A Game Theory Model for a Differentiated Service-Oriented Internet with Duration-Based Contracts

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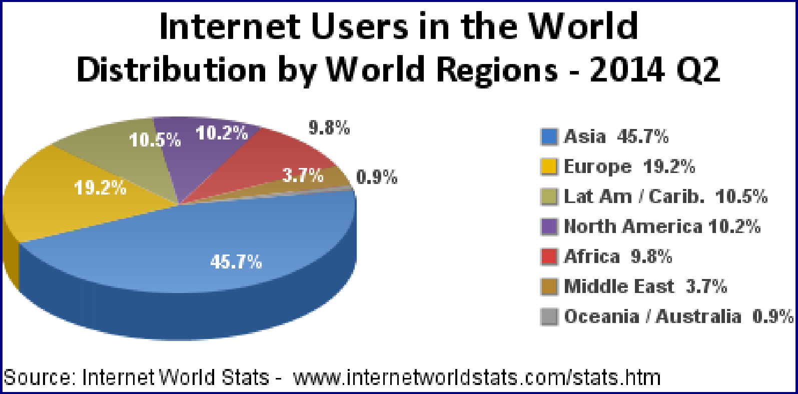
Outline

- Background and Motivation
- ChoiceNet
- The Competitive Duration-Based Differentiated Service-Oriented Internet Game Theory Model
- Variational Inequality Formulation
- The Algorithm
- Numerical Examples
- Summary and Conclusions

Global Internet

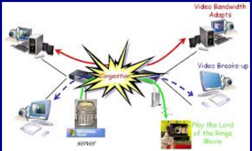
There are now 2.92 billion Internet users out of a global population of 7 billion.



Background

- Online video consumption almost doubled in the US from 2012 to 2013.

- As of March 2014, Netflix and Google, which owns Youtube, accounted for 47% of the Internet traffic during evening hours in the U.S.
- It may result in network congestion that leads to a degradation in the quality of transmission.



Current Internet Limitations

- Quality and price concerns

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- Customers are locked-in for extended periods of time

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- Pricing based on quality and the usage amount contracts of one to two year duration, may result in network congestion

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- Pricing based on quality and the usage amount contracts of one to two year duration, may result in network congestion
- Consumers may desire more flexibility and more choices
- Shorter duration contracts garnering greater interest

ChoiceNet

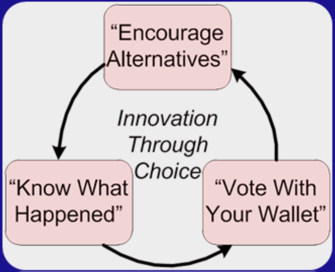
- This project is one of five NSF-sponsored Future Internet Architecture (FIA) projects, including:
  - NEBULA
  - eXpressive Internet Architecture
  - MobilityFirst
  - Named Data Networking
  - ChoiceNet
- Team:
  - University of Kentucky:
    Jim Griffioen, Ken Calvert
  - North Carolina State University:
    Rudra Dutta, George Rouskas
  - RENCI:
    Ilia Baldine
  - University of Massachusetts Amherst (lead):
    Tilman Wolf, Anna Nagurney

ChoiceNet Goals

- Expose choices throughout protocol stack
- Interactions between technological alternatives and economic interactions
  - Introduction of explicit “Economy Plane”
- “Network architecture”
  - Requires redesign of data and control plane
  - Possibly not a complete architecture
  - Aim to fit with existing and future architectures

ChoiceNet Principles

- Services are at core of ChoiceNet
  - Service provides a benefit, has a cost
  - Services are created, composed, sold, verified, etc.
- "Encourage alternatives"
  - Provide building blocks for different types of services
- "Know what happened"
  - Ability to evaluate services
- "Vote with your wallet"
  - Reward good services through "money protocol"



