Human Migration Networks: How Operations Research Can Assist with Refugees and Supply Chain Labor Shortages

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Acknowledgments

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I am grateful to my collaborators and students and to all the outstanding operations researchers for the impactful work.

Dedication

This talk is dedicated to essential workers, who have sustained us in the COVID-19 pandemic.

I also acknowledge all the freedom-loving people on the planet, including those fighting for their freedom in Ukraine.

Outline of Presentation

- **Motivation and Some Background**
- **Human Migration and Refugees**
- **Methodology - The Variational Inequality Problem**
- **The Refugee Migration Models and Variational Inequality Formulations**
- **Attracting International Migrant Labor to Alleviate Labor Shortages**
- **Summary and Conclusions**

Motivation and Some Background

I Work on the Modeling of Network Systems

Much of My Recent Research Has Been on Supply Chains

Some of My Books

It's All About People

A major research theme of ours in the COVID-19 pandemic has been the inclusion of labor in supply chains, using optimization and game theory, as well as expanding the scope of human migration networks. Such research is also very relevant to Russia's war against Ukraine with the immense refugee crisis.

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January 29, 2021 in Supply Chain Networks

In the End, It's All About People

COVID-19 vaccine production reveals dependency on supply chains, labor workforce in the U.S.

By Anna Nagurney

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The COVID-19 pandemic has dramatically revealed how dependent we are on supply chains and the availability of labor. Without the human element, meatpacking plants cannot function; fresh produce cannot be picked; grocery stores cannot be shelved; PPEs cannot be produced and distributed; and products cannot be delivered to our homes through e-commerce. Also, COVID-19 vaccine production may lack the human resources to ensure product quality and efficacy, as well as its distribution and ultimate administration into our arms. Without healthcare workers to

Human Migration and Refugees

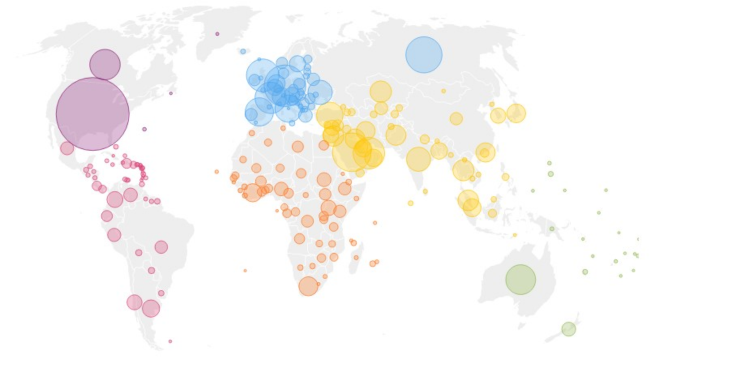
Human Migration and Refugees

• **Reasons for human migration are numerous**, from individuals **seeking better economic opportunities and enhanced prosperity for themselves and their families**, to **those fleeing conflict, violence, and persecution**. With climate change and the increasing number and severity of disasters, including hurricanes, floods, tornados, earthquakes, etc., some migrants are seeking locations of greater expected safety and security.

• **The current global estimate is that there were around 281 million international migrants in the world in 2020, about 3.6% of the global population. This is over three times the estimated number in 1970!**

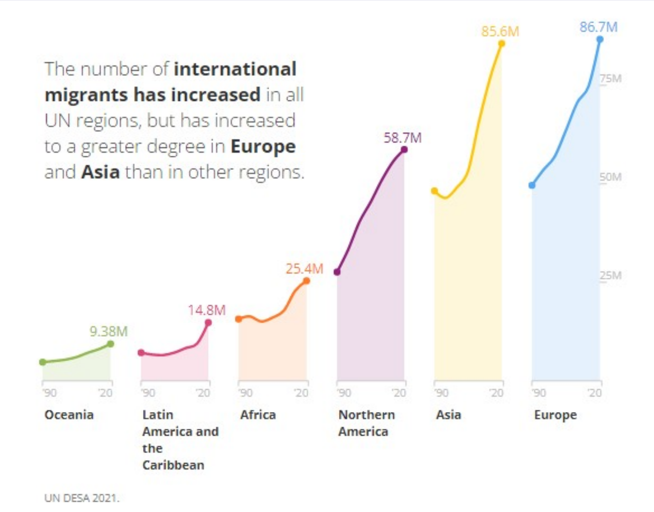
• **The number of international migrants is growing faster than the global population.**

The total number of international migrants within each country:



United Nations (2021)

International Migrants



United Nations (2021)

Definition of a Refugee

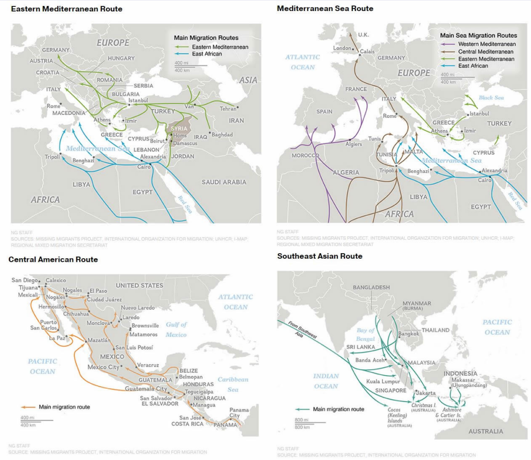
The United Nations Convention Relating to the Status of Refugees in 1951 defined a refugee as an individual living outside his or her country of nationality, who is unable or unwilling to return because of a well-substantiated fear of persecution due to race, religion, nationality, membership in a political social group, etc.

In our research, we also consider humans adversely affected by climate change, as refugees, and note that, as emphasized by Hebert, Perez, and Harati (2018), among the most studied causes of human migration are climate issues and conflicts, as well as economic reasons.

Refugees and the Pandemic

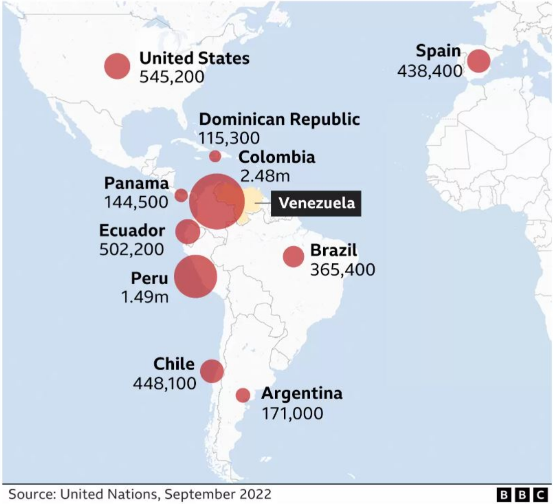
Refugees have historically always been part of human migration, seeking locations of greater safety and security for themselves and their families.

With the COVID-19 pandemic, declared by the World Health Organization on March 11, 2020, Dolmans et al. (2020) reported that COVID-19 likely exacerbated refugee flows and increased the difficulty in seeking asylum due to measures imposed by governments in response to the pandemic.



Source: National Geographic via IOM UN Migration Blog - 2015 data

More than seven million Venezuelans have left their homeland since 2015 amid an ongoing economic and political crisis, according to recent United Nations data.

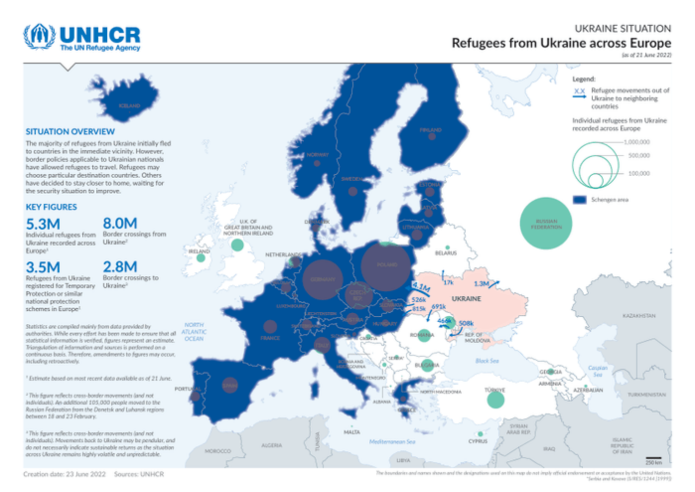


The Largest Refugee Crisis Since World War II

- **Russia's invasion of Ukraine on February 24, 2022, has resulted in the largest refugee crisis since World War II.**
- **In October 2022, the United Nations listed 7.6 million refugees across Europe from Ukraine.**
- **In all, nearly one-third of Ukrainians have been displaced. About 13 million are stranded in Ukraine due to fighting, impassable routes, or the lack of resources to move.**

- **About half of the refugees are children.**

Refugees from Ukraine



Source: UNHCR, June 27, 2022

Vivid depictions of people fleeing their origin locations permeate the news, whether attempting to escape the great strife and suffering in Syria; the violence in parts of Central America, the economic collapse of Venezuela, and even flooding in parts of Asia as well as droughts in parts of Africa.

Refugees

At times, refugees will travel in extremely dangerous conditions to escape the dire circumstances at their origin nodes.

In 2015, the UN Refugee Agency reported a maritime refugee crisis with, in the first half of that year, 137,000 refugees crossing the Mediterranean Sea to Europe, via very risky transport modes, and with many more unsuccessfully attempting such a passage. 800 died in the largest refugee shipwreck on record that April.

Refugees from Afghanistan

Prior to Russia's war on Ukraine, the largest number of refugees were from Syria, Venezuela, and Afghanistan.

Afghan evacuees boarding American aircraft during Operation Allies Refuge in August 2021.

Government Policies

Governments of various nations, hence, are increasingly being faced with multiple challenges associated with human migration flows. In response to challenges, they are adopting different regulations.

According to the United Nations (2013), **migration policies in both origin and destination countries play an important role in determining the migratory flows**. In managing international migration flows, governments usually focus on different types of migrants, of which the most salient are highly skilled workers, dependents of migrant workers, irregular migrants, and refugees and asylum seekers (cf. Karagiannis (2016)).

Between 11 March 2020, when the WHO declared COVID-19 a pandemic, and 22 February 2021, nearly 105,000 movement restrictions were implemented around the world, according to the International Organization for Migration.

