Timo Goßler[a], Tina Wakolbinger[a], Anna Nagurney[b], Patrizia Daniele[c]

How to increase the impact of disaster relief: a study of transportation rates, framework agreements and product distribution

WIRTSCHAFTS UNIVERSITÄT WIEN VIENNA UNIVERSITY OF ECONOMICS AND BUSINESS

EURO Conference 2018

9th of July 2018

Valencia, Spain

[a] Vienna University of Economics and Business

[b] Isenberg School of Management, University of Massachusetts

[c] University of Catania

This research is funded by the Austrian Science Fund (FWF): Project 26015

How to increase the impact of disaster relief: a study of transportation rates, framework agreements and product distribution

WIRTSCHAFTS UNIVERSITÄT WIEN VIENNA UNIVERSITY OF ECONOMICS AND BUSINESS

Agenda

How to increase the impact of disaster relief: a study of transportation rates, framework agreements and product distribution

WIRTSCHAFTS UNIVERSITÄT WIEN VIENNA UNIVERSITY OF ECONOMICS AND BUSINESS

Agenda

Motivation, approach and contribution

WIRTSCHAFTS UNIVERSITÄT WIEN VIENNA UNIVERSITY OF ECONOMICS AND BUSINESS

Motivation

- Limited funds of humanitarian organizations (HOs): **allocation of relief items** in order to maximize impact of disaster relief
- Frequent **misallocations**
- Extremely high **transportation rates** -> strong impact on allocation decisions
- **Competition** among HOs / among carriers
- **Negotiation** of framework agreements and **selection** of carriers critical for success
- **Research questions:**
  - How are transportation rates negotiated in framework agreements?
  - How do framework agreements influence allocation and distribution decisions?
  - How can policy makers intervene to mitgate the imposed limitations?

Approach

- Interaction of independent actors with divergent interests: **game-theoretic model** based on (Generalized) Nash Equilibrium
- Outcome of interactions from perspective of policy makers: analysis of **equilibrium values** through numerical simulations

Contribution

- Provides first comprehensive model of **transportation markets** in long-term relief operations (on level of individual actors)
- Addresses lack of quantitative models for humanitarian logistics taking into account **interdependencies** btw. independent actors
- Widens rare applications of **Generalized Nash Equilibrium** to supply chains

How to increase the impact of disaster relief: a study of transportation rates, framework agreements and product distribution

Agenda

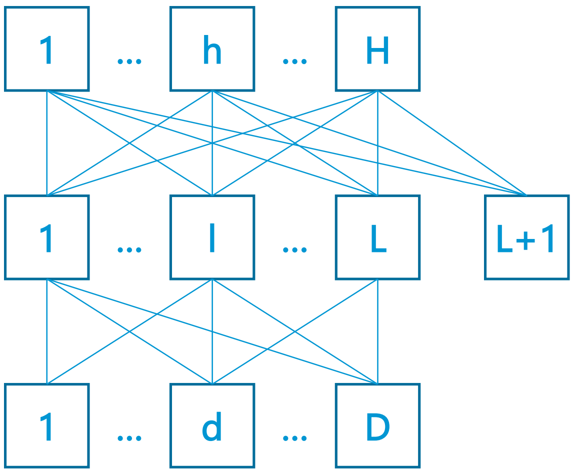
Model overview

Underlying network

Humanitarian organizations (HOs)

Pre-selected carriers & spot market

Distribution points



Game-theoretic model(s)

Sub-model 1

HOs sign **framework agreements** with preselected carriers

- Framework rates: fixed for all orders during the duration of the agreements
- Framework volumes: non-binding projections

Sub-model 2

HOs decide on

- **volume** of relief items to be purchased
- **distribution points** to be supplied
- **carriers** to be used for transportation

Sub-model 1
Behavior of humanitarian organizations

WIRTSCHAFTS UNIVERSITÄT WIEN VIENNA UNIVERSITY OF ECONOMICS AND BUSINESS

Optimization problem of one humanitarian organization

$$\underset{X_h}{\text{minimize}} \quad \boxed{C_h(X_h, P_h^*)} + \omega_h^R \cdot \boxed{R_h(X)}$$

Transportation cost

Dependency risk

$$\text{subject to} \quad \sum_{l=1}^{L} x_{hld} \geq M_{hd}, \quad d = 1, \ldots, D$$

Total volume of framework agreements must exceed target volume

$$\sum_{i=1}^{H} \sum_{d=1}^{D} x_{ild} \leq G_l, \qquad l = 1, \ldots, L$$

Total volume projected for carrier must not exceed its sustainability limit

$$x_{hld} \geq 0, \qquad l = 1, \ldots, L; d = 1, \ldots, D.$$

Non-negativity

Decision variables

x_{hld}	Volume projected by h for transport with l to d
$X_h \in \mathbb{R}_+^{L \cdot D}$	Vector of all x_{hld} for h
$X \in \mathbb{R}_+^{H \cdot L \cdot D}$	Vector of all x_{hld}

