A Game Theory Model for Freight Service Provision Security Investments for High-Value Cargo

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Freight Security

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- Background and Motivation
- ► The Game Theory Model for FSPs' Security Investments
- Variational Inequality Formulations
- The Algorithm and Case Study
- Summary and Conclusions

Background and Motivation

Freight and Security

- In a constantly and intricately connected world, security is imperative to not just the success but also the survival of businesses.
- Cargo theft is estimated to cost shippers and trucking companies at least \$30 billion a year in the US, according to the FBI.
- There is an average of 63 cargo thefts per month. The average loss value per incident in 2015 was almost \$190,000.
- In 2016, CargoNet reported an average loss value of \$206,837. CargoNet recorded eight separate thefts worth more than \$1 million and one shipment valued at \$8 million in the same 2016 quarter in the US.
- Cargo's value continues to increase and thieves are getting sophisticated.

According to Weiss (2016), cargo thefts in Europe, the Middle East, and Africa have almost tripled in the past five years.

In recent months, criminals have:

• stolen salmon worth 100,000 euros (\$112,000) from a trailer in Norway;

- taken 80 cases of whiskey from a vehicle near London, and
- absconded with truckloads of nuts worth more than \$10 million.

High-Value Cargo

High-value cargo, which can range from high tech equipment to precious metals and jewelry, alcohol and high-end food products, as well as pharmaceuticals, are especially attractive targets for theft while in transit.



As sourcing and marketing locations have become more dispersed globally, companies are faced with greater security challenges.

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Incident Heat Map



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Cargo Theft by Product Type



Cargo Theft



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Our Approach

- We develop a game theory model consisting of Freight Service Providers (FSPs) who compete with each other in terms of quantity of the high-value product and level of security investments.
- Shippers reflect their preferences through willingness to pay depending on the quantity and level of security provided by the FSPs.
- **FSPs** encumber the security investment costs.
- We also include probability of a successful attack on the logistics/transportation links, along with associated damages.
- FSPs try to maximize their utilities associated with quantities and security levels which may differ for different links.

- The shippers respond to the security investments of the FSPs, who compete for business, through the prices that they are willing to pay.
- We capture risk in that the level of security affects the probability of attack and the expected damages.
- The security levels in our model are continuous and have upper bounds.
- Our work is focused on high-value goods.

The Game Theory Model for FSPs' Security Investments



Figure 1: Network Structure of the Freight Security Game Theory Model

The Game Theory Model Features

Quantity of High-Value Cargo from Shipper *j* through FSP *i* to Node *k*:

 $0 \leq q_{ijk} \leq \bar{q}_{ijk}, \forall j, k.$

The above can be grouped into $q \in R^{mno}_+$. Security Level of FSP *i* for Shipping from *j* to *k*:

 $0 \leq s_{ijk} \leq \overline{s}_{ijk}, \forall j, k.$

The above can be grouped into $s \in R^{mno}_+$. Investment Cost Function h_{ijk} :

$$h_{ijk}(s_{ijk}) = lpha_{ijk}(rac{1}{\sqrt{(1-s_{ijk})}}-1), lpha_{ijk} > 0, orall i, j, k.$$

 α_{ijk} allows FSPs to have different investments based on needs and expert knowledge about any OD pair. $h_{ijk}(1) = \infty, h_{ijk}(0) = 0.$

The Game Theory Model Features

Probability of a Successful Attack on *i* **going from** *j* **to** *k*:

$$p_{ijk} = (1 - s_{ijk}), \forall i, j, k.$$

If there is no security on by *i* along (j, k), that is, $s_{ijk} = 0$, probability of an attack is equal to 1.

Price FSP *i* Charges the Shipper *j* to Carry Cargo to Node *k*:

$$\rho_{ijk} = \rho_{ijk}(q,s), \forall j, k.$$

Prices are continuously differentiable, increasing in quantities but decreasing in security levels. Total Cost Faced by FSP *i* in Transporting High-Value

Goods from *j* **to** *k*:

$$\hat{c}_{ijk} = \hat{c}_{ijk}(q), \forall j, k.$$

We assume that the total costs are continuously differentiable and convex. FSPs are affected by the quantities of other FSPs as well.