We can expect refugee migratory flows to continue to increase, posing a critical need for the provision of rigorous tools for policy-makers and decision-makers for the quantification of refugee migratory flows and the impacts of various regulations.

The construction of a relevant model with policy implications was one of our goals.

Methodology - The VI Problem

Methodology - The Variational Inequality Problem

We utilize the theory of variational inequalities for the formulation, analysis, and solution of both centralized and decentralized supply chain network problems.

Definition: The Variational Inequality Problem

The finite-dimensional variational inequality problem, $\mathrm{VI}(F, \mathcal{K})$*, is to determine a vector* $X^* \in \mathcal{K}$*, such that:*

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

where F *is a given continuous function from* $\mathcal{K}$ *to* R^N*,* $\mathcal{K}$ *is a given closed convex set, and* $\langle \cdot, \cdot \rangle$ *denotes the inner product in* R^N*.*

Methodology - The Variational Inequality Problem

The vector X consists of **the decision variables** – typically, the flows (products, prices, etc.).

$\mathcal{K}$ is the **feasible set representing how the decision variables are constrained** – for example, the flows may have to be nonnegative; budget constraints may have to be satisfied; similarly, quality and/or time constraints may have to be satisfied.

The function F that enters the variational inequality represents **functions that capture the behavior in the form of the functions such as costs, profits, risk, etc.**

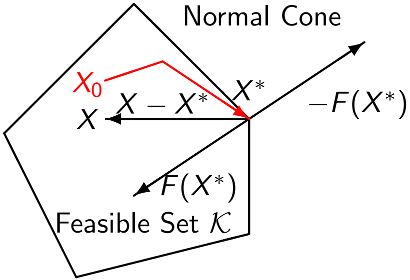
The variational inequality problem contains, as special cases, such mathematical programming problems as:

- systems of equations,
- optimization problems,
- complementarity problems,
- game theory problems, operating under Nash equilibrium,
- and is related to the fixed point problem.

Hence, it is a natural methodology for a spectrum of supply chain network problems from centralized to decentralized ones.

Geometric Interpretation of VI($F, \mathcal{K}$) and a Projected Dynamical System (Dupuis and Nagurney, Nagurney and Zhang)

In particular, $F(X^*)$ is "orthogonal" to the feasible set $\mathcal{K}$ at the point X^*.



Associated with a VI is a Projected Dynamical System, which provides the natural underlying dynamics.

To model the **dynamic behavior of complex networks**, including supply chains, we utilize *projected dynamical systems* (PDSs) advanced by Dupuis and Nagurney (1993) in *Annals of Operations Research* and by Nagurney and Zhang (1996) in our book *Projected Dynamical Systems and Variational Inequalities with Applications*.

Such nonclassical dynamical systems are now being used in:

evolutionary games (Sandholm (2005, 2011)),

ecological predator-prey networks (Nagurney and Nagurney (2011a, b)),

even **neuroscience** (Girard et al. (2008),

dynamic spectrum model for cognitive radio networks (Setoodeh, Haykin, and Moghadam (2012)),

Future Internet Architectures (Saberi, Nagurney, Wolf (2014); see also Nagurney et al. (2015), Marentes et al. (2016)).

The Refugee Migration Models and Variational Inequality Formulations

This part of my lecture is based on the paper:

A. Nagurney, P. Daniele, and L.S. Nagurney, "Refugee Migration Networks and Regulations: A Multiclass, Multipath Variational Inequality Framework," *Journal of Global Optimization* **78 (2022), pp 627-649.**

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Refugee migration networks and regulations: a multiclass, multipath variational inequality framework

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Abstract
In this paper, we take up the timely topic of the modeling, analysis, and solution of refugee migration networks. We construct a general, multiclass, multipath model, determine the governing equilibrium conditions, and provide alternative variational inequality formulations in path flows and in link flows. We also demonstrate how governmental imposed regulations associated with refugees can be captured via constraints. We provide qualitative properties and then establish, via a supernetwork transformation, that the model(s) are isomorphic to traffic network equilibrium models with fixed demands. Illustrative examples are given, along with numerical examples, inspired by a refugee crisis from Mexico to the United States, which are solved using the Euler method embedded with exact equilibration. The work sets the foundation for the development of additional models and algorithms and also provides insights as to who wins and who loses under certain refugee regulations.

Keywords Refugees · Human migration · Networks · Variational inequalities · Regulations

1 Introduction

Refugees have historically always been part of human migration, seeking locations of greater safety and security for themselves and their families. Now, with the increasing impacts of climate change, and associated increases in the severity, as well as the frequency, of many disasters, including hurricanes and floods, coupled with numerous conflicts, strife, and violence in many parts of the globe, the number of refugees and asylum seekers is escalating. According to the United Nations [52], the number of international migrants was approximately 258 million (3.4% of the global population) in the year 2017, with the total number of international migrants increasing by almost 50% since 2000 (United Nations [52]). Also,

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Literature Review with a Focus on Networks and Migration

• Nagurney (1989) introduced a multiclass migration equilibrium model, which did not include migration/movement costs, and **was isomorphic to a traffic network equilibrium with special structure**. The model was then extended to include flow-dependent migration costs and an expanded set of equilibrium conditions in Nagurney (1990).

• Nagurney, Pan, and Zhao (1992a) proposed **a multiclass human migration model**, which further generalized to include class transformations in Nagurney, Pan, and Zhao (1992b).

• Pan and Nagurney (1994) considered **chain migration (unlike the earlier work) and introduced a multi-stage (but single class) Markov chain model**. The authors established a connection between a sequence of variational inequalities and a non-homogeneous Markov chain. They also proved that, under certain assumptions, the stability of the one-step transition matrix guarantees the stability of the *n*-step transition matrix.

Literature Review with a Focus on Networks and Migration

• Pan and Nagurney (2006) utilized **evolution variational inequalities for the first time to model the dynamic adjustment of a socio-economic process in the context of human migration**. Convergence of algorithms in this framework, which is infinite-dimensional, was addressed (see Daniele (2006)).

• Interestingly, many of the network equilibrium models of human migration have **found application to the migration of animals in ecology with a focus on fish and maritime ecosystems** (see Mullon and Nagurney (2012), Mullon (2014), Mariani et al. (2016)).

• Kalashnikov et al. (2008) constructed a human migration model with a **conjectural variations equilibrium (CVE)**.

• Capello and Daniele (2019) developed **a Nash equilibrium model of human migration** with features of CV and provided examples on the flow of migrants from Africa through the Mediterranean sea to Italy in 2018.

The paper by Nagurney and Daniele (2020) was the first to include regulations within a human migration network framework.

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International human migration networks under regulations

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ABSTRACT

International human migration has transformed economies and societies. The new millennium, with climate change and its impacts, and increasing conflicts and displacements, has experienced a great increase in international migrants, with associated challenges faced by governments. In this paper, we advance the modeling, analysis, and solution of international human migration problems by developing a network model with regulations. The formalism uses the theory of variational inequalities, coupled with Lagrange analysis, in order to gain insights as to the impacts of the regulations on utilities of multiple classes of migrants, and on the equilibrium flows. Our results add to the literature on operations research for societal impact, inspired by the real world.

1. **Introduction**

Humans have engaged in migration since time immemorial. Reasons for human migration are numerous, from individuals seeking better economic opportunities and enhanced prosperity for themselves and their families, to those fleeing conflict, violence, and persecution. With climate change and the increasing number and severity of natural disasters, including hurricanes, floods, tornados, earthquakes, etc., some migrants are seeking locations of greater expected safety and security. In 2017, the number of international migrants was an estimated 258 million persons (3.4% of the global population), with the total number of international migrants increasing by almost 50% since 2000 (United Nations, 2017). As also reported therein, the number of international migrants is growing faster than the global population. Moreover, in the same almost two decade period, the number of refugees and asylum seekers increased from 16 to 26 million, comprising about 10% of the international migrants.

Vivid depictions of people fleeing their origin locations permeate the news, whether attempting to escape the great strife and suffering in Syria (United Nations Refugee Agency, 2019); the violence in parts of Central America (Bartenstein & McDonald, 2019), the economic collapse of Venezuela (Kennedy, 2019), and even flooding in parts of Asia (Dash & Paul, 2017), as well as droughts in parts of Africa (cf. Linke, Witmer, O'Loughlin, Mc Cabe, & Tir, 2018). Interestingly, since the onset of the new millennium, the countries experiencing the largest increase in their diaspora populations are the Syrian Arab Republic (872%), India (108%), and the Philippines (85%) (cf. United Nations, 2017). At times, refugees will travel in extremely dangerous conditions to escape the dire circumstances at their origin nodes. For example, the United Nations Refugee Agency (2015) reported a maritime refugee crisis with, in the first half of that year, 137,000 refugees crossing the Mediterranean Sea to Europe, via very risky transport modes, and with many more unsuccessfully attempting such a passage. 800 died in the largest refugee shipwreck on record that April. Clearly, the issue of human migration is one of multiple dimensions, including economic, political, sociological, and even environmental aspects.

Governments of various nations, hence, are increasingly being faced with multiple challenges associated with human migration flows. In response to challenges, they are adopting different regulations. According to Bertossi (2008), since the 1990s, unprecedented efforts have been made to control the movement of people across state borders. As argued eloquently by Helbling and Leblang (2019), migration regulations are core to national sovereignty since they directly impact the composition of the community that makes up the country. According to the informative report by the United Nations (2013), migration policies in both origin and destination countries play an important role in determining the migratory flows. Moreover, as noted therein, in managing international migration flows, governments usually focus on different types of migrants, of which the most salient are highly skilled workers, dependents of migrant workers, irregular migrants, and refugees and asylum seekers (cf. Karagiannis, 2016).

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Human migration networks and policy interventions: bringing population distributions in line with system optimization

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Abstract

In this paper, we demonstrate that, through policy interventions, in the form of subsidies, a system-optimum for a multiclass human migration network can be achieved, despite the migrants, who can be refugees, behaving in a user-optimized manner. The formulation and analysis are conducted using variational inequality theory. The policy intervention allows governmental decision-makers to moderate the flow of migrants while enhancing societal welfare. An algorithm is proposed and applied to compute the solutions to a series of numerical examples, with changes in initial populations and utility functions, inspired by a pandemic, followed by a natural disaster.