Sub-model 1
Behavior of carriers

WIRTSCHAFTS UNIVERSITÄT WIEN VIENNA UNIVERSITY OF ECONOMICS AND BUSINESS

Optimization problem of one carrier

$$\underset{P_l}{\text{maximize}} \quad \boxed{E_l(P, X^*)} + \omega_l^S \cdot \boxed{S_l(P)}$$

$$\text{subject to} \quad \boxed{c_{ld}^t \leq p_{hld} \leq p_{hld}^r, \quad h = 1, \ldots, H, d = 1, \ldots, D.}$$

Expected profit

Customer satisfaction (with prices)

Transportation rates must exceed transportation unit costs and must not exceed customer reservation price

Decision variables

p_{hld}	Rate for transportation from h to d with l
$P_l \in \mathbb{R}_+^{H \cdot D}$	Vector of all p_{hld} for l
$P \in \mathbb{R}_+^{H \cdot L \cdot D}$	Vector of all p_{hld}

Sub-model 2
Behavior of humanitarian organizations

WIRTSCHAFTS UNIVERSITÄT WIEN VIENNA UNIVERSITY OF ECONOMICS AND BUSINESS

Optimization problem of one humanitarian organization

$$\underset{Y_h}{\text{maximize}} \quad \boxed{I_h(Y)} + \omega_h^A \cdot \boxed{A_h(Y)}$$

Impact (reduced suffering of victims)

Activity signal (to donors)

$$\text{subject to} \quad \sum_{l=1}^{L+1} \sum_{d=1}^{D} (c_h^p + p_{hld}^*) \cdot y_{hld} \leq B_h$$

Expenses for purchasing and transportation must not exceed budget

$$\sum_{i=1}^{H} y_{ild} \leq K_{ld}, \quad l = 1, \ldots, L+1,\ d = 1, \ldots, D$$

Volume assigned to carrier must not exceed its available capacity

$$0 \leq y_{hld} \leq x_{hld}^*, \quad l = 1, \ldots, L,\ d = 1, \ldots, D$$

Framework volumes (from Sub-model 1) must not be exceeded

$$0 \leq y_{h(L+1)d}, \quad d = 1, \ldots, D$$

Non-negativity

Decision variables

y_{hld}	Volume transported by l to d on behalf of h
$Y_h \in \mathbb{R}_+^{(L+1)\cdot D}$	Vector of all y_{hld} for h
$Y \in \mathbb{R}_+^{H\cdot(L+1)\cdot D}$	Vector of all y_{hld}

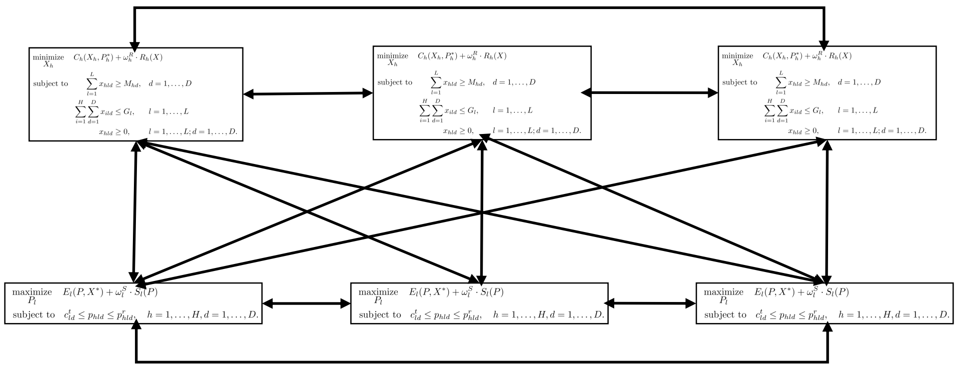
How to increase the impact of disaster relief: a study of transportation rates, framework agreements and product distribution

WIRTSCHAFTS UNIVERSITÄT WIEN VIENNA UNIVERSITY OF ECONOMICS AND BUSINESS

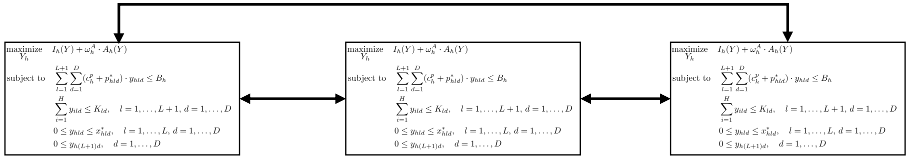
Agenda

Model overview and solution approach

Sub-model 1 (with 3 HOs and 3 carriers)



Sub-model 2 (with 3 HOs)



