b^2,\ \hat{c}_c(f) = .01f_c^2,\ \hat{c}_d(f) = .01f_d^2,$$

$$\hat{c}_e(f) = .02f_e^2,\ \hat{c}_f(f) = .05f_f^2, \quad \hat{c}_g(f) = .05f_g^2.$$

The total operational costs associated with Defense Firm 2's supply chain network links L^2 are:

$$\hat{c}_h(f) = .0075f_h^2,\ \hat{c}_i(f) = .008f_i^2,\ \hat{c}_j(f) = .005f_j^2,\ \hat{c}_k(f) = .005f_k^2,$$

$$\hat{c}_l(f) = .015f_l^2,\ \hat{c}_m(f) = .1f_m^2, \quad \hat{c}_n(f) = .1f_n^2.$$

Example 1 - Baseline

The hourly labor wages are:

$w_a = 10,\ w_b = 10,\ w_c = 15,\ w_d = 15,\ w_e = 20,\ w_f = 17,\ w_g = 18,$

$w_h = 11,\ w_i = 22,\ w_j = 15,\ w_k = 15,\ w_l = 18,\ w_m = 18,\ w_n = 18.$

The link labor productivity factors are:

$\alpha_a = 24,\ \alpha_b = 25,\ \alpha_c = 100,\ \alpha_d = 100,\ \alpha_e = 50,\ \alpha_f = 100,\ \alpha_g = 100,$

$\alpha_h = 23,\ \alpha_i = 24,\ \alpha_j = 100,\ \alpha_k = 100,\ \alpha_l = 70,\ \alpha_m = 100,\ \alpha_n = 100.$

The bounds on labor are:

$\bar{l}_a = 100,\ \bar{l}_b = 200,\ \bar{l}_c = 300,\ \bar{l}_d = 300,\ \bar{l}_e = 100,\ \bar{l}_f = 120,\ \bar{l}_g = 120,$

$\bar{l}_h = 800,\ \bar{l}_i = 90,\ \bar{l}_j = 200,\ \bar{l}_k = 200,\ \bar{l}_l = 300,\ \bar{l}_m = 100,\ \bar{l}_n = 100.$

Example 1 - Baseline

The demand price functions of Defense Firm 1 are:

$$\rho_{11}(d) = -.0001d_{11} - .00005d_{21} + 600,$$

$$\rho_{12}(d) = -.0002d_{12} - .0001d_{22} + 800.$$

The demand price functions of Defense Firm 2 are:

$$\rho_{21}(d) = -.0003d_{21} + 700, \quad \rho_{22}(d) = -.0002d_{22} + 700.$$

The paths are: $p_1 = (a, c, e, f)$, $p_2 = (b, d, e, f)$, $p_3 = (a, c, e, g)$, $p_4 = (b, d, e, g)$ for Defense Firm 1 and $p_5 = (h, j, l, m)$, $p_6 = (i, k, l, m)$, $p_7 = (h, j, l, n)$, and $p_8 = (i, k, l, n)$ for Defense Firm 2.

Example 1 - Baseline

All the Lagrange multipliers have a value of 0.00 at the equilibrium.

The defense product prices at equilibrium are:

$$\rho_{11} = 599.75, \quad \rho_{12} = 799.10, \quad \rho_{21} = 699.40, \quad \rho_{22} = 699.60,$$

with the equilibrium demands:

$$d^*_{11} = 1506.19, \quad d^*_{12} = 3494.12, \quad d^*_{21} = 1999.04, \quad d^*_{22} = 2001.03.$$

The utility for Defense Firm 1 is: 2,258,772.50 and that for Defense Firm 2 is: 1,649,827.75.

The efficiency of this supply chain network, $\mathcal{E} = 3.15$.

Example 2: Addition of Risk Functions Associated with Production Sites

Example 2 has the same data as that in Example 1, except that now we consider the situation that the production sites are suffering from geopolitical risk and, hence, we have:

$$r_a = f_a^2, \quad r_b(f) = f_b^2, \quad r_h(f) = f_h^2, \quad r_i(f) = f_i^2,$$

with the risk weights of the two firms: $\beta_1 = \beta_2 = 1$.
All the Lagrange multipliers, again, have a value of 0.00 at the equilibrium. In other words, the respective labor bounds are not reached in Example 2.
The defense product prices at equilibrium are now:

$$\rho_{11} = 599.98, \quad \rho_{12} = 799.83, \quad \rho_{21} = 699.91, \quad \rho_{22} = 699.94,$$

with the equilibrium demands:

$$d_{11}^* = 0.00, \quad d_{12}^* = 690.49, \quad d_{21}^* = 305.50, \quad d_{22}^* = 305.80.$$

Example 2: Addition of Risk Functions Associated with Production Sites

The utility for Defense Firm 1 now is: 275,793.59 and that for Defense Firm 2: 213,562.31.

One can see that the utilities of both firms have dropped precipitously, in comparison to the utilities that they earned in Example 1, when there was no risk.

The efficiency of this defense supply chain network, with risk functions associated with production sites, $\mathcal{E} = .43$. We see that this value is much lower than that in Example 1.

