Supply Chain Network Models for Humanitarian Logistics: Identifying Synergies and Vulnerabilities

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Contributions

- We build on supply chain network models with nonlinear costs that can also capture the reality of congestion, which may occur in humanitarian disaster relief operations.

- We built on the recent work of Nagurney (2008) who developed a system-optimization perspective for supply chain network integration in the case of horizontal mergers.
Contributions

- We also focus on supply chain network integration and we extend the contributions in Nagurney (2008) to include multiple products and with a humanitarian logistics perspective.

- We analyze the synergy effects associated with the “merging” of two humanitarian organizations, in the form of the integration of their supply chain networks, in terms of the operational synergy. Here we consider not only monetary costs but rather, generalized, costs that can include risk, environmental impacts associated with the humanitarian operations, etc.
Contributions

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Ethiopia’s Food Crisis

Source: BBC News
Flooding in Kenya

Source: www.alertnet.org
Famine in Southern Africa

Source: BBC News
In 2001 the total U.S. expenditure for humanitarian economic assistance was $1.46B, of which 9.7% represents a special supplement for victims of floods and typhoons in southern Africa (Tarnoff and Nowels (2001)).

The period between 2000-2004 experienced an average annual number of disasters that was 55% higher than the period of 1995-1999 with 33% more people affected in the more recent period (Balcik and Beamon (2008)).

According to ISDR (2006) 157 million people required immediate assistance due to disasters in 2005 with approximately 150 million requiring assistance the year prior (Balcik and Beamon (2008)).
The supply chain is a critical component not only of corporations but also of humanitarian organizations and their logistical operations.

At least 50 cents of each dollar’s worth of food aid is spent on transport, storage and administrative costs (Dugger (2005)).

The costs of provision may be divided among different products (food, clothing, fuel, medical supplies, shelter, etc.), which may increase efficiencies and enhance the organizations’ operational effectiveness.
The Integrated Multiproduct Humanitarian Supply Chain

- Humanitarian organizations may not only benefit from multiproduct supply chains (cf. Perea-Lopez, Ydtsil, and Grossman (2003)), but also from the integrated management and control of the entire supply chain (Thomas and Griffin (1996)).

- Coordination enables the sharing of information, which, according to Cachon and Fisher (2000), can reduce supply chain costs by approximately 2.2%.

- By offering services to enhance the World Food Programs existing logistics, TPG’s corporate socially responsible actions reduced operating and delivery costs enabling WFP to feed more people (Shister (2004)).
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Literature

- Cheng and Wu (2006)
- Davis and Wilson (2006)
- Soylu et al. (2006)
- Xu (2007)
- Thomas and Kopczak (2005)
- Thomas (2003)
- Van Wassenhove (2006)
- Haghani and Oh (1996)
- Balcik and Beamon (2008)
- Clark and Culkin (2007)
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Supply Chains of Humanitarian Organizations A and B Prior to the Integration

Organization A

\[ M_1^A \rightarrow \cdots \rightarrow M_{n_M}^A \]

\[ D_{1,1}^A \rightarrow \cdots \rightarrow D_{n_D,1}^A \]

\[ D_{1,2}^A \rightarrow \cdots \rightarrow D_{n_D,2}^A \]

\[ R_1^A \rightarrow \cdots \rightarrow R_{n_R}^A \]

Organization B

\[ M_1^B \rightarrow \cdots \rightarrow M_{n_M}^B \]

\[ D_{1,1}^B \rightarrow \cdots \rightarrow D_{n_D,1}^B \]

\[ D_{1,2}^B \rightarrow \cdots \rightarrow D_{n_D,2}^B \]

\[ R_1^B \rightarrow \cdots \rightarrow R_{n_R}^B \]
The Pre-integration Multiproduct Decision-making Optimization Problem (Case 0)

Let $L^0$ denote the links: $L_A \cup L_B$.

We assume that each organization can provide multiple, homogeneous products to the deserving populations at the respective demand points.

The demands for the products are assumed as given and are associated with each product, and each organization and demand pair, denoted by $d_{R_k}^j$ for product $j; j = 1, \ldots, J$, at demand point $R_k^i$ associated with organization $i; i = A, B; k = 1, \ldots, n^i_R$.

Let $x_p^j$ denote the nonnegative flow of product $j$, on path $p$. A path consists of a sequence of supply chain activities comprising supply, storage, and distribution of the products.

Let $P_{R_k}^0$ denote the set of paths joining an origin node $i$ with (destination) demand node $R_k^i$. 
The Pre-integration Multiproduct Decision-making Optimization Problem (Case 0)

Clearly, since we are first considering the two humanitarian organizations prior to any integration, the paths associated with a given organization have no links in common with paths of the other organization.