Literature Review

The importance of supporting various levels of quality of network service

- S. Saberi, A Nagurney, and T. Wolf. A Network Economic Game Theory Model of a Service-Oriented Internet with Price and Quality Competition in Both Content and Network Provision, *Service Science*, 6(4):229-250, 2014.
- A. Nagurney and T. Wolf. A Cournot-Nash-Bertrand game theory model of a service-oriented Internet with price and quality competition among network transport providers, *Computational Management Science*, 11(4):475-502, 2014.
- A. Nagurney, D. Li, S. Saberi, and T. Wolf. A dynamic network economic model of a service-oriented Internet with price and quality competition. In V.A. Kalyagin, P.M. Pardalos, and T.M. Rassias, editors, *Network Models in Economics and Finance*, Springer International Publishing Switzerland, 239-264, 2014.
- A. Nagurney, D. Li, T. Wolf, and S. Saberi. A network economic game theory model of a service-oriented Internet with choices and quality competition, *Netnomics*, 14:1-25, 2013.

Literature Review

Early mathematical models with duration and quality of services

- P.J. Bailey, I. Gamvros, and S. Raghavan. Ex-post Internet charging: an effective bandwidth model. In E.K. Baker, A. Joseph, A. Mehrotra, and M.A. Trick, editors, *Extending the Horizons: Advances in Computing, Optimization, and Decision Technologies*. Springer, US, 221-245, 2007.
- C. Courcoubetisaib and V.A. Siris. Managing and pricing service level agreements for differentiated services. *Proceedings of the Seventh International Workshop on Quality of Service*, London, England, 165-173, 1999.
- J. Hwang, H.J. Kim, and M.B.H. Weiss. Interprovider differentiated service interconnection management models in the internet bandwidth commodity markets. *Telematics and Informatics*, 19(4):351-369, 2002.
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However, they all consider a monopolistic provider.

Paper Contributions

Our model

- Formulates a competitive oligopoly market of Internet network providers
- Offers differentiated network services
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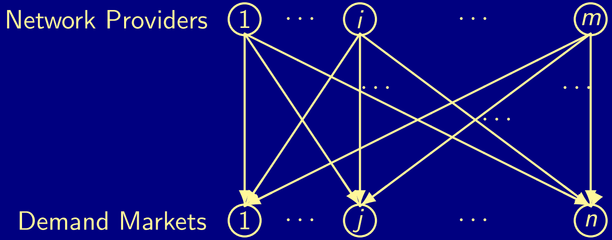
The users/demand markets select contracts based on three main criteria:

- The amount of usage contracted for per period of time (the usage rate) during the contract duration (d):
  (e.g. in Megabits/second or Kilobits/second)
- The quality level of service (q):
  (which ranges between 0 and 100, with 100 denoting perfect quality)
- The contract duration (T):
  (e.g. in seconds, minutes or hours)

The Bipartite Structure of the Competition Among the Network Providers



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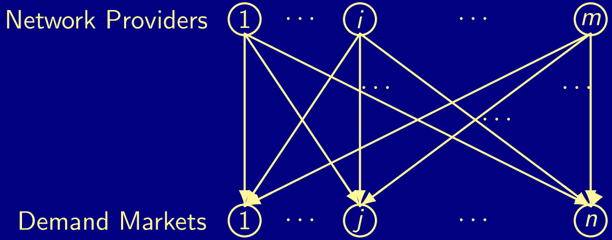

Due to technological limitations:

$$\underline{d}_{ij} \leq d_{ij} \leq \bar{d}_{ij}, \quad \forall i, j, \tag{1}$$

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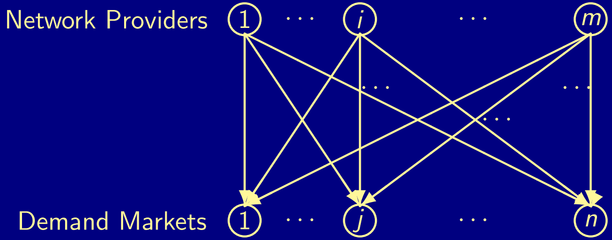