Keywords: human migration; networks; variational inequalities; policy interventions; system optimization; user optimization; subsidies

1. Introduction

Massive human migrations are posing major challenges to national governments across the globe. The reasons for recent migrations include violence, wars, and persecution, climate change, a variety of disasters (earthquakes, hurricanes, floods, and tornadoes), and poverty and economic inequality, with the latter driving humans to seek better lives for their families and themselves. According to the UNHCR (2020), 70.8 million have fled their homes worldwide, the highest level of displacement on record. The United Nations (2017) is reporting that the number of international migrants was an estimated 258 million individuals in 2017, with the total number of international migrants increasing by almost 50% since the new millennium. The number of refugees and asylum seekers during this period has increased from 16 to 26 million, approximately 10% of the international migrants.

The media has been filled with news and images of migrants, including refugees, often undertaking dangerous journeys on land and sea to flee their compromised situations. The economic

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INTERNATIONAL TRANSACTIONS IN OPERATIONAL RESEARCH

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Modeling of Covid-19 trade measures on essential products: a multiproduct, multicountry spatial price equilibrium framework

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Abstract

In this paper, we develop a unified variational inequality framework in the context of spatial price network equilibrium problems that handles multiple products with multiple demand and supply markets in multiple countries as well as multiple transportation routes. The model incorporates a plethora of distinct trade measures, which is particularly important in the pandemic, as PPEs and other essential products are in high demand, but short in supply globally. In the model, product flows as well as prices at the supply markets and the demand markets in different countries are variables that allows us to seamlessly introduce various trade measures, including tariffs, quotas, as well as price floors and ceilings. Qualitative properties are analyzed. Numerical examples are provided to illustrate the impacts of the trade measures on equilibrium product path and link flows, and on prices, and demand and supply quantities. Given the relevance of the trade measures in the world today and discussions concerning the impacts, the framework constructed in this paper is especially timely.

Keywords: Covid-19; essential supplies; trade measures; spatial price equilibrium; networks

1. Introduction

The World Health Organization (WHO) declared the Covid-19 pandemic on March 11, 2020 (WHO, 2020a). The ensuing global healthcare disaster has endangered and disrupted the lives of billions around the world, resulting in illnesses and deaths, and has also generated secondary crises. No one knows with certainty when the pandemic will end. According to Johns Hopkins

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Contributions in Our Paper

1. An international human migration network model is constructed, which allows for route choices by the migrants, which are refugees.
2. The routes consist of one or more links, with cost functions that capture congestion, a factor that has been seen in practice.
3. The model is then extended to include regulations that can be imposed by distinct multiple countries. In previous work (cf. Nagurney and Daniele (2020)), it was assumed that a single country imposes the regulations on migrants.
4. A supernetwork transformation into a traffic network equilibrium problem with fixed demands is constructed. This identification enables the transfer of algorithmic schemes for the TNE problem, which has had a long history, to the novel application domain of refugee/migration networks.
5. Theoretical results are presented plus an algorithm.
6. Numerical examples reveal insights for policy-makers and decision-makers.

The Multiclass, Multipath Refugee Migration Models

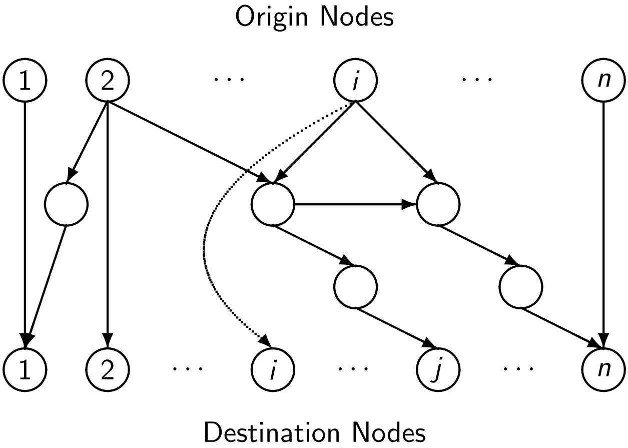


Figure: Sample Refugee Network Topology

Some Notation

Table: Common Notation for the Refugee Migration Models

Notation	Definition
x_r^k	flow of refugees of class k on route/path r. The $\{x_r^k\}$ elements are grouped into vector $x^k \in R_+^{n_P}$, where n_P denotes the number of paths in the migration network. We further group the x^k vectors; $k = 1, \ldots, J$, into vector $x \in R_+^{Jn_P}$.
f_a^k	flow of refugees of class k on link a. We group the link flows for class k for all links $a \in L$ into vector $f^k \in R^{n_L}$ where n_L is the number of links. We then group the link flows for all classes into vector $f \in R^{Jn_P}$.
p_i^k	nonnegative population of refugee class k at origin node i. We group the populations of class k; $k = 1, \ldots, J$, into vector $p^k \in R_+^n$. We further group all such vectors into vector $p \in R_+^{Jn}$.
$\bar{p}_i^k$	initial fixed population of class k at origin node i; $i = 1, \ldots, n$; $k = 1, \ldots, J$.
$u_i^k(p)$	utility perceived by refugee class k at node i; $i = 1, \ldots, n$; $k = 1, \ldots, J$. We group the utility functions for each k into vector $u^k \in R^n$ and then group all such vectors for all k into vector $u \in R^{Jn}$.
$c_a^k(f)$	migration cost associated with traversing link a by refugees of class k. Here we interpret the migration cost as a travel cost. We group link costs for each k into vector $c^k \in R^{n_L}$ and then group all such vectors into vector $c \in R^{Jn_L}$.
$C_r^k(x)$	cost of migration, that is, the travel cost, encumbered by class k in migrating on route r associated with an O/D pair w_{ij}; $i, j = 1, \ldots, n$; $k = 1, \ldots, J$.

Conservation of Flow

Since the route flows must be nonnegative, we have that

$$x_r^k \geq 0, \quad \forall r \in P, \forall k. \tag{1}$$

Furthermore, the refugee flows out of an origin node i must satisfy:

$$\bar{p}_i^k = \sum_{j=1}^{n} \sum_{r \in P_{w_{ij}}} x_r^k, \quad \forall i, \forall k. \tag{2}$$

The volume of population of each class k at each destination node j, after migration takes place, must satisfy the following equation:

$$p_j^k = \sum_{i=1}^{n} \sum_{r \in P_{w_{ij}}} x_r^k, \quad \forall j, \forall k. \tag{3}$$

The link flows are related to the route flows according to:

$$f_a^k = \sum_{r \in P} x_r^k \delta_{ar}, \quad \forall a, \forall k, \tag{4}$$

where $\delta_{ar} = 1$, if link a is contained in route r and 0, otherwise.

Additional Constructs

In view of the conservation of flow equations (4), we may define link cost functions in route/path flows, such that $\hat{c}_a^k = \hat{c}_a^k(x) \equiv c_a^k(f)$, for all links a and for all classes of refugees k.

The cost on a route r is equal to the sum of costs on the links that make up the route, that is,

$$C_r^k(x) = \sum_{a \in L} \hat{c}_a^k(x)\delta_{ar}, \quad \forall k, \forall r. \tag{5}$$

We define the feasible set
$K^1 \equiv \{(p,x)|x \geq 0, \text{ and } (2) \text{ and } (3) \text{ hold}\}$.

Equilibrium Conditions for the Multiclass, Multipath Refugee Migration Model without Regulations

Definition 1: Multiclass, Multipath Refugee Migration Equilibrium without Regulations

A vector of populations and refugee migration flows $(p^*, x^*) \in K^1$ *is in equilibrium if it satisfies the following conditions: For each class* k; $k = 1, \ldots, J$, *and each pair of origin/destination nodes* i, j; $i, j = 1, \ldots, n$, *and all routes* $r \in P_{w_{ij}}$ *we have that*

$$u_i^k(p^*) + C_r^k(x^*) \begin{cases} = u_j^k(p^*) - \lambda_i^{k*}, & \text{if} \quad x_r^{k*} > 0, \\ \geq u_j^k(p^*) - \lambda_i^{k*}, & \text{if} \quad x_r^{k*} = 0, \end{cases} \tag{6}$$

and

$$\lambda_i^{k*} \begin{cases} \geq 0, & \text{if} \quad \sum_{j=1}^n \sum_{r \in P_{w_{ij}}, j \neq i} x_r^{k*} = \bar{p}_i^k, \\ = 0, & \text{if} \quad \sum_{j=1}^n \sum_{r \in P_{w_{ij}}, j \neq i} x_r^{k*} < \bar{p}_i^k. \end{cases} \tag{7}$$

Variational Inequality Formulations

Theorem 1: Variational Inequality Formulation of the Refugee Migration Model without Regulations in Path Flows

A population and refugee flow pattern $(p^*, x^*) \in K^1$ is a refugee migration equilibrium without regulations according to Definition 1, if and only if it satisfies the variational inequality problem in path flows

$$-\langle u(p^*), p - p^* \rangle + \langle C(x^*), x - x^* \rangle \geq 0, \quad \forall (p, x) \in K^1, \qquad (8)$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in the appropriately dimensioned Euclidean space.

Existence of a solution follows from the standard theory of variational inequalities (see Kinderlehrer and Stampacchia (1980) Theorem 3.1) under the assumption of continuity of the utility functions u and the migration cost functions c, since the feasible convex set K^1 is compact.

Alternative variational inequality formulations can induce distinct algorithmic schemes. Hence, for completeness, we now provide a link flow variational inequality formulation equivalent to the path flow one in (8). We first define the feasible set
$K^2 \equiv \{(p, f) | \exists x \text{ such that } (1) - (4) \text{ hold}\}$.

Corollary 1: Variational Inequality Formulation of the Refugee Migration Model without Regulations in Link Flows

A population and refugee link flow pattern $(p^*, f^*) \in K^2$ is a refugee migration equilibrium without regulations according to Definition 1, if and only if it satisfies the variational inequality problem in link flows

$$-\langle u(p^*), p - p^* \rangle + \langle c(f^*), f - f^* \rangle \geq 0, \quad \forall (p, f) \in K^2. \tag{9}$$

Variational Inequality Formulation of the Refugee Migration Model with Regulations

We denote a specific country by h, where $h = 1, \ldots, H$ and define the set O^h consisting of origin nodes of refugees from countries/locations that the country h imposes a regulation on, and let D^h denote the set of destination nodes, which lie in country h. C^h denotes the set of refugee classes that country h imposes the regulations on and U^h is the nonnegative upper bound imposed by country h on refugee migratory flows.