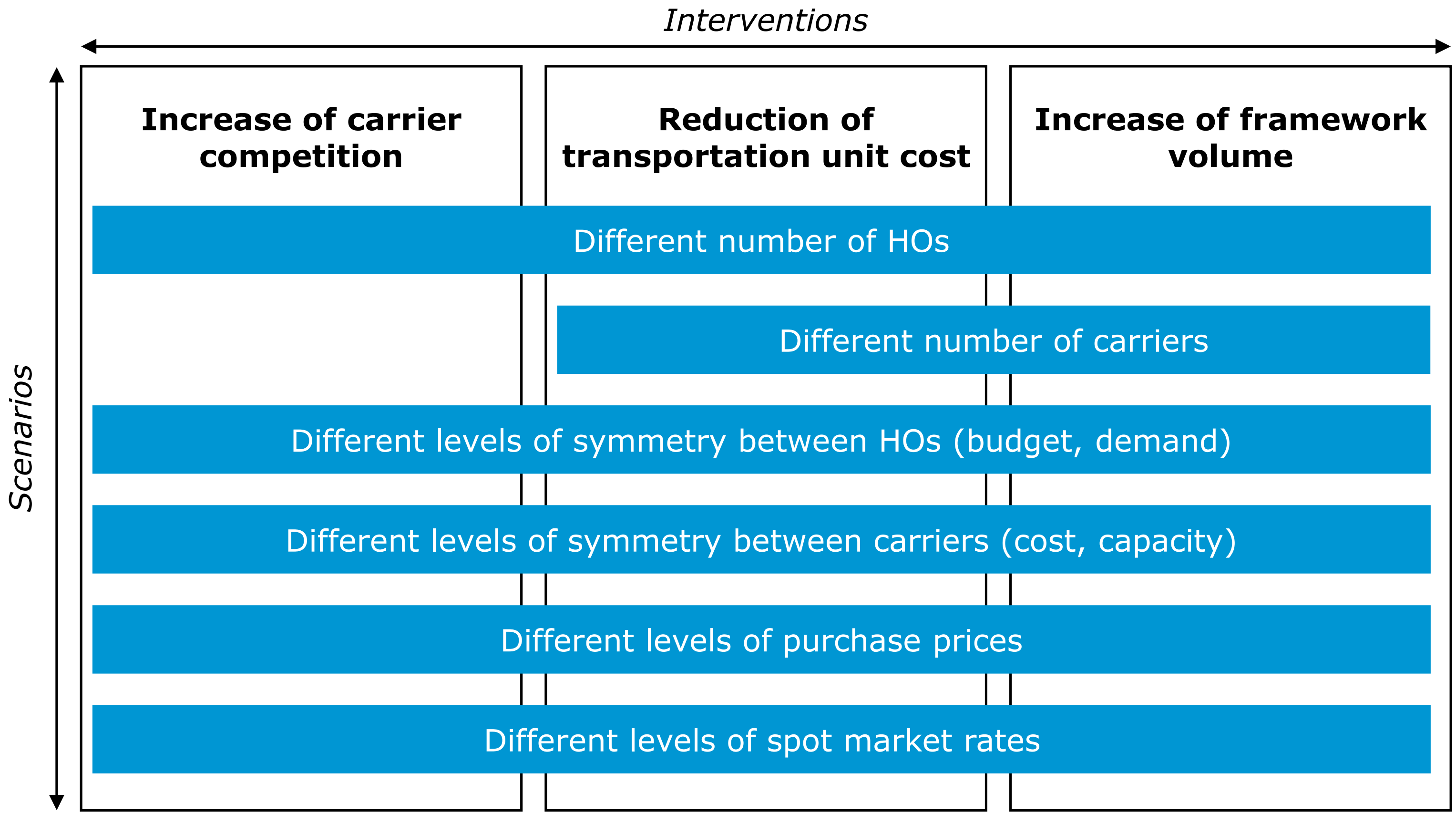
Solution approach

1. Nash Equilibrium (Nash, 1950, 1951) / Generalized Nash Equilibrium (Arrow and Debreu, 1954)
2. **Variational Equality** (Gabay & Moulin, 1980; **Nagurney**, 1999) / **Variational Equilibrium** (Kulkarni & Shanbhag, 2012; Nagurney et al., 2017)
3.
4. Existence and uniqueness of solution (Kinderlehrer & Stampacchia, 1980; Nagurney, 1999)
5. Iterative Projection Method (Solodov & Tseng, 1996)

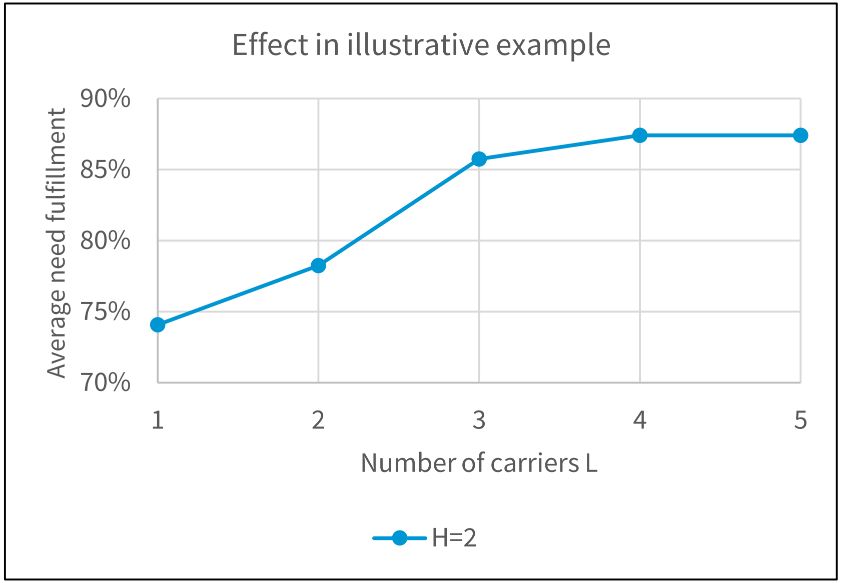
How to increase the impact of disaster relief: a study of transportation rates, framework agreements and product distribution