This changes (see also Nagurney (2008), Nagurney and Woolley (2008)) when the integration occurs, in which case the number of paths and the sets of paths also change, as do the number of links and the sets of links.

The following conservation of flow equations must hold for each organization $i$, each demand point $k$, and each product $j$:

$$\sum_{p \in P_{R_k}^i} x^j_p = d^j_{R_k}, \quad i = A, B; \quad j = 1, \ldots, J; \quad k = 1, \ldots, n^i_R$$

that is, the demand for each product must be satisfied at each demand point.
The Pre-integration Multiproduct Decision-making Optimization Problem (Case 0)

Links are denoted by $a$, $b$, etc. Let $f_a^j$ denote the flow of product $j$ on link $a$.

We must have the following conservation of flow equations satisfied:

$$f_a^j = \sum_{p \in P^0} x_p^j \delta_{ap}, \quad j = 1, \ldots, J; \quad \forall a \in L^0$$

where $\delta_{ap} = 1$ if link $a$ is contained in path $p$ and $\delta_{ap} = 0$, otherwise.

Here $P^0$ denotes the set of all paths, that is, $P^0 = \cup_{i=A,B;k=1,\ldots,n_R} R_i^0$. 
The Pre-integration Multiproduct Decision-making Optimization Problem (Case 0)

The path flows must be nonnegative, that is,

\[ x^j_p \geq 0, \quad j = 1, \ldots, J; \quad \forall p \in P^0. \]

Assume that there is a total cost associated with each link of the network corresponding to each organization \( i; i = A, B, \) and each product, \( j; j = 1, \ldots, J. \)

We assume that this cost is a \textit{generalized} cost and includes not only the monetary cost but also such costs as risk and even, if appropriate, the environmental cost, with all the associated costs weighted accordingly to produce the generalized cost.
The Pre-integration Multiproduct Decision-making Optimization Problem (Case 0)

We denote the total (generalized) cost on a link $a$ associated with product $j$ by $\hat{c}_a^j$. The total cost of a link associated with a product, be it a supply link, a shipment/distribution link, or a storage link is assumed to be a function of the flow of all the products on the link; see, for example, Dafermos (1972).

Hence, we have that

$$\hat{c}_a^j = \hat{c}_a^j(f_a^1, \ldots, f_a^J), \quad j = 1, \ldots, J; \quad \forall a \in L^0.$$  

We assume that the total cost on each link is convex, continuously differentiable, and has a bounded second order partial derivative.
The Pre-integration Multiproduct Decision-making Optimization Problem (Case 0)

Since the humanitarian organizations, pre-integration, have no links in common, their individual cost minimization problems can be formulated jointly as follows:

\[
\text{Minimize } \sum_{j=1}^{J} \sum_{a \in L^0} \hat{c}_a^j(f_a^1, \ldots, f_a^J) \\
\text{subject to the constraints presented earlier and the following capacity constraints:}
\]

\[
\sum_{j=1}^{J} \alpha_j f_a^j \leq u_a, \quad \forall a \in L^0.
\]

The term \(\alpha_j\) denotes the volume factor of product \(j\), whereas \(u_a\) denotes the nonnegative capacity of link \(a\).
Observe that this problem is, as is well-known in the transportation literature (cf. Beckmann, McGuire, and Winsten (1956), Dafermos and Sparrow (1969), and Dafermos (1972)), a *system-optimization* problem but in *capacitated* form.

We denote the associated optimal Lagrange multiplier by $\beta^*_a$. These terms may be interpreted as the price or value of an additional unit of capacity on link $a$. 
The Variational Inequality Formulation of the Pre-Integrated System-Optimized Problem

Theorem

The vector of link flows $f^* \in \mathcal{K}^0$ is an optimal solution to the pre-integration problem if and only if it satisfies the following variational inequality problem with the vector of nonnegative Lagrange multipliers $\beta^*$:

$$
\sum_{j=1}^{J} \sum_{k=1}^{J} \sum_{a \in L^0} \left[ \frac{\partial \hat{c}_k (f^1_a, \ldots, f^J_a)}{\partial f^j_a} (f^1_a, \ldots, f^J_a) + \alpha_j \beta^*_a \right] \times [f^j_a - f^j_a^*]
$$

$$
+ \sum_{a \in L^0} \sum_{j=1}^{J} \left[ u_a - \sum_{j=1}^{J} \alpha_j f^j_a^* \right] \times [\beta_a - \beta^*_a] \geq 0,
$$

$\forall f \in \mathcal{K}^0, \forall \beta \geq 0$.

Supply Chain Network after Humanitarian Organizations A and B Integrate their Supply Chains

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We associate total cost functions with the new links, for each product $j$.