The constraints can then be stated as follows:

$$\sum_{i \in O^h} \sum_{j \in D^h} \sum_{k \in C^h} \sum_{r \in P_{w_{ij}}} x_r^k \leq U^h, \quad h = 1, \ldots, H. \tag{10}$$

The set of constraints (10) is sufficiently general to capture specific, distinct migration regulations in practice. such as: an upper bound (which may be zero) of all classes from a certain country or countries; or an upper bound on a single class or several classes from a specific country or countries.

Variational Inequality Formulation of the Refugee Migration Model with Regulations

For the refugee migration model with regulations, the equilibrium conditions (6) and (7) are still relevant but with a new feasible set K^3 defined as below to include the constraints (10):

$$K^3 \equiv K^1 \cap \{x | (10) \text{ is satisfied}\}. \quad (11)$$

Theorem 2: Variational Inequality Formulation of the Refugee Migration Model with Regulations in Path Flows

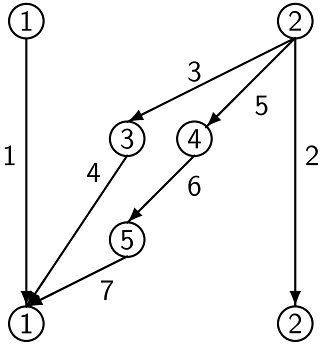
A population and refugee migration flow pattern $(p^*, x^*) \in K^3$ is a refugee migration equilibrium with regulations in path flows, if and only if it satisfies the variational inequality problem

$$-\langle u(p^*), p - p^* \rangle + \langle C(x^*), x - x^* \rangle \geq 0, \quad \forall (p, x) \in K^3. \quad (12)$$

Illustrative Examples

According to the network in the Figure below, refugees residing in country 1 are not interested in migrating to country 2. On the other hand, refugees residing in country 2 are interested in the possibility of migrating to country 1. There are two available paths joining country 2 with country 1.

Origin Nodes



Destination Nodes

Illustrative Example - No Regulation

The routes are comprised of links and are enumerated as follows:

$$r_1 = (1), \quad r_2 = (2), \quad r_3 = (3,4), \quad r_4 = (5,6,7).$$

We consider a single class of migrant. Hence, we suppress the superscript 1 in the notation. The data are: $\bar{p}_1 = 100$ and $\bar{p}_2 = 200$ with the utility functions: $u_1(p) = -p_1 + 1000$ and $u_2(p) = -p_2 + 500$.

The link migration cost functions are:

$$c_1 = c_2 = 0,$$

$$c_3(f) = f_3 + 200, \quad c_4(f) = f_4 + 100,$$

$$c_5(f) = f_5 + 30, \quad c_6(f) = .5f_6 + 40, \quad c_7(f) = f_7 + 30.$$

Illustrative Example - No Regulation

We first consider the case without regulations. It is easy to compute the equilibrium solution, using simple algebra. Indeed, we find that:

$$x_{r_1}^* = 100, \quad x_{r_2}^* = 75, \quad x_{r_3}^* = 25, \quad x_{r_4}^* = 100;$$

hence,

$$p_1^* = 225, \quad p_2^* = 75,$$

with associated utilities being:

$$\hat{u}_1(x^*) = u_1(p^*) = 775, \quad \hat{u}_2(x^*) = u_2(p^*) = 425,$$

and the incurred migration costs on the routes at equilibrium: $C_{r_1} = 0$, $C_{r_2} = 0$, $C_{r_3} = C_{r_4} = 350$. Moreover, $\lambda_1^* = \lambda_2^* = 0$.

It is clear that the equilibrium conditions (6) and (7) hold.

Illustrative Example - No Regulation

Observe that, initially, before the refugee migration takes place, $u_1(\bar{p}_1) = 900$ and $u_2(\bar{p}_2) = 300$.

Once equilibrium is achieved, those who have migrated from country 2 to country 1 more than double their utility (from 425 to 900), whereas those who remain in country 2 experience a gain in utility of over 33% (from 300 to 425).

Those in country 1, because of the increase in the number of refugees and that they are not migrating, suffer a reduction in utility of approximately 14% (from 900 to 775).

Illustrative Example with Regulation

We now suppose that a regulation is imposed on destination node 1 by country 1 of the following form:

$$x_{r_3} + x_{r_4} \leq U^1 = 25.$$

The new equilibrium solution is:

$$x^*_{r_1} = 100, \quad x^*_{r_2} = 175, \quad x^*_{r_3} = 0, \quad x^*_{r_4} = 25;$$

hence,

$$p^*_1 = 125, \quad p^*_2 = 175,$$

with associated utilities being:

$$\hat{u}_1(x^*) = u_1(p^*) = 875, \quad \hat{u}_2(x^*) = u_2(p^*) = 325,$$

and the migration costs on the routes: $C_{r_1} = 0$, $C_{r_2} = 0$, $C_{r_3} = 300$, $C_{r_4} = 162.50$. Moreover, $\lambda^*_1 = \lambda^*_2 = 0$. The optimal Lagrange multiplier associated with the regulation constraint is: $\mu^*_1 = 387.50$. One can see that route r_3 is too expensive and will not be used under the refugee migratory flow pattern.

Illustrative Example with Regulation

Now, refugees in country 1, at equilibrium, enjoy a higher utility of 875 than before the regulation was imposed (775), an increase of about 13%.

On the other hand, refugees in country 2 now experience a lower utility (325), than before the regulation was imposed (425), a drop of about 30%. They are no longer all free to migrate because of the imposed regulatory upper bound of 25 limiting the migration from country 2 to country 1.

Our numerical examples are inspired by the refugee flows from Mexico to the United States, an issue that has been receiving a lot of attention in the press.

The baseline network for the numerical examples is depicted in the next Figure.

We consider a single class of refugee.

Computation of Solutions to Larger Examples

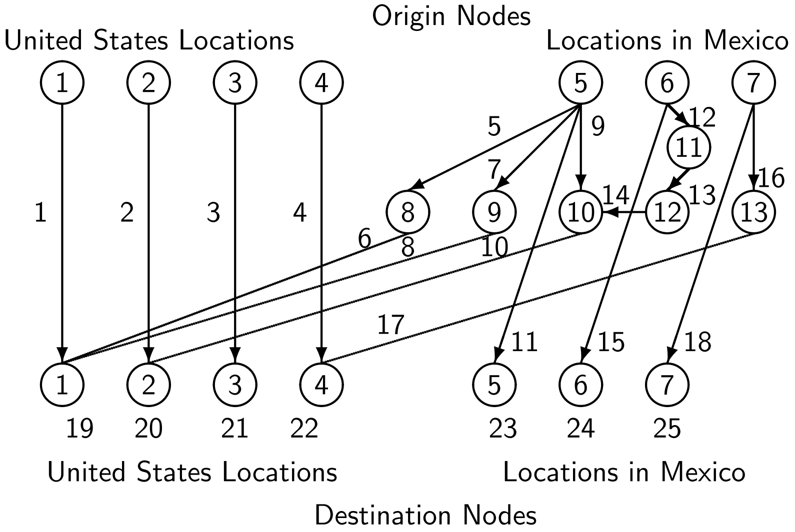


Figure: The Baseline Refugee Network Topology for the Larger Examples

We consider four examples: Example 1 through Example 4.

Example 1: No Regulations

The initial populations at the origin nodes are:

$$\bar{p}_1 = 1,400,000, \quad \bar{p}_2 = 20,000, \quad \bar{p}_3 = 70,000,$$

$$\bar{p}_4 = 260,000, \quad \bar{p}_5 = 50,000, \quad \bar{p}_6 = 225,000, \quad \bar{p}_7 = 30,000.$$

The utility functions associated with these locations are:

$$u_1(p) = -p_1 + 3,000,000, \; u_2(p) = -2p_2 + 200,000, \; u_3(p) = -p_3 + 1,500,$$

$$u_4(p) = 3p_4 + 900,000, \; u_5(p) = -p_5 + 100,000, \; u_6(p) = -p_6 + 300,000,$$

$$u_7(p) = -p_7 + 100,000.$$

From the above utility functions, one can see that the locations in the United States are more attractive than those in Mexico, due to the significantly larger fixed utility term in the corresponding utility functions.

The link costs associated with remaining at one's location at nodes 1 through 7, respectively, are, all equal to 0.00:

$$c_1(f) = c_2(f) = c_3(f) = c_4(f) = c_{11}(f) = c_{15}(f) = c_{18}(f) = 0.00.$$

The costs associated with the refugee migrations are, in turn, as follows:

$$c_5(f) = .00006f_5^4 + 6f_5 + 4f_6 + 200, \; c_6(f) = 7f_6 + 3f_8 + 300,$$

$$c_7(f) = .00008f_7^4 + 8f_7 + 2f_8 + 400,$$

$$c_8(f) = .00004f_8^4 + 5f_8 + 2f_{10} + 450, \; c_9(f) = .00001f_9^4 + 6f_9 + 2f_{10} + 300,$$

$$c_{10}(f) = 4f_{10} + f_{12} + 400,$$

$$c_{12}(f) = 8f_{12} + 2f_{13} + 100, \; c_{13}(f) = .00001f_{13}^4 + 7f_{13} + 3f_9 + 50,$$

$$c_{14}(f) = 8f_{14} + 3f_9 + 100,$$

$$c_{16}(f) = 3f_{16} + f_{12} + 100, \; c_{17}(f) = .00003f_{17} + 3f_{17} + 50.$$

The routes are enumerated as follows:

$$r_1 = (1), \quad r_2 = (2), \quad r_3 = (3), \quad r_4 = (4),$$

$$r_5 = (5, 6), \quad r_6 = (7, 8), \quad r_7 = (9, 10), \quad r_8 = (11),$$

$$r_9 = (12, 13, 14, 10), \quad r_{10} = (15),$$

$$r_{11} = (16, 17), \quad r_{12} = (18).$$

For the solution of the problem, we first construct the supernetwork equivalence with the supernetwork topology as in the Figure and the O/D pairs, the links and paths, the link costs on the new links, the demands, and the path costs, defined accordingly.