The Game Theory Model Features

Damages on *i* Traveling from *j* to *k*:

 $\sum_{j=1}^n \sum_{k=1}^o p_{ijk} D_{ijk}.$

Each FSP *i* Seeks to Maximize Profit $E(U_i)$:

$$E(U_i) = \sum_{j=1}^n \sum_{k=1}^o (1 - p_{ijk}) (\rho_{ijk}(q, s) q_{ijk} - \hat{c}_{ijk}(q))$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}p_{ijk}(\rho_{ijk}(q,s)q_{ijk}-\hat{c}_{ijk}(q)-D_{ijk})-\sum_{j=1}^{n}\sum_{k=1}^{o}h_{ijk}(s_{ijk}),\forall i.$$

Let K_i denote the feasible set corresponding to FSP *i*, where $K_i \equiv \{(q_i, s_i) | 0 \le q_{ijk} \le \overline{q}_{ijk}, \forall j, k \text{ and } 0 \le s_{ijk} \le \overline{s}_{ijk}, \forall j, k\}$. We also dene the feasible set corresponding to all FSPs: $K \equiv \prod_{i=1}^{m} K^i$.

The Game Theory Model

Definition 1: A Nash Equilibrium in High-Value Product Shipments and Security Levels

A high-value product shipment and security level pattern $(q^*, s^*) \in K$ is said to constitute a Nash equilibrium if for each FSP *i*:

$$\mathsf{E}(U_i(q_i^*,s_i^*,\hat{q}_i^*,\hat{s}_i^*)) \geq \mathsf{E}(U_i(q_i,s_i,\hat{q}_i^*,\hat{s}_i^*)), \quad orall(q_i,s_i) \in \mathsf{K}^i,$$

where

$$\hat{q_i^*} \equiv (q_1^*, \dots, q_{i-1}^*, q_{i+1}^*, \dots, q_m^*)$$
; and $\hat{s_i^*} \equiv (s_1^*, \dots, s_{i-1}^*, s_{i+1}^*, \dots, s_m^*)$.

An equilibrium is established if no FSP can unilaterally improve upon his expected profits by selecting an alternative vector of high-value product shipments and security levels.

Variational Inequality Formulations

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Variational Inequality Formulations

Theorem 1

Assume that for each FSP *i*; i = 1, ..., m, the expected profit function $E(U_i(q, s))$ is concave with respect to the variables $\{q_{i11}, ..., q_{ino}\}$ and $\{s_{i11}, ..., s_{ino}\}$, and is continuously differentiable. Then $(q^*, s^*) \in K$ is a Nash Equilibrium according to Definition 1 if and only if it satisfies the variational inequality

$$-\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o}\frac{\partial E(U_{i}(q^{*},s^{*}))}{\partial q_{ijk}}\times(q_{ijk}-q_{ijk}^{*})$$

 $-\sum_{i=1}^m\sum_{j=1}^n\sum_{k=1}^orac{\partial E(U_i(q^*,s^*))}{\partial s_{ijk}} imes (s_{ijk}-s^*_{ijk})\geq \overline{0,orall(q,s)\in K,}$

Variational Inequality Formulations

or, equivalently, $(q^*, s^*) \in K$ is a Nash Equilibrium high-value product shipment and security level pattern if and only if it satisfies the variational inequality

$$\begin{split} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{o} [\sum_{h=1}^{n} \sum_{l=1}^{o} \frac{\partial \hat{c}_{ihl}(q^*)}{\partial q_{ijk}} - \rho_{ijk}(q^*, s^*) - \sum_{h=1}^{n} \sum_{l=1}^{o} \frac{\partial \rho_{ihl}(q^*, s^*)}{\partial q_{ijk}} q^*_{ihl} \\ \times (q_{ijk} - q^*_{ijk}) \\ + \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{o} [-\sum_{h=1}^{n} \sum_{l=1}^{o} \frac{\partial \rho_{ihl}(q^*, s^*)}{\partial s_{ijk}} q^*_{ihl} - D_{ijk} + \frac{\partial h_{ijk}(s^*_{ijk})}{\partial s_{ijk}}] \\ \times (s_{ijk} - s^*_{ijk}) \ge 0, \quad \forall (q, s) \in K. \end{split}$$

Existence

A solution $(q^*, s^*) \in K$ to the variational inequalities is guaranteed to exist from the classical theory of variational inequalities since the feasible set K is compact.