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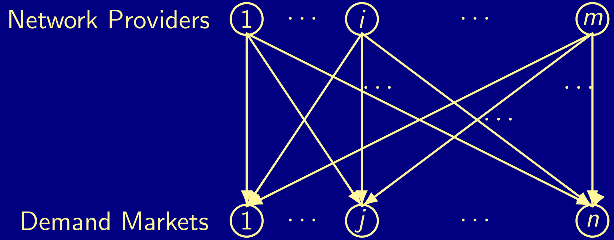

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$$\underline{T}_{ij} \leq T_{ij} \leq \bar{T}_{ij}, \quad \forall i, j. \tag{3}$$

Entities Behavior

The price of i's service provision to j, p_{ij} is:

$$p_{ij} = p_{ij}(d, q, T), \qquad \forall i, j. \tag{4}$$

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The utility or profit of network provider i is the difference between his revenue and his total cost:

$$U_i = \sum_{j=1}^{n} p_{ij} T_{ij} d_{ij} - \sum_{j=1}^{n} c_{ij}, \quad \forall i. \tag{6}$$

The Differentiated Service-Oriented Internet Network Equilibrium with Contract Durations

Definition 1

A service usage rate, quality, and contract duration pattern $(d^*, q^*, T^*) \in K$ is an equilibrium if, for each network provider i; $i = 1, \ldots, m$:

$$U_i(d_i^*, q_i^*, T_i^*, \hat{d}_i^*, \hat{q}_i^*, \hat{T}_i^*) \geq U_i(d_i, q_i, T_i, \hat{d}_i^*, \hat{q}_i^*, \hat{T}_i^*), \quad \forall (d_i, q_i, T_i) \in K^i, \tag{7}$$

where

$$\hat{d}_i^* = (d_1^*, \ldots, d_{i-1}^*, d_{i+1}^*, \ldots, d_m^*),$$

$$\hat{q}_i^* = (q_1^*, \ldots, q_{i-1}^*, q_{i+1}^*, \ldots, q_m^*),$$

and

$$\hat{T}_i^* = (T_1^*, \ldots, T_{i-1}^*, T_{i+1}^*, \ldots, T_m^*). \tag{8}$$

Variational Inequality Formulation

Theorem 1

Assume that the profit function $U_i(d, q, T)$ is concave with respect to the variables and is continuous and continuously differentiable for each network provider i;

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\Big[\sum_{l=1}^{n}\frac{\partial c_{il}(d^*,q^*,T^*)}{\partial d_{ij}} - p_{ij}(d^*,q^*,T^*)\times T^*_{ij}$$

$$-\sum_{l=1}^{n}\frac{\partial p_{il}(d^*,q^*,T^*)}{\partial d_{ij}}\times d^*_{il}\times T^*_{il}\Big]\times(d_{ij}-d^*_{ij})$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\Big[\sum_{l=1}^{n}\frac{\partial c_{il}(d^*,q^*,T^*)}{\partial q_{ij}} - \sum_{l=1}^{n}\frac{\partial p_{il}(d^*,q^*,T^*)}{\partial q_{ij}}\times d^*_{il}\times T^*_{il}\Big]\times(q_{ij}-q^*_{ij})$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\Big[\sum_{l=1}^{n}\frac{\partial c_{il}(d^*,q^*,T^*)}{\partial T_{ij}} - p_{ij}(d^*,q^*,T^*)\times d^*_{ij} - \sum_{l=1}^{n}\frac{\partial p_{il}(d^*,q^*,T^*)}{\partial T_{ij}}$$

$$\times d^*_{il}\times T^*_{il}\Big]\times(T_{ij}-T^*_{ij})\geq 0, \forall(d,q,T)\in K. \quad (9)$$

Variational Inequality Standard Form

Determine $X^* \in \mathcal{K} \subset R^N$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \qquad \forall X \in \mathcal{K}, \tag{10}$$

where F is a given continuous function from $\mathcal{K}$ to R^N, and $\mathcal{K}$ is a closed and convex set.