Supernetwork Transformation of Examples

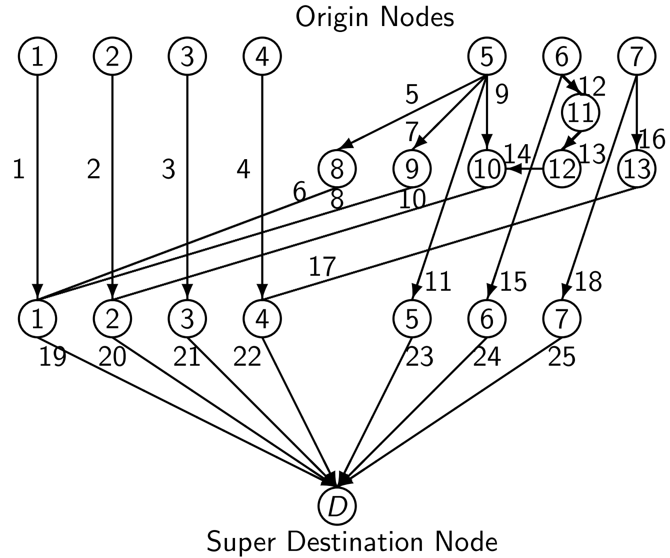


Figure: The Supernetwork Topology of the Examples

Computation of Solutions to Larger Examples

We implemented the Euler method, embedded with the exact equilibration algorithm as described in Nagurney and Zhang (1997); see also the book by Nagurney and Zhang (1996). We initialized the algorithm as follows. The initial populations were equally distributed among all the paths. The sequence $\{a_\mathcal{T}\}$ in the Euler method satisfied the conditions required for convergence and was set to: $.1\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \ldots\}$.

The algorithm was deemed to have converged if the absolute value of the difference between each successively computed path flow differed by no more than 10^{-7}. The computer system utilized was a Linux system at the University of Massachusetts Amherst.

Solution to Example 1

The computed equilibrium path flow pattern for Example 1 is:

$$x_{r_1}^* = 1,400,000.00, \; x_{r_2}^* = 20,000.00, \; x_{r_3}^* = 700,000.00,$$

$$x_{r_4}^* = 260,000.00, \; x_{r_5}^* = 400.25, \; x_{r_6}^* = 336.64, \; x_{r_7}^* = 317.17,$$

$$x_{r_8}^* = 48,945.95, \; x_{r_9}^* = 290.25, \; x_{r_{10}}^* = 224,709.75, \; x_{r_{11}}^* = 199.54,$$

$$x_{r_{12}}^* = 29,800.46.$$

The computed equilibrium populations at the four locations in the United States and at the three locations in Mexico are:

$$p_1^* = 1,400,736.88, \quad p_2^* = 20,607.42, \quad p_3^* = 700,000.00,$$

$$p_4^* = 260,199.55, \; p_5^* = 48,945.95, \; p_6^* = 224,709.75, \; p_7^* = 29,800.46.$$

Solution to Example 1

The associated incurred utilities at the equilibrium at the locations are:

$$u_1(p^*) = 1,599,263.13, \; u_2(p^*) = 158,785.17, \; u_3(p^*) = 800,000.00,$$

$$u_4(p^*) = 119,401.38, \; u_5(p^*) = 51,054.05, \; u_6(p^*) = 75,290.25,$$

$$u_7(p^*) = 70,199.55.$$

We also, for completeness, report the path costs at the computed equilibrium flows since it is easy to then verify that the equilibrium conditions, as stated in Definition 1, hold. Specifically, the incurred path costs at the equilibrium are:

$$C_{r_1}(x^*) = C_{r_2}(x^*) = C_{r_3}(x^*) = C_{r_4}(x^*) = 0.00,$$

$$C_{r_5}(x^*) = 1,548,207.50, \; C_{r_6}(x^*) = 1,548,196.38, \; C_{r_7}(x^*) = 107,729.39,$$

$$C_{r_8}(x^*) = 0.00, \quad C_{r_9}(x^*) = 83,500.69, \quad C_{r_{10}}(x^*) = 0.00,$$

$$C_{r_{11}}(x^*) = 49,200.66, \quad C_{r_{12}}(x^*) = 0.00.$$

Examples 2 through 4 are analogues of Example 1, but with regulations.

• In Example 2, we considered the following scenario: The US government has told the Mexican government that it is restricting the flow on route $r_5 = (5, 6)$ to zero; essentially resulting in the elimination of this path, since its processing facilities are experiencing delays due to congestion. The rest of the data remain as in Example 1.

• In Example 3, we studied the following scenario: Route $r_6 = (7, 8)$ is now unavailable to refugees, but the other routes and data remain as in Example 1.

• And, in Example 4, we investigate the scenario that the United States is concerned about the influx of refugees and both routes r_5 and r_6 from Mexico are, in effect, banned/eliminated.

Solutions to Examples 1 Through 4

In order to enable cross comparisons with the examples with regulations and with the baseline Example 1, in the following Tables, we report, respectively, the associated computed equilibrium populations, whereas the incurred utilities at the locations are reported in the final Table.

Solutions to Examples 1 Through 4

Node	Equilibrium Populations			
	Example 1	**Example 2**	**Example 3**	**Example 4**
1	1,400,736.88	1,400,336.63	1,400,400.38	1,400,000.00
2	20,607.42	20,607.72	20,607.67	20,607.67
3	700,000.00	700,000.00	700,000.00	700,000.00
4	260,199.55	260,199.55	260,199.55	260,199.55
5	48,945.95	49,345.84	49,282.21	49,682.27
6	224,709.75	224,709.75	224,709.75	224,709.75
7	29,800.46	224,709.75	224,709.75	29,800.46

Solutions to Examples 1 Through 4

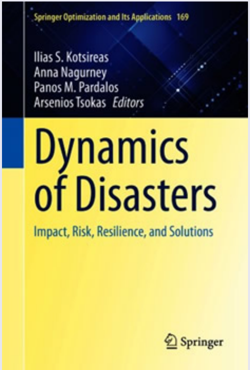
Node	Utility at Equilibrium			
	Example 1	**Example 2**	**Eample 3**	**Example 4**
1	1,599,263.13	1,599.663.38	1,599,599.63	1,600,000.00
2	158,785.17	158,784.56	158,784.66	158,784.00
3	800,000.00	800,000.00	800,000.00	800,000.00
4	119,401.38	119,401.38	119,401.38	119,401.38
5	51,054.05	50,654.16	50,717.79	50,317.73
6	75,290.25	75,290.25	75,290.25	75,290.25
7	70,199.55	70,199.55	70,199.55	70,199.55

One can see, from the preceding Table, that (as occurred also in the illustrative examples), under the regulations (as in Examples 2 through 4), the utility of those subject to regulations, as for those in node 5, is reduced.

On the other hand, those in location 1, which now has a lower flow of refugees, experience a higher utility.

Also, with both refugee routes blocked to the US, the population that remains at node 5 is the highest in Example 4, as compared to the value in Examples 2 and 3.

System-Optimization versus User-Optimization



In our edited volume, we have a paper with Daniele and Cappello, "Capacitated Human Migration Networks and Subsidization."

This work demonstrates that one can make system-optimized solutions for human migration sustainable through the imposition of subsidies at locations - in this way, migrants user-optimizing behavior with also be system-optimizing.

Attracting International Migrant Labor to Alleviate Labor Shortages

This part of the lecture is based on the paper:

A. Nagurney, "Attracting International Migrant Labor: Investment Optimization to Alleviate Supply Chain Labor," *Operations Research Perspectives* **9 (2022), 100233, open access.**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Attracting international migrant labor: Investment optimization to alleviate supply chain labor shortages

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ABSTRACT

The COVID-19 pandemic has disrupted supply chains globally with a major shortfall being that of labor shortages from production through distribution activities. In this paper, we construct a new supply chain network optimization model that includes both domestic labor and international migrant labor from multiple countries, with the latter made possible through investments in attracting labor subject to a budget constraint. We allow for different wage settings for domestic versus migrant labor and also have the flexibility of providing true information as to the wages of migrants or not. We derive variational inequality formulations of the model, along with qualitative properties, and present an algorithm that yields closed form expressions for the underlying problem variables at each iteration. The model is one of the very few variational inequality operations research models with nonlinear constraints. Three series of algorithmically solved numerical examples, motivated by a high value agricultural product — that of truffles, demonstrate the insights in terms of profits, prices, product path flows, and investments, with variations in the data including that of truthful and untruthful wages being used to attract migrant labor.

1. Introduction

The COVID-19 pandemic has demonstrated the importance of supply chains and their effective and efficient operation, with disruptions adversely affecting product prices and deliveries, as well as the prosperity of companies and the health and well-being of consumers [1]. The reasons for the disruptions have been multifaceted with shocks both on the demand side as well as on the supply side and challenges associated with transport [2–6]. One of the major characteristics of the pandemic has been that of labor shortages. Workers throughout the pandemic have been falling ill; some, sadly, have lost their lives, whereas others chose to switch jobs or to leave the labor force [7]. Furthermore, various countries imposed restrictions further impeding the flow of workers [8]. Even with vaccinations continuing, the challenges of ameliorating labor shortages throughout many supply chains remain [9]. Employers have had difficulties recruiting workers not only with advanced technical skills. but also those with low and middle level skills. Countries are increasingly looking towards immigration policy to mitigate the labor shortage crises (see [10,11] with the new variant Omicron adding to the complexities [12]. Attracting workers from other countries, who are known as international migrant laborers, may assist in alleviating labor shortages.

According to [13], noting the most recent estimate by the International Labour Organization, there are 164 million migrant laborers globally and, in many countries, they are a major proportion of the workforce, with contributions to economies both where they work and in terms of remittances to their countries of origins. Nevertheless, many migrant workers face inequality in terms of a wage gap (being paid less that the workers from the particular nation) among other discriminatory practices [14]. For example, migrant laborers in High Income Countries (HICs) earn, on the average, approximately 12.6% less than nationals, with the wages earned by migrants widening over the past 6 years or so in many HICs (see [13]). Furthermore, migrant laborers are among the most negatively affected workers by the economic recession due to the COVID-19 pandemic, in terms of job losses and a decrease in earnings for those who have been able to stay employed. This is happening despite the fact that the United Nations' Sustainable Development Goals (SDGs), in the framework of the UN agenda for 2030, have as their targets 8.5 and 8.8: having equal pay for work of equal value and protected labor rights for all workers, including migrant workers [13,15].