Uniqueness

Furthermore, if the function that enters the variational inequality is strictly monotone, then the solution is unique.

The Algorithm and Case Study

The Euler Method of Dupuis and Nagurney

Explicit Formulae for the Euler Method We have the following closed form expression for the high-value cargo shipments i = 1, ..., m; j = 1, ..., n; k = 1, ..., o:

$$egin{aligned} q_{ijk}^{ au+1} &= \max\{0,\min\{ar{q}_{ijk},q_{ij}^{ au}+a_{ au}(-\sum_{h=1}^n\sum_{l=1}^orac{\partial \hat{c}_{ihl}(q^{ au})}{\partial q_{ijk}}+
ho_{ijk}(q^{ au},s^{ au})\ &+\sum_{i=1}^n\sum_{l=1}^orac{\partial
ho_{ihl}(q^{ au},s^{ au})}{\partial q_{iik}}q_{ihl}^{ au})\}\} \end{aligned}$$

and the following closed form expression for the security levels i = 1, ..., m; j = 1, ..., n; k = 1, ..., o:

$$egin{aligned} s_{ijk}^{ au+1} &= \max\{0,\min\{ar{s}_{ijk},s_{ijk}^{ au}+a_{ au}(\sum_{h=1}^n\sum_{l=1}^orac{\partial
ho_{ihl}(q^{ au},s^{ au})}{\partial s_{ijk}}q_{ihl}^{ au}\ &+ D_{ijk}-rac{\partial h_{ijk}(s_{ijk}^{ au})}{\partial s_{ijk}})\}\}. \end{aligned}$$

<u>h=1</u> l=1

Case Study Focuses on Precious Metals



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We now apply the above Euler method to compute the high-value product shipments and security level investments in a series of numerical examples.

We implemented the algorithm in FORTRAN and used a LINUX system at the University of Massachusetts Amherst for the computations.

The convergence criterion was that the absolute value of the difference of the cargo shipment and security level iterates at two successive iterations was less than or equal to 10^5 .

All the variables (shipments and security levels) were initialized to 0.00.

The sequence $\{\alpha_{\tau}\} = \{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, ...\}.$

Example 1



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Example 1

The data are as follows. The total cost function is:

$$\hat{c}_{111} = q_{111}^2 + 5q_{111},$$

the demand price function is:

$$\rho_{111} = -2q_{111} + 10s_{111} + 100,$$

the upper bound on the security level is:

 $\bar{s}_{111} = .99,$

the upper bound on the cargo shipment is:

 $\bar{q}_{111} = 100.$

The damages, in order to reflect the high value of the cargo are: \$50,000,

so that, at a unit price of 500 and a maximum capacity of 100 for the shipment, we obtain \$50,000.

The security investment cost function has $\alpha_{111} = 10$. This reflects that the freight service provider does not have much security to begin with and, hence, the α_{111} is rather large.

The Euler method yields the equilibrium solution: $q_{111}^* = 17.48$ and $s_{111}^* = .99$. The demand price for shipping one unit, ρ_{111} , evaluated at the equilibrium pattern, is 74.93. The expected utility of freight service provider 1, $E(U_1)$, is 327.

FSP 1 invests in the maximum security level possible and still garners a positive expected utility.

Example 2

Example 2 introduces a competitor to the market in the form of a second FSP, as depicted in Figure 3.