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$$F^1_{ij}(X) \equiv -\frac{\partial U_i}{\partial d_{ij}}, \tag{11}$$

the (i,j)-th component, F^2_{ij}, of $F^2(X)$ given by

$$F^2_{ij}(X) \equiv -\frac{\partial U_i}{\partial q_{ij}}, \tag{12}$$

and the (i,j)-th component, F^3_{ij}, of $F^3(X)$ given by

$$F^3_{ij}(X) \equiv -\frac{\partial U_i}{\partial T_{ij}}. \tag{13}$$

Existence and Uniqueness

Theorem 2

A solution X^* to variational inequality (10) is guaranteed to **exist**.

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Theorem 3

If $F(X)$ is strictly monotone, that is:

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle > 0, \quad \forall X^1, X^2 \in \mathcal{K}, X^1 \neq X^2, \tag{14}$$

then the solution to variational inequality (10) is **unique***.*

The Algorithm

Euler Method

- Is induced by the general iterative scheme of Dupuis and Nagurney (1993)
- At iteration τ of the Euler method, one solves the following problem:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau} F(X^{\tau})). \tag{15}$$

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Explicit Formulae for the Euler Method Applied to the Internet Network Model with Contract Durations

$$d_{ij}^{\tau+1} = \max\left\{ \underline{d}_{ij}, \min\{\bar{d}_{ij}, d_{ij}^{\tau} - a_{\tau} F_{ij}^{1}(X^{\tau})\} \right\}, \tag{16}$$

$$q_{ij}^{\tau+1} = \max\left\{ 0, \min\{\bar{q}_{ij}, q_{ij}^{\tau} - a_{\tau} F_{ij}^{2}(X^{\tau})\} \right\}, \tag{17}$$

$$T_{ij}^{\tau+1} = \max\left\{ \underline{T}_{ij}, \min\{\bar{T}_{ij}, T_{ij}^{\tau} - a_{\tau} F_{ij}^{3}(X^{\tau})\} \right\}. \tag{18}$$

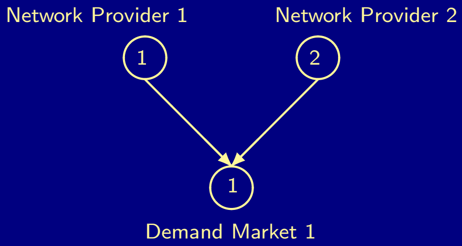
Numerical Examples

- 3 examples plus sensitivity analysis
- $\epsilon = 10^{-4}$
- $\{a_\tau\}$ is: $(1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \ldots)$
- We initialized the algorithm for all the examples by setting

$$d_{ij}^0 = \underline{d}_{ij}; q_{ij}^0 = \underline{q}_{ij}; T_{ij}^0 = \underline{T}_{ij}, \ \forall i, j.$$

- The contract durations, T_{ij}s, are in hours.
- The reserved service usage rates, d_{ij}s, are in Megabits/second.
- The prices p_{ij} are in cents/Megabit multiplied by 10^{-5}.

Example 1



The price functions at Demand Market 1 are:

$$p_{11} = 12 - .167\, d_{11} - .0334\, d_{21} + .032\, q_{11} - .0064\, q_{21} - .182\, T_{11} - .0546\, T_{21},$$

$$p_{21} = 12 - .0334\, d_{11} - .167\, d_{21} - .0064\, q_{11} + .032\, q_{21} - .0546\, T_{11} - .182\, T_{21}.$$