This paper aims to integrate and advance two streams of literature, which have received significant attention in the pandemic: that of

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Labor Shortages

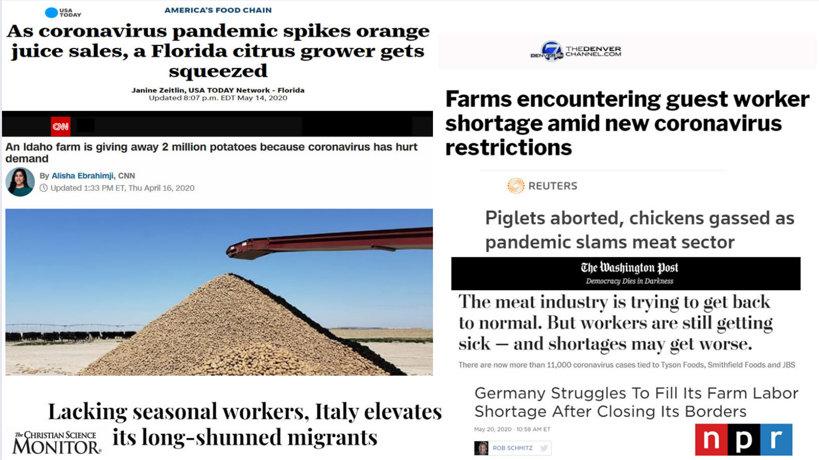
The COVID-19 pandemic has demonstrated the importance of supply chains and their effective and efficient operation. The reasons for disruptions have been multifaceted with shocks both on the demand side as well as on the supply side and challenges associated with transport.

A major characteristic of the pandemic has been that of labor shortages. Workers throughout the pandemic have been falling ill; some, sadly, have lost their lives, whereas others chose to switch jobs or to leave the labor force.

Various countries imposed restrictions further impeding the flow of workers.

Labor Shortages

Employers have had difficulties recruiting workers not only with advanced technical skills but also those with low and middle level skills.



Migrant Labor

There are 164 million migrant laborers globally and, in many countries, they are a major proportion of the workforce. Many migrant workers face inequality in terms of a wage gap among other discriminatory practices.

Countries are increasingly looking towards immigration policy to mitigate the labor shortage crises with the new variant Omicron adding to the complexities.

Migrant Labor

Migrant laborers are among the most negatively affected workers by the economic recession due to the COVID-19 pandemic.

This is happening despite the fact that the United Nations' Sustainable Development Goals (SDGs), in the framework of the UN agenda for 2030, have as their targets 8.5 and 8.8: having equal pay for work of equal value and protected labor rights for all workers, including migrant workers.

Addition to the Literature

This paper aims to integrate and advance two streams of literature, which have received significant attention in the pandemic:

• that of incorporating labor into supply chain network modeling, analysis, and computations (see Nagurney (2021a, b, c, d), (2022)) and

• problems of human migration, which have been exacerbated under COVID-19 (cf. Nagurney, Daniele, and Nagurney (2020), Cappello, Daniele, and Nagurney (2021), Nagurney, Daniele, and Cappello (2021a,b)).

Addition to the Literature

• **A supply chain network optimization model is constructed that captures the profit-maximizing behavior of a firm with respect to its supply chain network activities of production at multiple sites, the transport of the product to multiple storage sites, the storage at these facilities, and, finally, the ultimate distribution of the product to multiple points of demand.**

• **Associated with each of the supply chain network activities is a bound on domestic labor availability with possible investment in labor migration from other countries to attract workers.**

• **The migrants are responsive to the wages that they are told they will be paid for the respective supply chain network activities, as well as to the investments made.**

Addition to the Literature

In terms of the investigation of different wage scenarios, the model allows for the quantification of the impacts of:

1.paying international migrants the same wages for corresponding activities as the domestic workers are paid, under the scenario that the international migrant laborers are informed honestly of the wages that they will be paid;

2. paying international migrants less than the domestic workers are paid, but they are informed of this honestly before they migrate for work or

3. paying the international migrants less than the domestic workers but being dishonest as to what wages they will receive in order to attract them.

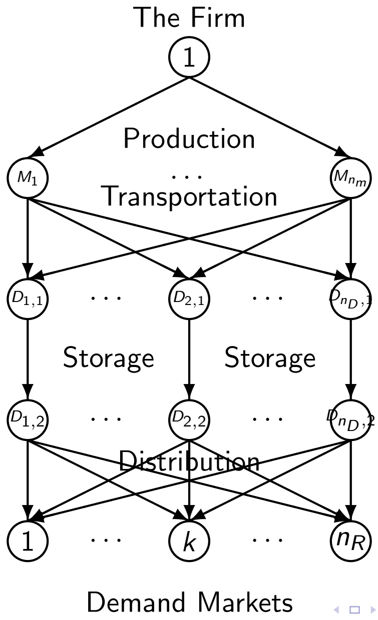
Addition to the Literature

4. The model can handle different wages for national/domestic labor and for international migrant labor. The model can also evaluate impacts of truthful versus untruthful wage information provided to potential international migrants, as implemented in the attraction functions.

5. The theoretical framework is that of variational inequalities and **this work is one of the very few that includes nonlinear constraints in the model and these can arise due to the form that the international migrant attraction functions take**.

6. The solution of a series of numerical examples, inspired by a **high value agricultural product - that of truffle**s, having a variety of the above features, via the proposed algorithm with nice features for implementation, yields interesting insights.

The Model - The Supply Chain Network Topology



The Model Notation - Parameters

w_a^1	hourly wage for a unit of labor on link a paid to domestic workers, $\forall a \in L$.
w_a^2	hourly wage for a unit of labor on link a paid to international migrant workers, $\forall a \in L$.
$\tilde{w}_a^j$	hourly wage for a unit of labor on link a that international migrant workers are told they will be paid, $\forall a \in L$, for all countries $j = 1, \ldots, J$.
α_a	the link productivity on link a, $\forall a \in L$, which maps labor hours into product flow.
δ_{ap}	indicator taking on the value 1 if link a is contained in path p and 0, otherwise.
$\bar{l}_a^1$	the maximum available domestic labor hours locally for work associated with link a, $a \in L$.
B	the amount of financing in the budget for investments in attracting migrant labor from different countries.

The Model Notation - Variables

d_k	demand for the product at demand market k; $k = 1, \ldots, n_R$. Group demands into vector $d \in R_+^{n_R}$.
x_p	product flow on path p, $\forall p \in P$. Group path flows into vector $x \in R_+^{n_P}$
f_a	product flow on link a, $\forall a \in L$. Group link flows into vector $f \in R_+^{n_L}$.
l_a^1	hours of domestic labor available for link a supply chain activity, $\forall a$.
l_a^2	hours of international migrant labor available for link a, $\forall a \in L$.
v_a^j	investment in attracting migrant labor from country j; $j = 1, \ldots, J$ for link a, $\forall a \in L$. Investments in attracting labor are grouped into vector $v \in R_+^{Jn_L}$.
η	nonnegative Lagrange multiplier associated with budget constraint.
λ_a	nonnegative Lagrange multiplier associated with bound on domestic labor hours on link a. Group Lagrange multipliers into vector $\lambda \in R_+^{n_L}$.
δ_a^1	nonnegative Lagrange multiplier associated with constraint guaranteeing that l_a^1 is nonnegative on link a. Group these Lagrange multipliers into vector $\delta^1 \in R_+^{n_L}$.
δ_a^2	nonnegative Lagrange multiplier associated with constraint guaranteeing that l_a^2 is nonnegative on link a. Group these Lagrange multipliers into vector $\delta^2 \in R_+^{n_L}$.

Conservation of Flow Equations

The sum of the product path flows to each demand market must be equal to the demand at the demand market:

$$\sum_{p \in P_k} x_p = d_k, \quad k = 1, \ldots, n_R, \tag{13}$$

with all the path flows being nonnegative:

$$x_p \geq 0, \quad \forall p \in P. \tag{14}$$

The amount of product flow on each link must be equal to the sum of product flows on paths that contain that link:

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L. \tag{15}$$

The Model

Labor Constraints

Liinear production functions are assumed, as in prior work, but extended to differentiate between domestic and international migrant labor:

$$f_a = \alpha_a(l_a^1 + l_a^2), \quad \forall a \in L. \tag{16}$$

Domestic worker labor hours available cannot exceed the bound on domestic labor hours of availability:

$$l_a^1 \leq \bar{l}_a^1, \quad \forall a \in L. \tag{17a}$$

The domestic labor hours available are nonnegative:

$$l_a^1 \geq 0, \quad \forall a \in L. \tag{17b}$$

Amount of international migrant labor hours available for a link a:

$$l_a^2 = \sum_{j=1}^{J} g_a^j(\tilde{w}_a^j, v_a^j), \quad \forall a \in L, \tag{18a}$$

$$l_a^2 \geq 0, \quad \forall a \in L. \tag{18b}$$

Constraints on Investments

The investments by the firm in attracting international migrant labor must be nonnegative:

$$v_a^j \geq 0, \quad j = 1, \ldots, J; \forall a \in L. \tag{19}$$

Finally, the firm's budget constraint in terms of attracting migrants is:

$$\sum_{j=1}^{J} \sum_{a \in L} v_a^j \leq B. \tag{20}$$

The Optimization Problem

The optimization problem faced by the firm in optimizing its supply chain network can now be stated. The firm seeks to maximize its objective function, denoted by U, which represents its profits, subject to the constraints (13) – (20):

$$\text{Maximize } U = \sum_{k=1}^{n_R} \rho_k(d)d_k - \sum_{a\in L} \hat{c}_a(f) - \sum_{j=1}^{J} \sum_{a\in L} v_a^j - \sum_{a\in L} (w_a^1 l_a^1 + w_a^2 l_a^2). \tag{21}$$

The demand price functions are assumed to be monotone decreasing and each $\rho_k(d)d_k$ is concave for each k, and the total operational link cost functions are convex with both the demand price functions and the operational cost functions being continuously differentiable. Ivestment functions $g_a^j(\tilde{w}_a^j, v_a^j)$ are assumed concave and continuously differentiable.