We calculated $\mathcal{R}^{\bar{I}\gamma}$ for $\gamma = .9, .7, .5, .3, .1$ and found that $\mathcal{R}^{\bar{I}\gamma} = 1$ for all the values of γ noted, except when $\gamma = .1$, where $\mathcal{R}^{\bar{I}.1} = .7$.

We can conclude that this defense supply chain network, with the data provided, is quite resilient to labor disruptions.

Equilibrium Product Path Flows

Equilibrium Product Path Flows	Ex. 1	Ex. 2
$x_{p_1}^*$	703.17	0.00
$x_{p_2}^*$	803.02	0.00
$x_{p_3}^*$	1696.82	345.41
$x_{p_4}^*$	1797.30	345.08
$x_{p_5}^*$	919.52	152.84
$x_{p_6}^*$	1079.51	152.66
$x_{p_7}^*$	920.51	152.99
$x_{p_8}^*$	1080.52	152.81

Equilibrium Labor Values

Equilibrium Link Labor Values	Ex. 1	Ex. 2
l_a^*	100.00	14.39
l_b^*	104.01	13.80
l_c^*	24.00	3.45
l_d^*	26.00	3.45
l_e^*	100.00	13.81
l_f^*	15.06	0.00
l_g^*	34.94	6.90
l_h^*	80.00	13.30
l_i^*	90.00	12.73
l_j^*	18.40	3.06
l_k^*	21.60	3.05
l_l^*	57.14	8.73
l_m^*	19.99	3.05
l_n^*	20.01	3.96

Efficiency when a Link is Removed

We now report the efficiency of the defense supply chain network for Example 2 when a link g is removed, along with the importance $I(g)$, for $g = a, \ldots, n$.

g	$\mathcal{E}(G-g)$	$I(g)$
a	2.43	-4.43
b	.90	-1.08
c	.89	-1.08
d	.89	-1.08
e	.77	-.79
f	1.01	-1.39
g	.99	-1.30
h	.99	-1.30
i	.34	.21
j	.34	.21
k	.34	.21
l	.22	.50
m	.46	-.06
n	.42	.03

Insights

- **Overall, one can see that the supply chain network of Defense Firm 2 is more important than that of Defense Firm 1 to this defense supply chain network economy and cognizant governments should make note of this.**

- Indeed, **five of the seven links of Defense Firm 2's supply chain network have positive values in terms of their importance.**

- **Furthermore, Defense Firm 2's link l, which corresponds to a storage link, has the highest importance value; therefore, every effort should be expended to preserve its functionality.**

• Also, the production link i of Defense Firm 2 merits maintenance and care as do the transportation links j and k.

• Finally, link m, a distribution link to Defense Demand Market 2, is also of importance.

• As for the supply chain network of Defense Firm 1, link e, which is a storage link, has the highest value in terms of importance for Defense Firm 1 and, interestingly, its production site associated with link a is of the lowest importance.

We emphasize that not only the absolute values in terms of importance of supply chain network components are relevant but also their relative values.

Summary and Conclusions

Summary and Conclusions

• A defense critical supply chain network game theory model was introduced, which includes labor and associated constraints, as well as risk, since current world events have heightened the importance of both risk management and well as resilience of supply chain networks to disruptions, including those associated with labor.

• The model consists of defense firms seeking to supply defense products, that are substitutable to demand markets, which can be associated with different governments that are not antagonistic to one another. The labor constraints are bounds on hours of labor available on the supply chain network links. The utility function of each firm captures revenue as well as weighted risk and the governing equilibrium concept is that of a Nash Equilibrium.

• The methodological framework for the modeling, analysis, and computations, made use of both variational inequality theory and the theory of projected dynamical systems.

Summary and Conclusions

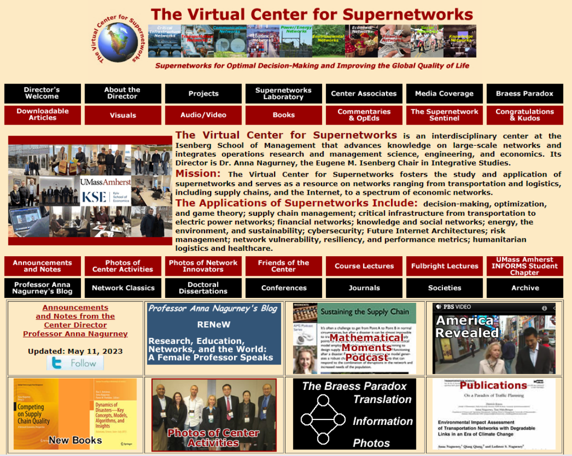
- The Euler Method was proposed and used in the solution of the numerical examples.

- The network efficiency / performance measure is constructed for the defense supply chain network economy, which can be applied to quantify the importance of supply chain network components, and then rank them. A resilience measure is also constructed to assess the impacts of disruptions to labor availability.

- In order to illustrate the defense supply chain network modeling framework, numerical examples are solved with input and output data reported.

- The information regarding the defense supply chain network economy, made possible with the tools in the paper, can be useful for decision-makers and policy-makers in governments that are concerned about defense.

Thank You Very Much!



More information on our work can be found on the Supernetwork Center site: https://supernet.isenberg.umass.edu/