We assume, for simplicity, that the corresponding functions on the links emanating from the supersource node are equal to zero.

A path $p$ now originates at the node 0 and is destined for one of the bottom demand nodes.

The set of paths $P^1 \equiv \bigcup_{i=A,B;k=1,\ldots,n_R^i} P^1_{R_k}$.

Let $x^j_p$, in the integrated network configuration, denote the flow of product $j$ on path $p$ joining (origin) node 0 with a (destination) demand node.
The Integrated Multiproduct Decision-making Optimization Problem (Case 1)

The following conservation of flow equations must hold:

\[
\sum_{p \in P^1_{R^i_k}} x^j_p = d^j_{R^i_k}, \quad i = A, B; \quad j = 1, \ldots, J; \quad k = 1, \ldots, n^i_R,
\]

where \( P^1_{R^i_k} \) denotes the set of paths connecting node 0 with demand node \( R^i_k \).

As before, let \( f^j_a \) denote the flow of product \( j \) on link \( a \). Hence, we must also have the following conservation of flow equations satisfied:

\[
f^j_a = \sum_{p \in P^1} x^j_p \delta_{ap}, \quad j = 1, \ldots, J; \quad \forall a \in L^1.
\]

Of course, we also have that the path flows must be nonnegative for each product \( j \), that is,

\[
x^j_p \geq 0, \quad j = 1, \ldots, J; \quad \forall p \in P^1.
\]
The Integrated Multiproduct Decision-making Optimization Problem (Case 1)

The supply chain network activities have nonnegative capacities, denoted as $u_a$, $\forall a \in L^1$, with $\alpha_j$ representing the volume factor for product $j$.

Hence, the following constraints must be satisfied:

$$\sum_{j=1}^{J} \alpha_j f^j_a \leq u_a, \quad \forall a \in L^1.$$

As in the pre-integration case, the post-integration optimization problem is also concerned with total cost minimization, in that the total cost is reflective of the generalized total cost. The following optimization problem for the integrated supply chain network is as follows:

Minimize $\sum_{j=1}^{J} \sum_{a \in L^1} \hat{c}^j_a(f^1_a, \ldots, f^J_a)$

subject to the constraints presented earlier.
The Variational Inequality Formulation of the Post-Integrated System-Optimized Problem

**Theorem**

The vector of link flows $f_1^* \in \mathcal{K}_1$ is an optimal solution to the post-integration problem if and only if it satisfies the following variational inequality problem with the vector of nonnegative Lagrange multipliers $\beta_1^*$:

$$
\sum_{j=1}^{J} \sum_{k=1}^{J} \sum_{a \in L_1} \left[ \frac{\partial \hat{c}_a^k(f_1^*, \ldots, f_j^*)}{\partial f_a^j} + \alpha_j \beta_a^* \right] \times [f_a^j - f_1^a] \\
+ \sum_{a \in L_1} \left[ u_a - \sum_{j=1}^{J} \alpha_j f_a^j \right] \times [\beta_a - \beta_a^*] \geq 0,
$$

$\forall f \in \mathcal{K}_1, \forall \beta \geq 0.$
Quantifying Synergy Associated with Multiproduct Decision-Making Organizations with Integration

Let $TC^0$ denote the total cost generated under solution $f^0*$ for the pre-integration case.

Let $TC^1$ denote the total cost generated under solution $f^1*$ for the post-integration case.

The synergy based on total costs and proposed by Nagurney (2008), but now in a multiproduct context, which we denote here by $S^{TC}$, can be calculated as the percentage difference between the total cost pre vs the total cost post the integration:

$$S^{TC} \equiv \left[ \frac{TC^0 - TC^1}{TC^0} \right] \times 100\%.$$
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Pre-Integration Humanitarian Logistics Network Topology for the Numerical Examples
Post-Integration Humanitarian Logistics Network Topology for the Numerical Examples
Numerical Examples

For all the numerical examples, we assumed that each organization $i; i = A, B,$ was involved in the production, storage, and distribution of two products.

For all the examples, we assumed that the pre-integration total cost functions and the post-integration total cost functions were nonlinear (quadratic), of the form:

$$\hat{c}_a^j(f_a^1, f_a^2) = \sum_{l=1}^{2} g_a^{jl} f_a^l + h_a^j f_a^j, \quad \forall a \in L^0, \forall a \in L^1; \quad j = 1, 2,$$

with strict convexity of the total cost functions being satisfied (except, where noted, for the top-most merger links from node 0).

The links post-integration that join the node 0 with nodes $A$ and $B$ had associated total costs equal to zero for each product $j = 1, 2,$ for Examples 1 through 3.
Numerical Example 1

Example 1 served as the baseline for our computations.