Figure 3: Example 2: Two Freight Service Providers, One Shipper, and Destination

Example 2

The data for FSP 1 remain as in Example 1 except that there is now a new demand price function due to competition.

The demand price functions for the FSPs are:

 $\rho_{111} = -2q_{111} - q_{211} + 10s_{111} + 100, \quad \rho_{211} = -3q_{211} - 2q_{111} + 10s_{211} + 110.$

Also, the total cost function for the second, new, FSP 2 is:

$$\hat{c}_{211} = .5q_{211}^2 + 5q_{211}.$$

The security investment cost function for FSP 2 has $\alpha_{211} = 10$ and the upper bound on the cargo shipment $\bar{q}_{211} = 120$.

The damage $D_{211} = 40,000$.

The Euler method converges to the following equilibrium shipment and security level pattern:

 $q_{111}^* = 15.49, \quad q_{211}^* = 11.99, \quad s_{111}^* = .99, \quad s_{211}^* = .99.$

The demand prices at the equilibrium solution are:

$$\rho_{111} = 66.94, \quad \rho_{211} = 52.96.$$

FSP 1 now has an expected utility, $E(U_1) = 129.36$, whereas FSP 2 has an expected utility $E(U_2) = 58.16$.

With increased competition, FSP 1 now has a lower expected utility than in Example 1 (129.36 vs. 327.00). Moreover, FSP 1 now charges a lower price for high-value cargo shipment than he did in Example 1 (66.94 vs. 74.93), when there was no competition.

The total volume of shipments from the shipper origin node to the destination node increases (from 17.48 to 27.48). This may be viewed as the shipper diversifying his risk.

Example 3

Example 3 introduces a new destination node.



Figure 4: Example 3: Two Freight Service Providers, One Shipper, and Two Destination Nodes

The data remain as in Example 2 but with the new data added as per below.

The added total cost functions are: $\hat{c}_{112} = 1.5q_{112}^2 + 5q_{112}$, $\hat{c}_{212} = q_{212}^2 + 5q_{212}$.

The added demand price functions are:

 $\rho_{112} = -3q_{112} - q_{212} + 5s_{112} + 270, \quad \rho_{212} = -2q_{212} - q_{112} + 5s_{212} + 200.$

The damages associated with transport to destination node 2 are:

$$D_{112} = 5,600, \quad D_{212} = 10,000.$$

The α_{iik} s of the added investment cost functions are:

$$\alpha_{112} = 12, \quad \alpha_{212} = 10.$$

The upper bounds on shipments on the new links are:

 $\bar{q}_{112} = 80, \quad \bar{q}_{212} = 100.$

The Euler method converges to the following equilibrium solution:

$$q_{111}^* = 15.49, q_{112}^* = 26.64, q_{211}^* = 11.99, q_{212}^* = 28.89,$$

 $s_{111}^* = .99, s_{112}^* = .46, s_{211}^* = s_{212}^* = .99.$

The demand prices at the computed equilibrium pattern are:

 $\rho_{111} = 66.94, \rho_{112} = 163.48, \rho_{211} = 52.96, \rho_{212} = 120.54.$

The expected utilities of the freight service providers are now:

 $E(U_1) = 237.83, \quad E(U_2) = 2371.25.$

With a new destination node to ship the high-value cargo to, both FSPs garner enhanced expected utilities in comparison to their values in Example 2. FSP 2 especially benefits from the new destination node requiring freight service provision (from 58.16 to 2371.25).

The prices that are paid for the freight service provision at destination node 2 are more than double those paid for at destination node 1 to a given FSP. This is due to the fact that the fixed components (intercepts) of the demand price functions to the new destination are higher than to destination node 1, demonstrating that shippers are willing to pay a higher price for delivery to destination node 2.

FSP 2 provides maximum security levels for transportation for both destinations and earns a higher expected utility than does FSP 1 who has a security level about one half that at destination node 2 than at destination node 1. This is due, in part, to FSP 1's lower damages as compared to those that would be accrued for FSP 2, given an attack, at destination node 2.