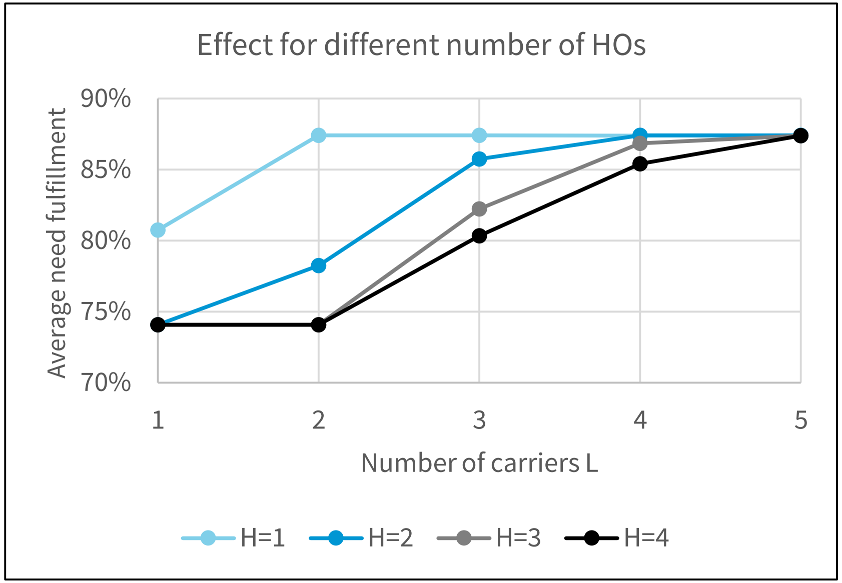
Agenda

Numerical simulations



Selected results
Intervention “Increase of carrier competition”

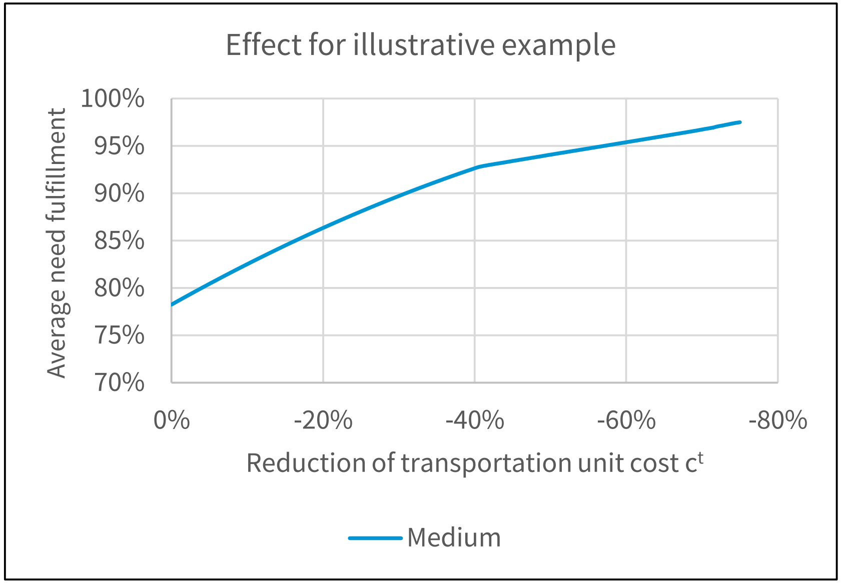


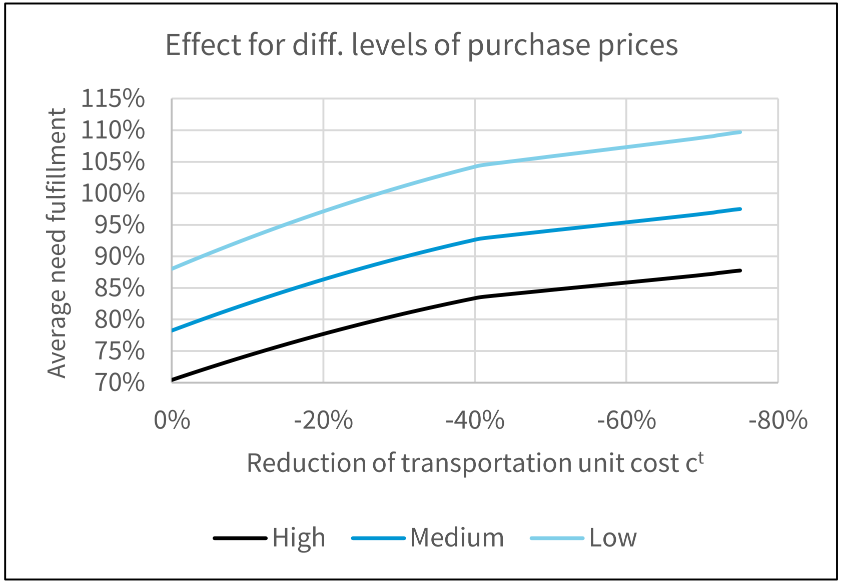


Considerable (S-shaped) improvements if number of pre-selected carriers is increased