In lieu of (13), we can define demand price functions $\tilde{\rho}_k(x) \equiv \rho_k(d)$, for $k = 1, \ldots, n_R$, and, in lieu of (15), we can define link operational total cost functions $\tilde{c}_a(x) \equiv \hat{c}_a(f)$, for $a \in L$. Using (16) and (18):

$$l_a^1 = \frac{\sum_{p \in P} x_p \delta_{ap}}{\alpha_a} - \sum_{j=1}^{J} g_a^j(\tilde{w}_a^j, v_a^j), \quad \forall a \in L. \tag{22}$$

The firm's optimization problem (21) can be expressed as:

$$\text{Maximize} \quad \tilde{U}(x,v) = \sum_{k=1}^{n_R} \tilde{\rho}_k(x) \sum_{p \in P_k} x_p - \sum_{a \in L} \tilde{c}_a(x) - \sum_{j=1}^{J} \sum_{a \in L} v_a^j$$

$$- \sum_{a \in L} w_a^1 \left(\frac{\sum_{p \in P} x_p \delta_{ap}}{\alpha_a}\right) + \sum_{a \in L} (w_a^1 - w_a^2) \sum_{j=1}^{J} g_a^j(\tilde{w}_a^j, v_a^j), \tag{23}$$

The Model

subject to:

$$\frac{\sum_{p\in P} x_p \delta_{ap}}{\alpha_a} - \sum_{j=1}^{J} g_a^j(\tilde{w}_a, v_a^j) \leq \bar{l}_a^1, \quad \forall a \in L, \tag{24a}$$

$$\frac{\sum_{p\in P} x_p \delta_{ap}}{\alpha_a} - \sum_{j=1}^{J} g_a^j(\tilde{w}_a, v_a^j) \geq 0, \quad \forall a \in L, \tag{24b}$$

$$\sum_{j=1}^{J} g_a^j(\tilde{w}_a, v_a^j) \geq 0, \quad \forall a \in L, \tag{25}$$

$$\sum_{j=1}^{J} \sum_{a\in L} v_a^j \leq B, \tag{26}$$

$$x_p \geq 0, \quad \forall p \in P, \tag{27}$$

$$v_a^j \geq 0, \quad j = 1, \ldots, j; \forall a \in L. \tag{28}$$

The Variational Inequality Formulations

$K^1 \equiv \{(x,v) \in R_+^{n_P + Jn_L}$ satisfying $(12a, b) - (14)\}$. It follows from the classical theory of variational inequalities that the optimal solution $(x^*, v^*) \in K^1$ satisfies the variational inequality problem:

$$-\sum_{p\in P} \frac{\partial \tilde{U}(x^*,v^*)}{\partial x_p} \times (x_p - x_p^*) - \sum_{j=1}^{J}\sum_{a\in L} \frac{\partial \tilde{U}(x^*,v^*)}{\partial v_a^j} \times (v_a^j - v_a^{j*}) \geq 0,$$

$$\forall (x,v) \in K^1, \qquad (29)$$

or: Determine $(x^*, v^*) \in K^1$, such that

$$\sum_{k=1}^{n_R}\sum_{p\in P_k}\left[\frac{\partial \tilde{C}_p(x^*)}{\partial x_p} + \sum_{a\in L}\frac{w_a^1}{\alpha_a}\delta_{ap} - \tilde{\rho}_k(x^*) - \sum_{l=1}^{n_R}\frac{\partial \tilde{\rho}_l(x^*)}{\partial x_p}\sum_{q\in P_l} x_q^*\right] \times \left[x_p - x_p^*\right]$$

$$+\sum_{j=1}^{J}\sum_{a\in L}\left[-(w_a^1 - w_a^2)\frac{\partial g_a^j(\tilde{w}_a^j, v_a^{j*})}{\partial v_a^j} + 1\right] \times \left[v_a^j - v_a^{j*}\right] \geq 0, \; \forall (x,v) \in K^1,$$

$$(30)$$

$$\frac{\partial \tilde{C}_p(x)}{\partial x_p} \equiv \sum_{a\in L}\sum_{b\in L}\frac{\partial \hat{c}_b(f)}{\partial f_a}\delta_{ap}, \; \forall p, \; \frac{\partial \tilde{\rho}_l(x)}{\partial x_p} \equiv \frac{\partial \rho_l(d)}{\partial d_k}, \forall p \in P_k, \forall k. \quad (31)$$

The Variational Inequality Formulations

A solution $(x^*, v^*) \in K^1$ to both variational inequalities (29) and (30) exists since the feasible set K^1 is compact and the underlying functions, under our imposed assumptions, are continuous.

The equivalent variational inequality to the one in (30) is: Determine $(x^*, \lambda^*, v^*, \eta^*, \delta^{1*}, \delta^{2*}) \in K^2$, where $K^2 \equiv \{(x, \lambda, v, \eta, \delta^1, \delta^2) | (x, \lambda, v, \eta, \delta^1, \delta^2) \in R_+^{n_P + n_L + Jn_L + 1 + 2n_L}\}$, such that

$$\sum_{k=1}^{n_R} \sum_{p \in P_k} \left[\frac{\partial \tilde{C}_p(x^*)}{\partial x_p} + \sum_{a \in L} \frac{w_a^1}{\alpha_a} \delta_{ap} - \tilde{\rho}_k(x^*) - \sum_{l=1}^{n_R} \frac{\partial \tilde{\rho}_l(x^*)}{\partial x_p} \sum_{q \in P_l} x_q^* + \sum_{a \in L} \frac{\lambda_a^*}{\alpha_a} \delta_{ap} - \frac{\delta_a^{1*}}{\alpha_a} \delta_{ap} \right] \times \left[x_p - x_p^* \right]$$

$$+ \sum_{a \in L} \left[\bar{l}_a^1 - \frac{\sum_{p \in P} x_p^* \delta_{ap}}{\alpha_a} + \sum_{j=1}^{J} g_a^j(\tilde{w}_a^j, v_a^{j*}) \right] \times [\lambda_a - \lambda_a^*]$$

$$+ \sum_{j=1}^{J} \sum_{a \in L} \left[1 + \eta^* - (w_a^1 - w_a^2 + \lambda_a^* - \delta_a^{1*} + \delta_a^{2*}) \frac{\partial g_a^j(\tilde{w}_a^j, v_a^{j*})}{\partial v_a^j} \right] \times \left[v_a^j - v_a^{j*} \right]$$

$$+ \left[B - \sum_{j=1}^{J} \sum_{a \in L} v_a^{j*} \right] \times [\eta - \eta^*]$$

$$+ \sum_{a \in L} \left[\frac{\sum_{p \in P} x_p^* \delta_{ap}}{\alpha_a} - \sum_{j=1}^{J} g_a^j(\tilde{w}_a^j, v_a^{j*}) \right] \times \left[\delta_a^1 - \delta_a^{1*} \right]$$

$$+ \sum_{a \in L} \left[\sum_{j=1}^{J} g_a^j(\tilde{w}_a^j, v_a^{j*}) \right] \times \left[\delta_a^2 - \delta_a^{2*} \right] \geq 0, \quad \forall (x, \lambda, v, \eta, \delta^1, \delta^2) \in K^2. \tag{32}$$

The Algorithm

The Modified Projection Method

Step 0: Initialization

Initialize with $X^0 \in \mathcal{K}$. Set $\tau := 1$, where τ is the iteration counter, and let β be a scalar such that $0 < \beta \leq \frac{1}{\omega}$, where ω is the Lipschitz constant.

Step 1: Computation

Compute $\bar{X}^\tau$ satisfying the variational inequality subproblem:

$$\langle \bar{X}^\tau + \beta F(X^{\tau-1}) - X^{\tau-1}, X - \bar{X}^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{33}$$

Step 2: Adaptation

Compute X^τ satisfying the variational inequality subproblem:

$$\langle X^\tau + \beta F(\bar{X}^\tau) - X^{\tau-1}, X - X^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{34}$$

Step 3: Convergence Verification

If $|X^\tau - X^{\tau-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then terminate the algorithm; else, set $\tau := \tau + 1$ and go to Step 1.

Numerical Examples

Numerical Examples

The numerical examples are inspired by recent issues surrounding agricultural supply chains in the UK. The UK has been pummeled with shortfalls in labor due to COVID-19 as well as Brexit. The numerical examples are focused on a very interesting agricultural product, now being grown in the UK - truffles.

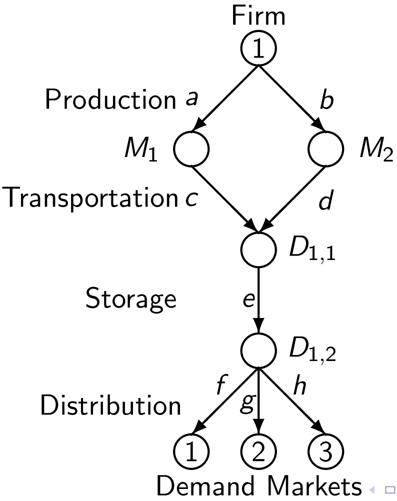
Numerical Examples

Truffles are a high value agricultural product, and are considered a delicacy, with challenges associated with production and harvesting.

In the Fall of 2021, due to a shortage, white truffle prices were about $4,500 a pound, whereas, in 2019, white truffle prices were in the range $1,100 to $1,200 a pound.

They are considered among the most expensive foods on the planet. The cost and price data, as well as the wages and the profits, in the numerical examples are in British pounds. The unit for the truffle product flows is a pound of weight. According to Elison (2021), vegetable pickers in the UK, due to shortages of labor are being paid 30 pounds an hour to pick the produce.