The cost functions for Network Providers 1 and 2 are, respectively:

$$c_{11} = (.0049\, q_{11}^2 + .001715\, q_{11} + .029\, d_{11})\, T_{11},$$

$$c_{21} = (.0037\, q_{21}^2 + .053\, d_{21}^2)\, T_{21}.$$

Example 1

The utility functions of the network providers are:

$$U_1 = p_{11} d_{11} T_{11} - c_{11}, \quad U_2 = p_{21} d_{21} T_{21} - c_{21}.$$

$$23 \leq d_{11} \leq 250, \qquad 0 \leq q_{11} \leq 100, \qquad 8 \leq T_{11} \leq 40,$$

$$15 \leq d_{21} \leq 200, \qquad 0 \leq q_{21} \leq 100, \qquad 11 \leq T_{21} \leq 40.$$

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Solution

$$d_{11}^* = 28.28, \qquad d_{21}^* = 20.97,$$
$$T_{11}^* = 17.83, \qquad T_{21}^* = 17.39,$$
$$q_{11}^* = 92.17, \qquad q_{21}^* = 90.63,$$
$$p_{11} = 4.75, \qquad p_{21} = 5.73.$$

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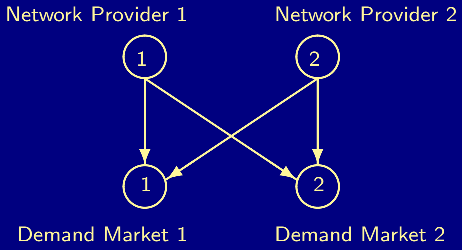
If the contract duration was 1 month, the revenue of a network provider per user would be approximately $35.

Example 2



The price functions for Demand Market 2 are:

$$p_{12} = 6 - .063\, d_{12} - .0126\, d_{22} + .026\, q_{12} - .0052\, q_{22} - .117\ T_{12} - .0351\ T_{22},$$

$$p_{22} = 6 - .0126\, d_{12} - .063\, d_{22} - .0052\, q_{12} + .026\, q_{22} - .0351\ T_{12} - .117\ T_{22}.$$

The cost functions for the network providers are:

$$c_{1j} = (.0049\, q_{1j}^2 + .001715\, q_{1j} + .029\, d_{1j})\, T_{1j}, \qquad j = 1, 2;$$

$$c_{2j} = (.0037\, q_{2j}^2 + .053\, d_{2j}^2)\, T_{2j}, \qquad j = 1, 2.$$

Example 2

The utilities of Network Providers 1 and 2 are, respectively:

$$U_1 = p_{11} d_{11} T_{11} + p_{12} d_{12} T_{12} - (c_{11} + c_{12}),$$

$$U_2 = p_{21} d_{21} T_{21} + p_{22} d_{22} T_{22} - (c_{21} + c_{22}).$$

$$23 \leq d_{1j} \leq 250, \quad 0 \leq q_{1j} \leq 100, \quad 8 \leq T_{1j} \leq 40, \quad j = 1, 2,$$

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$$23 \leq d_{1j} \leq 250, \quad 0 \leq q_{1j} \leq 100, \quad 8 \leq T_{1j} \leq 40, \quad j = 1, 2,$$

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Solution

$$d_{11}^* = 28.28, \quad d_{12}^* = 45.39, \quad d_{21}^* = 20.98, \quad d_{22}^* = 20.71,$$

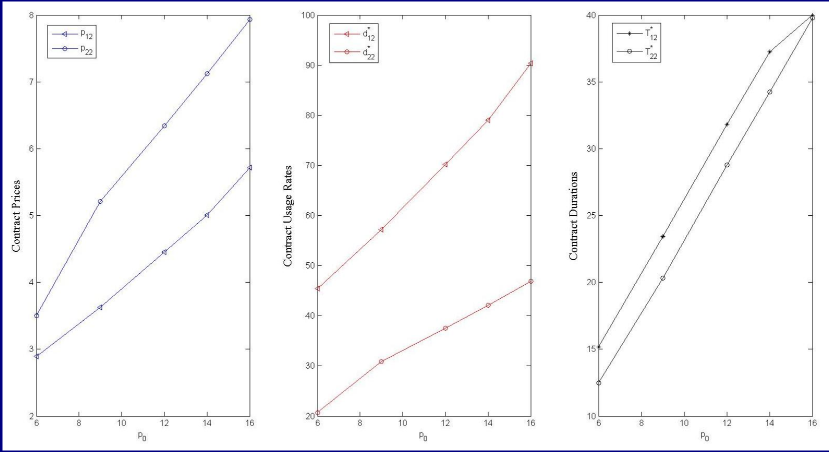
$$T_{11}^* = 17.83, \quad T_{12}^* = 15.18, \quad T_{21}^* = 17.39, \quad T_{22}^* = 12.47,$$