The less HOs exist in the market, the lower number of carriers is required

Selected results
Intervention “Reduction of transp. unit cost”





Consistently positive effect, but with decreasing marginal benefit

Qualitatively independent of purchase prices; interventions can cause inefficiencies (“waste”)

How to increase the impact of disaster relief: a study of transportation rates, framework agreements and product distribution

Agenda

Summary and outlook

WIRTSCHAFTS UNIVERSITÄT WIEN VIENNA UNIVERSITY OF ECONOMICS AND BUSINESS

Summary

- **Game-theoretic model** to
  - investigate influence of transportation rates & framework agreements on relief distribution
  - evaluate measures for increasing impact of disaster relief
- **Approach**: Analyze equilibrium states of model under different cond. (using Generalized Nash Equilibrium, Variational Equilibrium & Variational Inequalities)
- **Selected results:**
  - S-shaped improvements by increasing number of pre-selected carriers (with upper limit)
  - Improvements with decreasing marginal benefits by reducing transportation costs (w/o upper limit)

Outlook

- Model with **coordination**
- **Humanitarian** service providers
- **Interdependencies** between both sub-models
- Effects of conflicting **HO priorities**, asymmetric beneficiary **needs** and **media** attention
- Other **pricing** models

Thank you for your attention

WIRTSCHAFTS UNIVERSITÄT WIEN VIENNA UNIVERSITY OF ECONOMICS AND BUSINESS

VIENNA UNIVERSITY OF ECONOMICS AND BUSINESS

TIMO GOSSLER, DIPL. WI.-ING.

Research Associate

Research Institute for Supply Chain Management

Welthandelsplatz 1, 1020 Vienna, Austria

Phone: +43-1-313 36-6037
Mobile: +43-670-701 70 98
timo.gossler@wu.ac.at
www.wu.ac.at

BACKUP

(1) Generalized Nash Equilibrium
(2) Variational Equilibrium
(3) Variational Inquality Formulation of GNE

WIRTSCHAFTS UNIVERSITÄT WIEN VIENNA UNIVERSITY OF ECONOMICS AND BUSINESS

Definition 1 (Generalized Nash Equilibrium). *A vector of all transportation volumes projected by HOs, $X^* \in \mathbb{K}^1 \cap \mathcal{S}^1$, constitutes a Generalized Nash Equilibrium if for each HO h; $h = 1, \ldots, H$:*

$$U_h(X_h^*, \hat{X}_h^*) \geq U_h(X_h, \hat{X}_h^*), \quad \forall X_h \in \mathbb{K}_h^1 \cap \mathcal{S}^1, \tag{4}$$

where $\hat{X}_h^* \equiv (X_1^*, \ldots, X_{h-1}^*, X_{h+1}^*, \ldots, X_H^*)$ *and* $U_h(X) = -[C_h(X_h, P_h^*) + \omega_h^R \cdot R_h(X)]$; $h = 1, \ldots, H$.

Definition 2 (Variational Equilibrium). *A strategy vector X^* is said to be a variational equilibrium of the above Generalized Nash Equilibrium game if $X^* \in \mathbb{K}^1 \cap \mathcal{S}^1$ is a solution of the variational inequality:*

$$-\sum_{h=1}^{H} \langle \nabla_{X_h} U_h(X^*), X_h - X_h^* \rangle \geq 0, \quad \forall X \in \mathbb{K}^1 \cap \mathcal{S}^1. \tag{6}$$

Theorem 1 (VI Formulation of the GNE in Sub-model 1). *Specifically, we have that (6) is equivalent to the variational inequality: determine $X^* \in \mathbb{K}^1 \cap \mathcal{S}^1$, such that*

$$\sum_{h=1}^{H} \sum_{l=1}^{L} \sum_{d=1}^{D} \left[\frac{\partial C_h(X_h^*, P_h^*)}{\partial x_{hld}} + \omega_h^R \cdot \frac{\partial R_h(X^*)}{\partial x_{hld}} \right] \times [x_{hld} - x_{hld}^*] \geq 0, \quad \forall X \in \mathbb{K}^1 \cap \mathcal{S}^1. \tag{7}$$

Proof of the above follows through the use of the definition and the expansion of the gradient terms.

(1) Existence and uniqueness of solution
(2) Extended VI Formulation HOs Sub-model 1

WIRTSCHAFTS UNIVERSITÄT WIEN VIENNA UNIVERSITY OF ECONOMICS AND BUSINESS

Remark 1 (Existence and Uniqueness of Solution). *A solution to (7) is guaranteed to exist from the classical theory of variational inequalities (cf. Kinderlehrer & Stampacchia (1980) and Nagurney (1999)) since the function that enters the variational inequality is continuous and the feasible set is compact. Furthermore, since the function entering (7) is strictly monotone, the solution to the variational inequality (7) is unique.*