Example 1: Baseline Example: Migrant Workers Earn the Same Wage as Domestic Laborers and Migrants Are Told Their Truthful Wages in the Attraction Functions



Series 1 Numerical Examples

The total operational link cost functions are:

$$\hat{c}_a(f) = 2.5f_a^2, \quad \hat{c}_b(f) = 2.5f_b^2, \quad \hat{c}_c(f) = .5f_c^2, \quad \hat{c}_d(f) = .5f_d^2,$$

$$\hat{c}_e(f) = f_e^2 + 2f_e, \quad \hat{c}_f(f) = .5f_f^2, \quad \hat{c}_g(f) = .5f_g^2, \quad \hat{c}_h(f) = .5f_h^2.$$

The demand price functions are:

$$\rho_1(d) = -5d_1 + 800, \quad \rho_2(d) = -5d_2 + 850, \quad \rho_3(d) = -5d_3 + 900.$$

The α link parameters are:

$$\alpha_a = .55, \quad \alpha_b = .50, \quad \alpha_c = .35, \quad \alpha_d = .35, \quad \alpha_e = .60, \quad \alpha_f = .38,$$

We assume that the supply chain firm considers a single country to obtain international migrants from, but the specific country can differ from supply chain network activity to activity. The international migrant attraction functions are of the form: $g_a(\tilde{w}_a, v_a) = \tilde{w}_a v_a - \gamma_a v_a^2$, for all links $a \in L$. These functions are concave. The γ parameters in these functions are:

$$\gamma_a = .2, \quad \gamma_b = .2, \quad \gamma_c = .4, \quad \gamma_d = .4, \quad \gamma_e = .3 \quad \gamma_f = .4, \quad \gamma_g = .4,$$

Series 1 Numerical Examples

The wages in Example 1 are:

$$w_a^1 = w_a^2 = \tilde{w}_a = 30, \quad w_b^1 = w_b^2 = \tilde{w}_b = 20,$$

$$w_c^1 = w_c^2 = \tilde{w}_c = 18, \quad w_d^1 = w_d^2 = \tilde{w}_d = 18,$$

$$w_e^1 = w_e^2 = \tilde{w}_e = 17, \quad w_f^1 = w_f^2 = \tilde{w}_f = 19,$$

$$w_g^1 = w_g^2 = \tilde{w}_g = 19, \quad w_h^1 = w_h^2 = \tilde{w}_h = 19.$$

The bounds on domestic labor are: $\bar{l}_a^1 = 100$, for all links in the supply chain from a through h and the budget $B = 1,000$.

Example 2: Domestic Workers Earn a Higher Wage at Production Sites than Migrants and Are Told Their Truthful Wages
Example 2 has the same data as that in Example 1, except that now the domestic workers earn a higher wage at the two production sites with $w_a^1 = 40$ and $w_b^1 = 30$.

Example 3: Domestic Workers Earn a Higher Wage at Production Sites than Migrants but Migrants Are Told Untruthfully That They Will Be Paid the Same Wage as the Domestic Workers In Example 3, we investigate the impact of the firm being untruthful. Specifically, the firm now tells the international migrant laborers that it will pay them the same (higher) wage at each production site that it is paying its domestic laborers, but it actually will pay them less. The data, hence, are exactly as in Example 2, but now we have that $\tilde{w}_a = 40$ and $\tilde{w}_b = 30$.

The firm earns a profit of: 41,453.10 in Example 1; a profit of 41,444.64 in Example 2, and a profit of 41,447.33 in Example 3. Observe that the profit now increases suggesting that, without oversight, "cheating can pay.".

Series 1 Numerical Examples

Notation	Optimal Value		
	Example 1	Example 2	Example 3
f_a^*	38.86	38.85	38.85
f_b^*	41.28	41.27	41.27
f_c^*	38.86	38.85	38.85
f_d^*	41.28	41.27	41.27
f_e^*	80.13	80.12	80.13
f_f^*	20.00	20.00	20.00
f_g^*	24.30	24.29	24.29
f_h^*	35.84	35.83	35.83
l_a^{1*}	70.65	0.00	0.00
l_b^{1*}	82.56	0.00	0.00
l_c^{1*}	100.00	100.00	100.00
l_d^{1*}	100.00	100.00	100.00
l_e^{1*}	100.00	100.00	100.00
l_f^{1*}	52.64	52.63	52.63
l_g^{1*}	67.49	67.48	67.48
l_h^{1*}	89.59	89.58	89.58
l_a^{2*}	0.00	70.64	70.64
l_b^{2*}	0.00	82.54	82.55
l_c^{2*}	11.01	11.01	11.01
l_d^{2*}	17.94	17.91	17.92
l_e^{2*}	33.56	33.54	33.54
l_f^{2*}	0.00	0.00	0.00
l_g^{2*}	0.00	0.00	0.00
l_h^{2*}	0.00	0.00	0.00

Table: Optimal Link Flows and Domestic and International Migrant Labor Values for Examples 1, 2, 3

Series 1 Numerical Examples

Notation	Optimal Value		
	Example 1	Example 2	Example 3
v_a^*	0.00	2.39	1.78
v_b^*	0.00	4.31	2.80
v_c^*	0.62	0.62	0.62
v_d^*	1.02	1.02	1.02
v_e^*	2.05	2.05	2.05
v_f^*	0.00	0.00	0.00
v_g^*	0.00	0.00	0.00
v_h^*	0.00	0.00	0.00
λ_a^*	0.00	0.00	0.00
λ_b^*	0.00	0.00	0.00
λ_c^*	0.06	0.06	0.06
λ_d^*	0.06	0.06	0.06
λ_e^*	0.06	0.06	0.06
λ_f^*	0.00	0.00	0.00
λ_g^*	0.00	0.00	0.00
λ_h^*	0.00	0.00	0.00

Table: Optimal Link International Migrant Attraction Investments and Domestic Labor Bound Lagrange Multipliers for Examples 1, 2, and 3

Series 2 Numerical Examples

In Series 2 numerical examples, we use Example 1 as a baseline and we explore the impacts of increasing the prices that consumers are willing to pay.

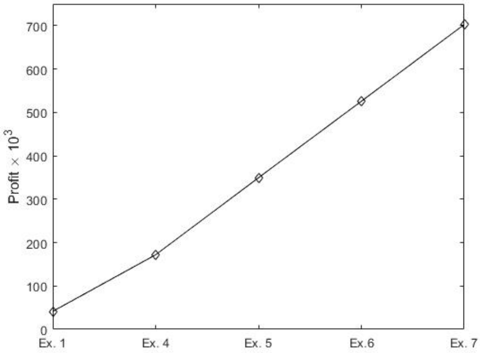


Figure: Effect on Profit of the Firm when the Demand Price Function Intercepts are Doubled, Tripled, and so on, with Example 1 Being the Baseline

Series 2 Numerical Examples

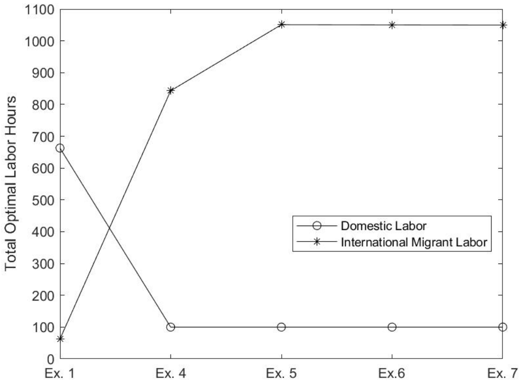


Figure: Effect on Optimal Total Labor Hours of Domestic Labor and of International Migrant Labor in the Supply Chain Network when the Demand Price Function Intercepts are Doubled, Tripled, and so on, with Example 1 Being the Baseline

Series 3 Numerical Examples

In the third, final series of numerical examples, we first, again, explore the impacts of being untruthful in terms of wages in recruiting international migrant laborers and then we investigate the impact of a tighter budget on investments. The results can be found in the paper.

We see a huge increase in the hiring of international migrant laborers, which are all attracted to the work; however, under false pretenses in the form of higher wages than either the domestic laborers or the migrant laborers are being paid! The firm increases its profit.

Series 3 Numerical Examples

The examples, cheating or not, demonstrate also the importance of having a sufficient budget in order to be able to attract the needed international migrant labor.

Furthermore, through the use of Lagrange multipliers, we can see the value of increasing the availability of domestic labor in this endeavor, as well as the budget for investing in attracting international migrant labor.

The above examples are stylized, but do provide insights and, importantly, demonstrate both the breadth of the model and the effectiveness of the computational procedure.

Some Additional Research

A. Nagurney, "Optimization of Investments in Labor Productivity in Supply Chain Networks," *International Transactions in Operational Research* **29(4) (2022), pp 2116-2144. This article was recognized with an Editor's Choice Award.**

A. Nagurney, "Supply Chain Networks, Wages, and Labor Productivity: Insights from Lagrange Analysis and Computations," *Journal of Global Optimization* **83 (2022), pp 615-638.**

A. Nagurney and A. Ermagun, "Resilience of Supply Chain Networks to Labor Disruptions," *Resilience Findings*, **June 16, 2022.**

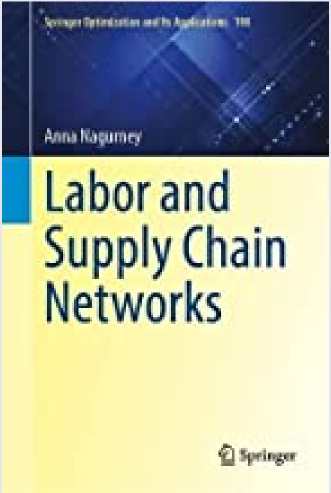
Some Additional Research

The numerical results in our papers clearly reveal the importance of a holistic approach to supply chain network modeling since decisions made by a specific firm can have unexpected impacts on other competing firms in the supply chain network economy.

Our results also strongly suggest that having wages and labor equilibrate without any wage ceilings can be beneficial for an individual firm and also for firms engaged in competition.

And, most importantly, taking care of workers is critical in times of peace and war!

New Book



Summary and Conclusions

Summary and Conclusions

- **In this lecture, I have highlighted some of our recent Operations Research contributions to address societal issues of human migration networks in the context of refugees as well as labor shortages in supply chains.**

- **The work is inspired by the need to include people in various network systems including supply chain networks.**

- **The world has benefited greatly from our models, methodologies, insights, and relevance to both practice and policy making.**

- **Together we can continue to accomplish what needs to be done in these challenging and, yet, fascinating times.**

Muchas Gracias!

More information on our work can be found on the Supernetwork Center site:
https://supernet.isenberg.umass.edu/