The demands at the demand markets for organization A and organization B were set to 5 for each product. Hence, $d_{R_i}^j = 5$ for $i = A, B; j = 1, 2$, and $k = 1, 2$.

The capacity on each link was set to 25 both pre and post integration, so that: $u_a = 25$ for all links $a \in L^0; a \in L^1$.

The values of $\alpha_j = 1$ were set to 1 for both products $j = 1, 2$, both pre and post-integration; thus, we assumed that the products are equal in volume.

Since none of the link flow capacities were reached, either pre- or post-integration, the vectors $\beta_{0*}$ and $\beta_{1*}$ had all their components equal to zero.
Numerical Example 1

It is interesting to note that, to optimize the reduction of costs, organization A’s original distribution center, after the integration/merger, stores the majority of the volume of product 1, while the majority of the volume of product 2 is stored, post-integration, at organization B’s original distribution center.

Additionally, post-integration, the majority of the production of product 1 takes place in organization B’s original supply plants, whereas the converse holds true for product 2.

This example, hence, vividly illustrates the types of supply chain cost gains that can be achieved in the integration of multiproduct supply chains.
Numerical Example 2

Ex. 2 was constructed from Ex. 1 but we now the total costs associated with the new integration links for each product were identically equal to zero.

It is interesting to note that now the second supply plant associated with the original organization B produces the majority of product 1 but the majority of product 1 is still stored at the original distribution center of organization A.

The zero costs associated with distribution between the original supply chain networks lead to further synergies as compared to those obtained for Ex. 1.

This obtained synergy is, in a sense, the maximum possible for this example since the total costs for both products on all the new links are all equal to zero.
Numerical Example 3

Ex. 3 was constructed from Ex. 2 but the capacities associated with the links that had zero costs between the two original firms had their capacities reduced from 25 to 5.

The computed vector of Lagrange multipliers $\beta^1*$ had all terms equal to zero except those for links formed post-merger, since the sum of the corresponding product flows on each of these links was equal to the imposed capacity of 5.

Even with substantially lower capacities on the new links, given the zero costs, the synergy associated with the supply chain network integration in Ex. 3 was quite high, although not as high as obtained in Ex. 2.
## Total Costs and Synergy Values for the Examples

<table>
<thead>
<tr>
<th>Measure</th>
<th>Example 1</th>
<th>Example 2</th>
<th>Example 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-Integration  $TC^0$</td>
<td>5,702.58</td>
<td>5,702.58</td>
<td>5,702.58</td>
</tr>
<tr>
<td>Post-Integration $TC^1$</td>
<td>4,240.86</td>
<td>2,570.27</td>
<td>3,452.34</td>
</tr>
<tr>
<td>Synergy $S_{TC}$</td>
<td>25.63%</td>
<td>54.93%</td>
<td>39.46%</td>
</tr>
</tbody>
</table>
We then proceeded to ask the following question: *When would the integration of the two humanitarian organizations not be beneficial, that is, not possess positive synergy for Examples 1, 2, and 3?*

We assume that the links, post-merger, joining node 0 to nodes A and B reflected a cost associated with the integration the two humanitarian organizations.

We further assumed that the cost was linear and of the specific form given by

$$\hat{c}_a^j = h_a^j f_a^j = h f_a^j, \quad j = 1, 2$$

for the upper-most links. Hence, we assumed that all the $h_a^j$ terms were identical and equal to an $h$. 
Additional Computations/Examples

Through computational experiments we were able to determine these values.

In the case of Example 1, if \( h = 36.52 \), then the synergy value would be approximately equal to zero since the new total cost would be approximately equal to \( TC^0 = 5,702.58 \). For any value larger than the above \( h \), one would obtain negative synergy.

This has clear implications for supply chain network integration and demonstrates that the total costs associated with the integration itself have to be carefully weighed against the cost benefits associated with the integrated supply chain activities.
In the case of Example 2, the $h$ value was approximately equal to 78.3. A higher value than this $h$ for each such integration link would result in the total cost exceeding $TC^0$ and, hence, negative synergy would result.

Finally, for completeness, we also determined the corresponding $h$ in the case of Example 3 and found the value to be $h = 78.3$, as in Example 2.
Conclusions

- We presented the pre-integration and the post-integration multiproduct humanitarian logistics supply chain network models, derived their variational inequality formulations, and then defined a total generalized cost synergy measure.

- The framework is based on a supply chain network perspective, in a system-optimization context, that captures the activities of a humanitarian organization such as supply, storage, as well as distribution.

- We are the first to analyze the synergy effects of integration as related to humanitarian efforts of the entire humanitarian supply chain.
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Thank you!

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