Remark 2 (Alternative Variational Inequality to (7)). *We now utilize the Lagrange multipliers associated with the constraints as defined in Table 1. Then, an equivalent variational inequality to that of (7), which we will use to construct the variational inequality for the complete supply chain network (see, e.g., Nagurney (In press)), is the following one:*

$$
\begin{aligned}
&\textit{Find } (X^*, \lambda^{M*}, \lambda^{G*}) \in \mathbb{R}_+^{H\cdot L\cdot D + H\cdot D + L}: \\
&\sum_{h=1}^{H}\sum_{l=1}^{L}\sum_{d=1}^{D}\left[\frac{\partial C_h(X_h^*, P_h^*)}{\partial x_{hld}} + \omega_h^R \cdot \frac{\partial R_h(X^*)}{\partial x_{hld}} - \lambda_{hd}^{M*} + \lambda_l^{G*}\right] \times [x_{hld} - x_{hld}^*] \\
&+ \sum_{h=1}^{H}\sum_{d=1}^{D}\left[-M_{hd} + \sum_{l=1}^{L} x_{hld}^*\right] \times \left[\lambda_{hd}^M - \lambda_{hd}^{M*}\right] \qquad (8) \\
&+ \sum_{l=1}^{L}\left[G_l - \sum_{i=1}^{H}\sum_{d=1}^{D} x_{ild}^*\right] \times \left[\lambda_l^G - \lambda_l^{G*}\right] \geq 0, \\
&\forall (X, \lambda^M, \lambda^G) \in \mathbb{R}_+^{H\cdot L\cdot D + H\cdot D + L}.
\end{aligned}
$$

(1) Nash Equilibrium Carriers Sub-model 1
(2) Sub-model 1 Network Equilibrium

Theorem 2 (VI Formulation of NE in Sub-model 1). *A price vector P^* is a Nash Equilibrium if and only if $P^* \in \mathbb{K}^2$ is a solution of the variational inequality:*

$$-\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{d=1}^{D}\left[\frac{\partial E_l(P^*,X^*)}{\partial p_{hld}} + \omega_l^S \cdot \frac{\partial S_l(P^*)}{\partial p_{hld}}\right] \times [p_{hld} - p^*_{hld}] \geq 0, \quad \forall P \in \mathbb{K}^2. \tag{12}$$

Theorem 3 (VI Formulation of SC Network Equilibrium in Sub-model 1). *A pattern of volume projections, transportation rates and Lagrange multipliers is a supply chain network equilibrium according to the above definition if and only if it satisfies the following variational inequality:*

$$\text{Find } (X^*, \lambda^{M*}, \lambda^{G*}, P^*) \in \mathbb{R}_+^{H\cdot L\cdot D + H\cdot D + L} \times \mathbb{K}^2 :$$

$$\begin{aligned}
&\sum_{h=1}^{H}\sum_{l=1}^{L}\sum_{d=1}^{D}\left[\frac{\partial C_h(X_h^*,P_h^*)}{\partial x_{hld}} + \omega_h^R \cdot \frac{\partial R_h(X^*)}{\partial x_{hld}} - \lambda_{hd}^{M*} + \lambda_l^{G*}\right] \times [x_{hld} - x^*_{hld}] \\
&+\sum_{h=1}^{H}\sum_{d=1}^{D}\left[-M_{hd} + \sum_{l=1}^{L} x^*_{hld}\right] \times \left[\lambda_{hd}^M - \lambda_{hd}^{M*}\right] \\
&+\sum_{l=1}^{L}\left[G_l - \sum_{i=1}^{H}\sum_{d=1}^{D} x^*_{ild}\right] \times \left[\lambda_l^G - \lambda_l^{G*}\right] \\
&-\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{d=1}^{D}\left[\frac{\partial E_l(P^*,X^*)}{\partial p_{hld}} + \omega_l^S \cdot \frac{\partial S_l(P^*)}{\partial p_{hld}}\right] \times [p_{hld} - p^*_{hld}] \geq 0, \\
&\qquad \forall (X, \lambda^M, \lambda^G, P) \in \mathbb{R}_+^{H\cdot L\cdot D + H\cdot D + L} \times \mathbb{K}^2.
\end{aligned} \tag{13}$$

(1) VI Formulation Sub-model 2
(2) Extended VI Formulation Sub-model 2

Theorem 4 (VI Formulation of Sub-model 2). *A strategy vector Y^* is said to be a variational equilibrium of the above Generalized Nash Equilibrium game if $Y^* \in \mathbb{K}^3 \cap \mathcal{S}^2$ is a solution of the variational inequality:*

$$\sum_{h=1}^{H}\sum_{l=1}^{L+1}\sum_{d=1}^{D}\left[-\frac{\partial I_h(Y^*)}{\partial y_{hld}} - \omega_h^A \cdot \frac{\partial A_h(Y^*)}{\partial y_{hld}}\right] \times [y_{hld} - y^*_{hld}] \geq 0, \quad \forall Y \in \mathbb{K}^3 \cap \mathcal{S}^2. \tag{18}$$