Example 4



Figure 5: Example 4: Two Freight Service Providers, Two Shippers, and Two Destination Nodes

Example 4

Example 4 is constructed from Example 3 and has the same data except that now we have an additional shipper who wishes to explore freight service provision from the two freight service providers.

The total cost functions associated with the second shipper are:

$$\hat{c}_{121} = q_{121}^2 + q_{121}, \quad \hat{c}_{122} = .5q_{122}^2 + q_{122},$$

 $\hat{c}_{221} = q_{221}^2 + 2q_{221}, \quad \hat{c}_{222} = 1.5q_{222}^2 + 3q_{222}.$

The demand price functions associated with transacting with the second shipper are:

 $\rho_{121} = -2q_{121} - q_{221} + s_{121} + 150, \quad \rho_{122} = -3q_{122} - q_{222} + 2s_{122} + 130,$ $\rho_{221} = -4q_{221} - q_{121} + 5s_{221} + 120, \quad \rho_{222} = -5q_{222} - q_{112} + 3s_{222} + 140.$ The additional α_{ijk} terms are:

$$\alpha_{121} = 5, \alpha_{122} = 4, \alpha_{221} = 3, \alpha_{222} = 12.$$

The additional damage terms are:

 $D_{121} = 20000, D_{122} = 15000, D_{221} = 25000, D_{222} = 2000.$

The additional upper bounds are: $\bar{q}_{121} = 100, \bar{q}_{122} = 80, \bar{q}_{221} = 70, \bar{q}_{222} = 60.$

Example 4

The Euler method converges to the following equilibrium shipment and security level pattern:

$$\begin{aligned} q_{111}^* &= 15.71, q_{112}^* = 26.64, q_{121}^* = 23.34, q_{122}^* = 17.78\\ q_{211}^* &= 10.65, q_{212}^* = 28.89, q_{221}^* = 9.96, q_{222}^* = 6.56.\\ s_{111}^* &= .99, s_{112}^* = .46, s_{121}^* = .99, s_{122}^* = .99,\\ s_{211}^* &= .99, s_{212}^* = .99, s_{221}^* = .99, s_{222}^* = .00. \end{aligned}$$

The demand prices incurred at the equilibrium pattern are:

$$\begin{split} \rho_{111}^* &= 67.83, \rho_{112}^* = 163.48, \rho_{121}^* = 94.35, \rho_{122}^* = 72.10, \\ \rho_{211}^* &= 47.61, \rho_{212}^* = 120.54, \rho_{221}^* = 61.77, \rho_{222}^* = 53.94. \end{split}$$

The expected utilities of the freight service providers are: $E(U_1) = 2567.49, E(U_2) = 708.97.$

With a second shipper node added, there is the potential for increased business for the two FSPs. Although FSP 1 now enjoys an expected utility that is more than tenfold higher than that in Example 3, FSP 2 experiences a high security investment cost function associated with destination node 2 and his security level associated with shipping from shipper 2 to destination node 2 is .00 at the equilibrium.

FSP 1 handles three times the volume of cargo from the two shippers to destination node 2. The lowest cargo shipment is q_{222}^* with security level $s_{222}^* = .00$.

Summary and Conclusions

Summary and Conclusions

- We quantify security investment cost functions which may differ for distinct FSP/shipper/destination node combinations.
- Shippers reveal their preferences and sensitivity to investments in security through the prices that they are will to pay for freight service provision and these also can be distinct for different freight service provider/ shipper/ destination node combinations.
- The FSPs seek to maximize their expected utilities, which capture the probability of an attack associated with different links and are a function of the security level associated with that link. Hence, risk is also captured in the competitors' objective functions.

Summary and Conclusions

- The model is not limited by the number of FSPs, shippers, and/or destination nodes.
- The equilibrium conditions, which correspond to a Nash Equilibrium, are formulated as a variational inequality problem for which a solution is guaranteed to exist.
- The model is computable and numerical examples reveal the equilibrium high-value cargo shipments plus security levels that the freight service providers deliver and invest in, respectively.

Thank You!

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