$$q_{11}^* = 92.16, \quad q_{12}^* = 100.00, \quad q_{21}^* = 90.72, \quad q_{22}^* = 72.64,$$

$$p_{11} = 4.75, \quad p_{12} = 2.89, \quad p_{21} = 5.73, \quad p_{22} = 3.50.$$

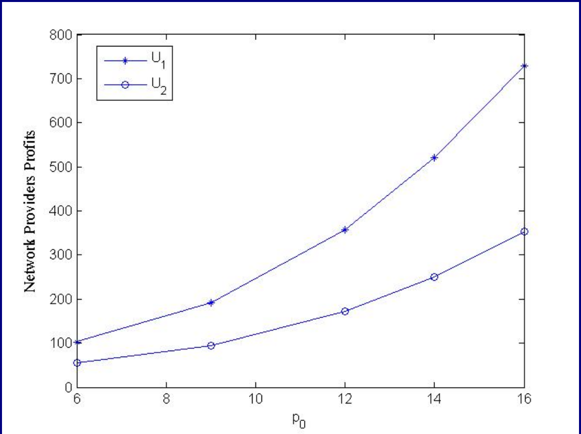
Example 2: the Impact of Changes in Price Functions on Price, Usage Rate, and Contract Duration

Vary p_0 from 6 (its initial value) in both p_{12} and p_{22} to 18 in increments of 2



Example 2: the Impact of Changes in Price Functions on Profit

Vary p_0 from 6 in both p_{12} and p_{22} to 18 in increments of 2



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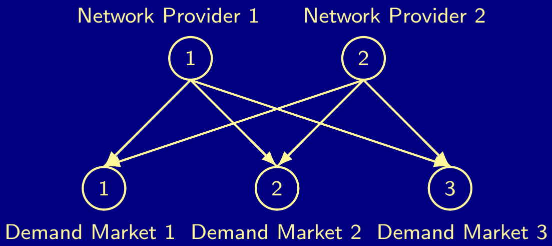

The price functions for Demand Market 3 are:

$$p_{13} = 9 - .115\, d_{13} - .023\, d_{23} + .028\, q_{13} - .0056\, q_{23} - .211\, T_{13} - .0633\, T_{23},$$

$$p_{23} = 9 - .023\, d_{13} - .115\, d_{23} - .0056\, q_{13} + .028\, q_{23} - .0633\, T_{13} - .211\, T_{23}.$$

The cost functions for Demand Market 3 are:

$$c_{13} = (.0049\, q_{13}^2 + .001715\, q_{13} + .029\, d_{13})\, T_{13}, \quad c_{23} = (.0037\, q_{23}^2 + .053\, d_{23}^2)\, T_{23},$$

with those for Demand Markets 1 and 2 as in Example 2.

Example 3

The utility functions of Network Providers 1 and 2 are:

$$U_1 = p_{11}d_{11}T_{11} + p_{12}d_{12}T_{12} + p_{13}d_{13}T_{13} - (c_{11} + c_{12} + c_{13}),$$

$$U_2 = p_{21}d_{21}T_{21} + p_{22}d_{22}T_{22} + p_{23}d_{23}T_{23} - (c_{21} + c_{22} + c_{23}).$$

$$23 \leq d_{1j} \leq 250, \quad 0 \leq q_{1j} \leq 100, \quad 8 \leq T_{1j} \leq 40, \quad j = 1, 2, 3,$$

$$15 \leq d_{2j} \leq 200, \quad 0 \leq q_{2j} \leq 100, \quad 11 \leq T_{2j} \leq 40, \quad j = 1, 2, 3.$$