Remark 5 (Alternative Variational Inequality to (18)). *Recall the Lagrange multipliers associated with the constraints as defined in Table 1. Then, an equivalent variational formulation of problem (15a) under constraints (15b) - (15e) is the following one:*

$$\begin{aligned}
&\textit{Find } (Y^*, \lambda^{B*}, \lambda^{K*}) \in \mathbb{R}_+^{H\cdot(L+1)\cdot D + H + (L+1)\cdot D}:\\
&\sum_{h=1}^{H}\sum_{l=1}^{L+1}\sum_{d=1}^{D}\left[-\frac{\partial I_h(Y^*)}{\partial y_{hld}} - \omega_h^A \cdot \frac{\partial A_h(Y^*)}{\partial y_{hld}} + (c_h^p + p^*_{hld}) \cdot \lambda_h^{B*} + \lambda_{ld}^{K*}\right] \times [y_{hld} - y^*_{hld}]\\
&+\sum_{h=1}^{H}\left[B_h - \sum_{l=1}^{L+1}\sum_{d=1}^{D}(c_h^p + p^*_{hld}) \cdot y^*_{hld}\right] \times \left[\lambda_h^B - \lambda_h^{B*}\right]\\
&+\sum_{l=1}^{L+1}\sum_{d=1}^{D}\left[K_{ld} - \sum_{i=1}^{H} y^*_{ild}\right] \times \left[\lambda_{ld}^K - \lambda_{ld}^{K*}\right] \geq 0,\\
&\forall (Y, \lambda^B, \lambda^K) \in \mathbb{R}_+^{H\cdot(L+1)\cdot D + H + (L+1)\cdot D}.
\end{aligned} \tag{19}$$

Functional forms for numerial simulations

Sub-model 1	
Transportation costs of h	$C_h(X_h, P_h^*) = \sum_{l=1}^{L} \sum_{d=1}^{D} p_{hld}^* \cdot x_{hld}$
Dependency risk of h	$R_h(X) = \sum_{l=1}^{L} \sum_{d=1}^{D} r_{hl} \cdot x_{hld}^2$
Expected profit of l	$E_l(P, X^*) = \sum_{h=1}^{H} \sum_{d=1}^{D} (p_{hld} - c_{ld}^t) \cdot x_{hld}^*$
Satisfaction with l	$S_l(P) = \sum_{h=1}^{H} \sum_{d=1}^{D} M_{hd} \cdot (1 - \frac{p_{hld}^2}{{p_{hld}^r}^2})$
Sub-model 2	
Impact of h	$I_h(Y) = \sum_{d=1}^{D} u_d \cdot (\sum_{l=1}^{L+1} y_{hld} - \frac{1}{2 \cdot n_d} \cdot \sum_{l=1}^{L+1} y_{hld} \cdot (2 \cdot \sum_{i=1}^{H} \sum_{l=1}^{L+1} y_{ild} - \sum_{l=1}^{L+1} y_{hld}))$
Activity signal of h	$A_h(Y) = \sum_{d=1}^{D} \left(i_{hd} \sum_{l=1}^{L+1} y_{hld} \right)$

Parameter values for numerical simulations

Notation	Description	Indices	Example	Scenarios
B_h	Budget of HO h	$(h=1,\ldots,H)$	5.000	2.500, ..., 7.500
c^t_{ld}	Unit cost of l for transport to d	$\binom{l=1,\ldots,L}{d=1,\ldots,D}$	0.300	0.075, ..., 0.300
c^p_h	Purchase price for h	$(h=1,\ldots,H)$	0.750	0.600,0.750,0.900
D	Number of distribution points	-	2	2
G_l	Volume limit for carrier l	$(l=1,\ldots,L)$	3.000	4.500
H	Number of HOs	-	2	1,...,4
i_{hd}	Relative importance of d for h	$\binom{h=1,\ldots,H}{d=1,\ldots,D}$	1.000	1.000
K_{ld}	Capacity of carrier l for transport to d	$\binom{l=1,\ldots,L}{d=1,\ldots,D}$	2.500	3.750
$K_{(L+1)d}$	Capacity of spot market for transport to d	$(d=1,\ldots,D)$	∞	∞
L	Number of carriers	-	2	1, ..., 5
M_{hd}	Target volume of h for d	$\binom{h=1,\ldots,H}{d=1,\ldots,D}$	1.500	1.500,...,2.498
n_d	Needs at d	$(d=1,\ldots,D)$	5.000	5.000
$p_{h(L+1)d}$	Spot market rates for transport by h to d	$\binom{h=1,\ldots,H}{d=1,\ldots,D}$	0.600	0.450,0.600,0.750
r_{hl}	Relative risk h associates with l	$\binom{h=1,\ldots,H}{l=1,\ldots,L}$	1.000	1.000
s_h	Surcharge accepted by h	$(h=1,\ldots,H)$	1.000	1.000
u_d	Relative urgency of d for h	$(d=1,\ldots,D)$	1.000	1.000
ω^A_h	Weight of signaling for h	$(h=1,\ldots,H)$	0.200	0.200
ω^R_h	Weight of risk for h	$(h=1,\ldots,H)$	0.200	0.200
ω^S_l	Weight of satisfaction for l	$(l=1,\ldots,L)$	0.400	0.400

Solution algorithm

The Solodov & Tseng (1996) method is an iterative projection-contraction method, where the second projection is a more general operator. The solution vector z_m^τ of sub-model $m \in \{1, 2\}$ in iteration τ of the algorithm is the result of the second projection and calculated as:

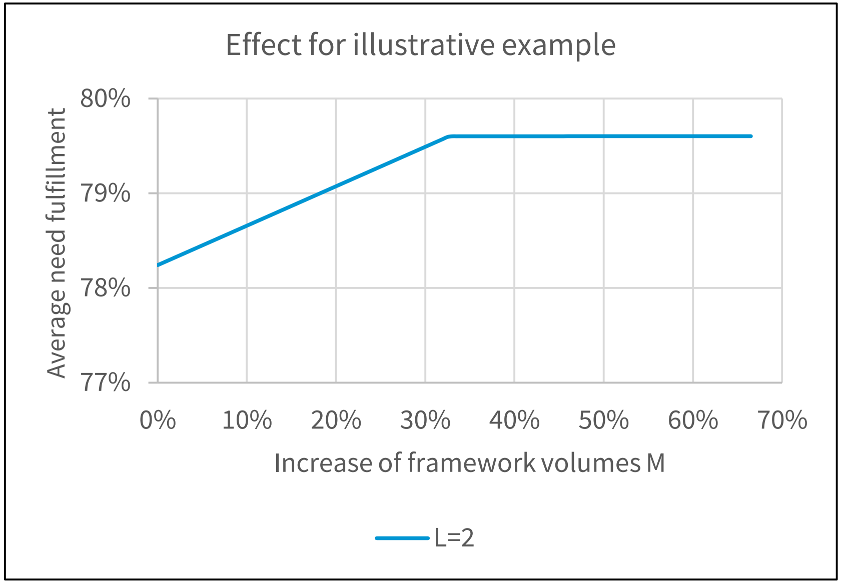
$$z_m^\tau = z_m^{\tau-1} - \gamma M^{-1}(T_{\alpha_\tau}(z_m^{\tau-1}) - T_{\alpha_\tau}(Pr_{\mathcal{K}}(z_m^{\tau^*})),$$

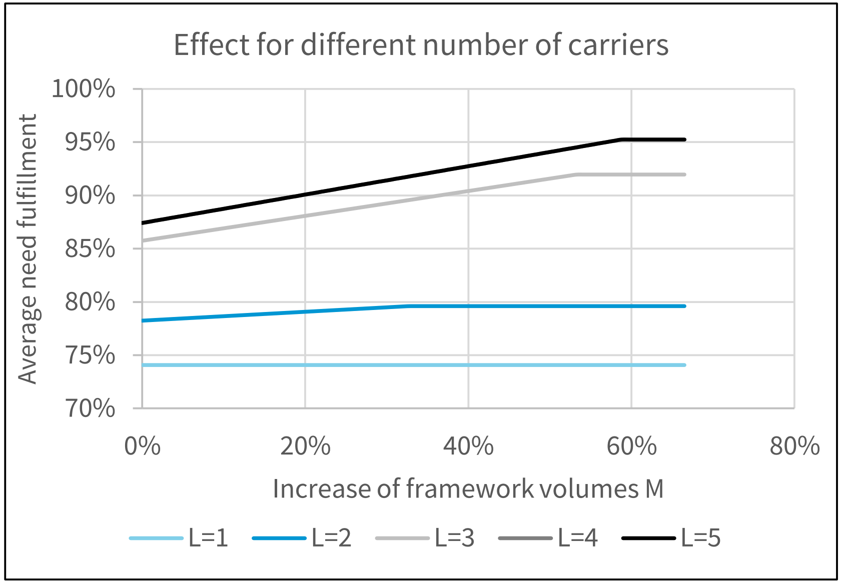
with $\gamma \in \mathbb{R}^+$. The scaling matrix M must be a symmetric positive matrix and is used to accelerate the convergence. Furthermore, $T_{\alpha_\tau} = I - \alpha_\tau F_m$, where I is the identity function, α_τ is chosen dynamically such that T_{α_τ} is strongly monotone, and F_m is the function entering the variational inequalities (13) and (19) when they are formulated in standard form. Finally, $z_m^{\tau^*}$ is the first projection in each iteration and is calculated as:

$$z_m^{\tau^*} = Pr_{\mathcal{K}}(z_m^{\tau-1} - \alpha_\tau F_m(z_m^{\tau-1})).$$

The Solodov & Tseng (1996) algorithm has less restrictive conditions for convergence than many variational inequality algorithms, and requires only monotonicity of the function F_m, with the rate of convergence also established in Solodov (2003) for this and related algorithms. We implemented the algorithm in MATLAB R2016a setting $\alpha_\tau = 0.3^{\tau-1}$, $\gamma = 1.0$ and $M = I$. We solved, in total, 4626 different instances on an Intel(R) Core(TM) i5 CPU with 2.60GHz and 8.00GB RAM. Let $d_m^\tau = z_m^\tau - z_m^{\tau-1}$ be the vector of differences between the solutions of two consecutive iterations for sub-model m. We stopped the algorithm when the Euclidean norm of d_m^τ fell below $\epsilon = 1 \cdot 10^{-5}$. Initializing all variables as zero, on average 390 iterations were required to solve Sub-model 1 and 433 iterations were required to solve Sub-model 2. On average, an iteration took 0.005 seconds.

Selected results
Intervention "Increase of framework volumes"





Linear improvement up to the limit imposed by the budget

Relevant improvements only if number of pre-selected carriers is sufficiently high