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Solution

$$d_{11}^* = 31.48,\ d_{12}^* = 45.39,\ d_{13}^* = 30.16,\ d_{21}^* = 23.55,\ d_{22}^* = 20.71,\ d_{23}^* = 19.87,$$

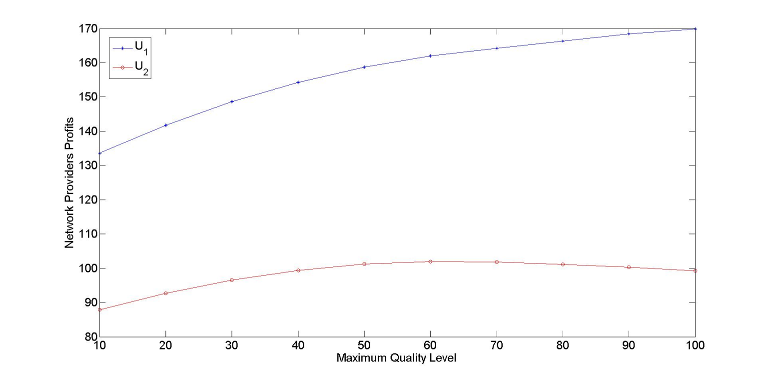
$$T_{11}^* = 20.31,\ T_{12}^* = 15.18,\ T_{13}^* = 13.49,\ T_{21}^* = 19.84,\ T_{22}^* = 12.47,\ T_{23}^* = 13.00,$$

$$q_{11}^* = 100.00,\ q_{12}^* = 100.00,\ q_{13}^* = 76.77,\ q_{21}^* = 100.00,\ q_{22}^* = 72.64,\ q_{23}^* = 67.11,$$

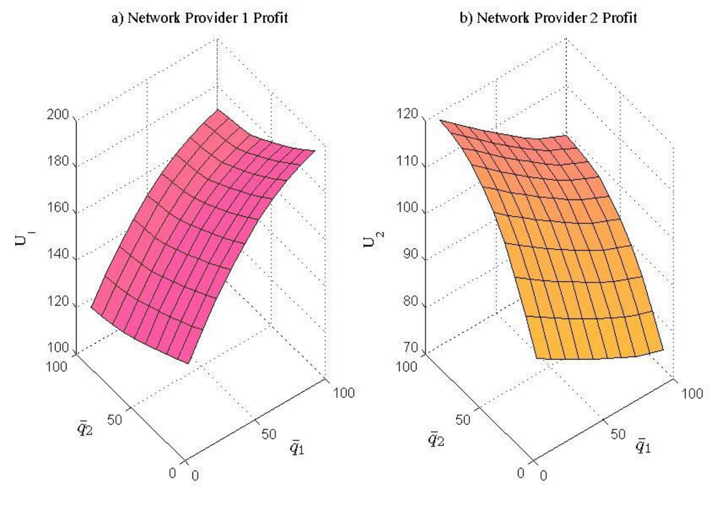
$$p_{11} = 5.29,\ p_{12} = 2.89,\ p_{13} = 3.77,\ p_{21} = 6.43,\ p_{22} = 3.50,\ p_{23} = 4.57.$$

Example 3: Effects of the Maximum Quality Level on Network Providers Profits

Varied the quality upper bounds from 10 through 100 in increments of 10 with both providers having the same quality upper bound



Example 3: Impact on Profits with Distinct Quality Level Upper Bounds for the Providers



Summary

- Developed a game theory model for a differentiated service-oriented Internet
- Formulated duration-based contracts
- Modeled quality competition
- Used variational inequalities for theoretical formalism
- Tested the model with numerical examples, supplemented with sensitivity analysis

THANK YOU!

For more information, see: http://supernet.isenberg.umass.edu